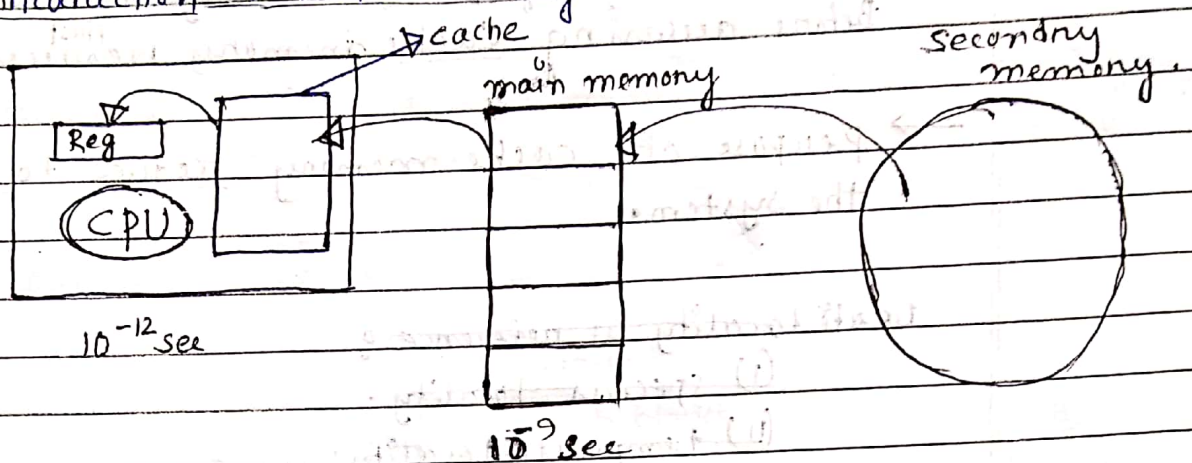


Cache memory

Introduction to cache memory:



→ cache memory faster than main memory.

• Cache hit: If required element present in cache, then it is called 'cache hit'.

• hit latency: Time taken to find out whether element present on the cache or not, that is called 'hit latency'.

• Cache miss: If required element not present in cache, that is called 'cache miss'.

• Miss latency: Time taken to get something from main memory and then place it into the cache and then read that's called 'miss latency'.

• Page hit: If required element present in main memory.

• Page fault: If required element not present in main memory.

• Tag directory: Tag directory says that required element present in tag or not.

→ Before ~~accessing~~ ^{first} main memory we ^{first} access page table,
Before accessing cache memory we access ^{first} Tags.

→ purpose of cache memory reduce to cost of the system.

locality of reference of

(i) spatial locality.

(ii) temporal locality.

- Locality of reference :- is a term for the phenomenon in which the same values or related storage locations are frequently accessed, depending on the memory access pattern.

(Locality of reference)

Temporal locality

Recently referenced items are likely to be referenced in the near future.

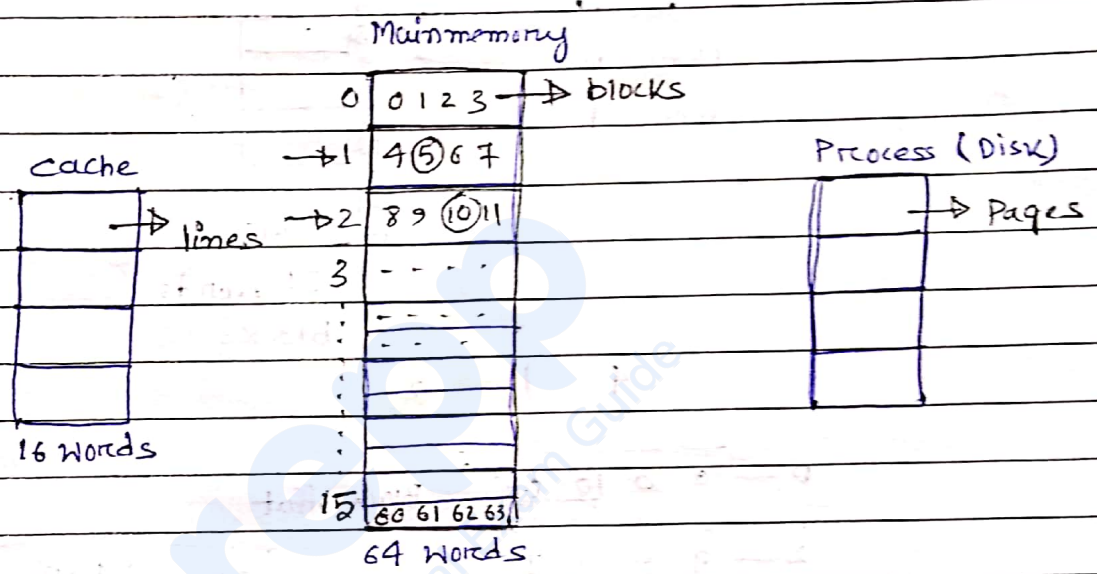
Spatial locality

Items with nearby addresses tend to be referenced close together in time.

• Introduction to Direct mapping

• Introduction to Direct mapping:

- talking about Disk and main memory He talking about paging.
- talking about cache and " " " " " Blocks.
- Block size = linesize.



→ smallest addressable unit ^{in the memory} called word.

let's

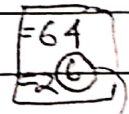
1W = 1B (means our system is byte addressable)

Block size = 4 words

No. of Block in main memory = $\frac{64}{4} = 16$ BLOCK.

No. of lines in cache = $\frac{16}{4} = 4$ lines.

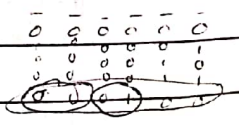
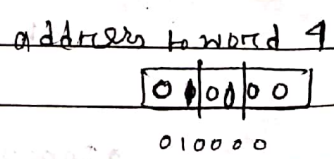
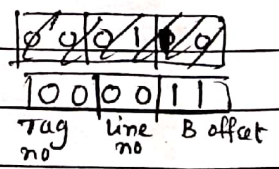
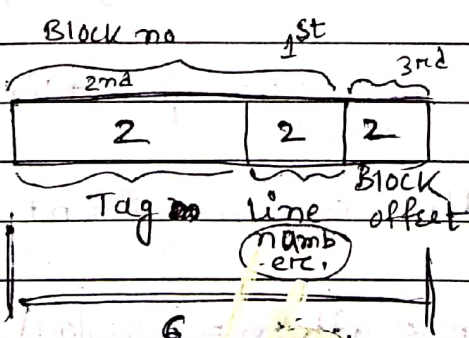
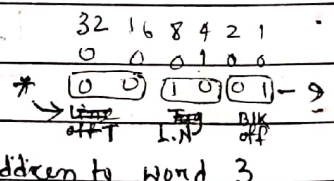
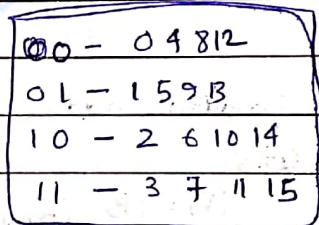
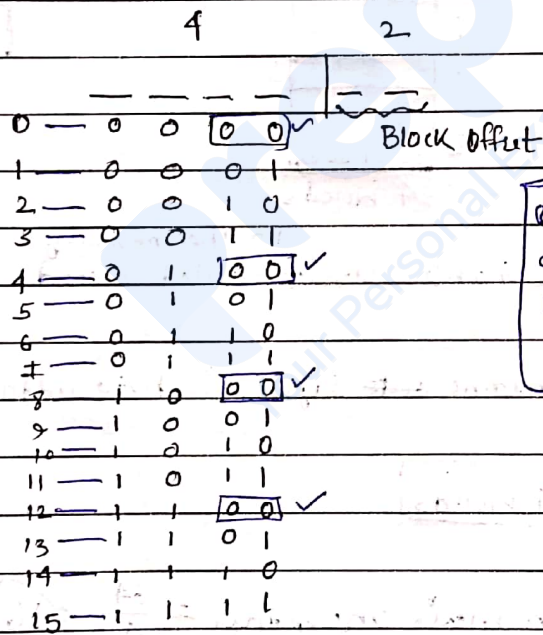
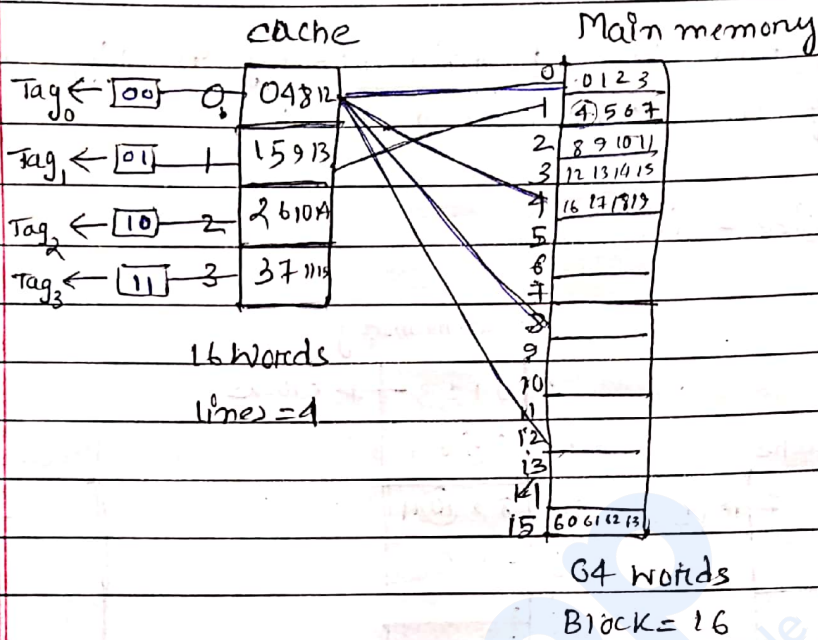
physical address contain = 6 bits



processor generate address = $\begin{matrix} \boxed{0001} & \boxed{01} \\ \text{Bik num} & \text{Bik offset} \end{matrix}$ (to word 5)

= $\begin{matrix} \boxed{0001010} \\ \text{B.no} & \text{B.off} \end{matrix}$ (to word 10)

• Direct Mapping :



1 bit ~~part~~ Byte = 8 bit
 KB = 2^{10} B
 MB = 2^{10} KB
 GB = 10^{10} MB

Direct Mapping Problem (1)

[Problem - 1]

	MM size	cache size	Block size	Tag Bits	Tag Directory size
✓ Q1	128 KB	16 KB	256 B	✓ 3 bit	✓ $(3 * 2^6)$ bit
✓ Q2	32 GB	32 KB	1 KB	✓ 20 bit	✓ $(20 * 2^5)$ "
✓ Q3	✓ 64 GB 2^{26} B	512 KB	1 KB	7	✓ $(7 * 2^9)$ "
✓ Q4	16 GB	✓ 16 KB 2^{14} B	1 KB	10	✓ $10 * 2^{14}$ "
Q5	64 MB	✓ 2^{16} B	can't guess	10	can't guess
✓ Q6	2^{26} B	512 KB	can't guess	7	can't guess

Assuming that memory is Byte addressable.

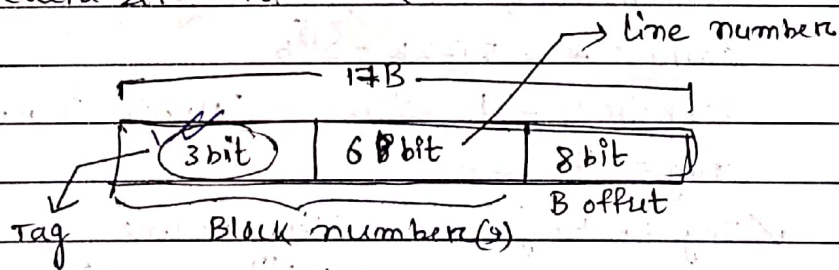
Q1:

Main memory size = 128 KB
 $= 2^{10} * 128$ B
 $= 2^{17}$ B

Block size = 256 B
 $= 2^8$ B

no. of Block = $\frac{2^{17}}{2^8} = 2^9$ - bit

Cache size = 16 KB = 2^{14} B



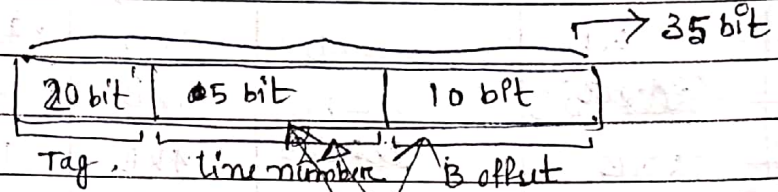
no. of lines = $\frac{\text{C size}}{\text{line size}} = \frac{2^{14}}{2^8} = 2^6$ - bit

Line = Block size

Tag Directory size = (Tag size * no. of lines)
 $= (3 * 2^6)$ bit

Q2:

$$\begin{aligned}
 \text{MM size} &= 32 \text{ GB} \\
 &= 2^{10} \times 32 \text{ MB} \\
 &= 2^{10} \times 2^{10} \times 32 \text{ KB} \\
 &= 2^{10} \times 2^{10} \times 2^{10} \times 32 \text{ B} \\
 &= 2^{30+5} \text{ B} = 2^{35} \text{ B}
 \end{aligned}$$



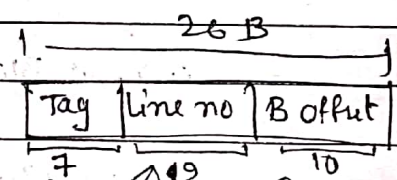
$$\begin{aligned}
 \text{Block size} &= 1 \text{ KB} \\
 &= 2^{10} \text{ B}
 \end{aligned}$$

(10 bit for Block offset)

$$\begin{aligned}
 \text{Cache size} &= 32 \text{ KB} \\
 &= 2^{10} \times 2^5 \text{ B} \\
 &= 2^{15} \text{ B}
 \end{aligned}$$

$$\begin{aligned}
 \text{no. of line} &= \frac{\text{C size}}{\text{line size}} \\
 &= \frac{2^{15}}{2^{10}} \\
 &= 2^5
 \end{aligned}$$

$$\begin{aligned}
 \text{Tag directory size} &= \text{Tag} \times \text{no. of lines} \\
 &= (20 \times 2^5)
 \end{aligned}$$



Q3:

$$\begin{aligned}
 \text{Main memory size} &= 9 \text{ GB} \\
 \text{Cache size} &= 512 \text{ KB} = 2^{19} \text{ B} \\
 \text{Block size} &= 1 \text{ KB} = 2^{10} \text{ B} \\
 \text{Tag} &= 7
 \end{aligned}$$

(10 bit Block offset)

$$\begin{aligned}
 \text{Tag directory size} &= 7 \times \text{no. of line} \\
 &= 7 \times \frac{2^{19}}{2^{10}} = (7 \times 2^9) \text{ B bit}
 \end{aligned}$$

$$\begin{aligned}
 \text{no. of line} \\
 \frac{2^{19}}{2^{10}} &= 2^9
 \end{aligned}$$

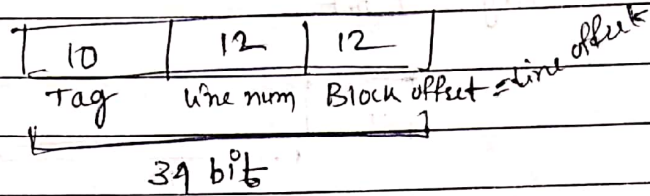
~~no. of block~~ ~~MM size~~ ~~block size~~

$$\text{address size} = 7 + 10 + 9 = 26$$

$$\begin{aligned}
 \text{main memory size} &= 2^{26} \text{ B} \\
 &= 2^{32} \text{ B} = 2^6 \text{ MB} = 64 \text{ MB} \\
 &= 2^{30} \times 2^2 \text{ B} = 4 \text{ GB}
 \end{aligned}$$

Q4

$$\text{MM size} = 16 \text{ GB} = 2^{30} \times 2^4 \text{ B} = 2^{34} \text{ B}$$



$$\begin{array}{r} 34 \\ - 22 \\ \hline 12 \end{array}$$

Cache size = ?

$$\text{Block size} = 4 \text{ KB} = 2^{10} \times 2^2 \text{ B} = 2^{12} \text{ B}$$

tag = 10

tag = Directory.

$$2^{12} \times 2^{12} = 2^{24} \text{ B}$$

$$\text{Cache size} = 2^{24} \text{ B} = 2^{24} \text{ B} = 16 \text{ MB}$$

$$\text{no of lines} = \frac{\text{c size}}{\text{line size}} = \frac{2^{24}}{2^{12}} = 2^{12}$$

$$\text{Tag directory} = 10 \times 2^{12} = 10 \text{ bits} \cdot (10 \times 2^{12}) \text{ bit}$$

Q5

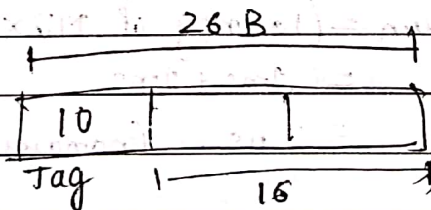
$$\text{MM size} = 64 \text{ MB} = 2^{20} \times 2^6 \text{ B} = 2^{26} \text{ B (PA)}$$

Cache size = ?

Block size = ?

tag = 10

$$\begin{array}{r} 2^{26} \\ - 26 \\ \hline \end{array}$$

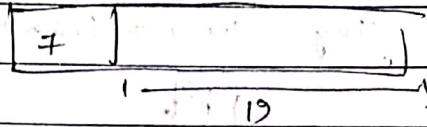


$$\text{Cache size} = 2^{16} \text{ B}$$

Q.6

Cache size = 512 KB = 2^{19} B

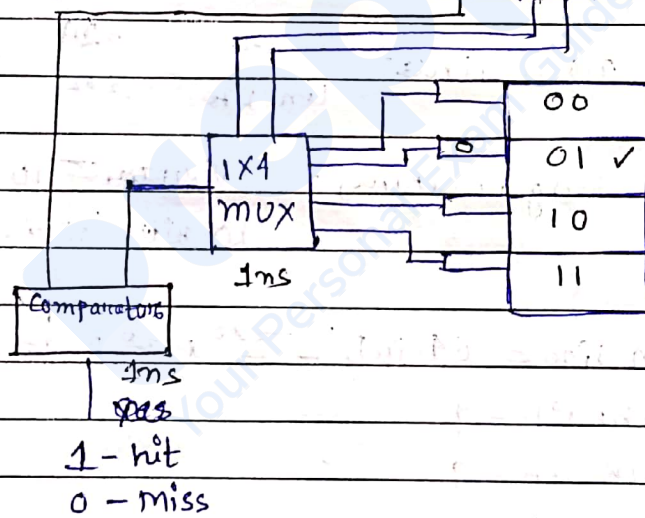
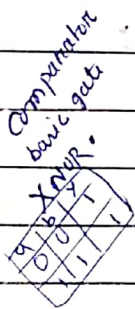
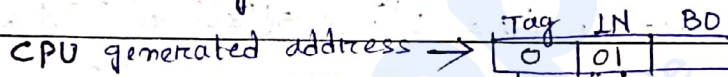
Tag = 7 bit



$19 + 7 = 26$

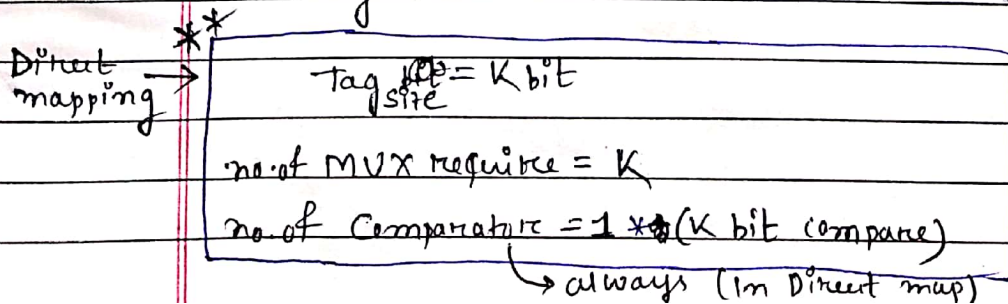
MM size = 2^{26} B

Direct Mapping HW Implementation



Total time taken = (latency of MUX + latency of comparator)
 = 1ns + 1ns
 = 2ns. (nanosecond)

→ no. of MUX and comparator depends on no. of Tag bits.



Q1:

MM size = 1 GB

Cache size = 1 MB

Comparator size = 10 Kms.

Hit Latency = ?

where latency
MUX is negligible

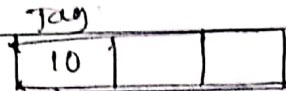
⇒ ~~Hit Latency~~

MM = 2^{30} B

** Tag = $\frac{\text{Main M size}}{\text{Cache size}}$

= $\frac{1 \text{ GB}}{1 \text{ MB}} \Rightarrow 2^{30} \text{ KB} \cdot 1 \text{ KB} \Rightarrow 2^{10} \text{ B}$

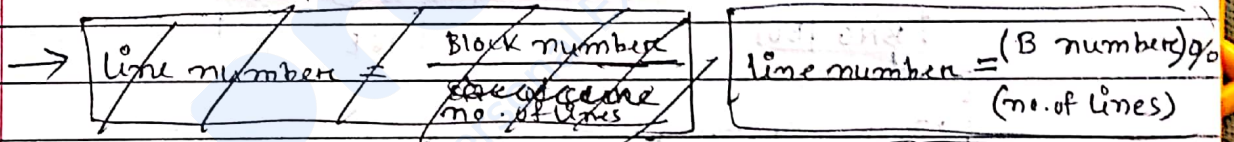
$K=10$



Hit Latency = ~~100 ns~~ 10 Kms

= $10 \cdot 10 \text{ ns}$

= 100 ns



$K = m \% n$

• Disadvantage of direct mapping =

→ Conflict miss problem.

ex:

Block req by CPU -

0	1	8, 12, 20
1	2	9
2	6	
3	15	

$L.N = (B.N) \% (NL)$

→ place was empty for long time, but
each time = line no. 0 used heavily, this
no. of lines = 4 is conflict miss problem.

ex:

Block request by CPU

0	4	8	12	16	20
1					
2					
3					

4 8 12 16 20

$K = m/n$

$n =$ num of lines in cache

Cache

$4 \div 4 = 0 \quad 4 \% 4 = 0$

$8 \div 4 = 0 \quad 8 \% 4 = 0$

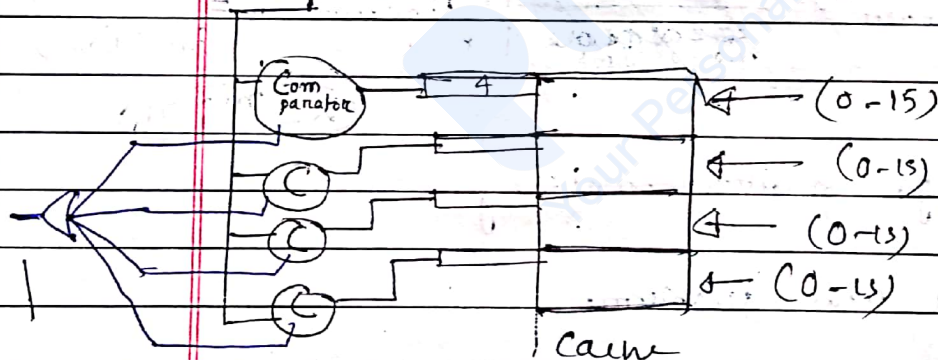
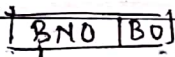
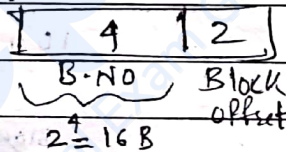
line no, 1, 2, 3, empty but then line 0 are heavily used
 it is conflict miss problem.

Associative mapping = (Intro)

can solve

→ By Associative mapping a conflict miss problem.

Physical address



- 1 - cache hit
- 0 - cache miss

→ 4 lines - 4 comparators need, so hardware cost increase along with freedom.

→ #no of comparator requires = no of cache lines.

→ We get freedom to place Block anywhere in cache, but requirement of comparator increases.

Q1:

Main memory size = 32 GB

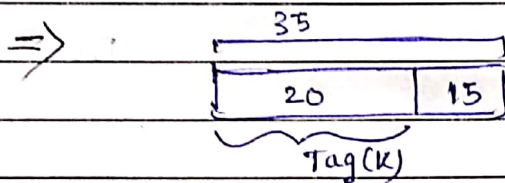
Block size = 32 KB

✓ Tag(K) = ?

Propagation delay (PD) of Comp = 10 Kns

PD of OR Gate = 10 ns

✓ Hit latency = ?



$$\begin{aligned}
 MM &= 32 \text{ GB} \\
 &= 2^{30+5} \text{ B} \\
 &= 2^{35} \text{ B}
 \end{aligned}$$

$$\begin{aligned}
 BS &= 32 \text{ KB} \\
 &= 2^{15} \text{ KB}
 \end{aligned}$$

✓ $k = 20$

$$\begin{aligned}
 \text{Hit latency} &= \text{PD of Comparator} + \text{PD of OR gate} \\
 &= 10 \text{ Kns} + 10 \text{ ns} \\
 &= (10 \times 20) \text{ ns} + 10 \text{ ns} \\
 &= 210 \text{ ns}
 \end{aligned}$$

Q2:

	MM size	cache size	Block size	Tag size	Tag directory size
✓ ① -	128 KB	16 KB	256 B	9 bits	(9 × 26) bits
✓ ② -	32 GB	32 KB	1 KB	25 bits	(25 × 32) bits
✓ ③ -	128 MB	512 KB	1 KB	17	(17 × 29) bits
✓ ④ -	16 GB	? (can't)	4 KB	22 bits	(can't guess)
✓ ⑤ -	64 MB	? (can't) (dir)	64 KB	10	(can't guess)
✓ ⑥ -	not possi	512 KB	not possi	7	not possible

① MM size = 128 KB

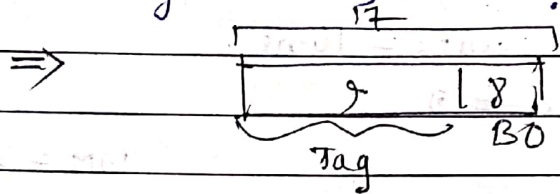
CS = cache size = 16 KB

Block size (BS) = 256 B

Tag size = ?

Tag Directory size = ?

$$\begin{aligned} \text{no of lines} &= \frac{CS}{BS} \\ &= \frac{16 \text{ KB}}{256 \text{ B}} \\ &= \frac{2^{14}}{2^8} \\ &= 2^6 \end{aligned}$$



$$\begin{aligned} BS &= 128 \\ &= 2^7 \times 2^{10} \text{ B} \\ &= 2^{17} \text{ B} \end{aligned}$$

BS = 256 B = 2⁸ B

Tag = 9

Tag directory size = Tag size * no of lines.
= (9 * 2⁶) bits

no of com req = 2⁶ = 64

②

MM = 2³⁵ B

CS = 2¹⁵ B

BS = 2¹⁰ B

Tag = (35 - 10) = 25

$$\begin{aligned} \text{no. of line} &= \frac{2^{15}}{2^{10}} \\ &= 2^5 \end{aligned}$$

Tag directory = Tag * no of lines
= 25 * 25

③

MM = ?

CS = 2¹⁹ B

BS = 2¹⁰ B

TS = 17

Tag directory = ?

$$\begin{aligned} \text{MM size} &= 2^{10+17} \\ &= 2^{27} \text{ B} = 2^7 \times 2^{20} \text{ B} \\ &= 128 \text{ MB} \end{aligned}$$

$$\text{no. of line} = \frac{2^{19}}{2^{10}} = 2^9$$

Tag D.S = (17 * 2⁹) bits.

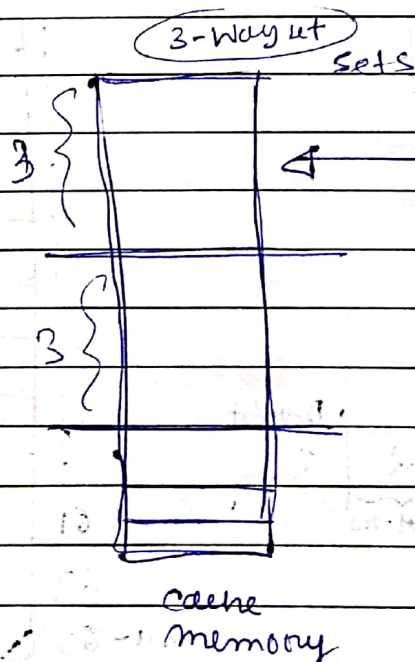
④ $MM = 2^{31} B$...
~~CS = 9 X~~
 $BS = 2^{12} B$
 $\sqrt{TS = 9}$
 $T.DS = ? X$

Tag size = $(34 - 12)$
 $= (22)$

⑤ $MM = 2^{26} B$
 $CS = 9$
 $BS = 9 (2^{16})$
 $Tag = 10$
 $Tag DS = 9$

$BS = 2^{26-10}$
 $= 2^{16} = 64 KB$

⑥ $MM = 9 X$
 $CS = 2^{19} B$
 $TS = 7$
 $BS = 9 X$
 $T.DS = 9 X$ not possible.



- set Associative mapping = (adv: no. of comparators reduced)

Ex: (how set associative work)

MM size = 64 B

cache s = 32 B

Block s = 4 B

✓ set size = 2 Blocks (lines)

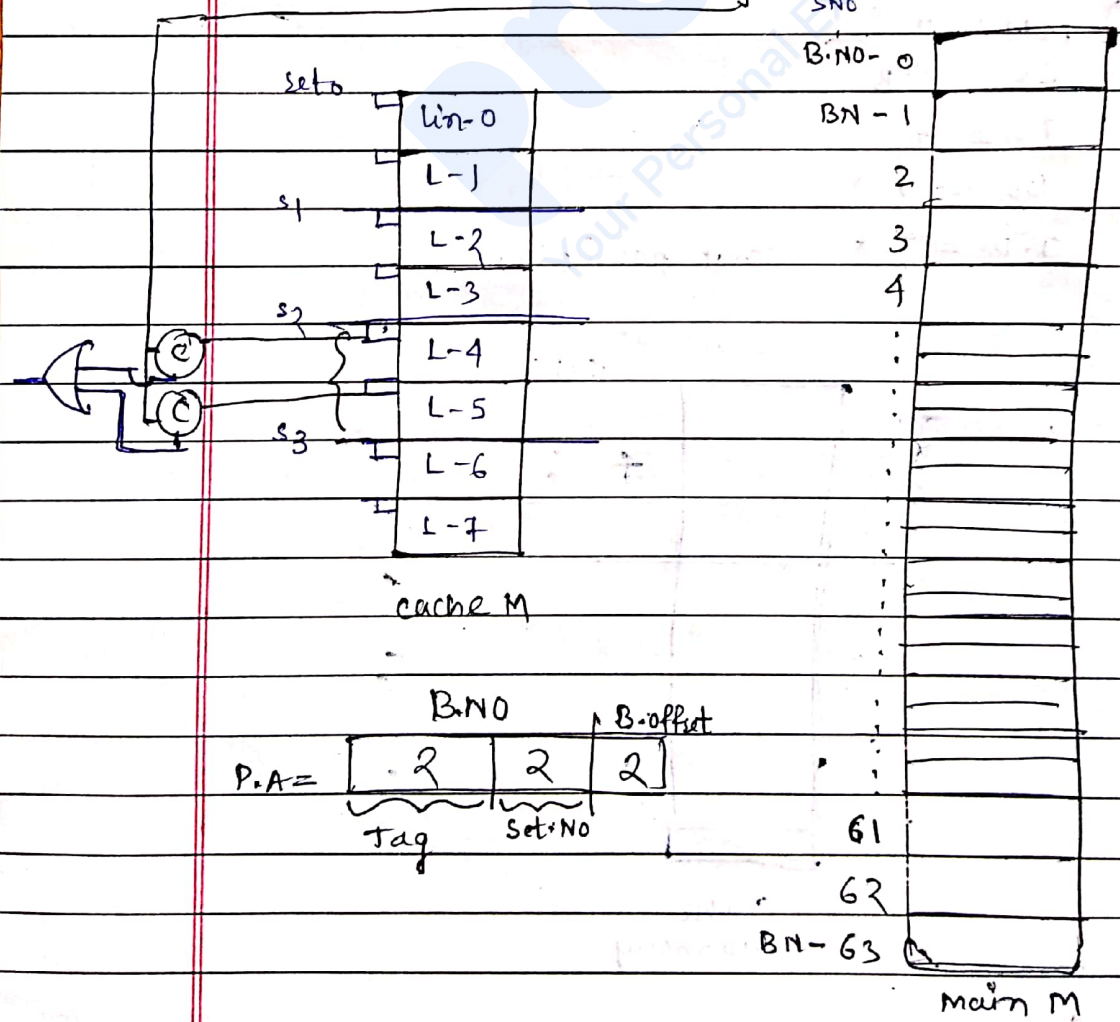
✓ (2-way set associative)

* K-way set associative required K-comparators

lines = $\frac{CS}{Bs} = \frac{32}{4} = 8$ lines

sets = $\frac{\text{lines}}{\text{sets}} = \frac{8}{2} = 4$ sets

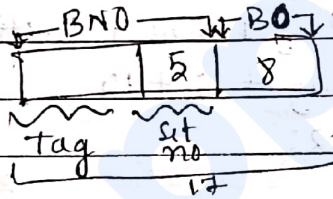
give address (P-A) = 01 | 10 | 11



Q1:

	MM size	Cache size	Block size	Tag bits	Tag directory size	set associative
①	128 KB	16 KB	256 B	4	(4×2^6) bits	2-Way set
②	32 GB	32 KB	1 KB	22	(22×2^5) bits	4
③	2 ³ MB	512 KB	1 KB	7	(7×2^9) bit	8
④	16 GB	64 MB	4 KB	10	(10×2^4) bit	4
⑤	64 MB	256 KB	X	10	X	4
⑥	8 MB	512 KB	X	7	X	8

①



$$MM S = 2^{17} B$$

$$CS = 2^{14} B$$

$$BS = 2^8 B$$

$$PA = 17 \text{ bit}$$

$$\text{no. of lines} = \frac{CS}{BS} = 2^6 \text{ lines}$$

$$\text{set no. of sets} = \frac{\text{no. of lines}}{\text{set size}}$$

$$= \frac{2^6}{2} = 2^5 = (32 \text{ sets})$$

$$PA = \text{Tag} + \text{set no} + B.\text{offset}$$

$$\Rightarrow 17 = \text{Tag} + 5 + 8$$

$$\Rightarrow \text{Tag} = 4$$

$$\text{Tag directory} = \text{Tag size} \times \text{no. of lines.}$$

$$= (4 \times 2^6) \text{ bits.}$$

②

$$MM = 32GB = 2^{35} B$$

$$CS = 32KB = 2^{15} B$$

$$BS = 1KB = 2^{10} B$$

$$T = ? (22)$$

$$T.D = 9$$

Set-ans = 4-way

$$\begin{aligned} \text{no. of line} &= \frac{CS}{BS} \\ &= \frac{2^{15}}{2^{10}} = 2^5 \end{aligned}$$

$$\text{no. of set} = \frac{\text{no. of line}}{\text{set size}} = \frac{2^5}{2^2} = 2^3$$

$$\rightarrow P.A = \text{Tag} + \text{s.no} + B.off$$

$$\rightarrow 35 = \text{Tag} + 3 + 10$$

$$\checkmark \rightarrow \text{Tag} = 22$$

$$\checkmark T.D = (22 * 2^5)$$

③

$$MM = ?$$

$$CS = 512KB = 2^{19} B$$

$$BS = 1KB = 2^{10} B$$

$$T = 7$$

$$T.D = 9$$

set-ans = 8-way

set³

7	6	10
---	---	----

Tag setno B.O

$$= 20 \text{ bit } \text{ or } 23 \text{ bit}$$

∴ (MCA) =

$$\text{no. of line} = \frac{2^{19}}{2^{10}} = 2^9$$

$$\checkmark MM \text{ size} = 2^{29} B = 2^3 MB$$

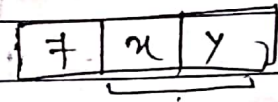
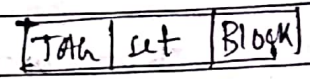
$$\text{no. of sets} = \frac{\text{no. of line}}{\text{size of sets}}$$

$$\begin{aligned} \checkmark T.D &= T * \text{no. of lines} \\ &= (7 * 2^9) \text{ bits} \end{aligned}$$

$$= \frac{2^9}{2^3} = 2^6$$

MM size	Cache size	Block size	Tag size	Tag directory	Set associative
64MB	-	-	16	-	4-way
-	512KB	-	7	-	8-way

(6) MM = 8 - (2^{23} B)
 CS = 512KB = 2^{19} B
 BS = -
 TS = 7
 TDS = -
 Set as = 8-way



cache size = no. of sets * lines per set * Block size

$$2^{19} = 2^x * 2^3 * 2^y$$

$$\Rightarrow 2^{16} = 2^{x+y}$$

$$\Rightarrow x+y = 16$$

$$\begin{aligned}
 P.A &= 7 + (x+y) \\
 &= 7 + 16 = 23
 \end{aligned}$$

$$MM = 2^{23} = 8 \text{ MB}$$

Questions gate

q-1995

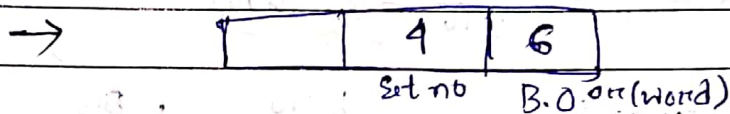
Q1

Cache size = 4K Words = 2^{12} words

Block size = 64 words = 2^6 words

set size = 4 blocks.

The numbers of bits in "set" and "word" field of MM address are -



$$\text{no of sets} = \frac{\text{no of lines}}{\text{set size}}$$

$$\text{no. of lines} = \frac{2^{12}}{2^6} = 2^6$$

$$= \frac{2^6}{2^2} = 2^4 \text{ (sets)}$$

Q2

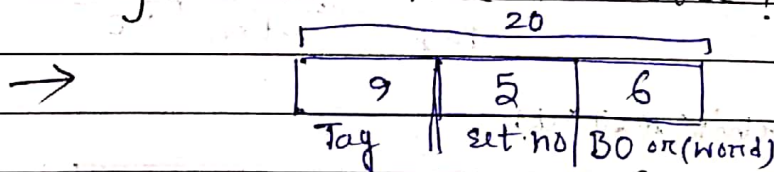
4-way set associative.

cache lines = 128

lines size = 64 words

P.A. = 20 bits

Tag, set and word fields are



$$\text{no. of lines} = \frac{\text{Cache size}}{\text{lines size}}$$

$$\text{Cache size} = 2^6 * 2^7 = 2^{13} \text{ words}$$

$$\text{sets} = \frac{\text{lines}}{\text{set size}} \Rightarrow \frac{2^7}{2^2} \Rightarrow 2^5$$

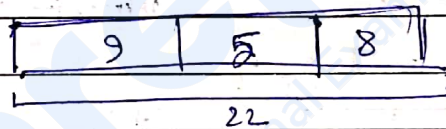
Tag = 9, set = 5, Words (B.O) = 6.

Q3) Blocks in cache = 128
 4-way set associative
 MM contains 2^{14} blocks.
 Block size is 256 eight bit words.

(i) How many bits are required for addressing MM? (22)

(ii) How many bits are needed to represent the Tag, set and word fields.
 (9) (5) (8)

→ cache lines = 2^7
 set size = 4
 MM blocks = 2^{14}
 Block size = 2^8 bit word.



$$\text{sets} = \frac{2^7}{2^2} = 2^5$$

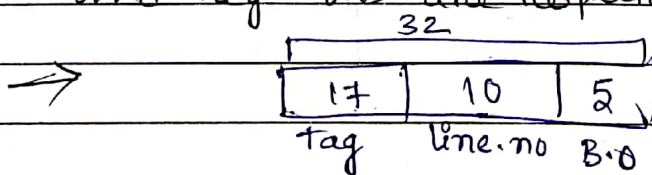
~~no. of lines~~

$$\text{MM size} = 2^{14} \times 2^8 \text{ words} \\ = 2^{22} \text{ words}$$

Q4)

Direct mapped cache
 cache size = 32 KB = 2^{15} B
 Block size = 32 B = 2^5 B
 PA = 32 bits

The number of bits needed for cache indexing and tag bits are respectively.



$$\text{no. of lines} = \frac{2^{15}}{2^5} = 2^{10}$$

Cache indexing = 10 bit

Tag bit = 17. Ans. (10, 17)

P-227

Q-2006 (85)
1.20-1.21

Consider two cache organizations

① first one

set associative

cache size = 32 KB = 2^{15} B

2-way set associative

Block size = 32 B = 2^5 B

② 2nd one

Direct

cache size same

Direct mapped

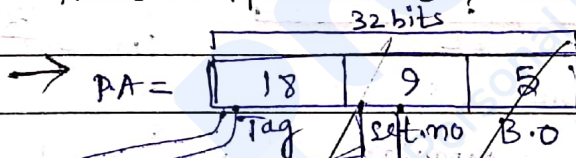
P.A = 32 bits

2-to-1 MUX latency = 0.6 ns.

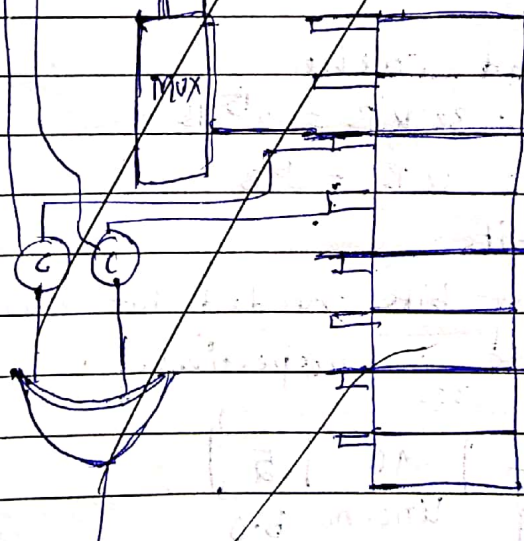
K bit comparator has latency = $K/10$ ns.

The hit latency of set associative organization is h_1 and " " Direct mapped is h_2 .

find h_1 and h_2



no of comparators requires = 2^1



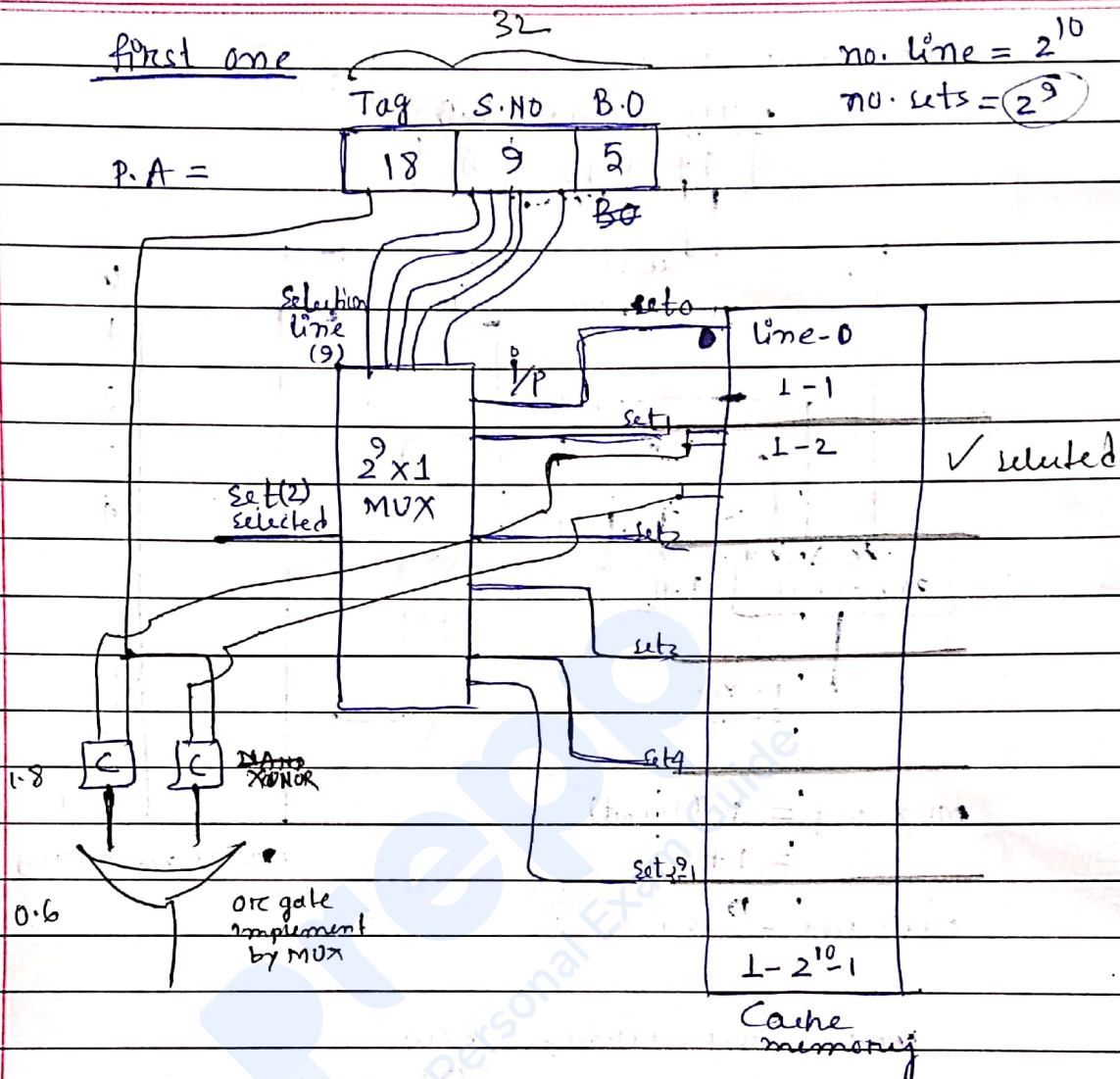
line no = $\frac{2^{15}}{2^5} = 2^{10}$

no. of set = $\frac{2^{10}}{2^1} = 2^9$

here, MUX required = $2 \times$ Tag bits = $K \times$ Tag bits

hit latency =

$\frac{32}{19} = 1.8$



MUX req = Tag bit * 2

K-way set associative (MUX req) = $T * K$

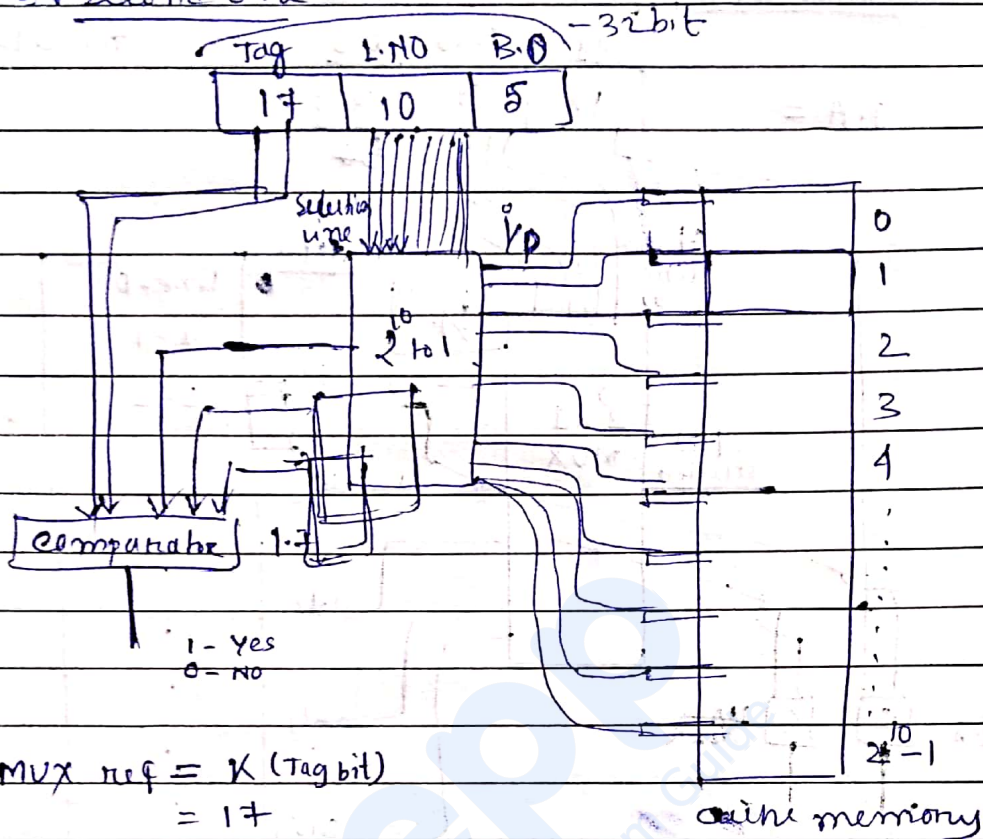
Size of each MUX = 2^S to 1

Comparator need = K

Comparator latency = $K/10 = 18/10 = 1.8$ ns.

$t_1 = 1.8 + 0.6$
 $= 2.4$ ns

Second one -



$$\text{MUX ref} = K (\text{Tag bit}) = 17$$

$$\text{each size} = 2^{10} \text{ to } 1$$

$$\text{comparator latency} = K/10 = 17/10 = 1.7$$

~~Total~~ $t_2 = 1.7 \text{ ns}$

9-2011

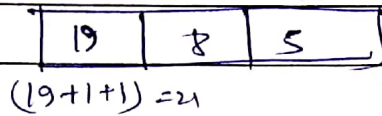
- (Q6) Direct mapping
cache size = 8KB
BS = 32 byte.
PA = 32 bits.

The cache controller maintains tag information for each cache block comprising of following.

Invalid bit, 1 modified bit and as many bits as the minimum needed to identify block mapped in the memory block mapped in the cache

What is the total size of memory needed at the cache controller of store meta-data (tags) for the cache?

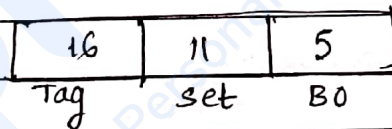
$$\text{lines} = \frac{CS}{BS} = \frac{2^{13}}{2^5} = 2^8$$



$$\begin{aligned} \text{T-Directory} &= (21 \times 2^8) \text{ bits} \\ &= 21 \times 256 \\ &= 5376 \text{ bits} \end{aligned}$$

P-2009
1-40-1-41
Q-2012

Q7 cache size = 256 KB = 2^{18} B
 set size = 4
 B.S = 32 B = 2^5 B
 P.A = 32 bits



$$\begin{aligned} \text{no of lines} &= \frac{BS \times CS}{BS} \\ &= 2^{13} \end{aligned}$$

① The number of bits in the tag field of an address — (16).

$$\begin{aligned} \text{sets} &= \frac{\text{lines}}{2^2} \\ &= \frac{2^{13}}{2^2} = 2^{11} \end{aligned}$$

② the size of the cache tag directory is — lines \times tag

$2^{13} \times (16+2+11)$	$= 2^{13} \times 29$
$= 2^{13} \times 20$	$= 2^{13} \times 4$
$= 2^3 \times 20 \text{ KB}$	$= 2^{17}$
$= 160 \text{ KB}$	$= 2^7 \text{ KB}$
	$= 128 \text{ KB}$

g-2019

88

4-way set associativity

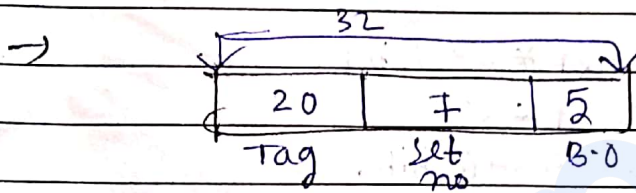
Cache size = 16KB = 2^{14} B

Block size = 8 Words = $(8 \times 32) \text{ bits} = \frac{2^8}{2^3} \text{ B} = 2^5 \text{ B}$

Word size = 32 bits

PA = 4GB = 2^{32} B

Tag bits = 9



Tag bits = 20

Lines = $\frac{CS}{Bs}$
 $= 2^9$

Set = $\frac{\text{lines}}{\text{set size}} = \frac{2^9}{2^2} = 2^7$

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Memory Interfacing

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Computer Architecture :

It deals with instructions, addressing modes, ALU, pipelining etc (Internal Design)

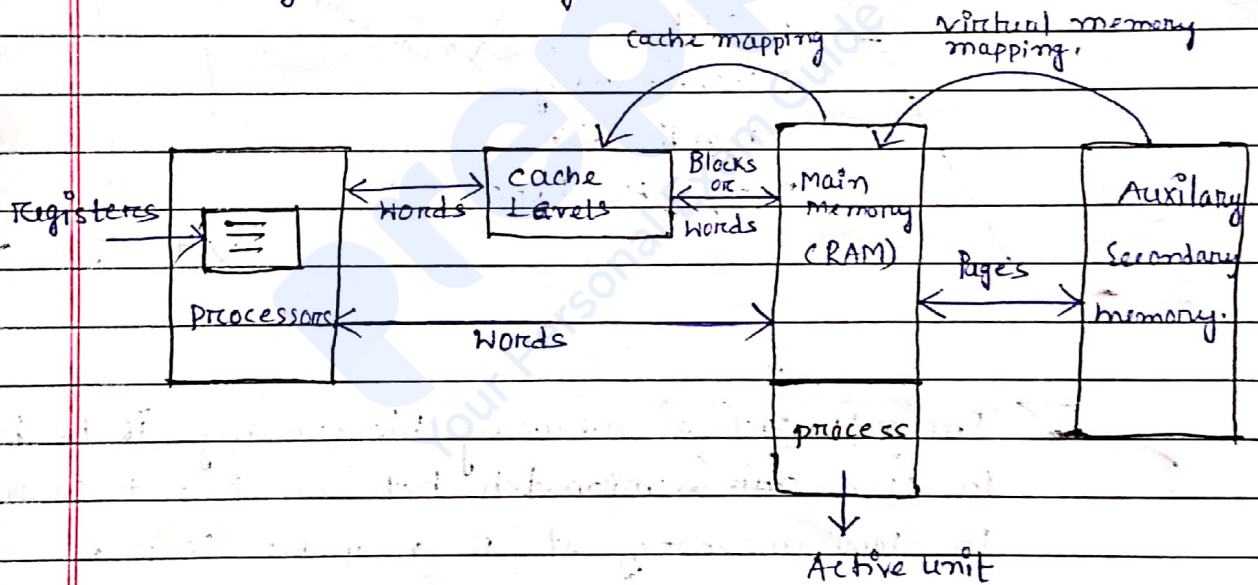
Computer Organization :

It deals with how various memory and I/O interact with a system.

Computer design :

It deals with hardware design.

• Memory Interfacing -



• Memory Hierarchy :

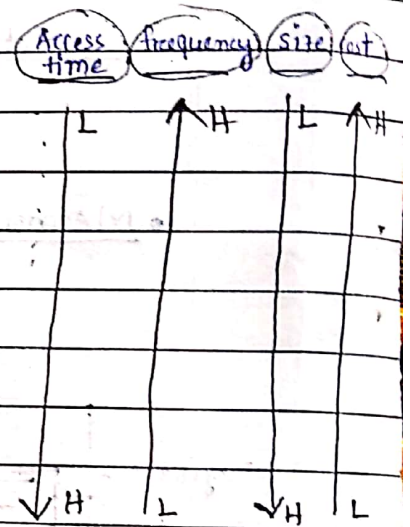
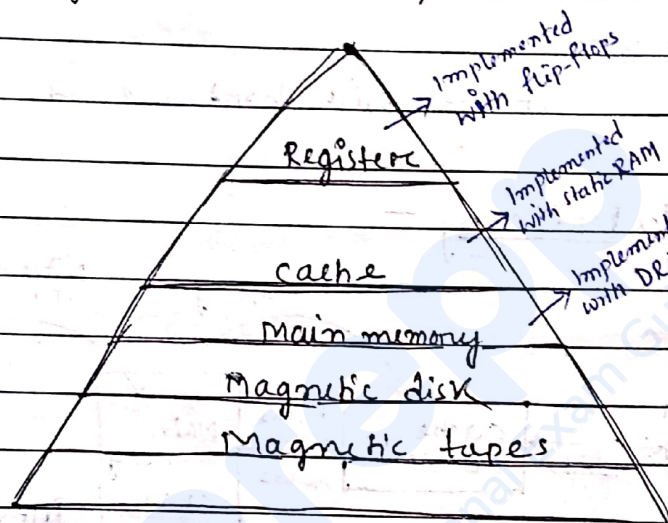
Cache levels }
 Main memory } Random Access.



Magnetic disk → semi Random Access.



Magnetic tapes → sequential Access.



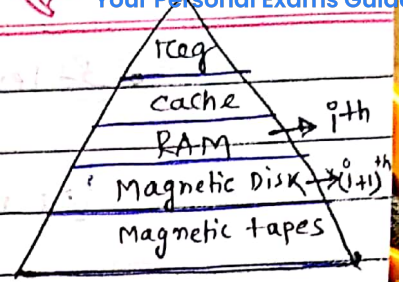
→ The purpose of memory hierarchy is to bridge the speed mismatch between fastest processor to slow memory at reasonable cost.

→ The goal of memory hierarchy is to minimize average access time of entire memory system.

• Level memory -

• 2 level memory -

Information in i^{th} ($(i+1)^{\text{th}}$) level

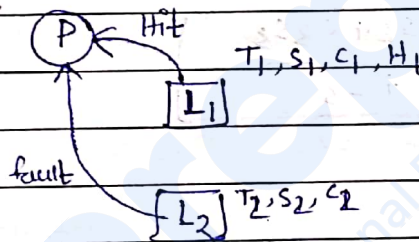


→ If processor refers to i^{th} level memory is found then "Hit" otherwise "Miss (or) fault".

→ There are two way in which the processor is connected to various levels of memory

Case 1:

Cases



$$\text{Hit rate} = \frac{x}{100}$$

$$\text{Miss rate} = (1 - \frac{x}{100})$$

T_1 → Time to access

S_1 → Size of level 1 memory.

C_1 → cost per bit.

H_1 → Hit rate.

$$T_{\text{avg}} = H_1 T_1 + (1 - H_1) T_2$$

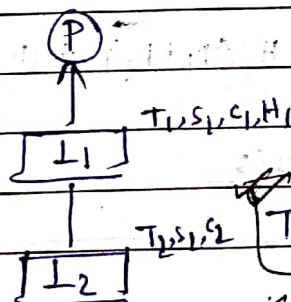
$$\text{Cost}_{\text{avg}} = \frac{C_1 S_1 + C_2 S_2}{S_1 + S_2}$$

100 → x

$$T_{\text{avg}} = \frac{x T_1 + (100 - x) T_2}{100}$$

$$= H_1 T_1 + (1 - H_1) T_2$$

Case 2:



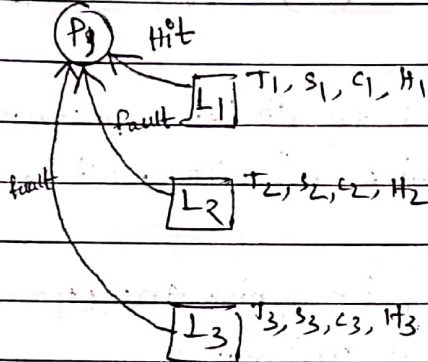
$$T_{\text{avg}} = H_1 T_1 + (1 - H_1) (T_1 + T_2)$$

$$\text{Cost}_{\text{avg}} = \frac{S_1 C_1 + S_2 C_2}{S_1 + S_2}$$

• 3-level memory -

Case-1

$$T_1 < T_2 < T_3$$



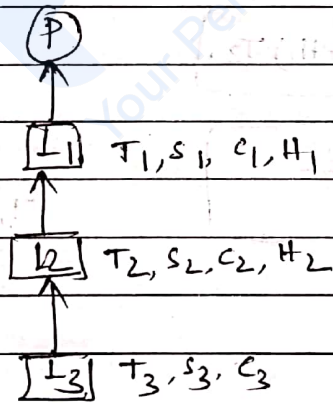
Best case time Worst time taken

$$T_1 \leq T_{avg} \leq T_3$$

$$T_{avg} = H_1 T_1 + (1-H_1) H_2 T_2 + (1-H_1)(1-H_2) T_3$$

$$C_{avg} = \frac{C_1 S_1 + C_2 S_2 + C_3 S_3}{S_1 + S_2 + S_3}$$

Case-2



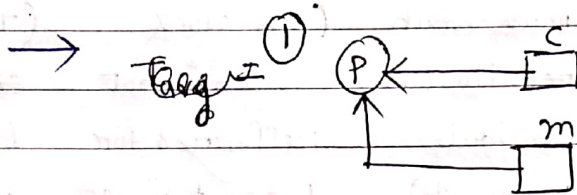
Best time taken Worst time taken

$$T_1 \leq T_{avg} \leq (T_1 + T_2 + T_3)$$

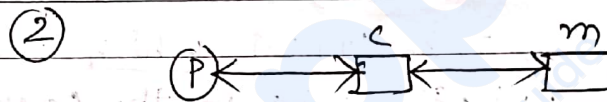
$$T_{avg} = H_1 T_1 + (1-H_1) H_2 (T_1 + T_2) + (1-H_1)(1-H_2) (T_1 + T_2 + T_3)$$

$$C_{avg} = \frac{S_1 C_1 + S_2 C_2 + S_3 C_3}{S_1 + S_2 + S_3}$$

Q: The average memory access time for a machine with a cache hit rate of 80% where the cache access time is 5ns and memory access time is 100ns is?



$$\begin{aligned}
 T_{avg} &= H_1 T_c + (1 - H_1) T_m \\
 &= (0.8)(5ns) + (1 - 0.8)(100ns) \\
 &= \boxed{24 ns}
 \end{aligned}$$



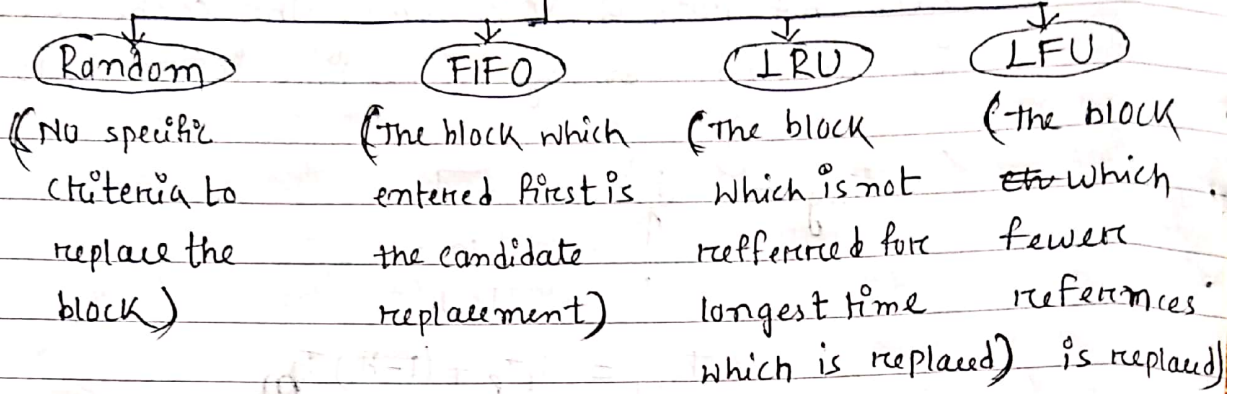
$$\begin{aligned}
 T_{avg} &= H_1 T_c + (1 - H_1) (T_c + T_m) \\
 &= (0.8)(5ns) + (1 - 0.8)(5ns + 100ns) \\
 &= 4ns + (0.2)(105ns) \\
 &= 4ns + 21.0ns \\
 &= \boxed{25 ns}
 \end{aligned}$$

• Cache replacement policy :-

↳ Replacement policy is required for associative mapping and set associative mapping but not for direct mapping.

↳ Replacement policies are aimed to minimize miss penalty for ~~future references~~ future references.

Replacement Policies



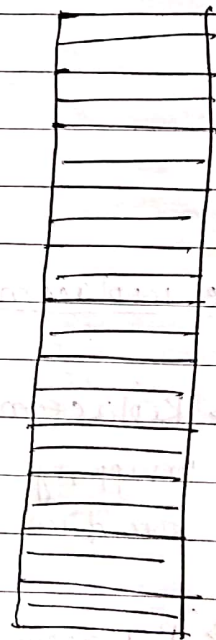
Q:1 Consider a direct mapped cache with 8 cache blocks (0-7), if the memory block requests are in the order 3, 5, 2, 8, 0, 6, 3, 9, 16, 20, 17, 25, 18, 30, 24, 2, 63, 5, 82, 17, 24 which one of the memory blocks will ~~be~~ not be in the cache at the end of the sequence?

- (a) 3 (b) 18 (c) 20 (d) 30

⇒

0	8 16 24
1	8 17 25 17
2	2 18 2 82
3	3 ✓
4	20 ✓
5	5
6	8 30 ✓
7	63

no. of Block (line) = 8
 cache



$$i = j \pmod 8$$

Line
 $i \rightarrow$ block no. of cache.
 $j \rightarrow$ block no. of MM.
 number of lines on cache (8).

main memory

Q:2 Consider a 1-way set associative mapping with 16 cache blocks, the memory block request are in the order (0, 255, 1, 1, 3, 8, 133, 159, 216, 219, 48, 32, 73, 92, 155) which one of the following memory block, will ~~not~~ be in the cache if LRU is used.

- 3
 8
 129
 216

⇒

S ₀	0, 1, 8, 216 , 48, 32, 92 8, 48, 32, 92	(LRU - Least recently used) Set-number = (i) mod 4 i → Block number.
S ₁	1, 133, 73,	
S ₂		
S ₃	(255, 1, 155) , 219, 216 , 63, 155	

cache memory (4-way set associative)
(means ~~one~~ each set contain 4 lines)

Q:3 Consider a small 2-way set associative mapping with a total of 4 blocks, for choosing the block to be replace use LRU scheme, the number of cache misses for the following sequence of block addresses 8, 12, 0, 12, 8 is 4 ?

M M M H M

⇒

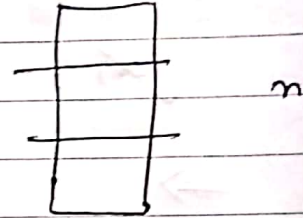
S ₀	8, 12 , 0, 12, 8	s.no = (i) mod 2
S ₁		

cache

✓ Miss rate = $(\frac{4}{5} \times 100) = 80\%$
 ✓ Hit rate = $(\frac{1}{5} \times 100) = 20\%$

Q: 9 Consider a 2-way set associative mapping consisting of 2^c memory blocks and $2c$ cache blocks, the cache location for the memory block 'K' is —

- a) $K \bmod 2c$
 b) $K \bmod 2^c$
 ✓ c) $K \bmod c$
 d) $K \bmod K$



no. of lines in cache = $2c$

2-way set-associative.

no. of set = $\frac{2c}{2} = c$

Set no = $K \bmod c$

Q: 5

Consider the cache has 4 blocks, for the memory reference (5, 12, 13, 17, 4, 12, 13, 17, 2, 13, 19, 13, 43, 61, 19) what is the hit ratio for the following cache replacement algorithms —

- (i) FIFO (ii) LRU (iii) Direct mapping
 (iv) 2-Way set Associate (LRU):

→ (i) FIFO. (5, 12, 13, 17, 4, 12, 13, 17, 2, 13, 19, 13, 43, 61, 19)

5	43
12	61
13	19
17	13

hit ratio = $\left(\frac{5}{15} \times 100\right)$

miss ratio = $\left(\frac{10}{15} \times 100\right)$

→ (ii) LRU. $(\overset{m}{5}, \overset{m}{12}, \overset{m}{13}, \overset{m}{17}, \overset{m}{4}, \overset{\checkmark}{12}, \overset{\checkmark}{13}, \overset{\checkmark}{17}, \overset{m}{2}, \overset{\checkmark}{13}, \overset{m}{19}, \overset{\checkmark}{13}, \overset{m}{43}, \overset{m}{61}, \overset{\checkmark}{19})$

$(\overset{\checkmark}{5}, \overset{\checkmark}{12}, \overset{\checkmark}{13}, \overset{\checkmark}{17}, \overset{\checkmark}{4}, \overset{\checkmark}{12}, \overset{\checkmark}{13}, \overset{\checkmark}{17}, \overset{\checkmark}{2}, \overset{\checkmark}{13}, \overset{\checkmark}{19}, \overset{\checkmark}{13}, \overset{\checkmark}{43}, \overset{\checkmark}{61}, \overset{\checkmark}{19})$

hit ratio = $(\frac{6}{15} \times 100)$ miss ratio = $(\frac{9}{15} \times 100)$

(iii) → Direct mapping $(\overset{m}{5}, \overset{m}{12}, \overset{m}{13}, \overset{m}{17}, \overset{m}{4}, \overset{m}{12}, \overset{m}{13}, \overset{m}{17}, \overset{m}{2}, \overset{m}{13}, \overset{m}{19}, \overset{\checkmark}{13}, \overset{m}{43}, \overset{m}{61}, \overset{m}{19})$

$4(6) / 15$
 $\frac{21}{30}$

0	12	17 2
1	5	13 17 13 4 61
2		
3	13	43 19

line no = (B.No) mod 4

cache hit ratio = $(\frac{1}{15} \times 100)$
miss ratio = $(\frac{14}{15} \times 100)$

→ (iv) 2-Way Set Associate (LRU) -

$(\overset{m}{5}, \overset{m}{12}, \overset{m}{13}, \overset{m}{17}, \overset{m}{4}, \overset{m}{12}, \overset{m}{13}, \overset{m}{17}, \overset{\checkmark}{2}, \overset{m}{13}, \overset{\checkmark}{19}, \overset{\checkmark}{13}, \overset{m}{43}, \overset{m}{61}, \overset{m}{19})$

S ₀	12 , 17 , 12, 2
S ₁	5 , 13 , 17 , 13 , 17 , 13 , 13 , 13 , 43 , 61, 19

line no = (i mod 2)

i → Block no.

hit ratio = $(\frac{5}{15} \times 100)$

miss ratio = $(\frac{10}{15} \times 100)$

Q:6

A hierarchical memory system has the following specification. 20MB main storage with access time of 300ns, 256 B cache with access time of 50ns, word size 4B, page size 8 words. What will be the hit ratio if the page address trace of a program has the pattern $0, 1, 2, 3, 0, 1, 3, 0, 1, 2, 4$ following LRU page replacement technique.



Cache size = 256 B

Word size = 4B

page size = 8 Words = $(8 \times 4) B = 32 B$

No. of cache page = $\frac{\text{size of cache}}{\text{cache page size}} = \frac{256}{32} = 2^3$

(0, 1, 2, 3, 0, 1, 3, 0, 1, 2, 4)

hit ratio = $(\frac{3}{8} \times 100)$

Q:7

consider an array A[100] and each element occupies 4 words, A 32-word cache is used and divided into 8-word blocks.

- a) What is the hit ratio for the statement.
 $for(i=0, i < 100, i++)$
 $A[i] = A[i] + 10.$



Cache size = 32 W

Block size = 8W

no. of Block in cache = $\frac{32}{8} = 4$ Block.

B ₀	A ₀	A ₁
B ₁	A ₂	A ₃
B ₂	A ₄	A ₅
B ₃	A ₆	A ₇

$$\frac{A_0}{m} \frac{A_0}{H} \frac{A_1}{H} \frac{A_1}{H}$$

$$\frac{A_2}{m} \frac{A_2}{H} \frac{A_3}{H} \frac{A_3}{H}$$

cache.

$$\text{hit ratio} = \left(\frac{3}{4} \times 100\right) \checkmark = 75\% \text{ hit rate.}$$

Q: 8

Consider an array has 100 elements and each element occupies 4 words. A 32 bit word cache is used and divided into a block of 8 words. What is the hit rate of.

for (i=0; i<10; i++)
for (j=0; j<10; j++)

$$A[i][j] = A[i][j] + 10.$$



Cache size = 32 word

Block size = 8 W

no. of block = 4.

i=0 10
i=1
i=2

B ₀	A ₀	A ₄
B ₁		
B ₂		
B ₃		

cache.

array = [A₀, A₁, A₂, ..., A₁₀₀]

RMO

00	01	02	...	09
10	11	12	...	19
20				
30				
:				
90	91	92	...	99

CMO

[00] [10] [20] [30]

RMO :

00	01

m	H	H	H
00	00	01	01
R	W	R	W

$$TM = (50)1$$

$$TH = (50)3$$

$$TR = (50)4$$

$$\text{Hit ratio} = \left(\frac{3}{4} \times 100\right) \\ = 75\%$$

(column major)

CMO :

m	H	m	H
00	00	01	01

$$TM = (50)2$$

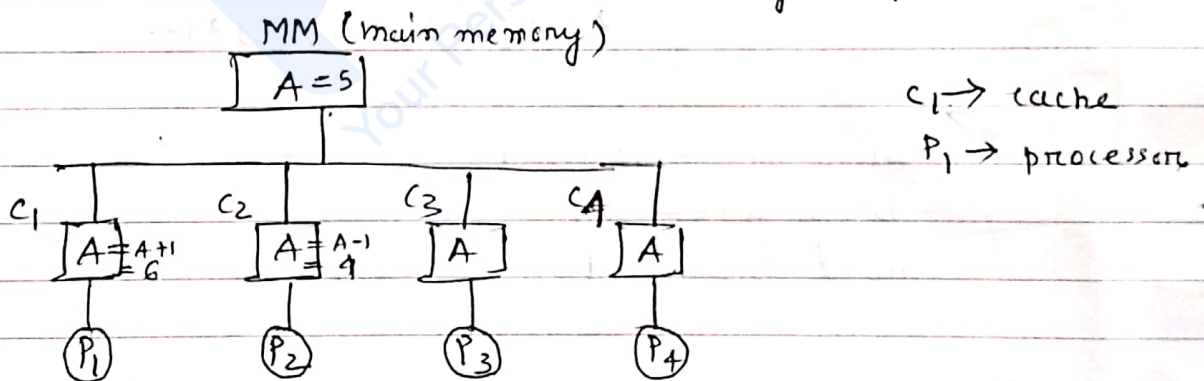
$$TH = (50)2$$

$$TR = (50)4$$

$$\text{Hit ratio} = \left(\frac{2}{4} \times 100\right) \\ = 50\%$$

• Cache Coherence Problem :

→ Cache Coherence Problem : Multiple copies of same data can exist in different caches simultaneously, and if processors are allowed to update their own copies freely an ~~an~~ inconsistent view of memory can result.

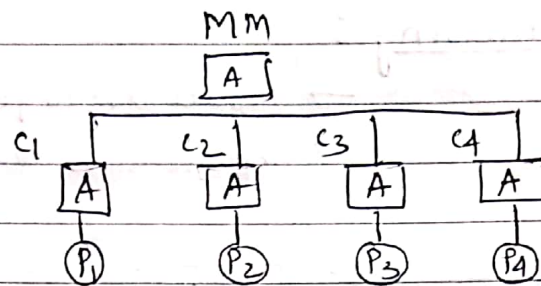


• Methods to avoid cache coherence problem -

→

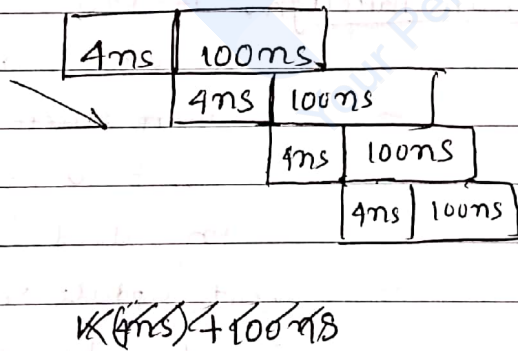
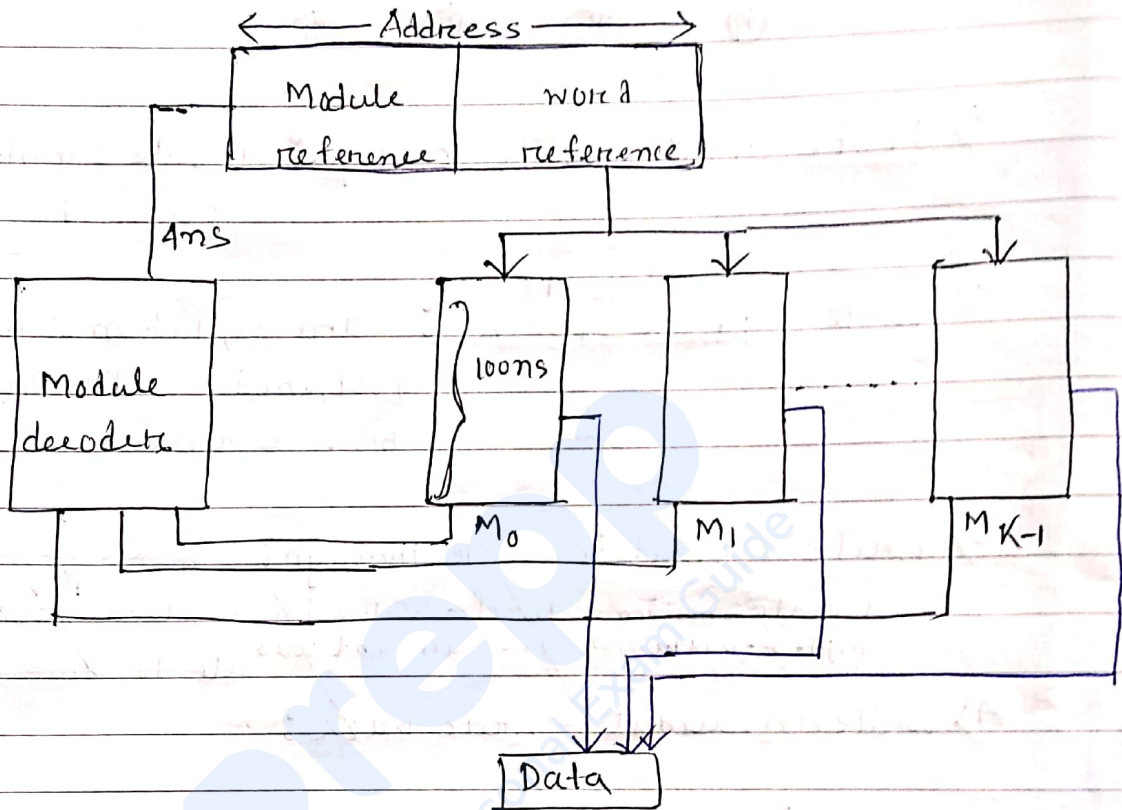
There are 4 methods to avoid cache coherence problem

- 1) write update → write through.
- 2) write update → write back.
- 3) write invalidate → write through.
- 4) write invalidate → write back.



- 1) Write update - write through: update simultaneously of a word in cache & - M memory
- 2) Write update - write Back: The updation of MM is postponed until the associated block is replaced.
- 3) Write Invalidate - write through: ~~changing a value in~~ update simultaneously of a value in ~~one cache~~ ^{the value} will be ~~invalid~~ MM. ~~other cache can't use~~ ~~ate to other cache.~~ ~~this value.~~
- 4) Write Invalidate - write Back:
 - 3) write invalidate - write through: if processor P₁ ~~changed~~ a value and change, then ~~other other~~ processor P₂ ~~can't~~ the value will be invalid for other processor and update simultaneously of value in ~~mm~~ MM.
 - 4) write invalidate - write Back: if P₁ ~~and~~ change a value then this value will be invalid to other processor and updation of MM is postponed until the associated block is replaced.

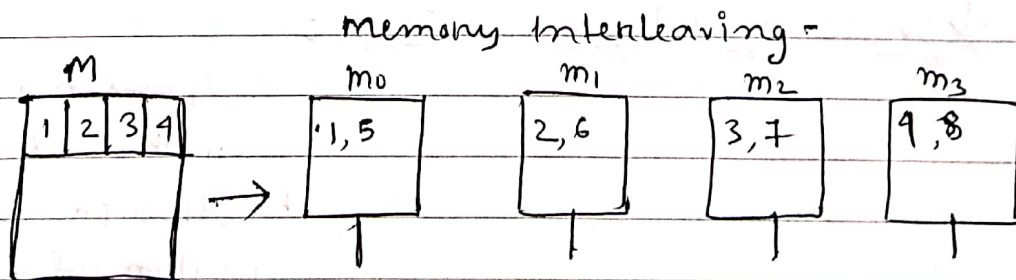
- Memory interleaving :
 - "Reduce average access time".
 - "Improve data transfer rate".



If read 'K' words then, total time taken, $T = K(4ns) + 100ns$.

AND

without interleaving concept total time, $T = K(100ns)$.



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