

**CSM – 53 / 15****Mathematics****Paper – II***Time : 3 hours**Full Marks : 300**The figures in the right-hand margin indicate marks.*

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and three of the remaining questions, selecting at least one from each Section.*

**Section – A**

1. Answer any **five** of the following :  $12 \times 5 = 60$
- (a) Find the lowest possible degree polynomial which assume the value 12, 3, –21, 15 when  $x$  has the values 2, 3, –1, 1. Hence, find the value of the function when  $x = 0$ .
- (b) Find by Newton-Raphson method the real root of  $3x - \cos x - 1 = 0$ , correct upto five decimal places.

(c) Define connected graph. If  $G$  is connected graph with  $n(\geq 3)$  vertices then at least one vertex must have degree  $> 1$ .

(d) Let  $G = (V, E)$  be a graph. Let  $\chi(G) =$  chromatic number of  $G$  and  $\Delta(G) =$

$$\max_{v_i \in V(G)} d_G(v_i). \text{ Then } \chi(G) \leq 1 + \Delta(G).$$

(e) Solve  $(D^2 + a^2)y = \sec ax$ .

(f) Solve  $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$ .

2. (a) Find  $y(1)$  by Euler's methods from the differential equation  $\frac{dy}{dx} = -\frac{y}{1+x}$  when  $y(0.3) = 2$ , correct upto four decimal places, taking step length  $h = 0.1$ . 20

(b) Compute Simpson one-third rule, the integral  $\int_0^1 x^2(1-x) dx$  correct upto three decimal places, step length is  $0.1$ . 20

(c) Determine the least square approximation of the type  $ax^2 + bx + c$  to the function  $2^x$  at the points  $x_i = 0, 1, 2, 3, 4$ . 20

3. (a)  $G = (V, E)$  be a graph with the vertices ordered as  $V = \{v_1, v_2, \dots, v_n\}$  and let  $A_G$  denotes the adjacency matrix of  $G$ . Then prove that the number of walks of length  $k$  between  $v_i$  and  $v_j$  is the  $(i, j)$ th entry of  $A_G^k$ .

20

- (b) If  $G$  is a connected graph with  $n$  vertices then prove that  $G$  has at least  $n - 1$  edge.

20

- (c) If a tree has a vertex of degree  $k$  then prove that it has at least  $k$  pendant vertices.

20

4. (a) Solve  $(2xz - yz) dx + (2yz - zx) dy - (x^2 - xy + y^2) dz = 0$ .

20

- (b) Using Charpit's method, find the complete integral of the equation  $2(z + xp + yq) = yp^2$ .

20

- (c) Solve the differential equation of Laplace

transformation  $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t}$ , given

$y(0) = y'(0) = 0$ .

20

## Section – B

5. Answer any **three** of the following :

(a) Forces P, Q, R act along the sides of a triangle formed by the lines  $x + y - 1 = 0$ ,  $x - y + 1 = 0$ ,  $y = 2$ . Find the magnitude and line of action of the resultant. 20

(b) Develop a flow chart to calculate the roots of  $ax^2 + bx + c = 0$  for various values of a, b, c. 20

(c) Find the graphical solution of the following LPP : 20

$$\text{Maximize } Z = 2x_1 - x_2$$

$$\text{Subject to } x_1 - x_2 \leq 1$$

$$x_1 \leq 3$$

$$x_1 \geq 0, x_2 \geq 0.$$

(d) A velocity field is given by  $q = -xi + (y + t)j$ . Find the stream function and the streamlines for this field for  $t = 2$ . 20

6. (a) Determine whether the motion specified by  $q = A(xj - iy) / (x^2 + y^2)$ , (A is a constant) is a possible motion for an incompressible fluid.

If so, determine the equation of streamlines. Also, show that the motion is of potential kind. Find the velocity potential. 30

- (b) A source S and a sink T of equal strengths  $m$  are situated within the space bounded by a circle whose centre is O. If S and T are equal distances from O on opposite sides of it and on the same diameter AOB, show that the velocity of the liquid at any point P is

$$2m \frac{OS^2 + OA^2}{OS} \cdot \frac{PA \cdot PB}{PS \cdot PS' \cdot PT \cdot PT'},$$

where S' and T' are the inverse points of S and T with regard to the circle. 30

7. (a) Find the moment of inertia of a uniform rectangular lamina about a line through its centre and perpendicular to its plane. 30
- (b) A semicircular disc rests in a vertical plane with its curved edge on a rough horizontal and equally rough vertical plane, the coefficient of friction being  $\mu$ . Show that the greatest angle that the bounding diameter can make with the horizontal plane is

$$\sin^{-1} \frac{(3\pi \frac{\mu(1+\mu)}{4})}{1+\mu^2} \quad 30$$

8. (a) Use Simplex method to solve the following LPP : 30

$$\text{Maximize } Z = 60x + 50y$$

Subject to

$$x + 2y \leq 40$$

$$3x + 2y \leq 60$$

$$x, y \geq 0.$$

- (b) Write a program to calculate the series

$$S = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \text{ for } x = 0.1. \text{ Use}$$

function subprogram to calculate the factorial. 30

