OJEE LE Tech (BSc) Sample Paper 01

A) Mode and Time: The exam is a Computer Based Test (CBT) lasting 60 minutes.
B) Total Questions: The exam contains 60 questions.
C) Marking Scheme & Rules:

- +4 marks for each correct answer; -1 mark for each incorrect answer.

Q1. What is the value of the integral $\int_{-\infty}^{\infty} e^{-x^2} dx$?

A. 0

B. 1

C. π

D. $\sqrt{\pi}$

Q2. If A is a real orthogonal matrix, then the determinant of A is:

A. 0

B. 1 or -1

C. Always 1

D. Always -1

Q3. The degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 - xy = 0$ are respectively:

A. 3 and 2

B. 2 and 3

C. 4 and 2

D. 2 and 4

Q4. By Liouville's theorem, a bounded entire function in the complex plane must be:

A. Zero

B. Constant

C. Analytic only at the origin

D. A polynomial of degree 1

Q5. The curl of the gradient of any scalar field $\phi(x, y, z)$ is always equal to:

A. $\nabla^2\phi$

B. $\mathbf{1}$

C. $\mathbf{0}$

D. $\nabla\phi$

Q6. For a square matrix A, if the characteristic equation is $\lambda^3 - 5\lambda^2 + 6\lambda = 0$, then the eigenvalues of A are:

A. $1, 2, 3$

B. $0, 2, 3$

C. $-2, -3, 0$

D. $0, 1, 6$

Q7. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$?

A. 0

B. 1

C. e

D. ∞

Q8. According to Cayley-Hamilton theorem, every square matrix satisfies its own:

A. Inverse equation

B. Minimal polynomial

C. Characteristic equation

D. Transpose

Q9. The integrating factor (I.F.) for the linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is:

A. $e^{\int Q dx}$

B. $e^{\int P dx}$

C. $\int e^P dx$

D. $\ln(\int P dx)$

Q10. The value of the limit $\lim_{x\to 0} \frac{\sin x - x}{x^3}$ is:

A. $1/6$

B. $-1/6$

C. 0

D. $-\infty$

Q11. If $u = x^2 - y^2$ and $v = 2xy$, then the functions u and v satisfy the Cauchy-Riemann equations. What is the corresponding analytic function $f(z) = u + iv$?

A. z^2

B. z^3

C. e^z

D. $2z$

Q12. The Wronskian $W(y_1, y_2)$ of two linearly dependent solutions of a second-order linear homogeneous differential equation is:

A. 1

B. A non-zero constant

C. 0

D. e^x

Q13. What is the Laplace transform of the function $f(t) = \sin(\omega t)$?

A. $\frac{s}{s^2+\omega^2}$

B. $\frac{\omega}{s^2-\omega^2}$

C. $\frac{s}{s^2-\omega^2}$

D. $\frac{\omega}{s^2+\omega^2}$

Q14. In numerical analysis, Simpson's 1/3 rule for numerical integration requires the number of sub-intervals (n) to be:

A. Odd

B. Prime

C. Even

D. A multiple of 3

Q15. The divergence of the position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is:

A. 0

B. 1

C. 2

D. 3

Q16. The Newton-Raphson iteration formula to find a root of the equation $f(x) = 0$ is given by:

A. $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

B. $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$

C. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

D. $x_{n+1} = x_n + \frac{f'(x_n)}{f(x_n)}$

Q17. The value of the Gamma function $\Gamma\left(\frac{1}{2}\right)$ is:

A. π

B. $\sqrt{\pi}$

C. $1/2$

D. 1

Q18. Gauss's Divergence Theorem relates which of the following integrals?

A. Line integral to Surface integral

B. Surface integral to Volume integral

C. Line integral to Volume integral

D. Contour integral to Line integral

Q19. If a function $f(x)$ is odd, then the Fourier coefficient a_0 in its Fourier series expansion over $[-\pi, \pi]$ is:

A. 1

B. π

C. 0

D. 2π

Q20. What is the order of the symmetric group S_n (the group of all permutations of n distinct symbols)?

A. n

B. n^2

C. 2^n

D. $n!$

Q21. The eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are always:

A. Linearly dependent

B. Parallel

C. Orthogonal

D. Complex conjugates

Q22. The area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is:

A. $\frac{16}{3}a^2$

B. $\frac{8}{3}a^2$

C. $4a^2$

D. $\frac{32}{3}a^2$

Q23. The partial differential equation $Pp + Qq = R$ is known as:

A. Charpit's equation

B. Lagrange's linear equation

C. Clairaut's equation

D. Euler-Cauchy equation

Q24. The residue of the complex function $f(z) = \frac{1}{z}$ at the pole $z = 0$ is:

A. 0

B. 1

C. $2\pi i$

D. -1

Q25. A necessary condition for a function $f(x, y)$ to have a local maximum at (a, b), where $r = f_{xx}$, $s = f_{xy}$, and $t = f_{yy}$, is:

A. $rt - s^2 > 0$ and $r > 0$

B. $rt - s^2 < 0$ and $r < 0$

C. $rt - s^2 > 0$ and $r < 0$

D. $rt - s^2 = 0$

Q26. The probability of a leap year having exactly 53 Sundays is:

A. 1/7

B. 2/7

C. 52/366

D. 1/366

Q27. The sum of the three cube roots of unity $(1, \omega, \omega^2)$ is:

A. 1

B. 0

C. 3

D. ω

Q28. If $\mathbf{A} \times \mathbf{B} = \mathbf{0}$ and neither $\mathbf{A}$ nor $\mathbf{B}$ is a zero vector, then the vectors are:

A. Orthogonal

B. Parallel

C. Coplanar but intersecting

D. Skew

Q29. The function $f(x) = |x|$ at $x = 0$ is:

A. Continuous and differentiable

B. Discontinuous and differentiable

C. Continuous but not differentiable

D. Neither continuous nor differentiable

Q30. The rank of a 3×3 matrix in which every element is 1 is:

A. 0

B. 1

C. 2

D. 3

Q31. According to Maxwell's equations, the statement $\nabla \cdot \mathbf{B} = 0$ physically implies that:

A. Magnetic fields are conservative

B. Magnetic monopoles do not exist

C. Electric charges do not exist

D. Time-varying electric fields produce magnetic fields

Q32. In quantum mechanics, the Heisenberg Uncertainty Principle states that $\Delta x \Delta p \geq$:

A. h

B. $\hbar/2$

C. $2\hbar$

D. $\hbar$

Q33. The direction of the Poynting vector $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$ represents the:

A. Direction of electric field

B. Direction of magnetic field

C. Direction of electromagnetic energy flow

D. Phase velocity of the wave

Q34. In the Compton effect, the shift in wavelength ($\Delta\lambda$) of the scattered X-ray photon depends only on the:

A. Intensity of the incident light

B. Scattering angle

C. Nature of the target material

D. Temperature of the target

Q35. In an intrinsic semiconductor at absolute zero temperature, the Fermi energy level lies:

A. Near the conduction band

B. Near the valence band

C. Exactly in the middle of the forbidden energy gap

D. Inside the valence band

Q36. According to Bragg's law of X-ray diffraction, constructive interference occurs when $2d\sin\theta$ is equal to:

A. $\lambda/2$

B. $n\lambda$

C. $(n+1/2)\lambda$

D. $1/\lambda$

Q37. The First Law of Thermodynamics, $dQ = dU + dW$, is essentially a statement of the conservation of:

A. Momentum

B. Mass

C. Energy

D. Entropy

Q38. The de Broglie wavelength λ associated with a particle of momentum p is given by:

A. $\lambda = hp$

B. $\lambda = h/p$

C. $\lambda = p/h$

D. $\lambda = h/p^2$

Q39. The relationship between the half-life ($T_{1/2}$) of a radioactive substance and its decay constant (λ) is:

A. $T_{1/2} = \frac{\ln 2}{\lambda}$

B. $T_{1/2} = \frac{\lambda}{\ln 2}$

C. $T_{1/2} = \lambda \ln 2$

D. $T_{1/2} = e^{-\lambda}$

Q40. According to the Stefan-Boltzmann law, the total energy radiated per unit surface area of a black body is proportional to:

A. T

B. T^2

C. T^3

D. T^4

Q41. The Hall effect can be experimentally used to determine the:

A. Mass of a charge carrier

B. Sign and density of the charge carriers

C. Speed of light in the material

D. Thermal conductivity of a semiconductor

Q42. The energy levels of a one-dimensional quantum simple harmonic oscillator are given by $E_n =$:

A. $n\hbar\omega$

B. $(n + \frac{1}{2})\hbar\omega$

C. $n^2\hbar\omega$

D. $\frac{1}{2}n\hbar\omega$

Q43. In Albert Einstein's photoelectric equation, $K_{max} = h\nu - \Phi$, the term Φ represents the:

A. Kinetic energy of the photon

B. Planck's constant

C. Work function of the material

D. Stopping potential

Q44. In special relativity, the mass-energy equivalence principle is mathematically expressed as:

A. $E = \frac{1}{2}mc^2$

B. $E = mc^2$

C. $E = mgh$

D. $E = p^2/2m$

Q45. If the group velocity of a wave packet is v_g and the phase velocity is v_p, the relationship for a non-dispersive medium is:

A. $v_g > v_p$

B. $v_g < v_p$

C. $v_g = v_p$

D. $v_g \cdot v_p = c^2$

Q46. The Crystal Field Stabilization Energy (CFSE) for a high-spin d^5 octahedral complex is:

A. $-0.4\Delta_o$

B. 0

C. $-1.2\Delta_o$

D. $-2.0\Delta_o$

Q47. According to VSEPR theory, the geometry of the SF_6 molecule is:

A. Tetrahedral

B. Trigonal bipyramidal

C. Square planar

D. Octahedral

Q48. For a chemical reaction, if the half-life period is completely independent of the initial concentration of the reactant, the order of the reaction is:

A. Zero order

B. First order

C. Second order

D. Third order

Q49. An S_N2 (Substitution Nucleophilic Bimolecular) organic reaction mechanism predominantly proceeds with:

A. Retention of configuration

B. Racemization

C. Inversion of configuration (Walden inversion)

D. Carbocation formation

Q50. In electrochemistry, the Nernst equation is used to calculate the:

A. Rate constant of a reaction

B. Electrode potential of a cell under non-standard conditions

C. Activation energy of a reaction

D. Enthalpy of formation

Q51. In the nitration of benzene using a mixture of concentrated HNO_3 and concentrated H_2SO_4, the active electrophile generated is:

A. NO_2^-

B. NO_3^-

C. NO^+

D. NO_2^+

Q52. The coordination number of an atom in a Face-Centered Cubic (FCC) crystal lattice structure is:

A. 6

B. 8

C. 12

D. 4

Q53. The Lanthanide contraction (the steady decrease in atomic and ionic radii of lanthanides) is primarily due to:

A. Increasing nuclear charge without effective shielding by $4f$ electrons

B. Strong shielding by $5d$ electrons

C. Decreasing effective nuclear charge

D. Increases in atomic mass

Q54. What is the hybridization of the central Xenon (Xe) atom in the XeF_4 molecule?

A. sp^3

B. sp^3d

C. sp^3d^2

D. dsp^2

Q55. The unit of the rate constant (k) for a second-order reaction is:

A. s^{-1}

B. $mol \cdot L^{-1} \cdot s^{-1}$

C. $L \cdot mol^{-1} \cdot s^{-1}$

D. $L^2 \cdot mol^{-2} \cdot s^{-1}$

Q56. Markownikoff's rule is observed in the electrophilic addition of hydrogen halides (HX) to:

A. Symmetrical alkenes

B. Unsymmetrical alkenes

C. Alkanes

D. Aromatic rings

Q57. To correct the ideal gas equation for the finite volume of molecules and intermolecular attractive forces, which equation is used?

A. Arrhenius equation

B. Van der Waals equation

C. Clausius-Clapeyron equation

D. Gibbs-Helmholtz equation

Q58. Tollens' reagent (ammoniacal silver nitrate solution) is a mild oxidizing agent used specifically to detect the presence of:

A. Ketones

B. Alcohols

C. Aldehydes

D. Carboxylic acids

Q59. The pH at which an amino acid carries no net electrical charge and does not migrate in an electric field is known as its:

A. Equivalence point

B. Isoelectric point (pI)

C. Neutralization point

D. Eutectic point

Q60. In the context of molecular orbital theory, HOMO stands for:

A. High Order Molecular Orbit

B. Highest Occupied Molecular Orbital

C. Heaviest Oxidized Molecular Orbital

D. Homogeneous Organic Molecular Orbital

Solutions

1. D
Explanation: The integral $\int_{-\infty}^{\infty} e^{-x^2} dx$ is the standard Gaussian integral, and its exact value is mathematically proven to be $\sqrt{\pi}$.

2. B
Explanation: For an orthogonal matrix A, $AA^T = I$. Taking the determinant on both sides gives $|A||A^T| = |I|$. Since $|A| = |A^T|$, we have $|A|^2 = 1$, which means $|A|$ can only be 1 or -1.

3. A
Explanation: The order of a differential equation is the highest derivative present, which is 2 (from d^2y/dx^2). The degree is the power of this highest derivative, which is 3. Thus, Order = 2, Degree = 3.

4. B
Explanation: Liouville's theorem in complex analysis states that every bounded entire function (analytic everywhere in the complex plane) must be a constant function.

5. C
Explanation: A fundamental vector identity states that the curl of the gradient of any scalar field is always the zero vector, mathematically $\nabla \times (\nabla \phi) = \mathbf{0}$.

6. B
Explanation: Factoring the characteristic equation $\lambda^3 - 5\lambda^2 + 6\lambda = \lambda(\lambda^2 - 5\lambda + 6) = \lambda(\lambda - 2)(\lambda - 3) = 0$ gives the roots (eigenvalues) as $0, 2, 3$.

7. D
Explanation: The given power series is the Maclaurin series expansion for the exponential function e^z. This series converges for all complex numbers, so its radius of convergence is ∞.

8. C
Explanation: The Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation.

9. B
Explanation: The standard formula for the integrating factor of a first-order linear ordinary differential equation in the form $dy/dx + P(x)y = Q(x)$ is $e^{\int P(x)dx}$.

10. B
Explanation: Expanding $\sin x$ using its Maclaurin series $(x - x^3/3! + x^5/5! - \dots)$, the expression becomes $(x - x^3/6 - x)/x^3 = -1/6$.

11. A
Explanation: Given $u = x^2 - y^2$ and $v = 2xy$, the complex function is $f(z) = u + iv = (x^2 - y^2) + i(2xy) = (x + iy)^2 = z^2$.

12. C
Explanation: If two solutions of a linear homogeneous differential equation are linearly dependent, their Wronskian determinant is mathematically zero everywhere.

13. D

Explanation: By definition, the Laplace transform of $\sin(\omega t)$ is derived as $\frac{\omega}{s^2+\omega^2}$.

14. C

Explanation: Simpson's 1/3 rule evaluates integrals by fitting parabolas through sets of 3 points (2 intervals). Thus, it requires the total number of sub-intervals (n) to be even.

15. D

Explanation: Divergence is $\nabla \cdot \mathbf{r} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = 3$.

16. C

Explanation: The Newton-Raphson method uses the tangent line to iteratively find roots. The formula is $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

17. B

Explanation: The Gamma function evaluated at one-half, $\Gamma(1/2)$, is a well-known mathematical constant equal to $\sqrt{\pi}$, derived from the Gaussian integral.

18. B

Explanation: Gauss's Divergence Theorem mathematically relates the flux of a vector field through a closed surface (Surface integral) to the divergence of the field in the enclosed volume (Volume integral).

19. C

Explanation: For an odd function $f(-x) = -f(x)$, the integral over a symmetric interval $[-\pi, \pi]$ is zero. Therefore, the DC component (or average value) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = 0$.

20. D

Explanation: The symmetric group S_n contains all possible permutations of n items. The total number of such permutations is $n!$ (n factorial).

21. C

Explanation: A fundamental property of real symmetric matrices is that eigenvectors corresponding to distinct eigenvalues are mutually orthogonal (their dot product is zero).

22. A

Explanation: Integrating to find the area between $y = \frac{x^2}{4a}$ and $y = \sqrt{4ax}$ from $x = 0$ to $x = 4a$ yields $\frac{16}{3}a^2$.

23. B

Explanation: A first-order partial differential equation of the form $P(x, y, z)p + Q(x, y, z)q = R(x, y, z)$ is standardly known as Lagrange's linear equation.

24. B

Explanation: The residue of a function at a simple pole $z = c$ is the coefficient a_{-1} in its Laurent series. For $1/z$ at $z = 0$, the coefficient of $1/z$ is 1.

25. C

Explanation: By the Second Derivative Test for functions of two variables, a local maximum occurs if $rt - s^2 > 0$ (meaning a local extremum exists) and $r < 0$ (meaning the surface is concave downward).

26. B

Explanation: A leap year has 366 days, which is exactly 52 weeks (364 days)

plus 2 extra days. These 2 days can be (Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun). Out of these 7 possibilities, 2 contain a Sunday.

27. B

Explanation: The roots of the equation $z^3 = 1$ are 1, $e^{i2\pi/3}(\omega)$, and $e^{i4\pi/3}(\omega^2)$. Their sum mathematically equals 0.

28. B

Explanation: The cross product magnitude is $|A||B|\sin\theta$. If the cross product is zero and neither vector is zero, then $\sin\theta = 0$, meaning the angle θ is 0 or 180° (they are parallel).

29. C

Explanation: The absolute value function is continuous everywhere. However, at $x = 0$, there is a sharp "corner", meaning the left-hand derivative (-1) does not equal the right-hand derivative $(+1)$. It is not differentiable at $x = 0$.

30. B

Explanation: Because all rows are identical to each other, row operations will eliminate the second and third rows entirely, leaving only one non-zero linearly independent row. Thus, the rank is 1.

31. B

Explanation: Gauss's law for magnetism $(\nabla \cdot \mathbf{B} = 0)$ states that the magnetic flux through any closed surface is zero, which physically implies that there are no isolated magnetic charges (magnetic monopoles).

32. B

Explanation: The formal statement of Heisenberg's Uncertainty Principle is $\sigma_x\sigma_p \geq \frac{\hbar}{2}$, meaning the product of uncertainties in position and momentum cannot be less than $\hbar/2$.

33. C

Explanation: The Poynting vector represents the directional energy flux (the rate of energy transfer per unit area) of an electromagnetic field.

34. B

Explanation: The Compton shift formula is $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$. Thus, the shift depends solely on the scattering angle θ.

35. C

Explanation: In an intrinsic (pure) semiconductor, there are an equal number of electrons and holes. At absolute zero, the Fermi level sits exactly midway between the conduction and valence bands.

36. B

Explanation: Bragg's law states that constructive interference occurs when the path difference between X-rays scattered from adjacent crystal planes $(2d\sin\theta)$ equals an integer multiple of the wavelength $(n\lambda)$.

37. C

Explanation: The First Law of Thermodynamics is the application of the conservation of energy principle to heat and thermodynamic processes.

38. B

Explanation: Louis de Broglie proposed that matter exhibits wave-like properties, with a wavelength inversely proportional to its momentum, formulated as $\lambda = h/p$.

39. A

Explanation: The half-life is the time required for half of the radioactive nuclei to decay. Solving $N_0/2 = N_0 e^{-\lambda T_{1/2}}$ yields $T_{1/2} = \frac{\ln 2}{\lambda}$.

40. D

Explanation: The Stefan-Boltzmann law states that the thermal energy radiated by a black body per second per unit area is proportional to the fourth power of its absolute temperature ($E = \sigma T^4$).

41. B

Explanation: The Hall effect generates a transverse voltage across a conductor placed in a magnetic field. The polarity of this voltage reveals whether the charge carriers are positive (holes) or negative (electrons), and its magnitude helps determine their density.

42. B

Explanation: Solving the Schrödinger equation for a simple harmonic oscillator yields quantized energy levels $E_n = (n + \frac{1}{2})\hbar\omega$, where $n = 0, 1, 2...$

43. C

Explanation: In the photoelectric equation, Φ (or W) is the work function: the minimum amount of energy required to eject an electron from the surface of the metal.

44. B

Explanation: Einstein's famous equation $E = mc^2$ establishes that mass and energy are interchangeable and equivalent.

45. C

Explanation: In a non-dispersive medium, all frequencies travel at the exact same speed. Consequently, the phase velocity of individual waves equals the group velocity of the wave packet ($v_p = v_g$).

46. B

Explanation: In a high-spin octahedral complex, the 5 d-electrons singly occupy the orbitals: three in t_{2g} (each $-0.4\Delta_o$) and two in e_g (each $+0.6\Delta_o$). Total CFSE $= 3(-0.4) + 2(0.6) = 0$.

47. D

Explanation: Sulfur hexafluoride (SF_6) has 6 bonding pairs and 0 lone pairs around the central sulfur atom. According to VSEPR theory, this corresponds to a perfect Octahedral geometry.

48. B

Explanation: For a first-order reaction, the half-life $t_{1/2} = 0.693/k$, which contains no concentration terms. It is entirely independent of initial concentration.

49. C

Explanation: The S_N2 reaction happens in a single concerted step. The nucleophile attacks from the backside of the leaving group, causing the molecule to flip like an umbrella in the wind (Walden inversion).

50. B
Explanation: The Nernst equation allows for the determination of cell potential at non-standard conditions by relating it to the standard electrode potential, temperature, and reaction quotient.

51. D
Explanation: Sulfuric acid is a stronger acid than nitric acid, forcing HNO_3 to accept a proton and dehydrate to form the highly reactive nitronium ion (NO_2^+), which acts as the electrophile.

52. C
Explanation: In an FCC (cubic close-packed) lattice, every atom touches 12 nearest neighbors (4 in its own plane, 4 above, and 4 below), making its coordination number 12.

53. A
Explanation: As atomic number increases across the lanthanide series, electrons enter the inner $4f$ subshell. These $4f$ electrons shield the increasing nuclear charge very poorly, causing the atomic radius to contract.

54. C
Explanation: Xenon in XeF_4 has 8 valence electrons. Four bond with Fluorine, leaving 4 electrons (2 lone pairs). 4 bonds + 2 lone pairs = 6 steric number, which dictates sp^3d^2 hybridization.

55. C
Explanation: The rate law for a second-order reaction is Rate $= k[A]^2$. Solving for k gives units of $(\text{concentration})^{-1}(\text{time})^{-1}$, which is $L \cdot mol^{-1} \cdot s^{-1}$.

56. B
Explanation: Markownikoff's rule predicts the regioselectivity of electrophilic addition to unsymmetrical alkenes, stating that the proton adds to the carbon with the most hydrogens to form the most stable carbocation intermediate.

57. B
Explanation: The Van der Waals equation $(P + \frac{an^2}{V^2})(V - nb) = nRT$ introduces parameters a and b to correct the ideal gas law for intermolecular forces and molecular volume, respectively.

58. C
Explanation: Tollens' reagent oxidizes aldehydes into carboxylic acids, reducing the silver ions in the process to form a characteristic "silver mirror" on the test tube wall. It does not react with ketones.

59. B
Explanation: The isoelectric point (pI) is the specific pH at which an amino acid exists primarily as a neutral zwitterion, carrying an equal number of positive and negative charges.

60. B
Explanation: In molecular orbital theory, HOMO stands for Highest Occupied Molecular Orbital. It represents the highest energy orbital in a molecule that currently holds electrons.