

JEE MAIN Sample Paper 03

Total Duration: 3 Hours | **Maximum Marks:** 300

Pattern: 75 Questions in total (25 per subject: 20 MCQs + 5 NVTs).

Marking Scheme:

- **Section A (MCQs):** +4 for correct, -1 for incorrect, 0 for unattempted.
- **Section B (Numerical):** +4 for correct, 0 for incorrect/unattempted.

Part A: Physics

Section A: Multiple Choice Questions (20 Questions)

Q.1 Match List I with List II:

List I (Physical Quantity)	List II (Dimensional Formula)
A. Spring Constant	I. $[ML^2T^{-2}]$
B. Angular Momentum	II. $[ML^0T^{-2}]$
C. Torque	III. $[ML^2T^{-1}]$
D. Gravitational Constant	IV. $[M^{-1}L^3T^{-2}]$

Choose the correct answer from the options given below:

- (A) A-II, B-III, C-I, D-IV
(B) A-II, B-I, C-III, D-IV
(C) A-I, B-III, C-II, D-IV

(D) A-IV, B-I, C-II, D-III

Q.2 Given below are two statements:

Statement I: Acceleration due to Earth's gravity decreases as you go 'up' or 'down' from Earth's surface.

Statement II: Acceleration due to Earth's gravity is the same at a height h and depth d from Earth's surface if $h = d$ (where $h, d \ll R_e$).

In the light of the above statements, choose the most appropriate answer:

- (A) Both Statement I and Statement II are correct
- (B) Both Statement I and Statement II are incorrect
- (C) Statement I is correct but Statement II is incorrect
- (D) Statement I is incorrect but Statement II is correct

Q.3 A body is projected at an angle of 30° with the horizontal. The ratio of kinetic energy at the highest point to the initial kinetic energy is:

- (A) 3 : 4
- (B) 1 : 4
- (C) 1 : 2
- (D) 4 : 3

Q.4 A block of mass m is connected to a spring of spring constant k . If the block is displaced by x from equilibrium, the time period of oscillation is T . If the mass is increased by $3m$, the new time period will be:

- (A) T
- (B) $2T$
- (C) $4T$
- (D) $T/2$

Q.5 Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: A purely capacitive circuit consumes zero average power.

Reason R: In a purely capacitive circuit, the current leads the voltage by a phase angle of $\pi/2$.

In the light of the above statements, choose the correct answer:

- (A) Both A and R are correct and R is the correct explanation of A
- (B) Both A and R are correct but R is NOT the correct explanation of A
- (C) A is correct but R is not correct
- (D) A is not correct but R is correct

Q.6 The electric potential V at any point (x, y, z) in space is given by $V = 3x^2$ volts. The electric field at the point $(1, 0, 3)$ is:

- (A) 6 V/m along negative x-axis
- (B) 6 V/m along positive x-axis
- (C) 1.5 V/m along negative x-axis
- (D) 1.5 V/m along positive x-axis

Q.7 An electromagnetic wave is propagating in a vacuum along the z-axis. The electric field is along the x-axis. The magnetic field will be along:

- (A) y-axis
- (B) x-axis
- (C) z-axis
- (D) between x and y axes

Q.8 Match List I with List II:

List I (EM Wave)	List II (Application)
A. Microwaves	I. Treatment of cancer

B. Infrared waves	II. TV communication
C. Gamma rays	III. Radar systems
D. Radio waves	IV. Night vision goggles

Choose the correct answer:

- (A) A-III, B-IV, C-I, D-II
- (B) A-II, B-IV, C-I, D-III
- (C) A-IV, B-II, C-I, D-III
- (D) A-III, B-I, C-IV, D-II

Q.9 The logic gate equivalent to the boolean expression $Y = \overline{A + B}$ is:

- (A) OR Gate
- (B) NOR Gate
- (C) AND Gate
- (D) NAND Gate

Q.10 Two coherent light sources of intensity ratio 81 : 1 produce interference fringes. The ratio of maximum to minimum intensity in the fringe system is:

- (A) 25 : 16
- (B) 10 : 8
- (C) 82 : 80
- (D) 25 : 1

Q.11 The de-Broglie wavelength of an electron having kinetic energy E is λ . If the kinetic energy is increased to $4E$, the new de-Broglie wavelength is:

- (A) $\lambda/4$
- (B) $\lambda/2$
- (C) 2λ

(D) 4λ

Q.12 An ideal gas undergoes an isothermal expansion. Which of the following is true?

- (A) The internal energy of the gas increases
- (B) Work done by the gas is zero
- (C) Heat is absorbed by the gas
- (D) Heat is released by the gas

Q.13 If the distance between two consecutive nodes in a stationary wave is 20 cm, the wavelength of the wave is:

- (A) 10 cm
- (B) 20 cm
- (C) 40 cm
- (D) 80 cm

Q.14 The magnetic field at the center of a circular current-carrying coil of radius R is B_0 . The magnetic field on its axis at a distance R from the center is:

- (A) $B_0/2$
- (B) $B_0/2\sqrt{2}$
- (C) $B_0/\sqrt{2}$
- (D) $B_0/4$

Q.15 In a photoelectric effect experiment, the stopping potential is V_0 when incident light of wavelength λ is used. If the wavelength is 2λ , the stopping potential is:

- (A) Exactly $V_0/2$
- (B) Less than $V_0/2$
- (C) Greater than $V_0/2$

(D) Exactly $2V_0$

Q.16 A parallel plate capacitor with air between the plates has a capacitance of 15 pF . If the separation is doubled and the space is filled with a dielectric of constant 6, the new capacitance is:

- (A) 15 pF
- (B) 30 pF
- (C) 45 pF
- (D) 90 pF

Q.17 Given below are two statements:

Statement I: A wire of resistance R is stretched to twice its length. Its new resistance becomes $4R$.

Statement II: The resistivity of the wire changes when it is stretched.

In the light of the above statements, choose the most appropriate answer:

- (A) Statement I is correct but Statement II is incorrect
- (B) Both Statement I and Statement II are correct
- (C) Statement I is incorrect but Statement II is correct
- (D) Both Statement I and Statement II are incorrect

Q.18 Two lenses of power $+12 \text{ D}$ and -2 D are combined together. The focal length of the equivalent lens is:

- (A) 10 cm
- (B) 12.5 cm
- (C) 16.6 cm
- (D) 8.33 cm

Q.19 The half-life of a radioactive substance is 20 days. The time taken for $3/4$ of its original mass to disintegrate is:

- (A) 20 days
- (B) 40 days
- (C) 60 days
- (D) 80 days

Q.20 A Carnot engine working between 300 K and 600 K has a work output of 800 J per cycle. The amount of heat energy supplied to the engine from the source per cycle is:

- (A) 800 J
- (B) 1600 J
- (C) 1200 J
- (D) 3200 J

Section B: Numerical Value Type Questions (5 Questions)

Q.21 A body of mass 2 kg initially at rest is subjected to a time-dependent force $\vec{F} = (4t\hat{i} + 6t^2\hat{j})$ N. The velocity of the body at time $t = 2$ s is v m/s. The square of the magnitude of velocity (v^2) is _____.

Q.22 In a meter bridge experiment, the balance point is found to be at 40 cm from end A when a resistor X is in the left gap and $Y = 12\ \Omega$ is in the right gap. The value of X in ohms is _____.

Q.23 A particle performs Simple Harmonic Motion with an amplitude of 5 cm. When the particle is at a distance of 3 cm from the mean position, the magnitude of its velocity is equal to its acceleration. The time period of the motion is $\frac{3\pi}{x}$ seconds. The value of x is _____.

Q.24 A uniform solid cylinder of mass 1 kg and radius 0.1 m is rolling without slipping on a horizontal surface with a velocity of 2 m/s. The total kinetic energy of the cylinder is _____ Joules.

Q.25 An object is placed at a distance of 30 cm from a convex mirror of focal length 15 cm. The magnitude of the magnification of the image is $\frac{1}{x}$. The value of x is _____.

Part B: Chemistry

Section A: Multiple Choice Questions (20 Questions)

Q.26 Which of the following sets of quantum numbers is NOT possible?

- (A) $n = 3, l = 2, m_l = 0, m_s = +1/2$
- (B) $n = 2, l = 2, m_l = -1, m_s = -1/2$
- (C) $n = 4, l = 3, m_l = -3, m_s = +1/2$
- (D) $n = 5, l = 0, m_l = 0, m_s = -1/2$

Q.27 Given below are two statements:

Statement I: O_2 molecule is paramagnetic.

Statement II: According to Molecular Orbital Theory, O_2 has two unpaired electrons in its anti-bonding π^* orbitals.

- (A) Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I.
- (B) Both Statement I and Statement II are correct but Statement II is NOT the correct explanation of Statement I.
- (C) Statement I is correct but Statement II is incorrect.
- (D) Statement I is incorrect but Statement II is correct.

Q.28 Match List I with List II:

List I (Species)	List II (Geometry/Shape)
A. SF_4	I. Square Planar

B. XeF_4	II. Tetrahedral
C. NH_4^+	III. See-Saw
D. BrF_3	IV. T-shape

- (A) A-III, B-I, C-II, D-IV
(B) A-III, B-II, C-I, D-IV
(C) A-IV, B-I, C-II, D-III
(D) A-I, B-III, C-II, D-IV

Q.29 The correct order of acid strength of the following compounds is:

- (A) Phenol
(B) p-Nitrophenol
(C) m-Nitrophenol
(D) p-Cresol

- (A) $B > C > A > D$
(B) $C > B > A > D$
(C) $B > A > C > D$
(D) $D > A > C > B$

Q.30 Which of the following acts as a primary standard in volumetric analysis?

- (A) $NaOH$
(B) $KMnO_4$
(C) $K_2Cr_2O_7$
(D) HCl

Q.31 Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Aniline does not undergo Friedel-Crafts reaction.

Reason R: Aniline reacts with the Lewis acid catalyst $AlCl_3$ to form a salt, strongly deactivating the ring.

- (A) Both A and R are correct and R is the correct explanation of A
- (B) Both A and R are correct but R is NOT the correct explanation of A
- (C) A is correct but R is not correct
- (D) A is not correct but R is correct

Q.32 The metal extracted by cyanide process is:

- (A) Fe
- (B) Cu
- (C) Ag
- (D) Al

Q.33 Which among the following is a homoleptic complex?

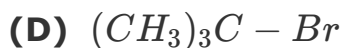
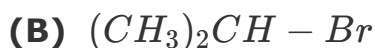
- (A) $[Co(NH_3)_4Cl_2]^+$
- (B) $[Ni(CN)_4]^{2-}$
- (C) $[Pt(NH_3)_2Cl_2]$
- (D) $[Co(en)_2Cl_2]^+$

Q.34 The correct sequence of decreasing bond angle is:

- (A) $NH_3 > PH_3 > AsH_3 > SbH_3$
- (B) $SbH_3 > AsH_3 > PH_3 > NH_3$
- (C) $NH_3 > AsH_3 > PH_3 > SbH_3$
- (D) $PH_3 > NH_3 > AsH_3 > SbH_3$

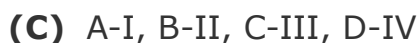
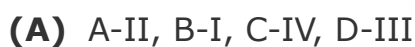
Q.35 In an S_N2 substitution reaction of the type: $R - Br + Cl^- \rightarrow R - Cl + Br^-$, which of the following has the highest rate of reaction?

- (A) $CH_3 - CH_2 - Br$

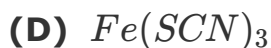
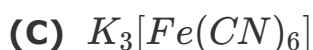
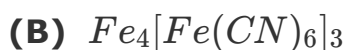
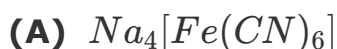


Q.36 Match List I with List II:

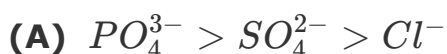
List I (Reaction Name)	List II (Reagents)
A. Reimer-Tiemann	I. $CO, HCl, Anhydrous AlCl_3$
B. Gattermann-Koch	II. $CHCl_3 + aq. NaOH$
C. Etard Reaction	III. $SnCl_2 + HCl, H_3O^+$
D. Stephen Reaction	IV. CrO_2Cl_2, H_3O^+

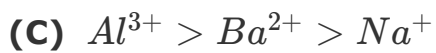


Q.37 The presence of nitrogen in an organic compound is detected by Lassaigne's test. The blue color formed is due to the formation of:

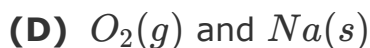
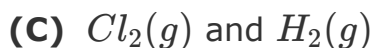
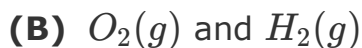
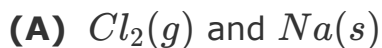


Q.38 The coagulation value of an electrolyte is defined as the minimum concentration required to cause precipitation. The coagulating power of ions for a positively charged sol follows the order:





Q.39 During the electrolysis of aqueous $NaCl$ solution, the products obtained at the anode and cathode respectively are:



Q.40 Which of the following vitamins is water-soluble?

(A) Vitamin A

(B) Vitamin D

(C) Vitamin C

(D) Vitamin E

Q.41 Which defect in ionic crystals lowers the density of the crystal?

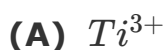
(A) Frenkel Defect

(B) Schottky Defect

(C) Interstitial Defect

(D) F-center

Q.42 Which transition metal ion is colorless in aqueous solution?



Q.43 The rate constant of a reaction is $k = 3.2 \times 10^{-4} \text{ s}^{-1}$. The order of the reaction is:

- (A) Zero order
- (B) First order
- (C) Second order
- (D) Third order

Q.44 Which among the following is the strongest reducing agent?

- (A) H_3PO_4
- (B) H_3PO_3
- (C) H_3PO_2
- (D) HNO_3

Q.45 Glucose upon prolonged heating with HI gives:

- (A) Hexanoic acid
- (B) n-Hexane
- (C) 1-Hexene
- (D) Gluconic acid

Section B: Numerical Value Type Questions (5 Questions)

Q.46 The number of chiral carbon atoms present in a β -D-glucopyranose molecule is _____.

Q.47 0.5 moles of an ideal gas expands isothermally and reversibly from 2 L to 20 L at 300 K. The magnitude of maximum work done is _____ Joules. (Given $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$, $\ln 10 = 2.303$) (Round off to the nearest integer).

Q.48 For a reaction $A \rightarrow \text{Products}$, the half-life is 50 minutes. If the reaction is first order, the time required for 87.5% completion of the reaction is _____ minutes.

Q.49 The elevation in boiling point of a solution containing 0.1 mole of a non-volatile, non-electrolyte solute in 500 g of water is $x \times 10^{-2}$ K. The molal elevation constant (K_b) for water is $0.52 \text{ K kg mol}^{-1}$. The value of x is _____.

Q.50 How many of the following compounds will give a positive Iodoform test?
Acetaldehyde, Acetone, Isopropyl alcohol, Ethyl alcohol, Benzophenone, Propan-1-ol.
Answer: _____

Part C: Mathematics

Section A: Multiple Choice Questions (20 Questions)

Q.51 Let R be a relation on the set of natural numbers $\mathbb{N}$ defined by $xRy \iff x^2 - y^2$ is divisible by 5. Then R is:

- (A) Reflexive and symmetric but not transitive
- (B) Equivalence relation
- (C) Symmetric and transitive but not reflexive
- (D) Reflexive and transitive but not symmetric

Q.52 Let $f(x) = \frac{x-1}{x+1}$, then $f(f(f(x)))$ is equal to:

- (A) x
- (B) $-1/x$
- (C) $-\frac{1+x}{1-x}$
- (D) $\frac{x-1}{x+1}$

Q.53 If $z = \frac{1+i}{1-i}$, then z^4 is equal to:

- (A) 1
- (B) -1
- (C) i
- (D) $-i$

Q.54 If α, β are the roots of the equation $x^2 - px + q = 0$, then the value of $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$ is:

- (A) $\frac{p^3 - 3pq}{q}$
- (B) $\frac{p^3 + 3pq}{q}$
- (C) $\frac{p^2 - 2q}{q}$
- (D) $\frac{p^2 + 2q}{q}$

Q.55 Let $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$. If $A + A^T = I$, then a possible value of θ is:

- (A) $\pi/6$
- (B) $\pi/3$
- (C) $\pi/2$
- (D) $\pi/4$

Q.56 The number of ways in which 5 boys and 4 girls can be seated in a row such that no two girls sit together is:

- (A) $5! \times 4!$
- (B) $5! \times {}^6P_4$
- (C) $5! \times {}^5C_4$
- (D) $9! - (5! \times 4!)$

Q.57 The term independent of x in the expansion of $\left(x^2 - \frac{1}{x}\right)^9$ is:

- (A) 84
 - (B) -84
 - (C) 36
 - (D) -36
-

Q.58 In an arithmetic progression, if the sum of the first n terms is $3n^2 + 5n$, then its n^{th} term is:

- (A) $6n + 2$
 - (B) $6n - 2$
 - (C) $3n + 2$
 - (D) $3n - 2$
-

Q.59 $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$ is equal to:

- (A) $1/6$
 - (B) $-1/6$
 - (C) $1/3$
 - (D) $-1/3$
-

Q.60 The function $f(x) = |x - 1| + |x - 2|$ is:

- (A) Continuous but not differentiable at $x = 1, 2$
 - (B) Differentiable everywhere
 - (C) Discontinuous at $x = 1, 2$
 - (D) Differentiable at $x = 1$ but not at $x = 2$
-

Q.61 The minimum value of $f(x) = e^x + e^{-x}$ is:

- (A) 1
- (B) 2

(C) e

(D) 0

Q.62 $\int \frac{dx}{e^x + e^{-x}}$ is equal to:

(A) $\tan^{-1}(e^x) + C$

(B) $\log(e^x + e^{-x}) + C$

(C) $e^x - e^{-x} + C$

(D) $\cot^{-1}(e^x) + C$

Q.63 The value of $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q.64 The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$ is:

(A) $\tan(y/2x) = cx$

(B) $\cot(y/2x) = cx$

(C) $\cos(y/2x) = cx$

(D) $\sin(y/2x) = cx$

Q.65 The perpendicular distance of the point $(3, -4)$ from the line $3x - 4y + 5 = 0$ is:

(A) 6

(B) 5

(C) 3

(D) 4

Q.66 The eccentricity of the ellipse $9x^2 + 16y^2 = 144$ is:

- (A) $\sqrt{7}/4$
- (B) $3/4$
- (C) $\sqrt{7}/3$
- (D) $4/5$

Q.67 If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, then $|\vec{a} + \vec{b} + \vec{c}|$ is:

- (A) 1
- (B) $\sqrt{3}$
- (C) 3
- (D) $\sqrt{2}$

Q.68 The direction cosines of a line equally inclined to the coordinate axes are:

- (A) 1, 1, 1
- (B) $\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}$
- (C) $1/\sqrt{2}, 1/\sqrt{2}, 0$
- (D) $1/\sqrt{2}, 0, 1/\sqrt{2}$

Q.69 A coin is tossed 4 times. The probability of getting exactly 2 heads is:

- (A) $1/2$
- (B) $3/8$
- (C) $1/4$
- (D) $5/8$

Q.70 The variance of first n natural numbers is:

- (A) $\frac{n^2-1}{12}$
- (B) $\frac{n^2+1}{12}$

(C) $\frac{n(n+1)}{2}$

(D) $\frac{n^2-1}{6}$

Section B: Numerical Value Type Questions (5 Questions)

Q.71 If the matrix $A = \begin{pmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $A^3 = \begin{pmatrix} 1 & 6 & 15 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then the value of x is _____.

Q.72 Let S be the sum of all coefficients in the expansion of $(1 + x - 3x^2)^{2143}$. The value of S is _____.

Q.73 The number of integer values of k in the interval $[-2, 2]$ for which the line $y = kx + 2$ intersects the circle $x^2 + y^2 = 1$ in two distinct points is _____.

Q.74 If $\int_0^1 (3x^2 + 2x + k)dx = 0$, then the value of k is _____.

Q.75 Two vectors $\vec{P}$ and $\vec{Q}$ have magnitudes 3 and 4 respectively. If the magnitude of their cross product is 6, then the angle between them in degrees is θ . The value of θ is _____.

Answer Key & Explanation

Part A: Physics

1. (A) Dimensional formulas: Spring constant $[ML^0T^{-2}]$, Angular momentum $[ML^2T^{-1}]$, Torque $[ML^2T^{-2}]$, Gravitational constant $[M^{-1}L^3T^{-2}]$.
2. (C) Acceleration due to gravity decreases both above and below the surface; it is equal when $d \approx 2h$, not $h = d$.
3. (A) Kinetic energy at the highest point is $\frac{1}{2}m(v \cos 30^\circ)^2 = K_{initial} \cos^2 30^\circ = K_{initial}(3/4)$, giving a ratio of 3:4.
4. (B) Time period $T \propto \sqrt{m}$; if mass is increased by $3m$, new mass is $4m$, making the new time period $\sqrt{4}T = 2T$.
5. (A) Average power is $V_{rms}I_{rms} \cos(\pi/2) = 0$, since current leads voltage by exactly $\pi/2$ in a purely capacitive circuit.
6. (A) Electric field $\vec{E} = -\frac{dV}{dx}\hat{i} = -6x\hat{i}$; at $x = 1$, $\vec{E} = -6\hat{i}$, which is 6 V/m along the negative x-axis.
7. (A) Propagation direction ($\hat{k}$) is given by $\vec{E} \times \vec{B}$; since $\vec{E}$ is along $\hat{i}$, $\vec{B}$ must be along $\hat{j}$ (y-axis) because $\hat{i} \times \hat{j} = \hat{k}$.
8. (A) Microwaves are used in Radar, Infrared in Night vision, Gamma rays in Cancer treatment, and Radio waves in TV communication.
9. (C) According to De Morgan's theorem, $Y = \overline{\overline{A} + \overline{B}} = \overline{\overline{A} \cdot \overline{B}} = A \cdot B$, representing an AND gate.
10. (A) $I_{max}/I_{min} = (\sqrt{I_1} + \sqrt{I_2})^2/(\sqrt{I_1} - \sqrt{I_2})^2 = (\sqrt{81} + \sqrt{1})^2/(\sqrt{81} - \sqrt{1})^2 = 100/64 = 25 : 16$.
11. (B) De-Broglie wavelength $\lambda = h/\sqrt{2mE}$; increasing energy to $4E$ decreases wavelength to $\lambda/\sqrt{4} = \lambda/2$.
12. (C) For isothermal expansion of an ideal gas, $\Delta U = 0$, so heat added equals work done by the gas ($Q = W > 0$), meaning heat is absorbed.
13. (C) The distance between two consecutive nodes is $\lambda/2$; thus $\lambda/2 = 20 \text{ cm} \implies \lambda = 40 \text{ cm}$.
14. (B) Magnetic field on the axis is $B = \frac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$; at $x = R$, it evaluates to $\frac{\mu_0 I}{2R(2\sqrt{2})} = \frac{B_0}{2\sqrt{2}}$.
15. (B) From Einstein's photoelectric equation, $eV_0 = \frac{hc}{\lambda} - W$; at 2λ , $eV' = \frac{hc}{2\lambda} - W = \frac{1}{2}(eV_0 + W) - W = \frac{eV_0}{2} - \frac{W}{2}$, which is less than $V_0/2$.
16. (C) New capacitance $C' = \frac{k\epsilon_0 A}{2d} = \frac{k}{2}C_0 = \frac{6}{2} \times 15 \text{ pF} = 45 \text{ pF}$.
17. (A) Resistance becomes $n^2 R$ when stretched to n times its length, but resistivity is a material property and remains unchanged.
18. (A) Equivalent power $P_{eq} = P_1 + P_2 = 12 - 2 = 10 \text{ D}$; Focal length $f = \frac{100}{P_{eq}} \text{ cm} = 10 \text{ cm}$.
19. (B) Disintegration of $3/4$ implies $1/4$ remains, which takes 2 half-lives ($2 \times 20 = 40$ days).
20. (B) Efficiency $\eta = 1 - \frac{300}{600} = 0.5$; Heat supplied $Q = \frac{W}{\eta} = \frac{800}{0.5} = 1600 \text{ J}$.
21. 80 Acceleration $\vec{a} = 2t\hat{i} + 3t^2\hat{j} \implies \vec{v} = t^2\hat{i} + t^3\hat{j}$; at $t = 2$, $\vec{v} = 4\hat{i} + 8\hat{j}$, so $|\vec{v}|^2 = 4^2 + 8^2 = 80$.
22. 8 For a balanced meter bridge, $\frac{X}{l} = \frac{Y}{100-l} \implies \frac{X}{40} = \frac{12}{60} \implies X = 8 \Omega$.

23. 2 Magnitude of velocity equals acceleration: $\omega\sqrt{A^2 - x^2} = \omega^2 x \implies \sqrt{25 - 9} = 3\omega \implies \omega = 4/3$; thus $T = 2\pi/\omega = 3\pi/2$, giving $x = 2$.

24. 3 Total kinetic energy for a rolling solid cylinder is $K = \frac{1}{2}mv^2(1 + \frac{k^2}{R^2}) = \frac{1}{2}(1)(4)(1 + \frac{1}{2}) = 3 \text{ J}$.

25. 3 Using mirror formula $\frac{1}{v} + \frac{1}{-30} = \frac{1}{15} \implies v = 10 \text{ cm}$; Magnification $m = -\frac{v}{u} = \frac{-10}{-30} = \frac{1}{3}$, so $x = 3$.

Part B: Chemistry

26. (B) The azimuthal quantum number l must strictly be less than the principal quantum number n , so $n = 2, l = 2$ is impossible.

27. (A) Molecular Orbital Theory explains the paramagnetism of O_2 by the presence of two unpaired electrons in its degenerate π^* anti-bonding orbitals.

28. (A) Shapes: SF_4 is see-saw, XeF_4 is square planar, NH_4^+ is tetrahedral, and BrF_3 is T-shaped.

29. (A) Electron-withdrawing groups increase acidity (p-Nitro > m-Nitro), while electron-donating groups decrease it (Phenol > p-Cresol).

30. (C) Potassium dichromate ($K_2Cr_2O_7$) acts as a primary standard due to its high purity, stability, and non-hygroscopic nature.

31. (A) Aniline's lone pair reacts with the Lewis acid $AlCl_3$ to form a salt, strongly deactivating the ring and preventing the Friedel-Crafts reaction.

32. (C) The Macarthur-Forrest cyanide process is widely used for leaching and extracting native silver (Ag) and gold (Au).

33. (B) A homoleptic complex contains only one type of ligand; $[Ni(CN)_4]^{2-}$ solely contains CN^- ligands.

34. (A) Moving down Group 15, central atom electronegativity decreases, reducing bond-pair bond-pair repulsion and thereby decreasing the bond angle.

35. (C) S_N2 mechanism favors primary over secondary/tertiary halides due to less steric hindrance, making $CH_3 - Br$ the fastest.

36. (A) Reimer-Tiemann uses $CHCl_3/NaOH$; Gattermann-Koch uses CO/HCl ; Etard uses CrO_2Cl_2 ; Stephen uses $SnCl_2/HCl$.

37. (B) The Prussian blue color in Lassaigne's test for nitrogen is due to the formation of Iron(III) ferrocyanide, $Fe_4[Fe(CN)_6]_3$.

38. (A) According to Hardy-Schulze rule, greater valency of the flocculating ion dictates greater coagulating power: $PO_3^{3-} > SO_3^{2-} > Cl^-$.

39. (C) In aqueous $NaCl$, Cl^- is oxidized to Cl_2 at the anode (due to overvoltage of O_2), and H_2O reduces to H_2 at the cathode.

40. (C) Vitamins B and C are water-soluble, whereas Vitamins A, D, E, and K are fat-soluble.

41. (B) A Schottky defect involves equal numbers of missing cations and anions, creating vacancies that lower the overall density of the crystal.

42. (C) The Cu^+ ion has a fully filled $3d^{10}$ configuration, meaning it lacks unpaired d-electrons to undergo d-d transitions, rendering it colorless.

43. (B) The unit of the rate constant k is s^{-1} , which corresponds exclusively to a first-order chemical reaction.

- 44. (C)** The reducing nature of phosphorus oxoacids depends on P-H bonds; H_3PO_2 has two P-H bonds and is therefore the strongest reducing agent.
- 45. (B)** Heating glucose with HI results in its complete reduction to n-hexane, confirming all six carbon atoms form a straight chain.
- 46. 5** The cyclic hemiacetal form of β -D-glucopyranose contains 5 chiral carbon atoms (C1 through C5).
- 47. 2872** Work done $W = 2.303nRT \log_{10}(V_2/V_1) = 2.303(0.5)(8.314)(300) \log_{10}(10) \approx 2872 \text{ J}$.
- 48. 150** 87.5% completion leaves 12.5% ($1/8$) remaining, which corresponds to 3 half-lives ($3 \times 50 = 150$ minutes).
- 49. 10.4** Molality $m = 0.1 \text{ mol}/0.5 \text{ kg} = 0.2$; Elevation $\Delta T_b = K_b \times m = 0.52 \times 0.2 = 0.104 \text{ K} = 10.4 \times 10^{-2} \text{ K}$.
- 50. 4** Acetaldehyde, Acetone, Isopropyl alcohol, and Ethyl alcohol will give a positive iodoform test due to the presence of CH_3CO- or $CH_3CH(OH)-$ groups.

Part C: Mathematics

- 51. (B)** The relation $x^2 - y^2 = 5k$ is reflexive ($x^2 - x^2 = 0$), symmetric ($y^2 - x^2 = -5k$), and transitive, hence an equivalence relation.
- 52. (C)** Substituting repeatedly: $f(f(x)) = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = -\frac{1}{x}$; applying it again gives $f(f(f(x))) = \frac{-1/x-1}{-1/x+1} = \frac{1+x}{1-x}$.
- 53. (A)** Rationalizing gives $z = \frac{1+i}{1-i} = \frac{(1+i)^2}{2} = \frac{2i}{2} = i$; thus $z^4 = i^4 = 1$.
- 54. (A)** Using sum and product of roots ($\alpha + \beta = p$, $\alpha\beta = q$), $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{p^3 - 3pq}{q}$.
- 55. (B)** $A + A^T = \begin{pmatrix} 2 \cos \theta & 0 \\ 0 & 2 \cos \theta \end{pmatrix} = I \implies 2 \cos \theta = 1 \implies \cos \theta = 1/2 \implies \theta = \pi/3$.
- 56. (B)** The 5 boys can be seated in $5!$ ways, creating 6 spaces around them; placing the 4 girls in these spaces takes 6P_4 ways, giving total $5! \times {}^6P_4$.
- 57. (A)** General term $T_{r+1} = {}^9C_r (x^2)^{9-r} (-x^{-1})^r = {}^9C_r (-1)^r x^{18-3r}$; setting $18 - 3r = 0 \implies r = 6$, making the term ${}^9C_6 = 84$.
- 58. (A)** The n^{th} term is given by $a_n = S_n - S_{n-1} = (3n^2 + 5n) - [3(n-1)^2 + 5(n-1)] = 6n + 2$.
- 59. (B)** Using Maclaurin series $\sin x = x - x^3/3! + \dots$, the expression simplifies to $\frac{-x^3/6}{x^3}$, giving a limit of $-1/6$.
- 60. (A)** A sum of modulus functions is continuous everywhere but non-differentiable at points where the expression inside the modulus is zero ($x = 1, x = 2$).
- 61. (B)** By Arithmetic Mean - Geometric Mean inequality, $\frac{e^x + e^{-x}}{2} \geq \sqrt{e^x \cdot e^{-x}} = 1 \implies e^x + e^{-x} \geq 2$.
- 62. (A)** Multiplying numerator and denominator by e^x yields $\int \frac{e^x}{e^{2x}+1} dx$; letting $t = e^x$ evaluates the integral to $\tan^{-1}(e^x) + C$.
- 63. (B)** Using the definite integral property $\int_{a0} f(x) dx = \int_{a0} f(a-x) dx$, adding the two expressions yields $2I = \int_{\pi/20} 1 dx = \pi/2 \implies I = \pi/4$.
- 64. (A)** A homogeneous equation; substituting $y = vx$ reduces it to $\csc v dv = \frac{dx}{x}$, which integrates to $\ln |\tan(v/2)| = \ln |cx| \implies \tan(y/2x) = cx$.

65. (A) The perpendicular distance from point (x_1, y_1) to $Ax + By + C = 0$ is $\frac{|3(3) - 4(-4) + 5|}{\sqrt{3^2 + (-4)^2}} = \frac{30}{5} = 6$.

66. (A) Rewriting as $\frac{x^2}{16} + \frac{y^2}{9} = 1$, we have $a^2 = 16$ and $b^2 = 9$; the eccentricity is $e = \sqrt{1 - b^2/a^2} = \sqrt{1 - 9/16} = \sqrt{7}/4$.

67. (B) $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 1 + 1 + 1 + 0 = 3 \implies |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$.

68. (B) Since the line is equally inclined to axes, direction cosines $\cos \alpha, \cos \beta, \cos \gamma$ are equal;
 $3 \cos^2 \alpha = 1 \implies \cos \alpha = \pm 1/\sqrt{3}$.

69. (B) Using binomial probability for $n = 4, p = 1/2, q = 1/2, r = 2$: $P(X = 2) = {}^4C_2(1/2)^2(1/2)^2 = 6/16 = 3/8$.

70. (A) The variance of the first n natural numbers is derived as $E(X^2) - [E(X)]^2 = \frac{(n+1)(2n+1)}{6} - (\frac{n+1}{2})^2 = \frac{n^2-1}{12}$.

71. 5 Through multiplication, $A^n = \begin{pmatrix} 1 & 2n & nx \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; for $A^3, 3x = 15 \implies x = 5$.

72. -1 The sum of all coefficients in a polynomial expansion is found by setting $x = 1$, which gives
 $(1 + 1 - 3)^{2143} = (-1)^{2143} = -1$.

73. 2 The line intersects the circle if perpendicular distance from origin is less than radius 1: $\frac{|2|}{\sqrt{k^2+1}} < 1 \implies k^2 > 3$, giving integer values ± 2 (2 values).

74. -2 Evaluating the integral yields $[x^3 + x^2 + kx]_{10} = 1 + 1 + k = 2 + k$; equating to 0 gives $k = -2$.

75. 30 Magnitude of cross product $|\vec{P} \times \vec{Q}| = |\vec{P}||\vec{Q}|\sin \theta \implies 6 = 3 \times 4 \times \sin \theta \implies \sin \theta = 0.5 \implies \theta = 30^\circ$.