

MENSURATION

Mensuration is mainly divided into three parts:

One is **Area**, second is **Volume** and third is **Perimeter**.

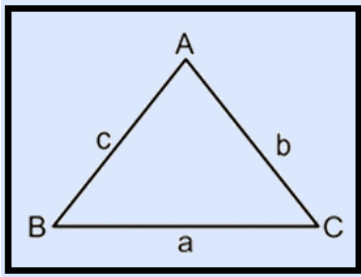
Triangle

Perimeter = $a + b + c$

Area of Triangle = $[\frac{1}{2} \times \text{base} \times \text{height}]$ or

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

Here, s is a semi perimeter



Q1. Sides of a triangle are 12m, 13m and 11m. Find the area of triangle and height with respect to the side of 12 cm.

Solution:

$$S = \frac{12+13+11}{2} = \frac{36}{2} = 18m$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\text{Area} = \sqrt{18 \times 6 \times 5 \times 6}$$

$$\text{Area} = 6\sqrt{105} \text{ m}^2$$

Side=12m (Base), height=?

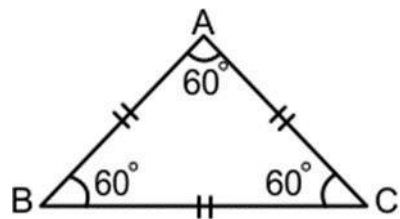
Area of triangle = $\frac{1}{2} \times \text{base} \times \text{Height}$

$$6\sqrt{105} = \frac{1}{2} \times 12 \times \text{height}$$

$$\text{Height} = \sqrt{105} \text{ m}$$

Properties of Equilateral Triangle

- Properties of equilateral triangle
- All three sides are equal.
- All three angles are 60°



Formulae

- Perimeter = $3 \times \text{side} = 3a$
- Area of equilateral triangle = $(\sqrt{3}/4)a^2$
- Height = Median = $(\sqrt{3}/2)a$
- Side of an equilateral triangle =
$$\frac{\text{The sum of distances of a point inside a triangle from all the sides} \times 2}{\sqrt{3}}$$

Q2. The area of an equilateral triangle is $49\sqrt{3} \text{ m}^2$, find the value of its height and side?

Solution:

The area of an equilateral triangle = $49\sqrt{3} \text{ m}^2$

$$(\sqrt{3}/4)a^2 = 49\sqrt{3}$$

$$a^2 = 49 \times 4$$

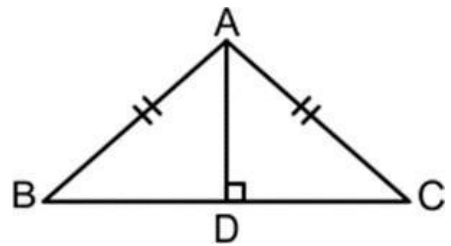
$$a = 14 \text{ m}$$

$$\text{Height} = (\sqrt{3}/2)a$$

$$\text{Height} = (\sqrt{3}/2) \times 14 = 7\sqrt{3} \text{ m}$$

Properties of Isosceles Triangle

- Two sides are equal.
- $AB = AC$



Formula:

Area of $\Delta = \frac{1}{2} \times \text{base} \times \text{height}$

Q3. In an isosceles triangle, if the ratio of equal side and unequal side is 5: 8 and its perimeter is 36 cm. Find the area of this triangle.

Solution:

Let the equal and unequal side be $5x$ and $8x$

Perimeter of this triangle = $5x + 5x + 8x$

$$36 = 18x$$

$$x = 2$$

Sides of this triangle are 10 cm, 10 cm and 16 cm

$$\text{Perpendicular on unequal side} = \sqrt{10^2 - (16/2)^2} = \sqrt{100 - 64}$$

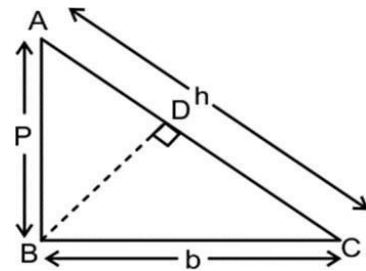
$$\text{Perpendicular on unequal side} = 6 \text{ cm}$$

Area of isosceles triangle = $\frac{1}{2} \times \text{unequal side} \times \text{perpendicular on unequal side}$

$$\text{Area of isosceles triangle} = \frac{1}{2} \times 16 \times 6 = 48 \text{ cm}^2$$

Properties of Right-Angle Triangle

- $\text{Hypotenuse}^2 = (\text{base})^2 + (\text{height})^2$
- $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times b \times p$
- $\text{Length of Perpendicular at hypotenuse} = \frac{\text{Height} \times \text{Base}}{\text{Hypotenuse}}$



Q4. The area of a right-angle triangle is 30 m and base is 7 m more than height, find the length of perpendicular on hypotenuse.

Solution:

Let the height of the triangle be x

$$\Rightarrow \text{Base} = x + 7$$

$$\Rightarrow \text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\Rightarrow 30 = \frac{1}{2} \times (x + 7) \times x$$

$$\Rightarrow 60 = x^2 + 7x$$

$$\Rightarrow x^2 + 7x - 60 = 0$$

$$\Rightarrow (x + 12)(x - 5) = 0$$

$$\Rightarrow x = 5 \text{ m}$$

$$\Rightarrow \text{Base} = x + 7 = 12 \text{ m}$$

$$\Rightarrow \text{Hypotenuse}^2 = (\text{base})^2 + (\text{height})^2$$

$$\Rightarrow \text{Hypotenuse}^2 = 144 + 25 = 169$$

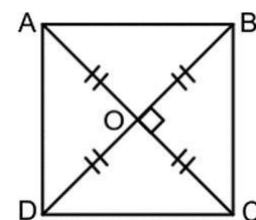
$$\Rightarrow \text{Hypotenuse} = 13 \text{ m}$$

The length of perpendicular on hypotenuse is $(60/13)$ m.

Square

Properties of square

- Opposite sides of a square are both parallel and equal in length.
- The diagonal of a square is equal. $AC = BD$
- The diagonals of a square bisect each other and meet at 90° . $AO = OC$, $BO = OD$ and $\angle BOC = 90^\circ$
- All four angles of a square are 90° .



Formula:

- Perimeter = $4 \times \text{side}$
- Area = $(\text{side})^2 = \frac{1}{2} \times (\text{diagonal})^2$
- Diagonal = $\sqrt{2} \times \text{side}$

Q5. Area of a square is 1352 m². Find the perimeter and the length of the diagonal.

Solution:

Area of a square = 1352

$$(\text{Side})^2 = 1352$$

$$\text{Side} = 26\sqrt{2} \text{ m}$$

$$\text{Perimeter} = 4 \times \text{side}$$

$$\text{Perimeter} = 4 \times 26\sqrt{2} = 104\sqrt{2} \text{ m}$$

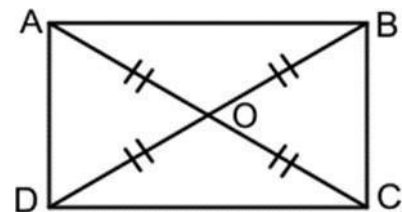
$$\text{Length of diagonal} = \sqrt{2} \times \text{side}$$

$$\text{Length of diagonal} = \sqrt{2} \times 26\sqrt{2} = 52 \text{ m}$$

Rectangle

Properties of rectangle

- Opposite sides of a rectangle are equal in length.
 $AB = DC, AD = BC$
- Opposite sides of a rectangle are parallel.
 $AB \parallel CD, AD \parallel BC$
- The diagonals of a rectangle bisect each other.
 $AO = OC, BO = OD$
- All four angles of a square are 90° .



Formulae:

$$\text{Area} = l \times b$$

$$\text{Perimeter} = 2(l + b)$$

$$\text{Diagonal} = \sqrt{l^2 + b^2}$$

Here, Length = l

Breadth = b

Q6. In a rectangle diagonal is 9 times of its length. Find the ratio of length and breath.

Solution:

Let the length of rectangle be x

Diagonal = $9x$

Diameter = $\sqrt{l^2 + b^2}$

$9x = \sqrt{x^2 + b^2}$

$x^2 + b^2 = 81 x^2$

$80 x^2 = b^2$

$b = (4\sqrt{5})x$

Length: breath = $x : (4\sqrt{5})x$

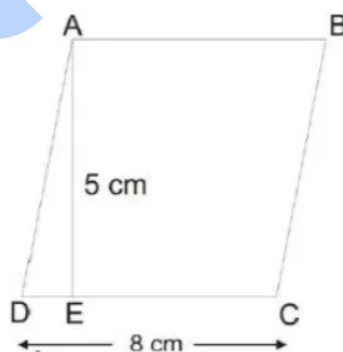
Length: breath = $1 : 4\sqrt{5}$

Parallelogram

In a parallelogram, the opposite sides are parallel and equal. But all 4 sides are not equal. Also, the angle of the corners is not at 90 degrees. Whereas in square and rectangle, the angle of corners is 90 degrees.

- Area of parallelogram = $l \times h$
- Perimeter of parallelogram = $2(l+b)$

Here, l = length or base of parallelogram. h = height of parallelogram.



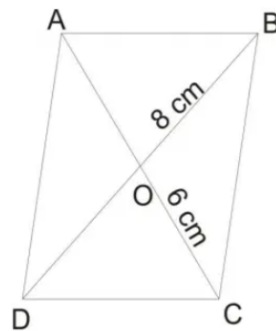
Rhombus

Rhombus is also a parallelogram. The main factor is that all 4 sides are equal in the rhombus. The mensuration formula chart for the rhombus is,

- Area of rhombus = $d_1 \times d_2 / 2$

- The perimeter of rhombus = $4l$

Here, d_1 and d_2 are the lengths of the diagonals. l is the side of the rhombus.

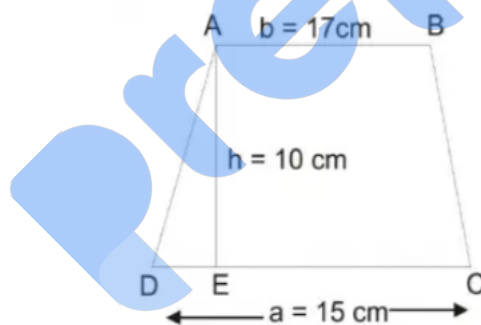


Trapezium

In trapezium, one set of opposite sides are parallel and unequal. The other set of opposite sides are not parallel.

- Area = $\frac{1}{2} h(a+b)$
- Perimeter = Sum of all sides

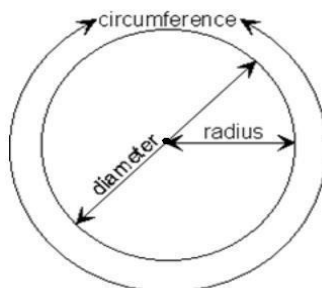
Here a and b are the top and bottom sides of the trapezium. h is the height of the trapezium.



Circle

- Circumference of a circle = $\pi \times \text{diameter}$
- Diameter of circle = $2r$
- Area of a circle = $\pi \times r \times r$

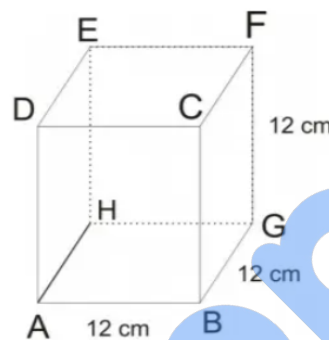
Here r is the radius of the circle.



Cube

Cube is a 3d shape of a square. In the cube all the length, breadth, and height are equal. The mensuration formula for bank exams also includes all 3d shapes. So, refer to the mensuration formulas for the cube here.

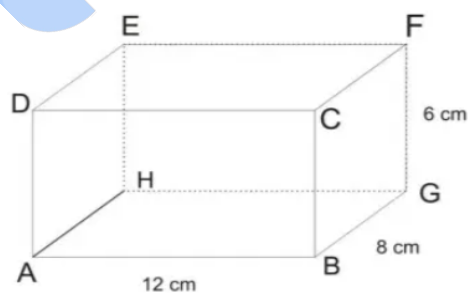
- Volume of a cube = (side)³
- Total surface area of a cube = $6 \times (\text{side})^2$
- Diagonal of cube = $\sqrt{3} \times (\text{side})$



Cuboid

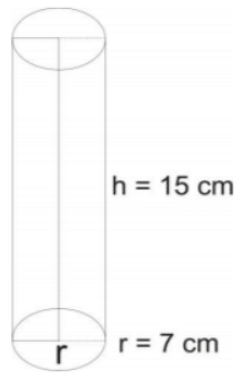
It is a 3d shape of a rectangle. The mensuration formulas for the cuboid are,

- Volume of a cuboid = (length $\times$ breadth $\times$ height) = lbh
- Total surface area of cuboid = $2(lb + bh + hl)$



Cylinder

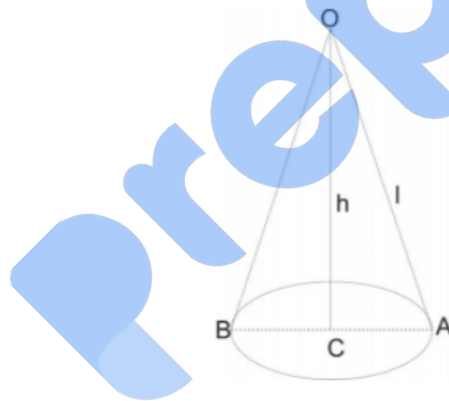
- Area of curved surface = (perimeter of base) $\times$ height = $2\pi rh$
- Total surface area = $2\pi r(r + h)$
- Volume = $\pi \times r \times r \times h$



Cone

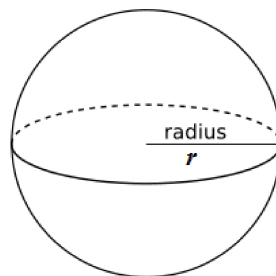
- Curved surface area = $\pi r l$
- Total surface area = $\pi r(r + l)$
- Volume of cone = $(1/3) \times \pi \times r \times r \times h$

Here l is the slant height of the cone.



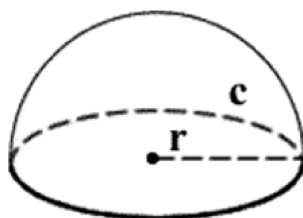
Sphere

- r = radius
- Volume: $V = \frac{4}{3} \pi r^3$
- Curved Surface Area = Total Surface Area = $4\pi r^2$



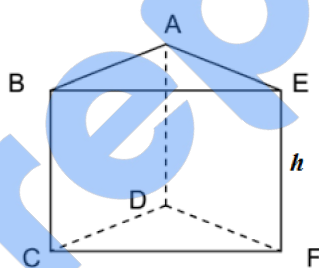
Hemisphere

- Volume = $\frac{2}{3} \pi r^3$
- Curved surface area = $2 \pi r^2$
- Total surface area = $3\pi r^2$



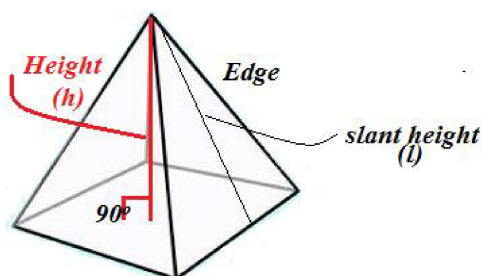
Prism

- Volume = Base area x height
- Lateral Surface area = perimeter of the base x height
- Total Surface area = LSA + 2(Area of base)



Pyramid

- Volume of a right pyramid = $\frac{1}{3}$ X area of the base X height
- Area of the lateral faces of a right pyramid = $\frac{1}{2}$ x perimeter of the base x slant height
- Area of whole surface of a right pyramid = Area of the lateral faces + Area of the base.



Practice Questions:

Q1. Three cubes of metal whose edges are in ratio of 3:4:5, are melted and one new cube is formed. If the diagonal of the cube is $18\sqrt{3}$ cm, then find the edge of the largest among three cubes.

- (A) 18 cm
- (B) 24 cm
- (C) 15 cm
- (D) 12 cm

Q2. What is the area of the largest triangle that can be fitted into a rectangle of length 'a' units and width 'b' units?

- (A) unit^2
- (B) unit^2
- (C) unit^2
- (D) unit^2

Q3. Find the perimeter and area of an isosceles triangle whose equal sides are 5 cm and the height is 4 cm.

- (A) 24 cm, 13 cm
- (B) 18 cm, 16 cm
- (C) 12 cm, 13 cm
- (D) 12 cm, 16 cm

Q4. A parallelogram has area A sq. mts. A second parallelogram is formed by joining the mid-points of its sides. A third parallelogram is formed by joining the mid-points of the sides of the second parallelogram. This process is continued upto infinite. What is the sum of area (in sq. mts) of all the parallelograms so formed?

- (A) A
- (B) $\frac{3A}{2}$
- (C) 2A
- (D) $\frac{A}{2}$

Q5. The radius of a cylindrical milk container is half its height and surface area of the inner part is 616 sq.cm. The amount of milk that the container can hold, approximately, is

[Use: $\sqrt{5} = 2.23$ and $\pi = \frac{22}{7}$]

- (A) 1.42 litres
- (B) 1.53 litres
- (C) 1.71 litres
- (D) 1.82 litres

Q6. From the four corners of a rectangular sheet of dimensions 25 cm x 20 cm, square of side 2 cm is cut off from four corners and a box is made. The volume of the box is-

- (A) 828 cm³
- (B) 672 cm³
- (C) 500 cm³

(D) 1000cm³

Q7. A rectangular paper sheet of dimensions 22 cm × 12 cm is folded in the form of a cylinder along its length. What will be the volume of this cylinder?

(A) 460 cm³

(B) 462 cm³

(C) 624 cm³

(D) 400 cm³

Q8. A copper rod of 1 cm diameter and 8 cm length is drawn into a wire of uniform diameter and 18 m length. The radius (in cm) of the wire is:

(A) $\frac{2}{15}$

(B) $\frac{1}{15}$

(C) $\frac{1}{30}$

(D) 15

Q9. A hollow iron pipe is 21 cm long and its exterior diameter is 8 cm. If the thickness of the pipe is 1 cm and iron weighs 8 gm/cm³, then the weight of the pipe is:

(A) 3.696 kg

(B) 3.6 kg

(C) 36 kg

(D) 36.9 kg

Q10. Water flows at the rate of 10 meters per minute from cylindrical pipe 5 mm in diameter how long it will take to fill up a conical vessel whose diameter at the base is 30 cm and depth 24 cm?

(A) 28 minutes 48 seconds

(B) 51 minutes 12 seconds

(C) 51 minutes 24 seconds

(D) 28 minutes 36 seconds

Solution:

Q1. (C)

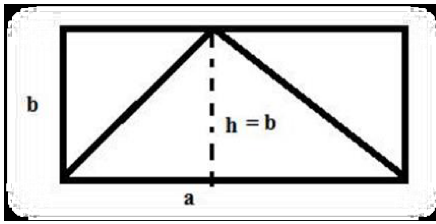
Total volume = $(3x)^3 + (4x)^3 + (5x)^3 = 216x^3$

Then side of new cube = $\sqrt[3]{216x^3} = 6x$

Then diagonal of new cube = $6x\sqrt{3} \rightarrow 18\sqrt{3} \times 3 = 54\sqrt{3}$

Then side of larger cube = $54\sqrt{3} \div \sqrt{3} = 54$

Q2. (B)

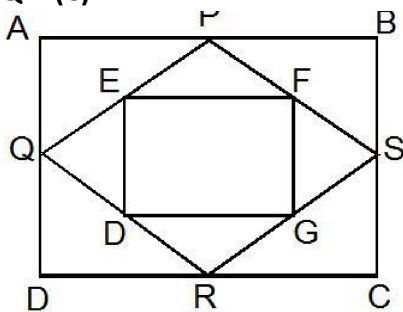


Area of triangle = $(1/2)$ base $\times$ height
 $= (1/2) ab$ unit²

Q3. (D)

$12 \text{ cm}^2, 16 \text{ cm}$

Q4. (C)



Sum of areas:

$$= A + \frac{1}{2}A + \frac{1}{4}A + \dots \infty$$

$$= \frac{A}{1 - \frac{1}{2}} = 2A$$

Q5. (B)

Let radius of the container is x cm.

Then height of the container is $2x$ cm.

According to question $2\pi r(2r) + \pi r^2 = 616$

$5\pi r^2 = 616$

$$r^2 = \frac{616 \times 7}{5 \times 22}$$

$$r = \frac{14}{\sqrt{5}}$$

$$V = \frac{22}{7} \times \frac{14}{\sqrt{5}} \times \frac{14}{\sqrt{5}} \times \frac{28}{\sqrt{5}}$$

$$= 1546.90 \text{ ml.}$$

$$= 1.54 \text{ lit.}$$

Q6. (B)

$$\text{Volume of the box} = 16 \times 21 \times 2 = 672 \text{ cm}^3$$

Q7. (B)

$$2\pi r = 22$$

$$R = \frac{7}{2} \text{ cm}$$

$$v = \pi R^2 h$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12$$

$$= 462 \text{ cm}^3$$

Q8. (C)

$$\pi r^2 h = \pi R^2 h = \frac{1}{4} \times 8 = 2$$

$$r^2 = \frac{2}{1800} = \frac{1}{900}$$

$$r = \frac{1}{30} \text{ cm.}$$

Q9. (A)

$$r = \pi(R^2 - r^2) \times h$$

$$= \frac{22}{7} \times (4^2 - 3^2) \times 21 = 462 \text{ cm}^3$$

$$\therefore 1 \text{ cm}^3 = 8 \text{ g}$$

$$462 \text{ cm}^3 = 462 \times 8 \text{ g}$$

$$= 2696 \text{ g} = 3.696 \text{ kg.}$$

Q10. (A)

Volume of water flowing from the pipe in 1 minute

$$\pi \times .25 \times .25 \times 1000$$

$$r = \frac{1}{3} \pi \times 15 \times 15 \times 24$$

$$T = \frac{1}{3} \times \frac{\pi \times 15 \times 15 \times 24}{\pi \times .25 \times .25 \times 1000} = 28 \frac{4}{5}$$

28 minutes 48 seconds

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