

KCET 2024

MATHEMATICS PAPER CODE-D4

1. The value of C in $(0, 2)$ satisfying the mean value theorem for the function $f(x) = x(x-1)^2$, $x \in [0, 2]$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans. B

Sol. $f(2) = 2, \quad f(0) = 0$

$$f'(c) = \frac{2-0}{2-0} = 1$$

$$f'(c) = 2c(c-1) + (c-1)^2$$

option verification

2. $\frac{d}{dx} \left[\cos^2 \left(\cot^{-1} \sqrt{\frac{2+x}{2-x}} \right) \right]$ is

(A) $-\frac{3}{4}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{1}{4}$

Ans. D

Sol. $x = 2 \cos \theta$

$$\frac{2+x}{2-x} = \frac{2(1+\cos \theta)}{2(1-\cos \theta)} = \cot^2 \frac{\theta}{2}$$

$$\therefore \frac{d}{dx} \frac{1}{2} (1 + \cos \theta) = \frac{1}{2} \frac{d}{dx} \left(1 + \frac{x}{2} \right)$$

$$= \frac{1}{4}$$

3. For the function $f(x) = x^3 - 6x^2 + 12x - 3$; $x = 2$ is

(A) a point of minimum
(B) a point of inflexion
(C) not a critical point
(D) a point of maximum

Ans. B

Sol. $f'(x) = 3x^2 - 12x + 12$

$$f''(x) = 6x - 12$$

$$f''(2) = 0$$

$$f'''(2) \neq 0$$

4. The function $f(x) = |\cos x|$ is
(A) Everywhere continuous and differentiable
(B) Everywhere continuous but not differentiable at odd multiples of $\frac{\pi}{2}$
(C) Neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$
(D) Not differentiable everywhere

Ans. B

Sol. Conceptual

5. If $y = 2x^{3x}$, then $\frac{dy}{dx}$ at $x = 1$ is

(A) 2 (B) 6 (C) 3 (D) 1

Ans. B

Sol. $\log y = \log(2 \cdot x^{3x})$

$$= \log 2 + 3x \log x$$

$$\frac{1}{y} \frac{dy}{dx} = 3 \left[x \cdot \frac{1}{x} + \log x \right]$$

$$\frac{dy}{dx} = 2 \cdot x^{3x} \cdot 3(1 + \log x)$$

$$\left(\frac{dy}{dx} \right)_{x=1} = 6$$

6. Let the function satisfy the equation $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$, where $f(0) \neq 0$. If $f(5) = 3$ and $f'(0) = 2$, then $f'(5)$ is

(A) 6 (B) 0 (C) 5 (D) -6

Ans. A

Sol. $f(x) = k^x$

$$f(5) = 3$$

$$k^5 = 3$$

$$\therefore f(x) = 3^{x/5}$$

$$f'(x) = 3^{x/5} \log_e 3 \cdot \frac{1}{5}$$

$$f'(0) = 2$$

$$\frac{\log_e 3}{5} = 2$$

$$\log_e 3 = 10$$

$$f'(x) = 2(3)^{x/5}$$

$$f'(5) = 2 \times 3 = 6$$

$$7. \int \frac{1}{x[6(\log x)^2 + 7\log x + 2]} dx =$$

$$(A) \frac{1}{2} \log \left| \frac{2\log x + 1}{3\log x + 2} \right| + C$$

$$(B) \log \left| \frac{2\log x + 1}{3\log x + 2} \right| + C$$

$$(C) \log \left| \frac{3\log x + 2}{2\log x + 1} \right| + C$$

$$(D) \frac{1}{2} \log \left| \frac{3\log x + 2}{2\log x + 1} \right| + C$$

Ans. B

Sol. Put $\log x = t \Rightarrow \frac{1}{x} dx = dt$

$$I = \int \frac{1}{(3t+2)(2t+1)} dt$$

$$\frac{1}{(3t+2)(2t+1)} = \frac{A}{3t+2} + \frac{B}{2t+1}$$

After solving $A = -3, B = 2$

$$\begin{aligned} \therefore I &= \int \frac{-3}{3t+2} dt + \int \frac{2}{2t+1} dt \\ &= -\log|3t+2| + \log|2t+1| + C \end{aligned}$$

$$8. \int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx =$$

$$(A) 2x + \sin x + 2\sin 2x + C$$

$$(B) x + 2\sin x + 2\sin 2x + C$$

$$(C) x + 2\sin x + \sin 2x + C$$

$$(D) 2x + \sin x + \sin 2x + C$$

Ans. C

$$\text{Sol. } \int \frac{2\sin\left(\frac{5x}{2}\right)\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}} dx$$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x} dx$$

$$= \int 3 - 4\sin^2 x + 2\cos x dx$$

$$= \int 1 + 2\cos 2x + 2\cos x dx$$

$$= x + \sin 2x + 2\sin x + C$$

$$9. \int_1^5 (|x-3| + |1-x|) dx =$$

$$(A) 12 \quad (B) \frac{5}{6} \quad (C) 21 \quad (D) 10$$

Ans. A

$$\text{Sol. } \int_1^5 (|x-3| + |x+1|) dx$$

$$= \int_1^3 2dx + \int_3^5 2x - 4dx$$

$$= 2(2) + (x^2 - 4x)_3^5$$

$$= 4 + [(25 - 20) - (9 - 12)] = 4 + 8 = 12$$

$$10. \lim_{n \rightarrow \infty} \left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{1}{5n} \right) =$$

$$(A) \frac{\pi}{4} \quad (B) \tan^{-1} 3 \quad (C) \tan^{-1} 2 \quad (D) \frac{\pi}{2}$$

Ans. C

$$\text{Sol. } \lim_{n \rightarrow \infty} \sum_{r=1}^{2n} \left(\frac{n}{n^2 + r^2} \right)$$

$$= \lim_{n \rightarrow \infty} \sum_{r=1}^{2n} \frac{n}{n^2 \left(1 + \left(\frac{r}{n} \right)^2 \right)}$$

$$= \int_0^2 \frac{1}{1+x^2} dx = \tan^{-1} 2$$

11. The area of the region bounded by the line $y = 3x$ and the curve $y = x^2$ in sq. units is

$$(A) 10$$

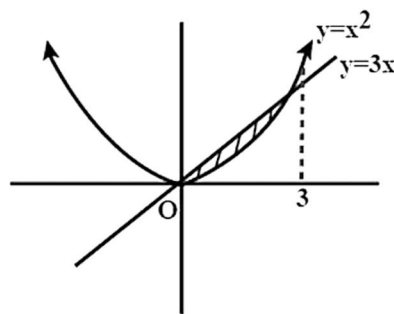
$$(B) \frac{9}{2}$$

$$(C) 9$$

$$(D) 5$$

Ans. B

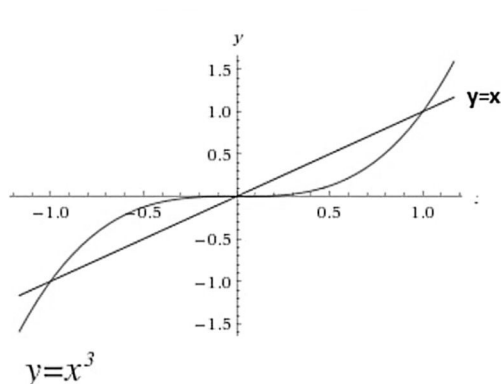
$$\text{Sol. } A = \int_0^3 (3x - x^2) dx = 3 \left(\frac{x^2}{2} \right) - \frac{x^3}{3} \Big|_0^3 = \frac{9}{2}$$



12. The area of the region bounded by the line $y = x$ and the curve $y = x^3$ is
 (A) 0.2 sq. units
 (B) 0.3 sq. units
 (C) 0.4 sq. units
 (D) 0.5 sq. units

Ans. D

Sol. $A = 2 \int_0^1 (x - x^3) dx = 2 \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = 0.5 \text{ sq. units}$



13. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a} \ \vec{b} \ \vec{c}]}$,

$$\vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a} \ \vec{b} \ \vec{c}]}, \text{ then}$$

$$(\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r} \text{ is}$$

- (A) 0 (B) 1 (C) 2 (D) 3

Ans. D

Sol. Since $\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a} \ \vec{b} \ \vec{c}]$,

$$(\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r} = 1 + 1 + 1 = 3$$

14. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and

$$\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5} \text{ are mutually}$$

perpendicular, then k is equal to

- (A) $-\frac{10}{7}$
 (B) $-\frac{7}{10}$
 (C) -10
 (D) -7

Ans. A

Sol. $(-3)(3k) + (2k)(1) + 2(-5) = 0$

15. The distance between the two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

- (A) 2 units (B) 8 units
 (C) $\frac{2}{\sqrt{29}}$ units (D) 4 units

Ans. C

Sol. $d = \frac{|6-4|}{\sqrt{4+9+16}} = \frac{2}{\sqrt{29}}$

16. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ and the plane

$$2x - 2y + z = 5 \text{ is}$$

- (A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{2}{5\sqrt{2}}$ (C) $\frac{3}{50}$ (D) $\frac{3}{\sqrt{50}}$

Ans. A

Sol. $\sin \theta = \frac{3(2) + 4(-2) + 5(1)}{\sqrt{9+16+25}\sqrt{4+4+1}}$

17. The equation $xy = 0$ in three-dimensional space represents

- (A) a pair of straight lines
 (B) a plane
 (C) a pair of planes at right angles
 (D) a pair of parallel planes

Ans. B

Sol. Conceptual

18. The plane containing the point (3, 2, 0) and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is

- (A) $x - y + z = 1$
 (B) $x + y + z = 5$
 (C) $x + 2y - z = 1$
 (D) $2x - y + z = 5$

Ans. A

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 4 \\ 1 & 5 & 4 \end{vmatrix}$

$$= \hat{i}(16-20) - \hat{j}(0-4) + \hat{k}(0-4)$$

$$= -4\hat{i} + 4\hat{j} - 4\hat{k}$$

$\therefore$ Eq. of plane is

$$-4(x-3) + 4(y-2) - 4(z-0) = 0$$

$$\Rightarrow -4x + 12 + 4y - 8 - 4z = 0$$

$$\Rightarrow x - y + z = 1$$

19. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $z = 4x + 6y$ be the objective function. The minimum value of z occurs at
 (A) Only (0, 2)
 (B) Only (3, 0)
 (C) The mid-point of the line segment joining the points (0, 2) and (3, 0)
 (D) Any point on the line segment joining the points (0, 2) and (3, 0)

Ans. D

Sol. Conceptual

20. A die is thrown 10 times. The probability that an odd number will come up at least once is
 (A) $\frac{11}{1024}$ (B) $\frac{1013}{1024}$ (C) $\frac{1023}{1024}$ (D) $\frac{1}{1024}$

Ans. C

Sol. $n = 10, p = \frac{1}{2}; q = \frac{1}{2}$
 $P(x \geq 1) = 1 - P(x = 0)$
 $= 1 - {}^{10}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{10}$

21. A random variable X has the following probability distribution :

X	0	1	2
P(X)	$\frac{25}{36}$	k	$\frac{1}{36}$

If the mean of the random variable X is $\frac{1}{3}$, then the variance is

- (A) $\frac{1}{18}$
 (B) $\frac{5}{18}$
 (C) $\frac{7}{18}$
 (D) $\frac{11}{18}$

Ans. B

Sol. $\sum p_i x_i = \frac{1}{3} \Rightarrow 0 + k + \frac{2}{36} = \frac{1}{3} \Rightarrow k = \frac{1}{3} - \frac{1}{18} = \frac{5}{18}$
 $\sigma^2 + \mu^2 = \frac{\sum p_i x_i^2}{\sum p_i} = \frac{0 + 6 + \frac{4}{36}}{1} = \frac{14}{36}$
 $\Rightarrow \sigma^2 = \frac{14}{36} - \mu^2 = \frac{14}{36} - \frac{1}{9} = \frac{5}{18}$

22. If a random variable X follows the binomial distribution with parameters $n = 5$, p and $P(X = 2) = 9P(X = 3)$, then p is equal to
 (A) 10 (B) $\frac{1}{10}$ (C) 5 (D) $\frac{1}{5}$

Ans. B

Sol. Given $n = 5, P(X = 2) = 9P(X = 3)$
 $\Rightarrow {}^n C_2 \cdot q^{n-2} \cdot p^2 = 9 \cdot {}^n C_3 \cdot q^{n-3} \cdot p^3$
 $\Rightarrow q = 3(n - 2) \cdot p$
 $\Rightarrow 1 - p = 3(3) \cdot p$
 $\Rightarrow p = \frac{1}{10}$

23. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are
 (A) 7, 6 (B) 5, 1 (C) 6, 3 (D) 8, 7

Ans. C

Sol. $2^m = 56 + 2^n$ then verification $m = 6, n = 3$

24. If $[x]^2 - 5[x] + 6 = 0$, where $[x]$ denotes the greatest integer function, then
 (A) $x \in [3, 4]$ (B) $x \in [2, 4]$
 (C) $x \in [2, 3]$ (D) $x \in (2, 3]$

Ans. B

Sol. $([x] - 2)([x] - 3) = 0$
 $\Rightarrow [x] = 2 \text{ or } [x] = 3$
 $\Rightarrow x \in [2, 4)$

25. If in two circles, arcs of the same length subtend angles 30° and 78° at the centre, then the ratio of their radii is
 (A) $\frac{5}{13}$ (B) $\frac{13}{5}$ (C) $\frac{13}{4}$ (D) $\frac{4}{13}$

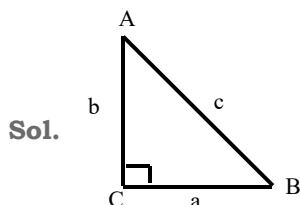
Ans. B

Sol. $l_1 = l_2, \theta_1 = 30^\circ, \theta_2 = 78^\circ$
 Then $\frac{l_1}{l_2} = \frac{r_1 \theta_1}{r_2 \theta_2} \Rightarrow \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{78^\circ}{30^\circ} = \frac{13}{5}$

26. If ΔABC is right angled at C, then the value of $\tan A + \tan B$ is

- (A) $a + b$ (B) $\frac{a^2}{bc}$
(C) $\frac{c^2}{ab}$ (D) $\frac{b^2}{ac}$

Ans. C



Since C is a right angle then

$$\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$$

27. The real value of ' α ' for which $\frac{1 - i \sin \alpha}{1 + 2i \sin \alpha}$ is purely real is

- (A) $(n+1)\frac{\pi}{2}, n \in \mathbb{N}$
(B) $(2n+1)\frac{\pi}{2}, n \in \mathbb{N}$
(C) $n\pi, n \in \mathbb{N}$
(D) $(2n-1)\frac{\pi}{2}, n \in \mathbb{N}$

Ans. C

Sol. $z = \frac{1 - i \sin \alpha}{1 + 2i \sin \alpha}$, after simplify
 $z = \frac{(1 - 2 \sin^2 \alpha) + i(-3 \sin \alpha)}{1 + 4 \sin^2 \alpha}$ and z is purely real
 then $\text{Im}(z) = 0$
 $\Rightarrow \sin \alpha = 0$
 $\Rightarrow \alpha = n\pi, n \in \mathbb{N}$

28. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then
- (A) Breadth ≤ 15 cm
(B) Breadth ≥ 15 cm
(C) Length ≤ 15 cm
(D) Length = 15 cm

Ans. B

Sol. Given $l = 5b$, $P \geq 180$
 $\Rightarrow 2(l + b) \geq 180$
 $\Rightarrow b \geq 15$

29. The value of ${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$ is

- (A) ${}^{50}C_4$
(B) ${}^{50}C_3$
(C) ${}^{50}C_2$
(D) ${}^{50}C_1$

Ans. A

Sol. Since $n_{C_r} + n_{C_{r-1}} = (n+1)_{C_r}$,
 ${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + ({}^{45}C_3 + {}^{45}C_4)$
 $= {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{46}C_4 \dots\dots$
 $= {}^{50}C_4$

30. In the expansion of $(1+x)^n$
 $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \dots\dots + n\frac{C_n}{C_{n-1}}$ is equal to

- (A) $\frac{n(n+1)}{2}$
(B) $\frac{n}{2}$
(C) $\frac{n+1}{2}$
(D) $3n(n+1)$

Ans. A

Sol. $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \dots\dots + n\frac{C_n}{C_{n-1}}$
 $= n + (n-1) + (n-2) + \dots\dots + 1 = \frac{n(n+1)}{2}$

31. If S_n stands for sum to n -terms of a G.P. with 'a' as the first term and 'r' as the common ratio then $S_n : S_{2n}$ is

- (A) $r^n + 1$
(B) $\frac{1}{r^n + 1}$
(C) $r^n - 1$
(D) $\frac{1}{r^n - 1}$

Ans. B

Sol. $\frac{S_n}{S_{2n}} = \frac{\frac{a(r^n - 1)}{r - 1}}{\frac{a(r^{2n} - 1)}{r - 1}} = \frac{r^n - 1}{(r^n - 1)(r^n + 1)} = \frac{1}{(r^n + 1)}$

32. If A.M. and G.M. of roots of a quadratic equation are 5 and 4 respectively, then the quadratic equation is

(A) $x^2 - 10x - 16 = 0$ (B) $x^2 + 10x + 16 = 0$
(C) $x^2 + 10x - 16 = 0$ (D) $x^2 - 10x + 16 = 0$

Ans. D

Sol. Given A.M. $= \frac{\alpha + \beta}{2} = 5 \Rightarrow \alpha + \beta = 10$,

G.M. $= \sqrt{\alpha\beta} = 4 \Rightarrow \alpha\beta = 16$

$\therefore$ The quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

33. The angle between the line $x + y = 3$ and the line joining the points (1,1) and (-3,4) is

(A) $\tan^{-1}(7)$ (B) $\tan^{-1}\left(-\frac{1}{7}\right)$
(C) $\tan^{-1}\left(\frac{1}{7}\right)$ (D) $\tan^{-1}\left(\frac{2}{7}\right)$

Ans. C

Sol. Slope of $x + y = 3$ is $m_1 = -1$ and

Slope of line joining the points (1,1), (-3,4) is

$m_2 = -\frac{3}{4}$

and $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \Rightarrow \theta = \tan^{-1} \frac{1}{7}$

34. The equation of parabola whose focus is (6,0) and directrix is $x = -6$ is

(A) $y^2 = 24x$ (B) $y^2 = -24x$
(C) $x^2 = 24y$ (D) $x^2 = -24y$

Ans. A

Sol. Focus = F = (a,0) = (6,0)

Equation of directrix is $x = -6$ then equation of parabola is of the form $y^2 = 4ax$, $a = 6$

35. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} \cos x - 1}{\cot x - 1}$ is equal to

(A) 2 (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Ans. C

Sol. By L.H. Rule

$$\text{Lt}_{x \rightarrow \frac{\pi}{4}} \frac{-\sqrt{2} \times \sin x - 0}{-\text{cosec}^2 x} = \frac{\sqrt{2} \times \frac{1}{\sqrt{2}}}{(\sqrt{2})^2} = \frac{1}{2}$$

36. The negation of the statement
“For every real number x ; $x^2 + 5$ is positive” is
(A) For every real number x ; $x^2 + 5$ is not positive.
(B) For every real number x ; $x^2 + 5$ is negative
(C) There exists at least one real number x such that $x^2 + 5$ is not positive
(D) There exists at least one real number x such that $x^2 + 5$ is positive

Ans. C

Sol. Conceptual

37. Let a, b, c, d and e be the observations with mean m and standard deviation S . The standard deviation of the observations $a + k, b + k, c + k, d + k$ and $e + k$ is

(A) kS (B) $S + k$ (C) $\frac{S}{k}$ (D) S

Ans. D

Sol. adding constant each observation of S.D does not effect.

38. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = \tan x$. Then $f^{-1}(1)$ is

(A) $\frac{\pi}{4}$
(B) $\left\{ n\pi + \frac{\pi}{4} : n \in \mathbb{Z} \right\}$
(C) $\frac{\pi}{3}$
(D) $\left\{ n\pi + \frac{\pi}{3} : n \in \mathbb{Z} \right\}$

Ans. B

Sol. $\tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}$

39. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2 + 1$. Then the pre images of 17 and -3 respectively are

(A) $\emptyset, \{4, -4\}$
(B) $\{3, -3\}, \emptyset$
(C) $\{4, -4\}, \emptyset$
(D) $\{4, -4\}, \{2, -2\}$

Ans. C

Sol. $f(x) = x^2 + 1 = 17 \Rightarrow x = \pm 4$

$x^2 + 1 = -3$ is not possible.

No preimage of -3

40. Let $(g \circ f)(x) = \sin x$ and $(f \circ g)(x) = (\sin \sqrt{x})^2$.

Then

(A) $f(x) = \sin^2 x$, $g(x) = x$

(B) $f(x) = \sin \sqrt{x}$, $g(x) = \sqrt{x}$

(C) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

(D) $f(x) = \sin \sqrt{x}$, $g(x) = x^2$

Ans. C

Sol. $g(f(x)) = g(\sin^2 x) = \sqrt{\sin^2 x} = \sin x$

$f(g(x)) = f(\sqrt{x}) = (\sin \sqrt{x})^2$

41. Let $A = \{2, 3, 4, 5, \dots, 16, 17, 18\}$. Let R be the relation on the set A of ordered pairs of positive integers defined by $(a, b) R (c, d)$ if and only if $ad = bc$ for all $(a, b), (c, d)$ in $A \times A$. Then the number of ordered pairs of the equivalence class of $(8, 2)$ is

- (A) 4 (B) 5 (C) 6 (D) 7

Ans. C

Sol. 6 Pairs

$\{(3, 2), (6, 4), (9, 6), (12, 8), (18, 12), (15, 10)\}$

42. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $x(y + z)$

$+ y(z + x) + z(x + y)$ equal to

- (A) 0 (B) 1 (C) 6 (D) 12

Ans. C

Sol. $x = y = z = -1$

$\Rightarrow x(y + z) + y(z + x) + z(x + y) = 6$

43. If $2 \sin^{-1} x - 3 \cos^{-1} x = 4$, $x \in [-1, 1]$ then $2 \sin^{-1}$

$x + 3 \cos^{-1} x$ is equal to

(A) $\frac{4 - 6\pi}{5}$

(B) $\frac{6\pi - 4}{5}$

(C) $\frac{3\pi}{2}$

(D) 0

Ans. B

Sol. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\cos^{-1} x = \frac{\pi - 4}{5}$

$\sin^{-1} x = \frac{3\pi + 8}{10}$

44. If A is square matrix such that $A^2 = A$, then $(I + A)^3$ is equal to

- (A) $7A - I$ (B) $7A$ (C) $7A + I$ (D) $I - 7A$

Ans. C

Sol. $(I + A)^3 = I + 3A + 3A + A = 7A + I$

45. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

- (A) $2^8 A$ (B) $2^9 A$ (C) $2^{10} A$ (D) $2^{11} A$

Ans. B

Sol. $A^2 = 2A$, $A^4 = A^3 A = A^{10} = 2^9 A$

46. If $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then

$f(1) \cdot f(3) + f(3) \cdot f(5) + f(5) \cdot f(1)$ is

- (A) -1 (B) 0 (C) 1 (D) 2

Ans. No option

Sol.

47. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2 \sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} =$

(A) -1

(B) 0

(C) 3

(D) 2

Ans. B

Sol. $f(x) = -x^2 \cos x + x \sin x$

$\therefore \lim_{x \rightarrow 0} \frac{f(x)}{x^2} = 0$

48. Which one of the following observations is correct for the features of logarithm function to and base $b > 1$?

(A) The domain of the logarithm function is R , the set of real numbers

(B) The range of the logarithm function is R^+ , the set of all positive real numbers.

(C) The point $(1, 0)$ is always on the graph of the logarithm function

(D) The graph of the logarithm function is decreasing as we move from left to right.

Ans. C

Sol. $\log 1 = 0$

49. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix

A and $|A| = 4$, then α is equal to

- (A) 4 (B) 5 (C) 11 (D) 0

Ans. C

Sol. $|P| = |A - A| = |A|^2 = 16 \Rightarrow 2\alpha - 6 = 16 \Rightarrow \alpha = 11$

50. If $A = \begin{bmatrix} x & 1 \\ 1 & x \end{bmatrix}$ and $B = \begin{bmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{bmatrix}$, then $\frac{dB}{dx}$ is

- (A) 3A (B) -3B (C) 3B+1 (D) 1-3A

Ans. A

Sol. $A = x^2 - 1, \frac{dB}{dx} = 3(x^2 - 1) = 3A$

51. If $f(x) = xe^{x(1-x)}$ then $f(x)$ is

- (A) increasing in $\mathbb{R}$
 (B) decreasing in $\mathbb{R}$
 (C) decreasing in $\left[-\frac{1}{2}, 1\right]$
 (D) increasing in $\left[-\frac{1}{2}, 1\right]$

Ans. D

Sol. $f'(x) = e^{x-x^2} (x - 2x^2)$
 $\Rightarrow f$ is increasing in $\left[-\frac{1}{2}, 1\right]$

52. $\int \frac{\sin x}{3 + 4\cos^2 x} dx =$

- (A) $-\frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{2\cos x}{\sqrt{3}} \right) + C$
 (B) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{\cos x}{3} \right) + C$
 (C) $\frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{\cos x}{3} \right) + C$
 (D) $-\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2\cos x}{3} \right) + C$

Ans. A

Sol. Put

$\cos x = t \Rightarrow \int \frac{-dt}{3 + (2t)^2} = -\frac{1}{\sqrt{3}} \times \frac{1}{2} \times \tan^{-1} \left(\frac{2t}{\sqrt{3}} \right)$

53. $\int_{-\pi}^{\pi} (1 - x^2) \sin x \cdot \cos^2 x dx =$

- (A) $\pi - \frac{\pi^2}{3}$ (B) $2\pi - \pi^3$
 (C) $\pi - \frac{\pi^3}{2}$ (D) 0

Ans. D

Sol. $f(x)$ is Odd function, then $I = 0$

54. The function $x^x; x > 0$ is strictly increasing at

- (A) $\forall x \in \mathbb{R}$ (B) $x < \frac{1}{e}$
 (C) $x > \frac{1}{e}$ (D) $x < 0$

Ans. C

Sol. $f'(x) = x^x (1 + \log x) \Rightarrow f'(x) > 0 \Rightarrow x > \frac{1}{e}$

55. The maximum volume of the right circular cone with slant height 6 units is

- (A) $4\sqrt{3}\pi$ cubic units
 (B) $16\sqrt{3}\pi$ cubic units
 (C) $3\sqrt{3}\pi$ cubic units
 (D) $6\sqrt{3}\pi$ cubic units

Ans. B

Sol. $V = \frac{1}{3} \pi r^2 h,$
 $l^2 = r^2 + h^2,$
 $r^2 = 36 - h^2,$
 $\Rightarrow x > 16\sqrt{3}\pi$
 $V_{\max} = 16\sqrt{3}\pi$

56. The vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ and $\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC . The length of the median through A is

- (A) $\sqrt{18}$ (B) $\sqrt{72}$ (C) $\sqrt{33}$ (D) $\sqrt{288}$

Ans. C

Sol. $\frac{1}{2} |\overline{AB} + \overline{AC}| = \frac{1}{2} |8\hat{i} - 2\hat{j} + 8\hat{k}| = \frac{1}{2} |4\hat{i} - \hat{j} + 4\hat{k}| = \sqrt{33}$

57. The volume of the parallelopiped whose co-terminous edges are $\hat{j} + \hat{k}$, $\hat{i} + \hat{k}$ and $\hat{i} + \hat{j}$ is

- (A) 6 cu.units (B) 2 cu.units
(C) 4 cu.units (D) 3 cu.units

Ans. B

Sol.
$$\begin{vmatrix} \hat{j} + \hat{k} & \hat{i} + \hat{k} & \hat{i} + \hat{j} \end{vmatrix} = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 2 \text{ cub units}$$

58. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

- (A) $\theta = \frac{\pi}{4}$ (B) $\theta = \frac{\pi}{3}$
(C) $\theta = \frac{2\pi}{3}$ (D) $\theta = \frac{\pi}{2}$

Ans. C

Sol. $|\vec{a} + \vec{b}|^2 = 1 \Rightarrow 1 + 1 + 2|\vec{a}||\vec{b}|\cos\theta = 1$
 $\Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

59. The solution of $e^{\frac{dy}{dx}} = x + 1, y(0) = 3$ is

- (A) $y - 2 = x \log x - x$
(B) $y - x - 3 = x \log x$
(C) $y - x - 3 = (x + 1) \log(x + 1)$
(D) $y + x - 3 = (x + 1) \log(x + 1)$

Ans. D

Sol. $\frac{dy}{dx} = \log(x + 1) \Rightarrow \int dy = \int \log(x + 1) dx$
 And $y(0) = 3$ then
 $\Rightarrow y + x - 3 = (x + 1) \log(x + 1)$

60. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

- (A) $xy = C$ (B) $x^2 + y^2 = C$
(C) $x^2 - y^2 = C$ (D) $\frac{y}{x} = C$

Ans. A

Sol. $\frac{x}{2x_1} + \frac{y}{2y_1} = 1 \Rightarrow xy = c$