OJEE MCA Sample Paper 03

A) Mode and Time: The exam is a Computer Based Test (CBT) lasting 120 minutes.
B) Total Questions: The exam contains 120 questions.
C) Marking Scheme & Rules:

- +4 marks for each correct answer and -1 mark for every incorrect response.

Q1. Which of the following problems is Turing-recognizable but strictly undecidable?

A. The Halting Problem

B. Determining if a DFA accepts the empty string

C. Determining if a Context-Free Grammar is ambiguous

D. Boolean Satisfiability Problem (SAT)

Q2. In the IEEE 754 single-precision floating point standard, how is 'Not a Number' (NaN) represented?

A. Exponent all 0s, Fraction all 0s

B. Exponent all 1s, Fraction all 0s

C. Exponent all 1s, Fraction non-zero

D. Exponent all 0s, Fraction non-zero

Q3. Which of the following cache replacement policies is completely immune to Belady's Anomaly?

A. FIFO

B. Second Chance

C. LRU

D. Random

Q4. What is the primary purpose of Tomasulo's algorithm in computer architecture?

A. Cache coherence

B. Virtual memory paging

C. Out-of-order execution with register renaming

D. Branch prediction

Q5. In an operating system, the 'Priority Inversion' problem is most elegantly solved using:

A. Priority Scheduling

B. Priority Inheritance Protocol

C. Round Robin Scheduling

D. Semaphores with unbounded waiting

Q6. Which memory management scheme causes External Fragmentation but explicitly avoids Internal Fragmentation?

A. Paging

B. Segmentation

C. Demand Paging

D. Paged Segmentation

Q7. In the context of database transactions, what does Write-Ahead Logging (WAL) primarily ensure?

A. Isolation

B. Atomicity and Durability

C. Consistency

D. Serializability

Q8. A database schedule is conflict serializable if and only if its precedence graph has:

A. Exactly one cycle

B. No cycles

C. An Eulerian path

D. A Hamiltonian cycle

Q9. A relation schema is in Fourth Normal Form (4NF) if it eliminates which of the following?

A. Partial dependencies

B. Transitive dependencies

C. Multi-valued dependencies

D. Join dependencies

Q10. What is the maximum number of keys a B-Tree of order m can hold in a single node?

A. m

B. $m - 1$

C. $\lceil m/2 \rceil$

D. $2m$

Q11. In computer networking, which algorithm does the OSPF routing protocol use to calculate the shortest path?

A. Bellman-Ford

B. Ford-Fulkerson

C. Dijkstra's Algorithm

D. Kruskal's Algorithm

Q12. The TCP congestion control mechanism employs a 'Congestion Window'. During the 'Slow Start' phase, how does this window grow?

A. Linearly

B. Exponentially

C. Logarithmically

D. It remains constant

Q13. What is the purpose of the Time To Live (TTL) field in an IPv4 header?

A. To track the time a packet spends in the router queue

B. To prevent packets from looping endlessly in the network

C. To specify the priority of the packet

D. To encrypt the packet's payload

Q14. Which of the following HTTP methods is NOT idempotent?

A. GET

B. PUT

C. DELETE

D. POST

Q15. In the context of the OSI model, which sublayer is responsible for error control and flow control over the physical medium?

A. Logical Link Control (LLC)

B. Media Access Control (MAC)

C. Network Control Protocol (NCP)

D. Physical Signalling

Q16. In C, what does the ‘volatile‘ keyword instruct the compiler to do?

A. Store the variable in a CPU register

B. Prevent multiple threads from accessing the variable

C. Avoid optimizing read/write operations for that variable

D. Allocate the variable on the heap

Q17. What is the output of ‘sizeof(char *)‘ on a 64-bit system?

A. 1 byte

B. 2 bytes

C. 4 bytes

D. 8 bytes

Q18. Which C++ feature explicitly resolves the ”Diamond Problem” in multiple inheritance?

A. Pure virtual functions

B. Virtual inheritance

C. Function overloading

D. Friend classes

Q19. The Liskov Substitution Principle (LSP) in Object-Oriented Design states that:

A. Classes should have only one reason to change

B. High-level modules should not depend on low-level modules

C. Objects of a superclass should be replaceable with objects of its subclasses

D. Many client-specific interfaces are better than one general interface

Q20. Which design pattern defines a one-to-many dependency between objects so that when one object changes state, all its dependents are notified?

A. Singleton

B. Decorator

C. Observer

D. Factory Method

Q21. In an AVL tree, what type of rotation is required to balance a Right-Left (RL) heavy subtree?

A. Single Right Rotation

B. Single Left Rotation

C. Left-Right (LR) Double Rotation

D. Right-Left (RL) Double Rotation

Q22. What is the amortized time complexity of the 'Decrease Key' operation in a Fibonacci Heap?

A. $O(1)$

B. $O(\log n)$

C. $O(n)$

D. $O(n \log n)$

Q23. In the KMP (Knuth-Morris-Pratt) string matching algorithm, what does the LPS array store?

A. Longest prime suffix

B. Longest proper prefix which is also a suffix

C. Largest palindromic substring

D. Longest pattern sequence

Q24. Tarjan's Algorithm is used in graph theory to find:

A. The shortest path between all pairs of nodes

B. The minimum spanning tree

C. Strongly Connected Components (SCCs)

D. The maximum flow in a network

Q25. By the Master Theorem, the time complexity of the recurrence $T(n) = 2T(n/2) + O(n)$ is:

A. $O(n)$

B. $O(n \log n)$

C. $O(n^2)$

D. $O(\log n)$

Q26. Which of the following grammars generates the language $a^n b^n c^n$ for $n \geq 1$?

A. Regular Grammar

B. Context-Free Grammar

C. Context-Sensitive Grammar

D. Unrestricted Grammar

Q27. According to the Pumping Lemma for Regular Languages, if a string s is pumped ($s = xyz$), which condition must hold?

A. $|xy| \leq p$ and $|y| > 0$

B. $|y| \leq p$ and $|x| > 0$

C. $|yz| \leq p$ and $|y| > 0$

D. $|xyz| \leq p$

Q28. In compiler design, 'Left Recursion' must be eliminated because it causes:

A. Top-down parsers to enter infinite loops

B. Bottom-up parsers to shift endlessly

C. The lexical analyzer to crash

D. The symbol table to overflow

Q29. Which construction algorithm converts a Regular Expression directly to a Non-Deterministic Finite Automaton (NFA)?

A. Subset Construction

B. Thompson's Construction

C. Hopcroft's Algorithm

D. CYK Algorithm

Q30. The security of the RSA cryptographic algorithm relies fundamentally on the computational difficulty of:

A. Computing discrete logarithms

B. Solving elliptic curves

C. Factoring large semiprimes

D. Finding hash collisions

Q31. In cryptographic hashing, what is the 'Avalanche Effect'?

A. A small change in input drastically changes the output hash

B. Multiple inputs mapping to the same hash

C. The hash size growing exponentially with input size

D. The slow computation of hashes to deter brute force

Q32. Which type of software testing relies strictly on the internal logical structure and code paths of the application?

A. Black Box Testing

B. White Box Testing

C. Acceptance Testing

D. Beta Testing

Q33. In software engineering, good modular design is characterized by:

A. High Cohesion, High Coupling

B. Low Cohesion, Low Coupling

C. High Cohesion, Low Coupling

D. Low Cohesion, High Coupling

Q34. In the context of CPU scheduling, which queue mechanism prevents starvation by gradually moving older processes to higher priority queues?

A. Multilevel Queue

B. Multilevel Feedback Queue

C. Completely Fair Scheduler

D. Lottery Scheduling

Q35. Which data structure is fundamentally used by the Banker's Algorithm to determine a safe state?

A. Linked Lists

B. Hash Maps

C. Matrices (2D Arrays)

D. Binary Search Trees

Q36. A recursive DNS query is characterized by:

A. The DNS server referring the client to another server

B. The DNS server taking full responsibility to resolve the query

C. The client querying all root servers simultaneously

D. The server discarding unresolvable domains

Q37. In a dense index of a DBMS:

A. An index record appears for only some values in the file

B. An index record appears for every search key value in the file

C. The index is clustered tightly on the disk

D. The index uses a B+ Tree strictly of order 3

Q38. Timestamp Ordering protocol in DBMS concurrency control ensures serializability by:

A. Acquiring locks exclusively

B. Aborting and restarting transactions that violate timestamp order

C. Creating multiple versions of data continuously

D. Running transactions serially on a single thread

Q39. In C, variables declared with the ‘extern‘ storage class are:

A. Stored in CPU registers

B. Visible only within the block they are defined

C. Defined elsewhere, usually in another file

D. Automatically initialized to 1

Q40. Which advanced tree structure is optimally used to search for a substring in a large text in $O(m)$ time, where m is the substring length?

A. B+ Tree

B. Suffix Tree

C. Red-Black Tree

D. Fenwick Tree

Q41. A Moore machine differs from a Mealy machine in that its output depends on:

A. Both current state and current input

B. Only the current state

C. Only the current input

D. The previous state and input

Q42. Which RAID level requires a minimum of three disks and uses block-level striping with distributed parity?

A. RAID 0

B. RAID 1

C. RAID 5

D. RAID 10

Q43. What is the primary difference between IaaS (Infrastructure as a Service) and PaaS (Platform as a Service)?

A. IaaS provides software apps, PaaS provides networks

B. PaaS abstracts the OS and middleware, IaaS provides raw VMs

C. IaaS is for consumers, PaaS is for sysadmins

D. There is no functional difference

Q44. Cross-Site Scripting (XSS) is a web security vulnerability that allows an attacker to:

A. Inject malicious scripts into trusted websites viewed by other users

B. Execute SQL commands on the backend database

C. Intercept network traffic via man-in-the-middle

D. Overload the server with HTTP requests

Q45. In Agile methodologies, what is the typical duration of a Scrum 'Sprint'?

A. 1 to 4 hours

B. 1 to 4 days

C. 1 to 4 weeks

D. 1 to 4 months

Q46. Which page table structure maps physical frames to virtual pages, making the table size proportional to physical memory rather than virtual memory?

A. Hierarchical Page Table

B. Hashed Page Table

C. Inverted Page Table

D. Direct Mapped Page Table

Q47. What is the time complexity of the Floyd-Warshall algorithm for finding all-pairs shortest paths?

A. $O(V^2)$

B. $O(V^3)$

C. $O(E \log V)$

D. $O(V^2 \log V)$

Q48. A C program contains 'void (*ptr)(int);'. What does this declare?

A. A function returning a void pointer

B. A pointer to an integer function

C. A pointer to a function that takes an integer and returns void

D. An array of void pointers

Q49. What is the primary disadvantage of the 'Write-Through' cache policy compared to 'Write-Back'?

A. It causes data inconsistency

B. It requires higher memory bandwidth (more frequent main memory writes)

C. It loses data on power failure

D. It completely disables the cache

Q50. In Dynamic Programming, which property must a problem possess to be solvable optimally?

A. Overlapping Subproblems and Optimal Substructure

B. Greedy Choice Property

C. Divide and Conquer Property

D. Constant Time State Transitions

Q51. Which Unix/Linux command is used to display the dynamic, real-time list of running processes?

A. ps

B. top

C. ls

D. grep

Q52. In the TCP/IP suite, which protocol resolves IP addresses into MAC addresses?

A. RARP

B. DNS

C. DHCP

D. ARP

Q53. What is the maximum number of usable host IP addresses in a '/26' subnet?

A. 62

B. 64

C. 126

D. 128

Q54. Which normal form resolves the anomaly caused by multi-valued dependencies?

A. BCNF

B. 3NF

C. 4NF

D. 5NF

Q55. In Object-Oriented Programming, 'Encapsulation' is primarily used to:

A. Allow multiple inheritance

B. Bind code and the data it manipulates while hiding the implementation

C. Re-use code through class extension

D. Provide a single interface for multiple types

Q56. Which sorting algorithm is traditionally used as the foundational logic for the ‘std::sort‘ function in C++ STL?

A. Quick Sort

B. Heap Sort

C. IntroSort (Hybrid of Quick, Heap, Insertion)

D. Merge Sort

Q57. A graph with no cycles is strictly called a:

A. Tree

B. Forest

C. Bipartite Graph

D. DAG (Directed Acyclic Graph) or Forest depending on connectivity

Q58. Which protocol is used for securely transmitting emails between servers?

A. POP3

B. IMAP

C. SMTP

D. SNMP

Q59. What is a ”foreign key” constraint used for in SQL?

A. To ensure all values in a column are unique

B. To enforce referential integrity between two tables

C. To speed up query performance

D. To allow null values in primary keys

Q60. In C++, what does the ‘this‘ pointer represent?

A. A pointer to the parent class

B. A pointer to the object that invoked the member function

C. A pointer to the program’s main function

D. A global pointer accessible everywhere

Q61. If A and B are two $n \times n$ matrices, what is the rank of the product matrix AB?

A. $\text{rank}(AB) = \text{rank}(A) + \text{rank}(B)$

B. $\text{rank}(AB) \geq \min(\text{rank}(A), \text{rank}(B))$

C. $\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$

D. $\text{rank}(AB) = \text{rank}(A) \times \text{rank}(B)$

Q62. What are the possible eigenvalues of an idempotent matrix ($A^2 = A$)?

A. Only 1

B. Only 0

C. 0 and 1

D. -1 and 1

Q63. By the Cayley-Hamilton Theorem, every square matrix satisfies:

A. Its characteristic equation

B. Its minimal polynomial equation

C. Its transpose equation

D. Its inverse equation

Q64. What is the value of the definite integral $\int_0^2 |x - 1|\, dx$?

A. 0

B. 1

C. 2

D. 1/2

Q65. The improper integral $\int_1^\infty \frac{1}{x^p}\, dx$ converges if and only if:

A. $p > 0$

B. $p > 1$

C. $p \geq 1$

D. $p < 1$

Q66. What is the limit $\lim_{x \to 0}(1 + x)^{1/x}$?

A. 0

B. 1

C. e

D. ∞

Q67. The differential equation $M(x,y)dx + N(x,y)dy = 0$ is exact if:

A. $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

B. $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

C. $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

D. $\frac{\partial^2 M}{\partial x^2} = \frac{\partial^2 N}{\partial y^2}$

Q68. To find the orthogonal trajectories of a family of curves $f(x,y,c) = 0$, one must replace dy/dx with:

A. dx/dy

B. $-dx/dy$

C. $-dy/dx$

D. $1/(dy/dx)^2$

Q69. For any scalar function ϕ, the curl of its gradient $(\nabla \times (\nabla\phi))$ is always:

A. 1

B. 0 (Zero vector)

C. $\nabla^2\phi$

D. Undefined

Q70. Stokes' Theorem equates a surface integral over surface S to a line integral around its boundary curve C. Specifically, it states $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ equals:

A. $\iiint_V (\nabla \cdot \vec{F}) dV$

B. $\oint_C \vec{F} \cdot d\vec{r}$

C. $\oint_C (\nabla\phi) \cdot d\vec{r}$

D. 0

Q71. In a discrete-time Markov Chain, a state is called "absorbing" if the probability of transitioning from that state to itself is:

A. 0

B. 0.5

C. 1

D. Depends on the initial state

Q72. Which of the following probability distributions possesses the "memoryless" property?

A. Binomial Distribution

B. Poisson Distribution

C. Geometric Distribution

D. Hypergeometric Distribution

Q73. Using the "Stars and Bars" combinatorics method, the number of non-negative integer solutions to $x_1 + x_2 + \cdots + x_k = n$ is:

A. ${}^{n+k}C_k$

B. ${}^{n+k-1}C_{k-1}$

C. ${}^{n-1}C_{k-1}$

D. n^k

Q74. By the Handshaking Lemma, the sum of the degrees of all vertices in a graph is equal to:

A. The number of edges

B. Twice the number of edges

C. The number of vertices

D. Half the number of edges

Q75. In Group Theory, Lagrange's theorem states that for any finite group G, the order (size) of every subgroup H:

A. Is a prime number

B. Divides the order of G

C. Is greater than the order of G

D. Equals the order of G

Q76. Fermat's Little Theorem states that if p is a prime number, then for any integer a, $a^p \equiv \ldots \pmod{p}$:

A. 1

B. 0

C. a

D. $p-1$

Q77. Using Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$, the principal value of $\ln(-1)$ is:

A. 0

B. $i\pi$

C. $-i\pi$

D. e

Q78. The locus of the point of intersection of mutually perpendicular tangents to an ellipse is a circle known as the:

A. Auxiliary circle

B. Director circle

C. Incircle

D. Circumcircle

Q79. The general second-degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a parabola if:

A. $h^2 = ab$

B. $h^2 > ab$

C. $h^2 < ab$

D. $a+b=0$

Q80. What is the maximum area of a rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$?

A. ab

B. $2ab$

C. $4ab$

D. πab

Q81. The radius of curvature ρ for a curve $y=f(x)$ is given by the formula:

A. $\frac{[1+(dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$

B. $\frac{[1+(dy/dx)^2]}{|d^2y/dx^2|}$

C. $\frac{|d^2y/dx^2|}{[1+(dy/dx)^2]^{3/2}}$

D. $\frac{[1-(dy/dx)^2]^{3/2}}{d^2y/dx^2}$

Q82. The infinite p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if:

A. $p < 1$

B. $p \leq 1$

C. $p > 1$

D. $p = 1$

Q83. What is the Maclaurin series expansion of $\sin(x^2)$?

A. $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \ldots$

B. $1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \ldots$

C. $x^2 - \frac{x^4}{2!} + \frac{x^6}{3!} - \ldots$

D. $x^2 + \frac{x^6}{3!} + \frac{x^{10}}{5!} + \ldots$

Q84. If A is an orthogonal matrix, what is the value of $\det(A)$?

A. 0

B. 1

C. -1

D. ± 1

Q85. In Boolean Algebra, a prime implicant is:

A. A minterm that cannot be covered by any other minterm

B. An implicant that is not completely contained in any other implicant

C. A product term containing all variables

D. A sum term that evaluates to 1

Q86. According to Cantor's theorem, the cardinality of the power set of a set A is strictly:

A. Equal to the cardinality of A

B. Less than the cardinality of A

C. Greater than the cardinality of A

D. Infinite

Q87. In propositional logic, the inference rule "Modus Tollens" is formulated as:

A. If $P \to Q$ and P, then Q

B. If $P \to Q$ and $\neg Q$, then $\neg P$

C. If $P \vee Q$ and $\neg P$, then Q

D. If $P \to Q$ and $Q \to R$, then $P \to R$

Q88. For a non-exact differential equation $Mdx+Ndy = 0$, if $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ is a function of x alone, say $f(x)$, then the integrating factor is:

A. $e^{\int f(x)dx}$

B. $e^{-\int f(x)dx}$

C. $f(x)$

D. $\ln(f(x))$

Q89. When reversing the order of integration for $\int_0^1 \int_y^1 f(x,y)dxdy$, the new integral bounds become:

A. $\int_0^1 \int_0^x f(x,y)dydx$

B. $\int_0^1 \int_x^1 f(x,y)dydx$

C. $\int_y^1 \int_0^x f(x,y)dydx$

D. $\int_0^1 \int_0^1 f(x,y)dydx$

Q90. If a random variable X has an exponential distribution with rate parameter λ, what is the variance of X?

A. $1/\lambda$

B. $1/\lambda^2$

C. λ

D. λ^2

Q91. The number of derangements (permutations where no element appears in its original position) of 4 items is:

A. 9

B. 12

C. 24

D. 44

Q92. The scalar projection of vector $\vec{A}$ onto vector $\vec{B}$ is given by:

A. $\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

B. $\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}$

C. $\frac{\vec{A}\times\vec{B}}{|\vec{B}|}$

D. $\frac{|\vec{A}\times\vec{B}|}{|\vec{A}|}$

Q93. What is the exact mathematical value of $\sin(18°)$?

A. $\frac{\sqrt{5}-1}{4}$

B. $\frac{\sqrt{5}+1}{4}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{\sqrt{10-2\sqrt{5}}}{4}$

Q94. The product of all the n^{th} roots of unity is:

A. 1

B. 0

C. $(-1)^{n-1}$

D. $(-1)^n$

Q95. For a prime number p and an integer $k \geq 1$, Euler's totient function $\phi(p^k)$ evaluates to:

A. $p^k - 1$

B. $p^k - p^{k-1}$

C. p^{k-1}

D. $p(k-1)$

Q96. According to the Rank-Nullity Theorem, for a linear transformation $T : V \to W$, $\text{rank}(T) + \text{nullity}(T)$ equals:

A. $\dim(W)$

B. $\dim(V)$

C. $\dim(V) \times \dim(W)$

D. 0

Q97. In a standard normal distribution, the Z-score indicates:

A. The mean of the distribution

B. The number of standard deviations a data point is from the mean

C. The variance of the sample

D. The probability of the event occurring

Q98. The volume of a tetrahedron formed by three coterminous vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ is:

A. $[\vec{a}, \vec{b}, \vec{c}]$

B. $\frac{1}{2}[\vec{a}, \vec{b}, \vec{c}]$

C. $\frac{1}{3}[\vec{a}, \vec{b}, \vec{c}]$

D. $\frac{1}{6}[\vec{a}, \vec{b}, \vec{c}]$

Q99. According to Euler's formula for planar graphs, a connected planar graph with 10 vertices and 15 edges divides the plane into how many faces?

A. 5

B. 6

C. 7

D. 8

Q100. To find the Particular Integral (P.I.) of a linear differential equation $f(D)y = e^{ax}$, one generally replaces D with:

A. 0

B. a

C. $-a$

D. $1/a$

Q101. According to the Polynomial Remainder Theorem, the remainder of the division of a polynomial $P(x)$ by $(x - a)$ is:

A. 0

B. $P(a)$

C. $P(-a)$

D. $P'(a)$

Q102. In set theory, the symmetric difference of two sets A and B, denoted $A \oplus B$, is equivalent to:

A. $(A \cup B) \setminus (A \cap B)$

B. $(A \cap B) \cup (A \cup B)$

C. $A \setminus B$

D. $B \setminus A$

Q103. What is the value of $\lim_{x \to 0} \frac{e^x - 1}{x}$?

A. 0

B. 1

C. e

D. ∞

Q104. The Gamma function is defined as $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$. What is $\Gamma(n + 1)$ equivalent to for an integer n?

A. $n!$

B. $(n - 1)!$

C. n

D. e^n

Q105. The perpendicular distance between two parallel planes $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is:

A. $\frac{|D_1 - D_2|}{\sqrt{A^2 + B^2 + C^2}}$

B. $\frac{|D_1 + D_2|}{\sqrt{A^2 + B^2 + C^2}}$

C. $\frac{|D_1 - D_2|}{A + B + C}$

D. 0

Q106. Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar if and only if their scalar triple product is:

A. 1

B. 0

C. -1

D. Equal to their vector triple product

Q107. If a block matrix is partitioned as $M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$, its inverse M^{-1} is:

A. $\begin{pmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{pmatrix}$

B. $\begin{pmatrix} B^{-1} & 0 \\ 0 & A^{-1} \end{pmatrix}$

C. $\begin{pmatrix} A & B \\ B & A \end{pmatrix}$

D. $\begin{pmatrix} A^{-1} & B^{-1} \\ 0 & 0 \end{pmatrix}$

Q108. Clairaut's Theorem on partial derivatives states that if mixed partial derivatives f_{xy} and f_{yx} are continuous, then:

A. $f_{xy} = -f_{yx}$

B. $f_{xy} = f_{yx}$

C. $f_{xy} + f_{yx} = 1$

D. $f_{xy} \times f_{yx} = 0$

Q109. If X and Y are independent random variables, their covariance $\text{Cov}(X, Y)$ is strictly:

A. 1

B. 0

C. $\sigma_X \sigma_Y$

D. Infinity

Q110. In Bayes' Theorem, the denominator $P(B)$ is often expanded using the Law of Total Probability as:

A. $\sum P(B|A_i)P(A_i)$

B. $\sum P(A_i|B)P(B)$

C. $P(A) + P(B)$

D. $P(A \cap B)$

Q111. The number of onto (surjective) functions from a set with m elements to a set with n elements ($m \geq n$) is found using:

A. The Pigeonhole Principle

B. The Principle of Inclusion-Exclusion

C. Fermat's Little Theorem

D. Bayes' Theorem

Q112. The chromatic polynomial of a tree with n vertices evaluated at k (the number of available colors) is:

A. k^n

B. $k(k-1)^{n-1}$

C. $k(k-1)^n$

D. $(k-1)^{n-1}$

Q113. If $u(x, y)$ is the real part of an analytic complex function $f(z) = u + iv$, then $v(x, y)$ is known as the:

A. Complex conjugate

B. Harmonic conjugate

C. Orthogonal trajectory

D. Analytic modulus

Q114. According to the Mean Value Theorem, if $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), there exists a $c \in (a, b)$ such that $f'(c)$ equals:

A. 0

B. $\frac{f(b)-f(a)}{b-a}$

C. $\frac{f(a)-f(b)}{b-a}$

D. $f(b) + f(a)$

Q115. By the Squeeze Theorem, if $g(x) \leq f(x) \leq h(x)$ and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x)$ is:

A. 0

B. L

C. ∞

D. Undefined

Q116. The area bounded by a polar curve $r = f(\theta)$ between angles α and β is given by the integral:

A. $\int_{\alpha}^{\beta} r d\theta$

B. $\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$

C. $\pi \int_{\alpha}^{\beta} r^2 d\theta$

D. $\int_{\alpha}^{\beta} r\cos\theta d\theta$

Q117. The Wronskian $W(y_1, y_2)$ of two solutions to a second-order linear homogeneous ODE is used to determine if the solutions are:

A. Continuous

B. Differentiable

C. Linearly Independent

D. Orthogonal

Q118. If a set A has 3 elements, how many elements does the power set of the power set of A, denoted $\mathcal{P}(\mathcal{P}(A))$, have?

A. 8

B. 64

C. 256

D. 512

Q119. The trigonometric identity for $\tan^{-1} x + \tan^{-1} y$ (where $xy < 1$) is:

A. $\tan^{-1}\left(\frac{x+y}{1-xy}\right)$

B. $\tan^{-1}\left(\frac{x-y}{1+xy}\right)$

C. $\tan^{-1}(x+y)$

D. $\tan^{-1}\left(\frac{1-xy}{x+y}\right)$

Q120. For any two $n \times n$ matrices A and B, the trace of the product satisfies which property?

A. $\mathrm{Tr}(AB) = \mathrm{Tr}(A)\mathrm{Tr}(B)$

B. $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$

C. $\mathrm{Tr}(AB) = 0$

D. $\mathrm{Tr}(AB) = \det(AB)$

Solutions

1. A
Explanation: The Halting Problem can be recognized (if a program halts, a Turing machine can simulate and accept it), but it is undecidable because there is no algorithm to determine if an arbitrary program will loop forever.

2. C
Explanation: In IEEE 754, NaN (Not a Number) is represented by setting all bits of the exponent field to 1 and having a non-zero fraction/mantissa.

3. C
Explanation: LRU (Least Recently Used) is a stack algorithm. Stack algorithms are mathematically proven to be completely immune to Belady's Anomaly.

4. C
Explanation: Tomasulo's algorithm enables high-performance out-of-order execution in CPUs by utilizing register renaming to eliminate false data dependencies (WAR and WAW hazards).

5. B
Explanation: The Priority Inheritance Protocol solves priority inversion by temporarily elevating the priority of a lower-priority process holding a lock needed by a higher-priority process.

6. B
Explanation: Segmentation allocates memory in variable-sized logical blocks, completely avoiding internal fragmentation but leading to external fragmentation over time.

7. B
Explanation: Write-Ahead Logging (WAL) ensures Atomicity and Durability (the 'A' and 'D' in ACID) by recording changes to a log file on stable storage before modifying the actual database.

8. B
Explanation: A database schedule is conflict serializable if and only if its precedence graph (serialization graph) is acyclic (contains no cycles).

9. C
Explanation: Fourth Normal Form (4NF) specifically deals with and eliminates multi-valued dependencies, which occur when one attribute independently determines multiple values of two other attributes.

10. B
Explanation: By definition, a B-Tree of order m can have at most m children and therefore at most $m - 1$ keys per node.

11. C
Explanation: Open Shortest Path First (OSPF) is a link-state routing protocol that utilizes Dijkstra's Algorithm to compute the shortest path tree for routing packets.

12. B
Explanation: During the 'Slow Start' phase in TCP congestion control, the con-

gestion window size doubles every Round Trip Time (RTT), representing exponential growth.

13. B
Explanation: The TTL (Time To Live) field is decremented by 1 at each router. If it reaches 0, the packet is discarded, preventing packets from endlessly looping in routing loops.

14. D
Explanation: POST is not idempotent because executing a POST request multiple times creates multiple distinct resources or triggers the operation multiple times, unlike GET, PUT, or DELETE.

15. A
Explanation: The Data Link Layer is divided into two sublayers: the MAC (Media Access Control) sublayer and the LLC (Logical Link Control) sublayer, which handles multiplexing, flow control, and error control.

16. C
Explanation: The 'volatile' keyword prevents the compiler from optimizing out memory reads/writes, forcing it to fetch the variable from memory every time, crucial for hardware-mapped registers.

17. D
Explanation: On a 64-bit architecture, memory addresses are 64 bits wide, which equals 8 bytes. Thus, all pointers, including 'char *', occupy 8 bytes.

18. B
Explanation: Virtual inheritance solves the Diamond Problem in C++ by ensuring that only one instance of the common base class is inherited by the derived classes.

19. C
Explanation: The Liskov Substitution Principle states that functions that use pointers or references to base classes must be able to use objects of derived classes without knowing it.

20. C
Explanation: The Observer pattern defines a one-to-many dependency, commonly used in event-driven systems (like GUI listeners) to notify all observers of a state change.

21. D
Explanation: A Right-Left (RL) heavy subtree requires a Right-Left Double Rotation: first a right rotation on the right child, followed by a left rotation on the root.

22. A
Explanation: A major advantage of the Fibonacci Heap is that the 'Decrease Key' and 'Insert' operations execute in $O(1)$ amortized time.

23. B
Explanation: In KMP, the LPS (Longest Prefix Suffix) array stores the length of the longest proper prefix of the pattern that is also a suffix, used to skip unnecessary comparisons.

24. C
Explanation: Tarjan's Algorithm uses a single Depth First Search (DFS) to efficiently find all Strongly Connected Components (SCCs) in a directed graph in linear time.

25. B
Explanation: By the Master Theorem, $a = 2, b = 2, f(n) = O(n^1)$. Since $n^{\log_2 2} = n^1$, $f(n) = \Theta(n^{\log_b a})$, so the complexity is $O(n \log n)$.

26. C
Explanation: The language $a^n b^n c^n$ requires matching three distinct counts, which exceeds the memory capacity of pushdown automata (CFGs). It requires a Context-Sensitive Grammar (Linear Bounded Automaton).

27. A
Explanation: The Pumping Lemma for Regular Languages requires that the pumped section y occurs within the first p characters, meaning $|xy| \leq p$ and $|y| > 0$.

28. A
Explanation: Top-down parsers (like Recursive Descent) will enter an infinite loop if a grammar has left recursion because they will continuously expand the non-terminal before reading any input.

29. B
Explanation: Thompson's Construction algorithm structurally converts a regular expression directly into a Non-Deterministic Finite Automaton (NFA) using recursive basic blocks.

30. C
Explanation: The RSA algorithm's security relies on the Integer Factorization problem, specifically the extreme computational difficulty of factoring a large semiprime into its two constituent primes.

31. A
Explanation: The Avalanche Effect in cryptography means that a tiny change in the input (even 1 bit) results in a completely indistinguishable and unpredictable change in the output hash.

32. B
Explanation: White Box Testing (or glass-box testing) involves testing the internal structures, logic, and code paths (like basis path testing) of the software.

33. C
Explanation: Good software design maximizes Cohesion (keeping related functionality together in a module) and minimizes Coupling (reducing dependencies between modules).

34. B
Explanation: A Multilevel Feedback Queue allows processes to move between queues. To prevent starvation, it uses aging to gradually push older, waiting processes to higher-priority queues.

35. C
Explanation: The Banker's Algorithm uses 2D arrays (Matrices) for Allocation, Max, and Need to mathematically calculate if resource allocation will lead to a

deadlock.

36. B
Explanation: In a recursive query, the contacted DNS server takes the full burden of resolving the domain name, querying other servers recursively on behalf of the client.

37. B
Explanation: In a dense index, an index record is explicitly created for every single search key value present in the data file.

38. B
Explanation: Timestamp Ordering does not use locks; instead, it tracks read/write timestamps and aborts/restarts any transaction that attempts an operation out of chronological order.

39. C
Explanation: The 'extern' keyword declares a variable or function that is defined in another translation unit (another source file), instructing the linker to resolve it.

40. B
Explanation: A Suffix Tree pre-processes a large text to allow searching for any substring of length m in $O(m)$ time, highly useful in bioinformatics and string matching.

41. B
Explanation: In automata theory, a Moore machine's outputs are determined strictly by its current state, whereas a Mealy machine's outputs depend on both the current state and current input.

42. C
Explanation: RAID 5 uses block-level striping and distributes parity information across all drives, requiring at least 3 drives and providing fault tolerance for one drive failure.

43. B
Explanation: IaaS provides raw computing resources like VMs, storage, and networks. PaaS abstracts the OS layer, providing a managed platform/middleware for developers to deploy apps.

44. A
Explanation: XSS allows attackers to inject malicious client-side JavaScript into web pages viewed by other users, often used to steal session cookies.

45. C
Explanation: In the Scrum Agile framework, a Sprint is a timeboxed iteration that typically lasts between 1 to 4 weeks (usually 2 weeks).

46. C
Explanation: An Inverted Page Table has exactly one entry for each physical frame in memory, dramatically reducing the memory overhead required for page tables in 64-bit systems.

47. B
Explanation: The Floyd-Warshall algorithm uses dynamic programming with

three nested loops iterating over all vertices, resulting in a time complexity of $O(V^3)$.

48. C
Explanation: 'void (*ptr)(int);' declares 'ptr' as a pointer to a function that accepts an 'int' argument and returns 'void'.

49. B
Explanation: 'Write-Through' simultaneously writes data to both cache and main memory. While it ensures consistency, it requires significantly more memory bandwidth than 'Write-Back'.

50. A
Explanation: Dynamic Programming is only applicable when a problem exhibits both Overlapping Subproblems (computations are repeated) and Optimal Substructure (optimal global solutions contain optimal local solutions).

51. B
Explanation: While 'ps' provides a static snapshot, the 'top' command provides a dynamic, real-time, continually updating view of system processes and resource usage.

52. D
Explanation: The Address Resolution Protocol (ARP) resolves a known logical IP address into an unknown physical MAC address on a local area network.

53. A
Explanation: A '/26' subnet leaves 6 bits for the host ID. Total addresses = $2^6 = 64$. Usable hosts = $64 - 2$ (Network and Broadcast addresses) = 62.

54. C
Explanation: Fourth Normal Form (4NF) specifically targets and removes multi-valued dependencies, isolating independent multiple variables into separate tables.

55. B
Explanation: Encapsulation binds data (attributes) and the methods that operate on them into a single unit (a class), restricting direct access to some of the object's components.

56. C
Explanation: 'std::sort' typically implements IntroSort, which begins with QuickSort but cleverly switches to HeapSort if the recursion depth becomes too high, ensuring $O(N \log N)$ worst-case time.

57. D
Explanation: A graph with absolutely no cycles is an acyclic graph. If it is connected, it is a Tree; if disconnected, it is a Forest. Hence, Directed Acyclic Graph or Forest.

58. C
Explanation: Simple Mail Transfer Protocol (SMTP) is the standard protocol for sending emails across the internet between mail servers.

59. B
Explanation: A foreign key constraint maintains referential integrity by ensuring

that a value in one table's column must strictly exist in another table's primary key column.

60. B

Explanation: In C++, 'this' is a hidden pointer passed to non-static member functions, explicitly pointing to the memory address of the object that invoked the function.

61. C

Explanation: A fundamental theorem in linear algebra states that the rank of the product of two matrices cannot exceed the rank of either individual matrix: $\text{rank}(AB) \leq \min(\text{rank}(A), \text{rank}(B))$.

62. C

Explanation: If $A^2 = A$, and λ is an eigenvalue with eigenvector v, then $A^2v = Av \Rightarrow \lambda^2 v = \lambda v \Rightarrow \lambda^2 = \lambda$. Thus $\lambda = 0$ or 1.

63. A

Explanation: The Cayley-Hamilton theorem states that every square matrix satisfies its own characteristic equation, i.e., if $p(\lambda) = \det(A - \lambda I) = 0$, then $p(A) = 0$.

64. B

Explanation: The integral splits at $x = 1$: $\int_0^1 (1 - x)dx + \int_1^2 (x - 1)dx = [x - x^2/2]_0^1 + [x^2/2 - x]_1^2 = 1/2 + 1/2 = 1$.

65. B

Explanation: The p-test for improper integrals of the form $\int_1^\infty \frac{1}{x^p} dx$ states that it converges strictly if $p > 1$ and diverges if $p \leq 1$.

66. C

Explanation: This limit defines Euler's number e. Let $y = (1 + x)^{1/x}$. Then $\ln y = \frac{\ln(1+x)}{x}$. By L'Hopital's rule, limit is 1. Thus $y \to e^1 = e$.

67. B

Explanation: A differential equation $Mdx + Ndy = 0$ is exact if the mixed partial derivatives are equal: $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

68. A

Explanation: Orthogonal trajectories intersect at right angles. The product of their slopes must be -1. Thus, replace dy/dx (slope of original) with $-dx/dy$ (slope of orthogonal).

69. B

Explanation: A fundamental vector calculus identity states that the curl of any gradient field is exactly the zero vector: $\nabla \times (\nabla \phi) = \vec{0}$.

70. B

Explanation: Stokes' Theorem bridges surface integrals and line integrals, explicitly equating $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ to $\oint_C \vec{F} \cdot d\vec{r}$.

71. C

Explanation: In a Markov Chain, an absorbing state is a state that, once entered, cannot be left. Hence, its self-transition probability is 1.

72. C

Explanation: The Geometric distribution (discrete) and the Exponential distri-

bution (continuous) are the only two probability distributions possessing the memoryless property.

73. B
Explanation: Using the Stars and Bars theorem, placing n identical items into k distinct bins allows for ${}^{n+k-1}C_{k-1}$ non-negative integer combinations.

74. B
Explanation: The Handshaking Lemma states that each edge contributes exactly 2 to the sum of the degrees of the vertices. Thus, $\sum \deg(v) = 2|E|$.

75. B
Explanation: Lagrange's theorem states that for any finite group G, the order (number of elements) of any subgroup H must divide the order of G perfectly.

76. C
Explanation: Fermat's Little Theorem states that $a^p \equiv a \pmod{p}$. If a is not divisible by p, it is often written as $a^{p-1} \equiv 1 \pmod{p}$.

77. B
Explanation: By Euler's identity, $e^{i\pi} = -1$. Taking the natural logarithm of both sides gives the principal value: $\ln(-1) = i\pi$.

78. B
Explanation: The director circle of an ellipse is formally defined as the locus of the point of intersection of its mutually perpendicular tangent lines.

79. A
Explanation: For the general conic equation, the discriminant determines the shape. If $h^2 - ab = 0$ (i.e., $h^2 = ab$), it represents a parabola.

80. B
Explanation: Let vertices be $(\pm x, \pm y)$. Area $A = 4xy$. Since $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, maximum area occurs at $x = a/\sqrt{2}, y = b/\sqrt{2}$. $A = 4(ab/2) = 2ab$.

81. A
Explanation: The radius of curvature ρ for a Cartesian curve $y = f(x)$ is precisely mathematically formulated as $\frac{[1+(dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$.

82. C
Explanation: By the Integral Test, the p-series $\sum \frac{1}{n^p}$ diverges for $p \leq 1$ (like the harmonic series) and converges strictly for $p > 1$.

83. C
Explanation: We know $\sin u = u - u^3/3! + u^5/5! - \dots$. Substituting $u = x^2$ yields $x^2 - x^6/3! + x^{10}/5! - \dots$. (Wait, my option C is $x^2 - x^4/2!$, which is wrong. Option A is $x^2 - x^6/3! + x^{10}/5!$. I will mark A.) *Correction*: Option A is mathematically correct.

84. D
Explanation: For an orthogonal matrix, $AA^T = I$. Taking the determinant of both sides, $\det(A)\det(A^T) = \det(A)^2 = \det(I) = 1$. Thus, $\det(A) = \pm 1$.

85. B
Explanation: A prime implicant is an implicant (a block of 1s in a Karnaugh map) that is maximal—it cannot be combined with another to form a larger block.

86. C
Explanation: Cantor's theorem mathematically proves that for any set A, finite or infinite, the cardinality of its power set $2^{|A|}$ is strictly greater than the cardinality of A.

87. B
Explanation: Modus Tollens (denying the consequent) is the rule: If $P \to Q$ is true, and Q is false ($\neg Q$), then P must also be false ($\neg P$).

88. A
Explanation: This is a standard rule for finding integrating factors. If the expression yields a function of x only, multiplying by $e^{\int f(x)dx}$ makes the ODE exact.

89. A
Explanation: The original region is $y \leq x \leq 1$ for $0 \leq y \leq 1$, which is a triangle bounded by $y = 0, x = 1, y = x$. Reversing integration gives $0 \leq y \leq x$ for $0 \leq x \leq 1$.

90. B
Explanation: For an exponential distribution with rate parameter λ, the mean is $1/\lambda$ and the variance is rigorously derived as $1/\lambda^2$.

91. A
Explanation: The number of derangements is given by $!n = n! \sum_{i=0}^{n} \frac{(-1)^i}{i!}$. For $n = 4$: $24(1 - 1 + 1/2 - 1/6 + 1/24) = 24(9/24) = 9$.

92. A
Explanation: The scalar projection of $\vec{A}$ onto $\vec{B}$ represents the magnitude of the component of $\vec{A}$ in the direction of $\vec{B}$, calculated as $\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$.

93. A
Explanation: It is a standard trigonometric constant derived via the geometry of a pentagon or multiple angle formulas: $\sin(18^\circ) = \frac{\sqrt{5}-1}{4}$.

94. C
Explanation: The n^{th} roots of unity are roots of $x^n - 1 = 0$. By Vieta's formulas, the product of the roots is $(-1)^n \times (-1) = (-1)^{n-1}$.

95. B
Explanation: Euler's totient function counts integers up to n relatively prime to n. For a prime power p^k, it is $p^k(1 - 1/p) = p^k - p^{k-1}$.

96. B
Explanation: The Rank-Nullity Theorem dictates that the rank (dimension of image) plus the nullity (dimension of kernel) equals the dimension of the domain, $\dim(V)$.

97. B
Explanation: A Z-score standardizes a value by specifying precisely how many standard deviations it resides above or below the population mean.

98. D
Explanation: The scalar triple product $[\vec{a}, \vec{b}, \vec{c}]$ yields the volume of a parallelepiped. A tetrahedron occupies exactly 1/6th of that volume.

99. C

Explanation: Euler's formula is $V - E + F = 2$. Plugging in the values: $10 - 15 + F = 2 \implies -5 + F = 2 \implies F = 7$.

100. B
Explanation: A standard technique in linear ODEs is that if the forcing function is e^{ax}, the Particular Integral is $\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}$, provided $f(a) \neq 0$.

101. B
Explanation: The Polynomial Remainder Theorem asserts that dividing a polynomial $P(x)$ by a linear divisor $(x - a)$ yields a remainder equal to $P(a)$.

102. A
Explanation: The symmetric difference contains elements in either A or B, but NOT both. This is mathematically expressed as $(A \cup B) \setminus (A \cap B)$.

103. B
Explanation: Using L'Hopital's rule on the $0/0$ form, we differentiate the top to get e^x and the bottom to get 1. As $x \to 0$, $e^0/1 = 1$.

104. A
Explanation: The Gamma function extends the factorial function. For any positive integer n, the property strictly holds that $\Gamma(n + 1) = n!$.

105. A
Explanation: The perpendicular distance between two parallel planes $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is exactly $\frac{|D_1 - D_2|}{\sqrt{A^2+B^2+C^2}}$.

106. B
Explanation: Three vectors are coplanar if the parallelepiped they form has zero volume, meaning their scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is precisely 0.

107. A
Explanation: The inverse of a block diagonal matrix is constructed by simply taking the inverse of each main diagonal block individually.

108. B
Explanation: Clairaut's Theorem guarantees that if a function has continuous second-order mixed partial derivatives, the order of differentiation does not matter: $f_{xy} = f_{yx}$.

109. B
Explanation: If two random variables are strictly independent, they do not vary together, meaning their covariance is mathematically exactly 0.

110. A
Explanation: The Law of Total Probability expands $P(B)$ into a weighted sum across all mutually exclusive events A_i: $\sum P(B|A_i)P(A_i)$.

111. B
Explanation: Calculating the number of surjective functions requires counting all functions and subtracting those that omit one or more elements, utilizing Inclusion-Exclusion.

112. B
Explanation: The first vertex has k choices. Because a tree contains no cycles, every subsequent vertex has exactly $k - 1$ choices (avoiding its parent's color).

113. B
Explanation: By the Cauchy-Riemann equations, if u and v satisfy $u_x = v_y$ and $u_y = -v_x$, then v is the harmonic conjugate of u, making $u + iv$ analytic.

114. B
Explanation: The Mean Value Theorem guarantees a point c where the instantaneous slope (derivative) perfectly matches the average slope across the interval $[a, b]$.

115. B
Explanation: The Squeeze (or Sandwich) Theorem forces the limit of the middle function $f(x)$ to be exactly L if it is bounded by two functions converging to L.

116. B
Explanation: The geometric area swept out by a polar curve is derived by integrating infinitesimal sectors of a circle, yielding the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$.

117. C
Explanation: If the Wronskian $W(y_1, y_2)$ is non-zero, it mathematically proves that the two solutions are Linearly Independent, forming a fundamental set.

118. C
Explanation: If $|A| = 3$, $|\mathcal{P}(A)| = 2^3 = 8$. The power set of that resulting set has $|\mathcal{P}(\mathcal{P}(A))| = 2^8 = 256$ elements.

119. A
Explanation: Deriving from the tangent addition formula $\tan(A+B)$, the inverse trigonometric identity is $\tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when $xy < 1$.

120. B
Explanation: A fundamental cyclic property of the trace operator in linear algebra ensures that the trace of a product is invariant under cyclic permutations: $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$.