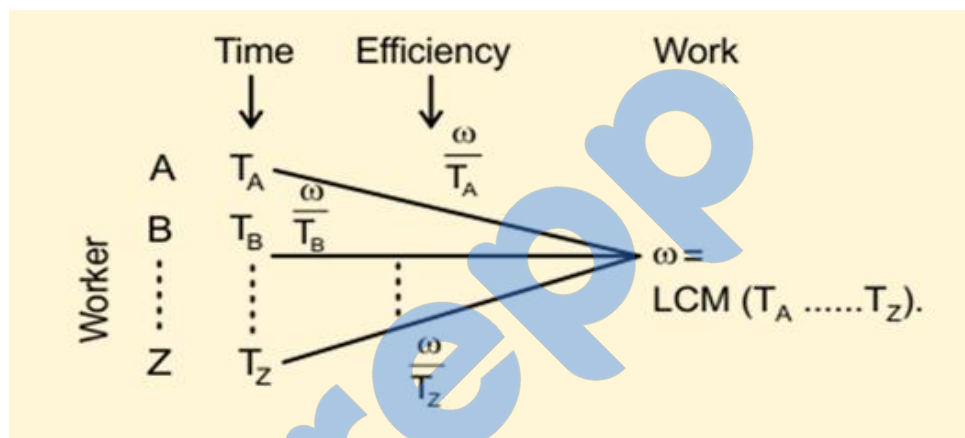


PIPES AND CISTERNS

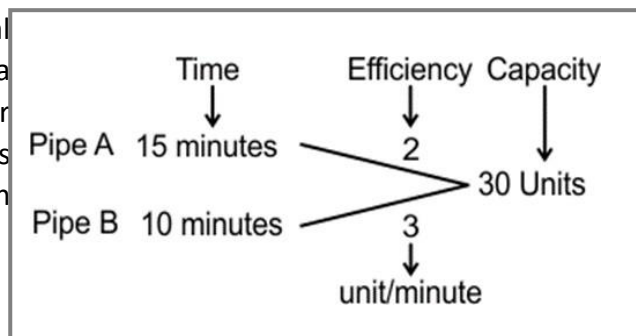
Concept of 'Pipe and Cistern'

- Same as 'Time and Work'
- Here, Time of filling cistern = Time to do a work
- Volume of cistern = Total work
- Speed of filling cistern = Efficiency of work

Trick



The above diagram shows the general structure of how we can quickly analyze a question based on Time and Work. Similar concept is applicable for Pipe and Cistern as well. The figure beside shows its application in Pipe and Cistern Problems.



Points to Remember

- If a pipe can fill a tank in x hours and another pipe can fill it in y hours, then the fraction of tank filled by both pipes together in 1 hour

$$\frac{1}{X} + \frac{1}{Y} = \frac{X + y}{XY}$$

- If a pipe can fill a tank in x hours and another pipe can empty it in y hours, then the fraction of tank filled by both the pipes together in 1 hour

$$\frac{1}{X} - \frac{1}{Y} = \frac{Y - X}{XY}$$

Basic Questions

Q1. Two pipes P and Q, when opened alone can fill the tank in 20 and 30 hours respectively. If both pipes are opened together, then in how many hours will the tank be filled?

Solution:

Part of tank filled by pipe P in 1 hour = $\frac{1}{20}$ Part

of tank filled by pipe Q in 1 hour = $\frac{1}{30}$

$\Rightarrow$ Tank filled by both pipes in 1 hour = $\frac{1}{20} + \frac{1}{30} = \frac{5}{60} = \frac{1}{12}$

$\therefore$ Complete tank will be filled by both in 12 hours

Q2. A tank can be filled by pipe A in 5 hours and emptied by pipe B in 8 hours respectively. How much time will it take for the tank to be half full?

Solution:

Pipe alone can fill the tank in = 5 hrs.

Pipe alone can empty the tank in = 8 hrs.

Let tank to be half full in x hrs.,

then

According to the question

$$x/5 - x/8 = \frac{1}{2}$$

$$(8x - 5x)/40 = \frac{1}{2}$$

$$3x/40 = \frac{1}{2}$$

$$3x = \left(\frac{1}{2}\right) \times 40 = 20$$

$$x = 20/3 \text{ hrs.}$$

Work Alternately

In this type of questions two pipes work on alternate hours

Q3. Two pipes A and B can fill a tank in 6 hours and 4 hours respectively. If they are opened on alternate hours and if pipe A is opened first, then the tank shall be filled in:

Solution:

Let the filling rate of A be a, and that of B be b. Let the capacity of the tank be C. We have

$$C = a \times 6 = b \times 4$$

$$\Rightarrow a = C/6 \text{ and } b = C/4$$

Given, pipe A is opened in the first hour. Hence, $C/6$ is filled in the first hour. Similarly, $C/4$ is filled in the second hour. In two hours, we have a capacity of $5C/12$ filled.

Hence, in 4 hrs.

We have $10C/12$ capacity filled. There is $1/6^{\text{th}}$ of the capacity to be filled.

Pipe A is filling the tank now. It will fill this tank in an hour. Hence, it takes a total of 5 hours to fill this tank.

Solved Examples

Q1. Three taps P, Q and R can fill a tank in 20, 30 and 40 minutes respectively. If all the three taps are opened, then how much time (in minutes) it will take to completely fill the tank?

Solution:

Time taken by tap P to fill the tank = 20 minutes

$$\Rightarrow \text{Part of tank filled in 1 minute} = 1/20$$

Similarly, tank filled by Q and R in 1 min = $1/30$ and $1/40$ respectively.

$$\Rightarrow \text{Part of tank filled by all three taps in 1 minute} = 1/20 + 1/30 + 1/40 = 13/120$$

$$\therefore \text{Tank will be completely filled in } 120/13 \text{ minutes} = 9\frac{3}{13} \text{ Min}$$

Q2. Pipe A can fill in 20 minutes and Pipe B in 30 mins and Pipe C can empty the same in 40 mins. If all of them work together, find the time taken to fill the tank?

Solution:

In one min work done by A = $1/20$

In one min work done by B = $1/30$

In one min work done by C = $1/40$

$\therefore$ A and B both fill the tank and C empty the tank

$$\therefore \text{Total work done in one min to fill the tank} = \frac{1}{20} + \frac{1}{30} - \frac{1}{40} = \frac{7}{120}$$

$$\therefore \text{Time taken to fill the tank} = 120/7 \text{ mins}$$

Q3. Two taps A and B fill a tank in 24 minutes and 32 minutes respectively. If both the taps are opened simultaneously, after how many minutes tap B should be closed so that the tank fills up in 18 minutes.

Solution:

Tap A fills $1/24$ of the tank in 1 minute

Tap B fills $1/32$ of the tank in 1 minute

Let,

After x minutes tap B should be closed

So,

$$\Rightarrow \frac{18}{24} + \frac{x}{32} = 1$$

$$\Rightarrow \frac{x}{32} = 1 - \frac{18}{24}$$

$$x = \frac{6}{24} \times 32 = 8 \text{ minutes}$$

prepp