GATE 2024 Mechanical Engineering Question Paper with Solution

Time Allowed :3 Hours	**Maximum Marks :**100	**Total Questions :**65

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The GATE Exam will be structured with a total of 100 marks.
2. The exam mode is Online CBT (Computer Based Test)
3. The total duration of Exam is 3 Hours.
4. It will include 65 questions , divided in 3 sections.
5. Section 1 : General Aptitude.
6. Section 2 : Engineering Mathematics.
7. Section 3 : Subject Based Questions.
8. The marking scheme is as such : 1 and 2 marks Questions. Each correct answer will carry marks as specified in the question paper. Incorrect answers may carry negative marks, as indicated in the question paper.
9. Question Types: The exam will include a mix of Multiple Choice Questions (MCQs), Multiple Select Questions (MSQs), and Numerical Answer Type (NAT). questions.

GENERAL APTITUDE

Question 1-5 carry one mark each

1. If '→' denotes increasing order of intensity, then the meaning of the words [smile → giggle → laugh] is analogous to [disapprove → ____ → chide]. Which one of the given options is appropriate to fill the blank?

(1) reprove

(2) praise

(3) reprise

(4) grieve

Correct Answer: (1) reprove

Solution: Step 1: Analyze the pattern of increasing intensity in the first set.

The words 'smile', 'giggle', and 'laugh' show increasing levels of amusement.

Step 2: Apply the same pattern to the second set.

'Disapprove', followed by a word of greater intensity, would logically be 'reprove', and the highest level would be 'chide'.

Conclusion: The correct option is **(1) reprove**.

Quick Tip

For analogy-based questions, identify the relationship in the first set and apply it consistently to the second.

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2. Find the odd one out in the set: {19, 37, 21, 17, 23, 29, 31, 11}.

(1) 21

(2) 29

(3) 37

(4) 23

Correct Answer: (1) 21

Solution: Step 1: Observe the properties of the numbers.

All numbers except 21 are prime.

Step 2: Identify the odd one out.

21 is a composite number (divisible by 3 and 7), whereas all others are prime.

Conclusion: The correct option is **(1) 21**.

> **Quick Tip**
>
> To identify the odd one out, check for distinct properties like primality, divisibility, or patterns in the given set.

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3. In the following series, identify the number that needs to be changed to form the Fibonacci series: 1, 1, 2, 3, 6, 8, 13, 21,...

(1) 8

(2) 21

(3) 6

(4) 13

Correct Answer: (3) 6

Solution: Step 1: Understand the Fibonacci sequence.

Each term is the sum of the two preceding terms.

Step 2: Compare the given series with the Fibonacci sequence.

The series should be: $1, 1, 2, 3, 5, 8, 13, 21$. The number 6 needs to be replaced by 5.

Conclusion: The correct option is **(3) 6**.

> **Quick Tip**
>
> For sequence-based questions, verify each term against the rule defining the sequence.

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4. The real variables x, y, z, and the real constants p, q, r satisfy

$$\frac{x}{pq - r^2} = \frac{y}{qr - p^2} = \frac{z}{rp - q^2}.$$

Given that the denominators are non-zero, the value of px + qy + rz is:

(1) 0

(2) 1

(3) pqr

(4) $p^2 + q^2 + r^2$

Correct Answer: (1) 0

Solution: The given equation is:

$$\frac{x}{pq - r^2} = \frac{y}{qr - p^2} = \frac{z}{rp - q^2} = k$$

Step 1: Express x, y, and z in terms of k:

$$x = k(pq - r^2), \quad y = k(qr - p^2), \quad z = k(rp - q^2)$$

Step 2: Substitute x, y, and z into the expression $px + qy + rz$:

$$px + qy + rz = k\left[p(pq - r^2) + q(qr - p^2) + r(rp - q^2)\right]$$

Step 3: Simplify the expression inside the brackets:

$$= k\left[p^2q - pr^2 + q^2r - qp^2 + r^2p - rq^2\right]$$

Step 4: Group terms and simplify further:

$$= k\left[p^2q - qp^2 + q^2r - rq^2 + r^2p - pr^2\right]$$

Step 5: Note that the terms cancel out:

$$= k(0) = 0$$

Conclusion: px + qy + rz = 0

Quick Tip

In equations with proportional relationships, assume a common constant and simplify expressions systematically.

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5. Take two long dice (rectangular parallelepiped), each having four rectangular faces labelled as 2, 3, 5, and 7. If thrown, the long dice cannot land on the square faces and has $1/4$ probability of landing on any of the four rectangular faces. The label on the top face of the dice is the score of the throw. If thrown together, what is the probability of getting the sum of the two long dice scores greater than 11?

(1) $\frac{3}{8}$

(2) $\frac{1}{8}$

(3) $\frac{1}{16}$

(4) $\frac{3}{16}$

Correct Answer: (4) $\frac{3}{16}$

Solution: Step 1: Identify the possible outcomes of rolling two dice. Each die has four possible outcomes: $2, 3, 5, 7$. The total number of outcomes is $4 \times 4 = 16$.

Step 2: Find combinations where the sum exceeds 11. The valid pairs are:

$$(5, 7), (7, 5), (7, 7).$$

This gives 3 favorable outcomes.

Step 3: Calculate the probability.

$$\text{So, the probability} = \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4}$$

Conclusion: The correct option is **(4)** $\frac{3}{16}$.

Quick Tip

For probability questions, ensure the total and favorable outcomes are clearly identified before calculating.

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6. In the given text, the blanks are numbered (i)–(iv). Select the best match for all the blanks. Prof. P ___ (i) merely a man who narrated funny stories. ___ (ii) in his blackest moments he was capable of self-deprecating humor. Prof. Q ___ (iii) a man who hardly narrated funny stories. ___ (iv) in his blackest moments was he able to find humor.

(1) (i) was (ii) Only (iii) wasn't (iv) Even

(2) (i) wasn't (ii) Even (iii) was (iv) Only

(3) (i) was (ii) Even (iii) wasn't (iv) Only

(4) (i) wasn't (ii) Only (iii) was (iv) Even

Correct Answer: (2) (i) wasn't (ii) Even (iii) was (iv) Only

Solution: Step 1: Analyze the context of each sentence.

Prof. P was not merely a storyteller, hence the correct verb for (i) is **"wasn't"**.

"Even" fits logically in (ii) to emphasize humor in dark moments.

Prof. Q was a man with rare humor, so (iii) uses **"was"**.

"Only" appropriately concludes (iv).

Conclusion: The correct option is **(2)**.

Quick Tip

For fill-in-the-blanks questions, match each option logically with the sentence context.

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7. How many combinations of non-null sets A, B, C are possible from the subsets of $\{2, 3, 5\}$, satisfying the conditions: (i) A is a subset of B, and (ii) B is a subset of C?

(1) 28

(2) 27

(3) 18

(4) 19

Correct Answer: (2) 27

Solution: Step 1: Understand the conditions.

Each element of $\{2, 3, 5\}$ must satisfy:

$$A \subseteq B \subseteq C.$$

Step 2: Use set theory rules.

For each element, there are 3 possibilities:

- The element belongs to A, B, C,

- The element belongs to B, C (but not A),

- The element belongs to C only.

Thus, the total combinations are:

$$3^3 = 27.$$

Conclusion: The correct option is **(2) 27**.

Quick Tip

Use the subset relationship and count possibilities systematically for each element.

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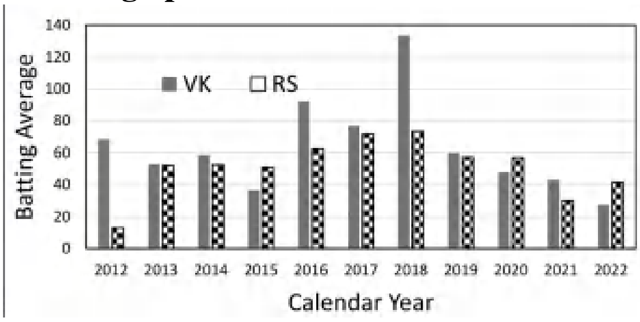
8. The bar chart gives the batting averages of VK and RS for 11 calendar years from 2012 to 2022. Considering that 2015 and 2019 are world cup years, which one of the following options is true?



(1) RS has a higher yearly batting average than that of VK in every world cup year.

(2) VK has a higher yearly batting average than that of RS in every world cup year.

(3) VK's yearly batting average is consistently higher than that of RS between the two world cup years.

(4) RS's yearly batting average is consistently higher than that of VK in the last three years.

Correct Answer: (3) VK's yearly batting average is consistently higher than that of RS between the two world cup years.

Solution: Step 1: Analyze the batting averages from 2015 to 2019.

VK's averages are higher than RS in all the years between the two world cups.

Step 2: Validate other options.

- Option (1) is false as RS does not have a higher average in every world cup year.
- Option (2) is false as VK does not have a higher average in every world cup year.
- Option (4) is false because VK's average is higher in the last three years.

Conclusion: The correct option is **(3)**.

Quick Tip

For data interpretation questions, focus on key years and compare specific metrics systematically.

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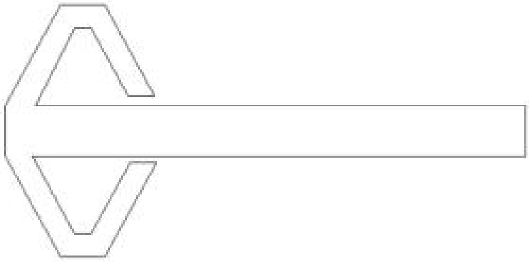
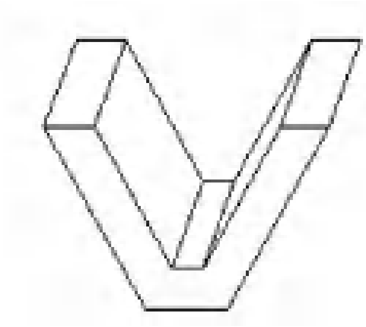
9. A planar rectangular paper has two V-shaped pieces attached as shown below. This piece of paper is folded to make the following closed three-dimensional object. The number of folds required to form the above object is:





(1) 9

(2) 7

(3) 11

(4) 8

Correct Answer: (1) 9

Solution: Step 1: Analyze the geometry of the object.

The paper includes a main rectangular strip and two V-shaped sections. Each section requires folds along its edges to form the closed 3D structure.

Step 2: Breakdown of folds required:

- The long rectangular strip requires two folds, one for each end.

- Each V-shaped section requires 3 folds (two for the arms and one at the vertex).

Step 3: Total folds calculation: The plane rectangular paper when folded to form the 3-D shave the number of folding required is = 2 + 1 + 3 + 3 = 9

Conclusion: The correct number of folds is **(1) 9**.

Quick Tip

For folding questions, count folds at each joint and corner of the structure systematically.

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10. Four equilateral triangles are used to form a regular closed three-dimensional

object by joining along the edges. The angle between any two faces is:

(1) $30°$

(2) $60°$

(3) $45°$

(4) $90°$

Correct Answer: (2) $60°$

Solution: Step 1: Analyze the geometry.

The four equilateral triangles form a regular tetrahedron. A tetrahedron has 4 triangular faces and 6 edges.

Step 2: Dihedral angle formula.

The dihedral angle between two triangular faces of a tetrahedron is given by:

$$\text{Dihedral Angle} = \cos^{-1}\left(-\frac{1}{3}\right).$$

Step 3: Simplify the calculation:

$$\cos^{-1}\left(-\frac{1}{3}\right) \approx 60°.$$

Conclusion: The angle between any two faces is **(2)** $60°$.

Quick Tip

For polyhedral shapes, apply geometric properties and symmetry to determine angles.

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11. In order to numerically solve the ordinary differential equation $\frac{dy}{dt} = -y$ for $t > 0$, with an initial condition $y(0) = 1$, the following scheme is employed:

$$\frac{y_{n+1} - y_n}{\Delta t} = -\frac{1}{2}(y_{n+1} + y_n).$$

Here, Δt is the time step and $y_n = y(n\Delta t)$ for $n = 0, 1, 2, ...$ This numerical scheme will yield a solution with non-physical oscillations for $\Delta t > h$. The value of h is:

(1) $\frac{1}{2}$

(2) 1

(3) $\frac{3}{2}$

(4) 2

Correct Answer: (4) 2

Solution: Step 1: Stability condition for the numerical scheme.

The stability of the scheme is governed by:

$$\left|1 - \frac{\Delta t}{2}\right| < 1.$$

Step 2: Solve the inequality.

Expand the inequality:

$$\frac{dy}{dt} = -y \quad \text{and} \quad y(0) = 1$$

Given scheme is $\quad \dfrac{y_{n+1} - y_n}{\Delta t} = -\dfrac{1}{2}(y_{n+1} + y_n)$

$$\Rightarrow 2y_{n+1} - 2y_n = -\Delta t y_{n+1} - \Delta t y_n$$

$$\Rightarrow (2 + \Delta t)y_{n+1} = (2 - \Delta t)y_n$$

$$\Rightarrow y_{n+1} = \left(\frac{2 - \Delta t}{2 + \Delta t}\right) y_n$$

For the scheme to yield physical oscillations $\quad \left|\dfrac{2 - \Delta t}{2 + \Delta t}\right| < 1$

$$\Rightarrow |2 - \Delta t| < |2 + \Delta t|$$

Simplify each part:

$$\text{On solving} \quad |\Delta t| < 2$$

scheme yield no physical solution for —$\Delta t| > 2$

Conclusion: The value of h is **(4) 2**.

Quick Tip

For stability conditions, derive bounds on the time step Δt using inequalities.

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12. The value of the surface integral $\iint_S z\,dx\,dy$, where S is the external surface of the sphere $x^2 + y^2 + z^2 = R^2$, is:

(1) 0

(2) $4\pi R^3$

(3) $\frac{4\pi}{3} R^3$

(4) πR^3

Correct Answer: (3) $\frac{4\pi}{3}R^3$

Solution: The equation of a sphere is given by:

$$S : x^2 + y^2 + z^2 = R^2$$

Step 1: Volume of the Sphere Using Triple Integration:

To find the volume of the sphere, we use triple integration:

$$\iint_S z\,dx\,dy = \iiint_V dz\,dx\,dy$$

Here: - The left-hand side represents the surface integral over the sphere S. - The right-hand side represents the volume integral over the volume V enclosed by the sphere.

Step 2: Formula for the Volume of the Sphere:

Using the standard volume formula for a sphere, we get:

$$\text{Volume of sphere} = \frac{4}{3}\pi R^3$$

Explanation:

1. The equation $x^2 + y^2 + z^2 = R^2$ represents a sphere centered at the origin with radius R.

2. The volume is computed using triple integration, converting the surface integral to a volume integral.

3. The volume of the sphere is calculated using the well-known formula $\frac{4}{3}\pi R^3$, which is derived through this method.

Quick Tip

For symmetric surfaces, integrals involving odd functions often evaluate to zero.

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13. Let $f(z)$ be an analytic function, where $z = x + iy$. If the real part of $f(z)$ is $\cosh x \cos y$, and the imaginary part of $f(z)$ is zero for $y = 0$, then $f(z)$ is:

(1) $\cosh x \exp(-iy)$

(2) $\cosh z \exp z$

(3) $\cosh z \cos y$

(4) $\cosh z$

Correct Answer: (4) $\cosh z$

Solution: Step 1: Understand the given function properties.

The function $f(z)$ is analytic, so it satisfies the Cauchy-Riemann equations. The real part of $f(z)$ is $\text{Re}(f(z)) = \cosh x \cos y$.

Step 2: Find the imaginary part of $f(z)$. From analytic function theory:

$$\text{Im}(f(z)) = \sin y \sinh x.$$

Step 3: Derive the complete function. Combine the real and imaginary parts:

$$f(z) = \cosh x \cos y + i \sinh x \sin y.$$

Step 4: Simplify using exponential form. Expressing in terms of $\cosh z$:

$$f(z) = \cosh(x + iy) = \cosh z.$$

Conclusion: The correct option is **(4)** $\cosh z$.

Quick Tip

For analytic functions, use the Cauchy-Riemann equations and exponential forms of trigonometric functions to simplify the expressions.

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14. Consider the system of linear equations:

$$x + 2y + z = 5, \quad 2x + ay + 4z = 12, \quad 2x + 4y + 6z = b.$$

The values of a and b such that there exists a non-trivial null space and the system admits infinite solutions are:

(1) $a = 8, b = 14$

(2) $a = 4, b = 12$

(3) $a = 8, b = 12$

(4) $a = 4, b = 14$

Correct Answer: (4) $a = 4, b = 14$

Solution: Step 1: Represent the system as a matrix equation. The coefficient matrix is:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 4 \\ 2 & 4 & 6 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 12 \\ b \end{bmatrix}.$$

Step 2: Use the condition for infinite solutions. For the system to admit infinite solutions, the rank of the augmented matrix $[A|b]$ must equal the rank of A, which should be less than 3.

Step 3: Apply row operations to simplify A. To determine the conditions under which the system has infinite solutions, we simplify the coefficient matrix A using elementary row operations. Starting with:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & a & 4 \\ 2 & 4 & 6 \end{bmatrix},$$

we aim to reduce it to row echelon form.

1. First, eliminate the first element of R_2 and R_3 below A_{11}:

$$R_2 \to R_2 - 2R_1 \quad \text{and} \quad R_3 \to R_3 - 2R_1.$$

2. Applying these operations, we get:

$$A' = \begin{bmatrix} 1 & 2 & 1 \\ 0 & a-4 & 2 \\ 0 & 0 & b-14 \end{bmatrix}.$$

This matrix represents the simplified form of A. Next, we analyze the conditions for the rank of A and the augmented matrix $[A|b]$.

Step 4: Solve for a and b. For the system to have infinite solutions, the rank of the coefficient matrix A must equal the rank of the augmented matrix $[A|b]$, and this rank must be less than the number of variables (3).

1. The rank of A will drop below 3 if the determinant of A becomes zero, which happens when the last pivot element $b - 14 = 0$. Solving this gives:

$$b = 14.$$

2. Furthermore, for the second row to not introduce an additional pivot, the second pivot

must also be zero. This occurs when:

$$a - 4 = 0.$$

Solving this gives:

$$a = 4.$$

Thus, the conditions for a and b to ensure infinite solutions are $a = 4$ and $b = 14$.

Conclusion: The correct option is **(4)** $a = 4, b = 14$.

Quick Tip

For infinite solutions in a linear system, ensure the determinant of the coefficient matrix is zero and analyze the augmented matrix's rank.

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15. Let $f(\cdot)$ be a twice-differentiable function from $\mathbb{R}^2 \to \mathbb{R}$. If $p, x_0 \in \mathbb{R}^2$, where $\|p\|$ is sufficiently small (here $\|\cdot\|$ is the Euclidean norm or distance function), then:

$$f(x_0 + p) = f(x_0) + \nabla f(x_0)^T p + \frac{1}{2} p^T \nabla^2 f(\psi) p,$$

where $\psi \in \mathbb{R}^2$ is a point on the line segment joining x_0 and $x_0 + p$. If x_0 is a strict local minimum of $f(x)$, which one of the following statements is true?

(A) $\nabla f(x_0)^T p > 0$ and $p^T \nabla^2 f(\psi) p = 0$

(B) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p > 0$

(C) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p = 0$

(D) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p < 0$

Correct Answer: (B) $\nabla f(x_0)^T p = 0$ and $p^T \nabla^2 f(\psi) p > 0$

Solution: Step 1: First-order necessary condition for a minimum. At a strict local minimum, the gradient $\nabla f(x_0)$ must satisfy:

$$\nabla f(x_0) = 0.$$

Thus, the term $\nabla f(x_0)^T p = 0$.

Step 2: Second-order condition for a strict local minimum. For x_0 to be a strict local minimum:

$$p^T \nabla^2 f(\psi) p > 0 \quad \forall p \neq 0.$$

This indicates that the Hessian matrix $\nabla^2 f(\psi)$ is positive definite.

Conclusion: The correct option is **(B)**.

Quick Tip

For optimization problems, always check both first-order and second-order conditions to classify critical points.

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16. The velocity field of a two-dimensional, incompressible flow is given by:

$$\vec{V} = 2\sinh x\,\hat{i} + v(x,y)\,\hat{j},$$

where $\hat{i}$ and $\hat{j}$ denote the unit vectors in the x- and y-directions respectively. If $v(x,0) = \cosh x$, find $v(0,-1)$.

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution: Step 1: Use the incompressibility condition. For incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

Substituting $u = 2\sinh x$:

$$\frac{\partial}{\partial x}(2\sinh x) + \frac{\partial v}{\partial y} = 0 \quad \Rightarrow \quad 2\cosh x + \frac{\partial v}{\partial y} = 0.$$

Step 2: Solve for $v(x,y)$. Integrate $\frac{\partial v}{\partial y} = -2\cosh x$ with respect to y:

$$v(x,y) = -2y\cosh x + C(x).$$

Step 3: Apply the boundary condition. From $v(x,0) = \cosh x$:

$$v(x,0) = C(x) = \cosh x.$$

Thus:

$$v(x,y) = \cosh x - 2y\cosh x = \cosh x(1-2y).$$

Step 4: Evaluate $v(0,-1)$. Substitute $x=0$ and $y=-1$:

$$v(0,-1)=\cosh(0)(1-2(-1))=1(1+2)=3.$$

Conclusion: The correct option is **(C)**.

Quick Tip

For incompressible flow, apply the continuity equation to relate velocity components.

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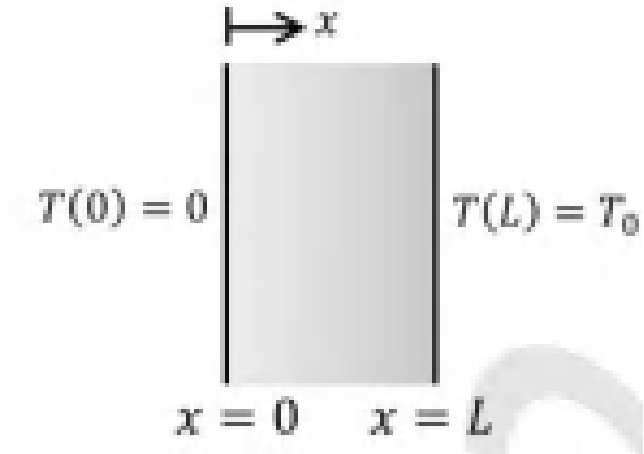
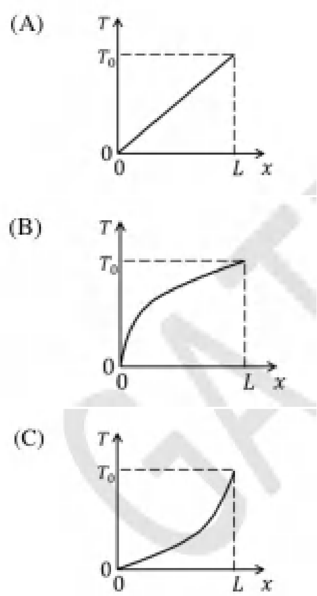
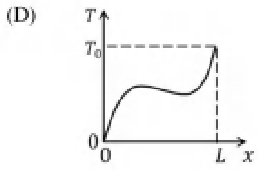
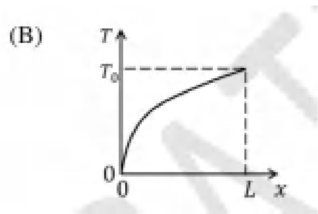
17. A plane, solid slab of thickness L has thermal conductivity $k=A+Bx$, where $A>0$ and $B>0$. The slab walls are maintained at $T(0)=0$ and $T(L)=T_0$. Which plot qualitatively depicts the steady-state temperature distribution?









Correct Answer:

Solution: Step 1: Use Fourier's law of heat conduction.

The heat flux is:

$$q = -k\frac{dT}{dx} = -(A + Bx)\frac{dT}{dx}.$$

In steady state, $\frac{dq}{dx} = 0$, so:

$$(A + Bx)\frac{dT}{dx} = C, \quad \text{where } C \text{ is constant.}$$

Step 2: Integrate to find $\frac{dT}{dx}$.

$$\frac{dT}{dx} = \frac{C}{A + Bx}.$$

Step 3: Integrate again to find $T(x)$.

$$T(x) = \int \frac{C}{A + Bx}dx = \frac{C}{B}\ln(A + Bx) + C_1.$$

Step 4: Determine the qualitative shape. Since $\ln(A + Bx)$ grows slower than x, the temperature profile $T(x)$ is concave up.

Conclusion: The correct option is **(B)**.

Quick Tip

For varying thermal conductivity, solve Fourier's law step-by-step to determine the temperature distribution.

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18. Consider incompressible laminar flow over a flat plate with freestream velocity of u_∞. The Nusselt number corresponding to this flow velocity is Nu_1. If the freestream velocity is doubled, the Nusselt number changes to Nu_2. Choose the correct option for Nu_2/Nu_1.

(1) $\sqrt{2}$

(2) 2

(3) 1.26

(4) 1

Correct Answer: (1) $\sqrt{2}$

Solution: Step 1: Nusselt number relation for laminar flow.

The Nusselt number for a flat plate in laminar flow is proportional to the Reynolds number raised to a power:

$$Nu \propto Re^{1/2}.$$

Step 2: Reynolds number dependency on freestream velocity.

The Reynolds number Re is given by:

$$Re \propto u_\infty.$$

If u_∞ is doubled, then:

$$Re_2 = 2Re_1.$$

Step 3: Calculate the change in Nu. Using the proportional relationship:

$$Nu_2/Nu_1 = \left(\frac{Re_2}{Re_1}\right)^{1/2} = \left(\frac{2Re_1}{Re_1}\right)^{1/2} = \sqrt{2}.$$

Conclusion: The correct option is **(1)** $\sqrt{2}$.

Quick Tip

For convective heat transfer problems, relate the Nusselt number to Reynolds and Prandtl numbers to analyze flow changes.

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19. Consider a hydrodynamically fully developed laminar flow through a circular pipe with the flow along the axis (i.e., z direction). In the following statements, T is the temperature of the fluid, T_w is the wall temperature, and T_m is the bulk mean temperature of the fluid. Which one of the following statements is TRUE?

(1) For a thermally fully developed flow, $\frac{\partial T}{\partial z} = 0$, always.

(2) For constant wall temperature of the duct, $\frac{dT_m}{dz} =$ constant.

(3) Nusselt number varies linearly along the z-direction for a thermally fully developed flow.

(4) For constant wall temperature ($T_w > T_m$) of the duct, $\frac{dT_m}{dz}$ increases exponentially with distance along the z-direction.

Correct Answer: (4) For constant wall temperature ($T_w > T_m$) of the duct, $\frac{dT_m}{dz}$ increases exponentially with distance along the z-direction.

Solution: Step 1: Understand the thermally fully developed flow.

For such a flow, the wall temperature remains constant, and the temperature gradient depends on heat transfer through the fluid.

Step 2: Analyze the bulk temperature gradient.

For constant wall temperature ($T_w > T_m$), the temperature gradient $\frac{dT_m}{dz}$ is proportional to the heat flux at the wall. In laminar flow, this flux increases exponentially with z as the fluid absorbs more heat.

Step 3: Validate other options.

- Option (1) is false because $\frac{\partial T}{\partial z} \neq 0$ in fully developed thermal flows.
- Option (2) is false as $\frac{dT_m}{dz}$ is not constant but changes along z.
- Option (3) is false because the Nusselt number is constant for thermally fully developed flow.

Conclusion: The correct option is **(4)**.

Quick Tip

For thermally fully developed flows, remember that the Nusselt number remains constant, but temperature gradients vary depending on flow conditions.

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20. A furnace can supply heat steadily at $1200\,K$ at a rate of 24000 kJ/min. The maximum amount of power (in kW) that can be produced by using the heat supplied by this furnace in an environment at $300\,K$ is:

(1) 300

(2) 150

(3) 18000

(4) 0

Correct Answer: (1) 300

Solution: Step 1: Determine the Carnot efficiency. The Carnot efficiency η for a heat engine operating between temperatures T_H and T_C is:

$$\eta = 1 - \frac{T_C}{T_H}.$$

Substitute $T_H = 1200\,K$ and $T_C = 300\,K$:

$$\eta = 1 - \frac{300}{1200} = 1 - 0.25 = 0.75.$$

Step 2: Calculate the maximum power output. The heat supplied per second is:

$$\dot{Q}_{\text{in}} = \frac{24000}{60} = 400\,\text{kJ/s}.$$

The maximum power output is:

$$W_{\text{max}} = \eta \dot{Q}_{\text{in}} = 0.75 \times 400 = 300\,\text{kW}.$$

Conclusion: The correct option is **(1)** 300.

Quick Tip

For Carnot engines, always calculate efficiency using the absolute temperatures in Kelvin.

---

21. Which one of the following statements regarding a Rankine cycle is FALSE?

(1) Superheating the steam in the boiler increases the cycle efficiency.

(2) The pressure at the turbine outlet depends on the condenser temperature.

(3) Cycle efficiency increases as condenser pressure decreases.

(4) Cycle efficiency increases as boiler pressure decreases.

Correct Answer: (4) Cycle efficiency increases as boiler pressure decreases.

Solution: Step 1: Analyze the effect of boiler pressure on efficiency.

In a Rankine cycle, increasing the boiler pressure increases the average temperature of heat addition, which improves efficiency.

Step 2: Evaluate other statements.

Superheating increases efficiency by reducing moisture content at the turbine outlet.

The turbine outlet pressure depends on the condenser temperature due to the saturation curve.

Lower condenser pressure increases efficiency by reducing the heat rejection.

Conclusion: The correct option is **(4)**, as decreasing boiler pressure reduces efficiency.

Quick Tip

For Rankine cycles, higher boiler pressure and lower condenser pressure lead to increased efficiency.

---

22. For a ball bearing, the fatigue life in millions of revolutions is given by $L = \left(\frac{C}{P}\right)^n$, where P is the constant applied load and C is the basic dynamic load rating. Which one of the following statements is TRUE?

(1) $n = 3$, assuming that the inner racing is fixed and outer racing is revolving.

(2) $n = \frac{1}{3}$, assuming that the inner racing is fixed and outer racing is revolving.

(3) $n = 3$, assuming that the outer racing is fixed and inner racing is revolving.

(4) $n = \frac{1}{3}$, assuming that the outer racing is fixed and inner racing is revolving.

Correct Answer: (3) $n = 3$, assuming that the outer racing is fixed and inner racing is revolving.

Solution: Step 1: Understand the relationship.

The exponent n depends on the type of rolling contact:

- For ball bearings, $n = 3$.

- For roller bearings, $n = 10/3$.

Step 2: Analyze the assumptions.

When the outer race is fixed and the inner race revolves, the fatigue life equation holds true for $n = 3$.

Conclusion: The correct option is **(3)**.

Quick Tip

For ball bearings, the exponent $n = 3$, and for roller bearings, $n = 10/3$.

---

23. The change in kinetic energy ΔE of an engine is $300\ J$, and minimum and maximum shaft speeds are $\omega_{\text{min}} = 220$ rad/s and $\omega_{\text{max}} = 280$ rad/s, respectively.Assume that the

torque-time function is purely harmonic. To achieve a coefficient of fluctuation of speed 0.05, the moment of inertia (in kg $\cdot$ m^2) of the flywheel to be mounted on the engine shaft is

(1) 0.113

(2) 0.096

(3) 0.071

(4) 0.053

Correct Answer: (2) 0.096

Solution: Step 1: The given energy fluctuation is:

$$\Delta E = 300\,\text{J}$$

The angular velocities are:

$$\omega_{\text{max}} = 280\,\text{rad/s}, \quad \omega_{\text{min}} = 220\,\text{rad/s} \quad \text{(Without mounting flywheel)}$$

The coefficient of fluctuation of speed is:

$$K_s = 0.05$$

Let:

$$I_s = \text{Inertia of shaft}, \quad I_f = \text{Inertia of flywheel to be mounted to ensure } K_s = 0.05$$

The total inertia after mounting the flywheel becomes:

$$I = I_s + I_f$$

Before Mounting the Flywheel:

$$\Delta E = \frac{1}{2} I_s \left(\omega_{\text{max}}^2 - \omega_{\text{min}}^2\right)$$

Substituting the values:

$$300 = \frac{1}{2} I_s \left(280^2 - 220^2\right)$$

Simplify:

$$I_s = 0.02\,\text{kgm}^2$$

After Mounting the Flywheel:

The energy fluctuation is expressed as:

$$\Delta E = I\omega^2 K_s \tag{i}$$

The mean angular velocity is:

$$\omega = \frac{\omega_{\text{max}} + \omega_{\text{min}}}{2} = \frac{280 + 220}{2} = 250\,\text{rad/s}$$

Substitute the values in equation (i):

$$300 = I \times 250^2 \times 0.05$$

Solve for I:

$$I = 0.096\,\text{kgm}^2$$

From the total inertia:

$$I = I_s + I_f$$

Substitute the values:

$$0.096 = 0.02 + I_f$$

Solve for I_f:

$$I = 0.096\,\text{kgm}^2$$

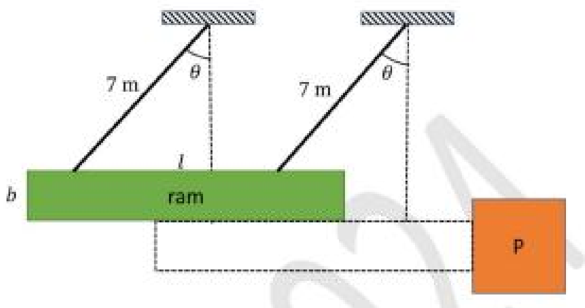
Conclusion: The correct option is **(2)** 0.096.

Quick Tip

For flywheel problems, always use the fluctuation of speed to calculate kinetic energy.

---

24. A ram in the form of a rectangular body of size $l = 9\,$m and $b = 2\,$m is suspended by two parallel ropes of lengths $7\,$m. Assume the center of mass is at the geometric center and $g = 9.81\,$m/s^2. For striking the object P with a horizontal velocity of $5\,$m/s, what is the angle θ with the vertical from which the ram should be released from rest?



(1) 67.1°

(2) 40.2°

(3) 35.1°

(4) 79.5°

Correct Answer: (3) 35.1°

Solution: Step 1: Use conservation of energy. At the release point:

$$mgh = \frac{1}{2}mv^2.$$

Substitute $v = 5\,\text{m/s}$ and solve for h:

$$gh = \frac{v^2}{2} \quad \Rightarrow \quad h = \frac{5^2}{2 \times 9.81} = 1.275\,\text{m}.$$

Step 2: Relate h to θ. From geometry, $h = L(1 - \cos\theta)$, where $L = 7\,\text{m}$. Solve for θ:

$$1.275 = 7(1 - \cos\theta) \quad \Rightarrow \quad \cos\theta = 1 - \frac{1.275}{7} = 0.8179.$$

$$\theta = \cos^{-1}(0.8179) \approx 35.1^\circ.$$

Conclusion: The correct option is **(3)** 35.1°.

Quick Tip

For pendulum-based problems, use energy conservation and geometric relations to find the release angle.

---

25. A linear spring-mass-dashpot system with a mass of 2 kg is set in motion with viscous damping. If the natural frequency is 15 Hz, and the amplitudes of two successive cycles measured are 7.75 mm and 7.20 mm, the coefficient of viscous damping (in N.s/m) is:

(1) 4.41

(2) 7.51

(3) 2.52

(4) 6.11

Correct Answer: (1) 4.41

Solution: Step 1: Calculate the logarithmic decrement δ. The logarithmic decrement is defined as:

$$\delta = \ln\left(\frac{x_1}{x_2}\right),$$

where $x_1 = 7.75\,\text{mm}$ and $x_2 = 7.20\,\text{mm}$. Substitute the values:

$$\delta = \ln\left(\frac{7.75}{7.20}\right) = \ln(1.076) \approx 0.0734.$$

Step 2: Determine the damping ratio ζ. The damping ratio is related to δ by:

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}.$$

Assuming ζ is small, $(1-\zeta^2) \approx 1$:

$$\zeta = \frac{\delta}{2\pi} = \frac{0.0734}{2\pi} \approx 0.0117.$$

Step 3: Calculate the coefficient of viscous damping c. The natural frequency ω_n is:

$$\omega_n = 2\pi f = 2\pi \times 15 = 94.25\,\text{rad/s}.$$

The damping coefficient is:

$$c = 2m\omega_n\zeta = 2 \times 2 \times 94.25 \times 0.0117 \approx 4.41\,\text{N.s/m}.$$

Conclusion: The correct option is **(1)** 4.41.

Quick Tip

For damped systems, use the logarithmic decrement to estimate the damping ratio and damping coefficient.

---

26. Which one of the following failure theories is the most conservative design approach against fatigue failure?

(1) Soderberg line

(2) Modified Goodman line

(3) Gerber line

(4) Yield line

Correct Answer: (1) Soderberg line

Solution: Step 1: Understand the Soderberg criterion.

The Soderberg criterion is based on the most conservative design principle, considering both yield strength and fatigue strength.

Step 2: Compare with other criteria.

- The Modified Goodman line is less conservative than the Soderberg line as it does not consider the yield strength directly.
- The Gerber line is the least conservative and considers parabolic relations.
- The Yield line is unrelated to fatigue and applies only to static loads.

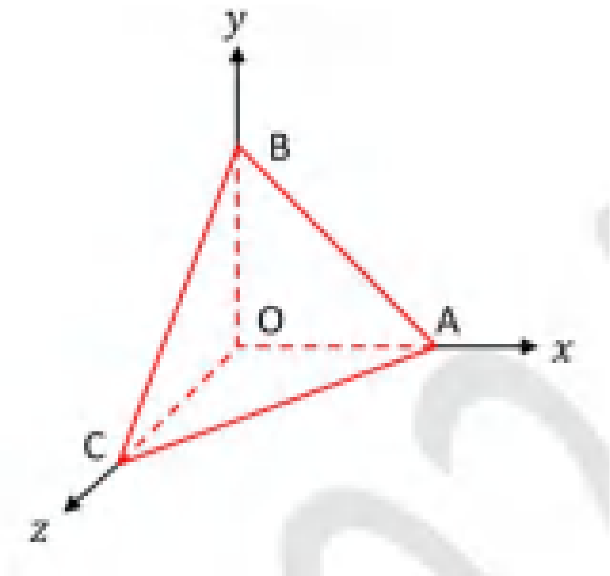
Conclusion: The correct option is **(1) Soderberg line**.

Quick Tip

For fatigue failure, the Soderberg line provides the most conservative design limits by considering yield strength.

---

27. A rigid massless tetrahedron is placed such that vertex O is at the origin and the other three vertices A, B and C lie on the co-ordinate axes as shown in the figure. The body is acted on by three point loads, of which one is acting at A along x-axis and other at point B along y-axis. for the body to be in equilibrium, the third point load acting at point O must be



(1) along z-axis

(2) in $x - y$ plane but not along x or y-axis

(3) in $y - z$ plane but not along y or z-axis

(4) in $z - x$ plane but not along z or x-axis

Correct Answer: (2) in $x - y$ plane but not along x or y-axis

Solution: Step 1: Apply equilibrium conditions.

For equilibrium, the sum of forces and moments about any axis must be zero.

Step 2: Analyze the given configuration.

The loads at A (along x-axis) and B (along y-axis) require a balancing component in the $x - y$ plane.

Step 3: Determine the orientation.

The load at O must lie in the $x - y$ plane but cannot align with either the x- or y-axis to ensure proper moment balancing.

Conclusion: The correct option is **(2)**.

Quick Tip

For equilibrium of rigid bodies, ensure both force and moment equilibrium conditions are satisfied.

---

28. The phases present in pearlite are:

(1) austenite and ferrite

(2) cementite and austenite

(3) ferrite and cementite

(4) martensite and ferrite

Correct Answer: (3) ferrite and cementite

Solution: Step 1: Understand the composition of pearlite.

Pearlite is a two-phase microstructure consisting of alternating layers of ferrite (α-iron) and cementite (Fe_3C).

Step 2: Analyze the phases.

Pearlite forms during the slow cooling of eutectoid steel and represents a balanced composition of ferrite and cementite at the eutectoid temperature.

Conclusion: The correct option is **(3) ferrite and cementite**.

Quick Tip

Pearlite is a common microstructure in steels and provides a balance between strength and ductility.

---

29. The ”Earing” phenomenon in metal forming is associated with:

(1) deep drawing

(2) rolling

(3) extrusion

(4) forging

Correct Answer: (1) deep drawing

Solution: Step 1: Understand the Earing phenomenon. Earing occurs during the deep drawing process due to anisotropy in the material. It results in the formation of wavy edges (ears) on the drawn part.

Step 2: Determine the cause.

The anisotropy arises due to non-uniform mechanical properties along different directions in the rolled sheet, causing variations in flow during drawing.

Conclusion: The correct option is **(1) deep drawing**.

Quick Tip

To minimize earing, ensure uniform grain structure and material properties during sheet metal processing.

---

30. The grinding wheel used to provide the best surface finish is:

(1) A36L5V

(2) A54L5V

(3) A60L5V

(4) A80L5V

Correct Answer: (4) A80L5V

Solution: Step 1: Understand grinding wheel specifications.

The number in the grinding wheel designation (e.g., 36, 54, 60, 80) indicates the grit size. Higher numbers correspond to finer grit sizes, which provide smoother surface finishes.

Step 2: Choose the finest grit size.

Among the given options, $A80L5V$ has the finest grit, making it most suitable for achieving the best surface finish.

Conclusion: The correct option is **(4) A80L5V**.

Quick Tip

For high-quality surface finishes, use grinding wheels with finer grit sizes and ensure proper dressing of the wheel.

---

31. The allowance provided to a pattern for easy withdrawal from a sand mold is:

(1) finishing allowance

(2) shrinkage allowance

(3) distortion allowance

(4) shake allowance

Correct Answer: (4) shake allowance

Solution: Step 1: Understand shake allowance.

Shake allowance is added to a pattern to account for the ease of withdrawal from the mold. It ensures that the pattern can be easily removed without damaging the mold cavity.

Step 2: Evaluate other allowances.

Finishing allowance is added for machining operations.

Shrinkage allowance compensates for metal contraction during cooling.

Distortion allowance accounts for changes in shape due to uneven cooling.

Conclusion: The correct option is **(4) shake allowance**.

Quick Tip

In casting, different allowances are provided to patterns to address specific issues like shrinkage, machining, and distortion.

---

32. The most suitable electrode material used for joining low alloy steels using Gas Metal Arc Welding (GMAW) process is:

(1) copper

(2) cadmium

(3) low alloy steel

(4) tungsten

Correct Answer: (3) low alloy steel

Solution: Step 1: Understand the requirement.

In GMAW, the electrode material should be compatible with the base material to ensure good weld quality and prevent cracking or other defects.

Step 2: Evaluate electrode materials.

For low alloy steels, matching electrodes (e.g., low alloy steel electrodes) are preferred to maintain mechanical properties and avoid metallurgical incompatibility.

Conclusion: The correct option is **(3) low alloy steel**.

Quick Tip

Always use electrodes compatible with the base material to achieve optimal weld strength and quality.

---

33. The preparatory functions in Computer Numerical Controlled (CNC) machine programming are denoted by the alphabet:

(1) G

(2) M

(3) P

(4) O

Correct Answer: (1) G

Solution: Step 1: Understand preparatory functions.

In CNC programming, preparatory functions are used to specify machining operations like linear interpolation, circular interpolation, and tool positioning. These are denoted by G-codes.

Step 2: Evaluate other codes.

- M-codes are used for miscellaneous functions like spindle on/off.

- P-codes and O-codes are used for subroutines and program numbering.

Conclusion: The correct option is **(1) G**.

Quick Tip

Learn common G-codes for CNC operations, such as $G01$ for linear interpolation and $G02/G03$ for circular interpolation.

---

34. A set of jobs U, V, W, X, Y, Z arrive at time t = 0 to a production line consisting of two workstations in series. Each job must be processed by both workstations in sequence (i.e., the first followed by the second). The process times (in minutes) for each job on each workstation in the production line are given below.

Job	U	V	W	X	Y	Z
Workstation 1	5	7	3	4	6	8
Workstation 2	4	6	6	8	5	7

(1) $W - X - Z - V - Y - U$

(2) $W - X - V - Z - Y - U$

(3) $W - U - Z - V - Y - X$

(4) $U - Y - V - Z - X - W$

Correct Answer: (1) $W - X - Z - V - Y - U$

Solution: Step 1: Apply Johnson's rule.

To minimize makespan, schedule the jobs based on the shortest processing times in ascending order, alternating between Workstation 1 and Workstation 2.

Step 2: Evaluate the given sequence.

The sequence $W - X - Z - V - Y - U$ minimizes idle time and ensures optimal job scheduling.

Conclusion: The correct option is **(1)** $W - X - Z - V - Y - U$.

Quick Tip

Use Johnson's rule for two-machine flow shop scheduling to minimize the total makespan.

---

35. A queueing system has one single server workstation that admits an infinitely long queue. The rate of arrival of jobs to the queueing system follows the Poisson distribution with a mean of 5 jobs/hour. The service time of the server is exponentially distributed with a mean of 6 minutes. The probability that the server is not busy at any point in time is:

(1) 0.20

(2) 0.17

(3) 0.50

(4) 0.83

Correct Answer: (3) 0.50

Solution: Step 1: Compute the service rate μ. The service rate is the reciprocal of the mean service time:

$$\mu = \frac{1}{6/60} = 10\,\text{jobs/hour}.$$

Step 2: Compute the server utilization ρ. Utilization is the ratio of arrival rate to service rate:

$$\rho = \frac{\lambda}{\mu} = \frac{5}{10} = 0.50.$$

Step 3: Compute the probability of the server being idle. The probability that the server is not busy is:

$$P_0 = 1 - \rho = 1 - 0.50 = 0.50.$$

Conclusion: The correct option is **(3)** 0.50.

Quick Tip

For queueing systems, use the utilization factor to determine server idle probabilities.

---

36. The matrix $\begin{bmatrix} 1 & a \\ 8 & 3 \end{bmatrix}$ (where $a > 0$) has a negative eigenvalue if a is greater than:

(1) $\frac{3}{8}$

(2) $\frac{1}{8}$

(3) $\frac{1}{4}$

(4) $\frac{1}{5}$

Correct Answer: (1) $\frac{3}{8}$

Solution: Step 1: To find the condition for a negative eigenvalue, we use the property of eigenvalues and determinants. Condition for a negative eigenvalue

$$\text{For a negative eigenvalue, the product of eigenvalues must be less than 0.}$$

This means:

$$\text{Determinant of the matrix} < 0$$

Step 2: Determinant condition

$$\text{Given determinant: } 3 - 8a$$

The condition becomes:

$$3 - 8a < 0$$

Step 3: Solve for a Rearrange the inequality:

$$8a > 3$$

Divide both sides by 8:

$$a > \frac{3}{8}$$

Conclusion: The parameter a must satisfy $a > \frac{3}{8}$ for the determinant to be negative, ensuring the presence of a negative eigenvalue.

Quick Tip

For eigenvalues, analyze the determinant and trace of the matrix.

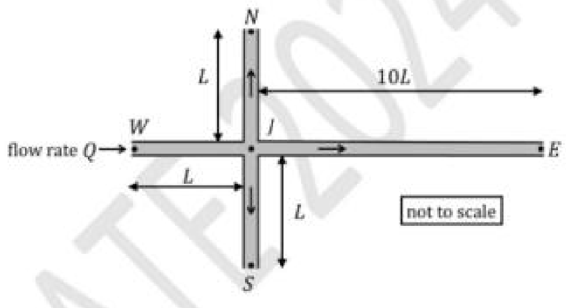
---

37. In the pipe network as shown in figure, all pipes have the same cross section areas and can be assumed to have the same friction factor. The pipes connecting points W, N

and S with the joint J have an equal length L. The pipe connecting points J and E has a length 10L. The pressures at the ends N, E and S are equal. The flow rate in the pipe connecting W and J is Q. Assume that the fluid flow is steady, incompressible, and the pressure losses at the pipe entrance and the junction are negligible, Consider the following statements.

I. The flow rate in pipe connecting J & E is Q/21

II. The pressure difference between J & N is equal to the pressure difference between J & E. Which one of the following options is CORRECT?



(1) I is True and II is False

(2) I is False and II is True

(3) Both I and II are True

(4) Both I and II are False

Correct Answer: (3) Both I and II are True

Solution: Step 1: The given pressure difference is expressed as:

$$P_N - P_J = P_E - P_J$$

Dividing throughout by ρg:

$$\frac{P_N - P_J}{\rho g} = \frac{P_E - P_J}{\rho g}$$

This implies:

$$(h_f)_{NJ} = (h_f)_{EJ}$$

Using the Hagen-Poiseuille equation for head loss:

$$\frac{32\mu v_1 L}{\rho g d^2} = \frac{32\mu v_2 (10L)}{\rho g d^2}$$

From the above:

$$v_1 = 10v_2$$

Using the continuity equation:

$$Q = 2Q_1 + Q_2$$

Substitute discharge equations:

$$\frac{\pi}{4}d^2 v = \frac{\pi}{4}d^2[2v_1 + v_2]$$

From the above:

$$v = 2v_1 + v_2 \tag{ii}$$

Using equations (i) and (ii):

$$v_2 = \frac{1}{21}v \quad \text{and} \quad v_1 = \frac{10}{21}v$$

For discharge Q_2:

$$Q_2 = \frac{\pi}{4}d^2 v_2 = \frac{\pi}{4}d^2\left(\frac{1}{21}v\right) = \frac{\pi}{4}d^2 v \cdot \frac{1}{21} = \frac{Q}{21}$$

Similarly, for Q_1:

$$Q_1 = \frac{\pi}{4}d^2 v_1 = \frac{\pi}{4}d^2\left(\frac{10}{21}v\right) = \frac{\pi}{4}d^2 v \cdot \frac{10}{21} = \frac{10Q}{21}$$

Clearly:

$$Q_2 = \frac{Q}{21}$$

Hence, Statement I is correct.

For the pressure difference:

$$(\text{Pressure})_N = (\text{Pressure})_E$$

This implies:

$$P_N = P_E$$

From the equation:

$$P_J - P_N = P_J - P_E$$

Hence, the pressure difference between J and N is equal to the pressure difference between J and E.

Thus, Statement II is true.

Quick Tip

For pipe flow problems, apply the principles of flow distribution and pressure drop.

38. A company orders gears in conditions identical to those considered in the Economic Order Quantity (EOQ) model in inventory control. The annual demand is 8000 gears, the cost per order is 300 rupees, and the holding cost is 12 rupees per month per gear.The company uses an order size that is 25% more than the optimal order quantity determined by the EOQ model. The percentage change in the total cost of ordering and holding inventory from that associated with the optimal order quantity is:

(1) 2.5

(2) 5

(3) 0

(4) 12.5

Correct Answer: (1) 2.5

Solution: Step 1: Given:

$$D = 8000\,\text{units/year}, \quad C_0 = \text{Rs. } 300/\text{order}, \quad C_h = \text{Rs. } 12 \times 12 = \text{Rs. } 144/\text{year}$$

Now, the EOQ is calculated as:

$$\text{EOQ} = \sqrt{\frac{2C_0D}{C_h}} = \sqrt{\frac{2 \times 300 \times 8000}{144}}$$

$$\therefore Q^* = 182.5\,\text{units}$$

Now, the actual quantity:

$$q = 1.25Q^* = 1.25 \times 182.5 = 228.125\,\text{units}$$

The total cost at EOQ, TIC_1, is:

$$TIC_1 = \sqrt{2C_0C_hD} = \sqrt{2 \times 300 \times 144 \times 8000}$$

$$TIC_1 = \text{Rs. } 26290.68$$

Now, the total cost at $q = 228.125$ units:

$$TIC_2 = \frac{q}{2} \times C_h + \frac{D}{q} \times C_0$$

Substituting the values:

$$TIC_2 = \frac{228.125}{2} \times 144 + \frac{8000}{228.125} \times 300$$

Simplify:

$$TIC_2 = \text{Rs. } 26945.54$$

The percentage increase is:

$$\% \text{ increase} = \frac{TIC_2 - TIC_1}{TIC_1} \times 100$$

$$\% \text{ increase} = \frac{26945.54 - 26290.68}{26290.68} \times 100 = 2.5\%$$

Conclusion: The percentage increase in the total cost of inventory is 2.5%.

Quick Tip

For inventory models, cost changes scale with square deviations in order size.

---

39. At the current basic feasible solution (bfs) v_0 ($v_0 \in \mathbb{R}^5$), the simplex method yields the following form of a linear programming problem in standard form:

Minimize $z = -x_1 - 2x_2$

Subject to:

$$x_3 = 2 + x_1 - x_2$$

$$x_4 = 7 + x_1 - 2x_2$$

$$x_5 = 3 - x_1$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Here the objective function is written as a function of the non-basic variables. If the simplex method moves to the adjacent bfs v_1 ($v_1 \in \mathbb{R}^5$) that best improves the objective function, which of the following represents the objective function at v_1, assuming that the objective function is written in the same manner as above?

(1) $z = -4 - 5x_1 + 2x_3$

(2) $z = -3 + x_5 - 2x_2$

(3) $z = -4 - 5x_1 + 2x_4$

(4) $z = -6 - 5x_1 + 2x_3$

Correct Answer: (1) $z = -4 - 5x_1 + 2x_3$

Solution: Step 1: The given objective function and constraints are:

$$z = -x_1 - 2x_2$$

$$x_3 = 2 + 2x_1 - x_2 \tag{i}$$

$$x_4 = 7 + x_1 - 2x_2 \tag{ii}$$

$$x_5 = 3 - x_1 \tag{iii}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0 \tag{iv}$$

Solution:

From equation (i):

$$x_2 = 2 + 2x_1 - x_3$$

Substituting this value in the objective function:

$$z = -x_1 - 2(2 + 2x_1 - x_3)$$

Simplify:

$$z = -4 - 5x_1 + 2x_3$$

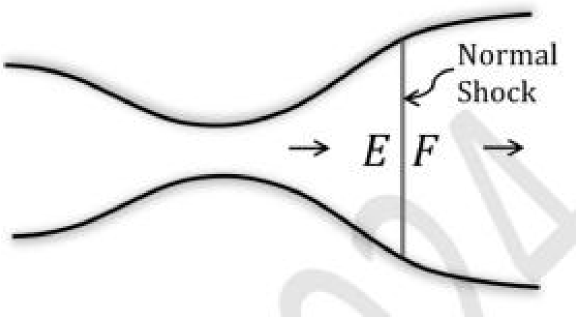
Hence, option (a) is correct.

Quick Tip

For linear programming, use pivot operations to track objective improvements.

---

40. Steady, compressible flow of air takes place through an adiabatic converging-diverging nozzle, as shown in the figure. For a particular value of pressure difference across the nozzle, a stationary normal shock wave forms in the diverging section of the nozzle. If E and F denote the flow conditions just upstream and downstream of the normal shock, respectively, which of the following statement(s) is/are TRUE?



(1) Static pressure at E is lower than at F

(2) Density at E is lower than at F

(3) Mach number at E is lower than at F

(4) Specific entropy at E is lower than at F

Correct Answer: 1 Static pressure at E is lower than at F

, 2 Density at E is lower than at F

, 4. Specific entropy at E is lower than at F

Solution: Step 1: Explanation of Options:

(a) $\rho_E < \rho_F$

This is a **correct statement** because the density of the fluid increases across a normal shock wave. This is a fundamental property of normal shock waves, where the upstream density (ρ_E) is always less than the downstream density (ρ_F).

(b) $P_E \leq P_F$

This is a **correct statement** because the pressure increases across a normal shock wave. The downstream pressure (P_F) is always greater than or equal to the upstream pressure (P_E) as per the shock wave relations.

(c) $M_E \leq M_F$

This is a **wrong statement**. The Mach number (M) decreases across a shock wave, as the velocity of the flow reduces. Thus, the upstream Mach number (M_E) is greater than the downstream Mach number (M_F), which invalidates the statement.

(d) Specific entropy increases across the shock wave

This is a **correct statement** because a normal shock wave is an irreversible process. As a result, the entropy of the system increases across the shock wave. The increase in entropy is a hallmark of the irreversibility of the phenomenon.

Conclusion: Statements (a), (b), and (d) are correct, while statement (c) is incorrect.

> **Quick Tip**
>
> For shock waves, analyze changes in pressure, density, and entropy for upstream and downstream conditions.

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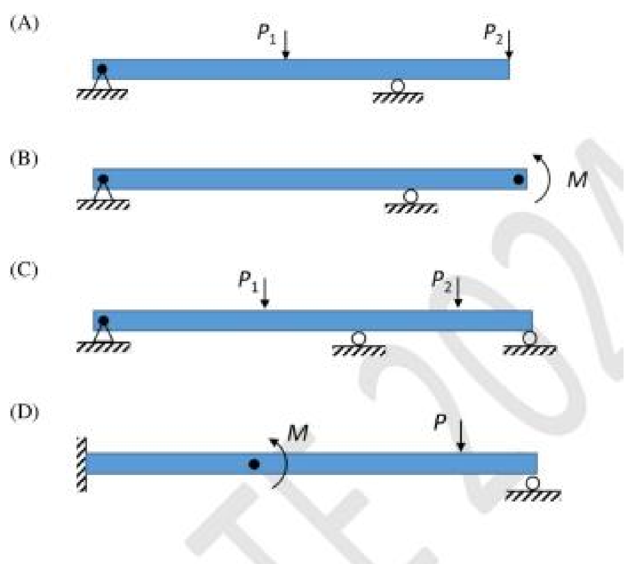
41. Which of the following beam(s) is/are statically determinate?



Correct Answer: (3, 4)

Solution: Step 1: For a beam to be statically determinate or indeterminate, the number of reactions (R) is compared to the number of independent static equilibrium equations available (E).

$$\text{Condition:} \quad R > E \quad \text{(Statically Indeterminate Beam)}$$

$$\text{Condition:} \quad R = E \quad \text{(Statically Determinate Beam)}$$

Below is the analysis of each case:

(A) **Three reactions Two static equilibrium equations**

The beam in this case has three reactions ($R = 3$) and only two useful static equilibrium equations ($E = 2$). Therefore:

$$R > E \implies \text{The beam is statically indeterminate.}$$

(B) **Three reactions   Two static equilibrium equations**

In this case, the beam again has three reactions ($R = 3$) and two useful static equilibrium equations ($E = 2$). Thus:

$$R > E \implies \text{The beam is statically indeterminate.}$$

(C) **Two reactions = Two static equilibrium equations**

The beam in this case has two reactions ($R = 2$) and two useful static equilibrium equations ($E = 2$). Therefore:

$$R = E \implies \text{The beam is statically determinate.}$$

(D) **Two reactions = Two static equilibrium equations**

In this case as well, the beam has two reactions ($R = 2$) and two useful static equilibrium equations ($E = 2$). Thus:

$$R = E \implies \text{The beam is statically determinate.}$$

Conclusion:

- Options (A) and (B) represent **statically indeterminate beams** because the number of reactions exceeds the number of useful static equilibrium equations.
- Options (C) and (D) represent **statically determinate beams** because the number of reactions is equal to the number of useful static equilibrium equations.

Conclusion: The correct options are **(3, 4)**.

Quick Tip

For statically determinate structures, check the balance of equations of equilibrium and reaction forces.

---

42. If the value of double integral

$$x = \int_{x=3}^{4} \int_{y=1}^{2} \frac{dy}{dx} \left((x+y)^2\right)$$

is $\log_e\left(\frac{a}{24}\right)$**, then** a **is** ________ **(Answer in integer).**

Correct Answer: 25

Solution: Step 1:

Solving the inner integral with respect to x

$$= \int_{x=3}^{4} \left[-\frac{1}{x+y} \right]_{y=1}^{2} dx$$

$$= \int_{x=3}^{4} \left[-\frac{1}{x+2} + \frac{1}{x+1} \right] dx$$

$$= \{ -\ln(x+2) + \ln(x+1) \}_{x=3}^{4}$$

$$= -\ln(6) + \ln(5) - (-\ln(5) + \ln(4))$$

$$= \ln(25) - \ln(24)$$

$$= \ln\left(\frac{25}{24}\right)$$

Now comparing with $\ln\left(\frac{a}{24}\right)$, we get $a = 25$.

Quick Tip

For approximate solutions to integrals, analyze the range of the function within the bounds.

---

43. If $x(t)$ satisfies the differential equation

$$t\frac{dx}{dt} + (t - x) = 0$$

subject to the condition $x(1) = 0$, then the value of $x(2)$ is _______ (rounded off to 2 decimal places).

Correct Answer: 1.39

Solution: Step 1:Given:

$$t\frac{dx}{dt} + (t - x) = 0, \quad x(1) = 0, \quad x(2) = ?$$

We can write the equation as:

$$t\frac{dx}{dt} = (x - t) \quad \Rightarrow \quad t\,dx = (x - t)\,dt \quad \Rightarrow \quad t\,dx - x\,dt = -t\,dt$$

Now, divide both sides by t^2:

$$\frac{t\,dx - x\,dt}{t^2} = \frac{-t\,dt}{t^2} \quad \Rightarrow \quad \frac{d(x/t)}{dt} = -\frac{1}{t}$$

Next, integrate both sides:

$$\int \frac{d(x/t)}{dt} = \int -\frac{1}{t}\,dt \quad \Rightarrow \quad \frac{x}{t} = -\ln t + C$$

Now, apply the boundary condition $x(1) = 0$:

$$\frac{x}{t} = -\ln t + C \quad \Rightarrow \quad \frac{0}{1} = -\ln(1) + C \quad \Rightarrow \quad C = 0$$

Thus, the equation becomes:

$$\frac{x}{t} = -\ln t$$

Now, at $t = 2$:

$$x = -t\ln t = -2\ln 2$$

Finally, we get:

$$x = -2\ln 2 \approx -1.386$$

Thus, the value of $x(2)$ is approximately $\boxed{-1.386}$. .

Quick Tip

For second-order linear differential equations, use the characteristic equation to find solutions.

---

44. Let X be a continuous random variable defined on $[0, 1]$ such that its probability density function $f(x) = 1$ for $0 \leq x \leq 1$ and 0 otherwise. Let $Y = \ln_e(X + 1)$. Then the expected value of Y is: *(rounded off to 2 decimal places)*.

Correct Answer: 0.39

Solution: Step 1: The expected value $E[Y]$ is given by:

$$E[Y] = \int_0^1 \ln_e(x + 1) \cdot f(x)\, dx.$$

Since $f(x) = 1$, the expression simplifies to:

$$E[Y] = \int_0^1 \ln_e(x + 1)\, dx.$$

Step 2: Solve the integral: Using integration by parts:

$$\int \ln_e(x + 1)\, dx = (x + 1)\ln_e(x + 1) - x + C.$$

Evaluate from 0 to 1:

$$E[Y] = \left[(x + 1)\ln_e(x + 1) - x\right]_0^1.$$

At $x = 1$:

$$(2)\ln_e(2) - 1 \approx 1.386 - 1 = 0.386.$$

At $x = 0$:

$$(1)\ln_e(1) - 0 = 0.$$

Hence:

$$E[Y] \approx 0.39.$$

Conclusion: The expected value is 0.39.

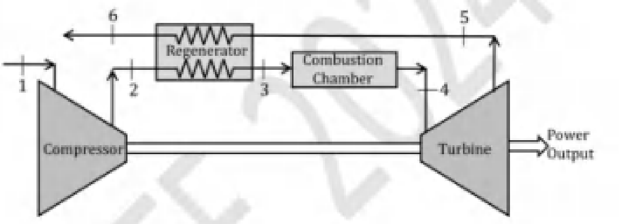
Quick Tip

Use properties of logarithms and integration by parts for solving such problems.

---

45. Consider an air-standard Brayton cycle with adiabatic compressor and turbine, and a regenerator, as shown in the figure. Air enters the compressor at 100 kPa and 300 K and exits the compressor at 600 kPa and 550 K. The air exits the combustion chamber at 1250 K and exits the adiabatic turbine at 100 kPa and 800 K. The exhaust

air from the turbine is used to preheat the air in the regenerator. The exhaust air exits the regenerator (state 6) at 600 K. There is no pressure drop across the regenerator and the combustion chamber. Also, there is no heat loss from the regenerator to the surroundings. The ratio of specific heats at constant pressure and volume is $c_p/c_v = 1.4$. The thermal efficiency of the cycle is ______ % (answer in integer). *(answer in integer)*.



Correct Answer: 40%

Solution: Step 1: The thermal efficiency η of a Brayton cycle is given by:

$$\eta = 1 - \frac{1}{r_c^{\gamma-1}},$$

where $r_c = \frac{P_2}{P_1}$ and $\gamma = \frac{C_p}{C_v}$.

Step 2: Compute η: Given $\gamma = 1.4$, and the pressure ratio $r_c = \frac{600}{100} = 6$:

$$\eta = 1 - \frac{1}{6^{1.4-1}} = 1 - \frac{1}{6^{0.4}} \approx 1 - 0.6 = 0.4.$$

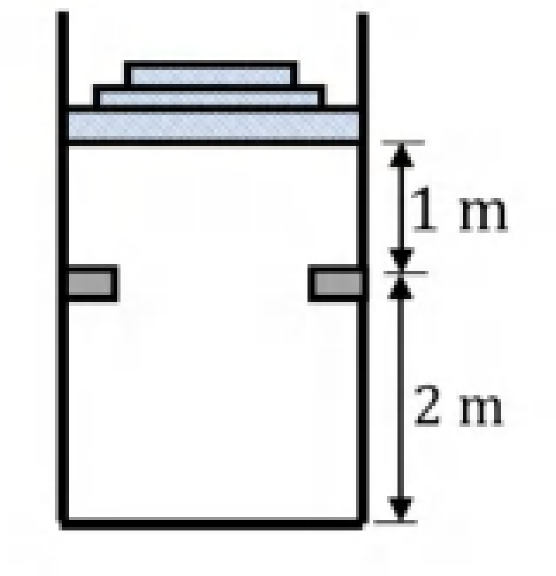
Conclusion: The thermal efficiency is 40%.

Quick Tip

For Brayton cycles, efficiency improves with higher pressure ratios.

---

46. A piston-cylinder arrangement shown in the figure has a stop located 2 m above the base. The cylinder initially contains air at 140 kPa and 350°C, and the piston is resting in equilibrium at a position that is 1 m above the stops. The system is now cooled to the ambient temperature of 25°C. Consider air to be an ideal gas with a value of gas constant $R = 0.287$ kJ/(kg·K). The absolute value of specific work done during the process is ____ kJ/kg *(rounded off to 1 decimal place)*.



Correct Answer: $59.60\,\text{kJ/kg}$

Solution: For a constant pressure process, the work done is given by:

$$W_{1\to a} = P_a V_a - P_1 V_1$$

Since $P_1 = P_a$ for a constant pressure process:

$$W_{1\to a} = mR(T_1 - T_a)$$

Step 1: Relating Temperatures and Volumes

From the ideal gas relation, for a constant pressure process:

$$\frac{T_a}{T_1} = \frac{V_a}{V_1}$$

It is given that:

$$\frac{V_a}{V_1} = \frac{2}{3}$$

Therefore:

$$\frac{T_a}{T_1} = \frac{2}{3}$$

Substituting $T_1 = 623\,\text{K}$:

$$T_a = \frac{2}{3} \cdot 623 = 415.33\,\text{K}$$

Step 2: Calculate Work Done for $W_{1\to a}$

Substituting into the work formula:

$$W_{1\to a} = mR(T_1 - T_a)$$

Given $m = 1\,\text{kg}$, $R = 0.287\,\text{kJ/(kg.K)}$, $T_1 = 623\,\text{K}$, and $T_a = 415.33\,\text{K}$:

$$W_{1\to a} = 1 \cdot 0.287 \cdot (623 - 415.33)$$

Simplifying:

$$W_{1\to a} = 0.287 \cdot 207.67 \approx 59.60\,\text{kJ/kg}$$

Step 3: Work Done for $I_{a\to 2}$

For the process $I_{a\to 2}$, the work done:

$$W \to 0$$

Total Work Done:

Therefore, the total work done is:

$$\text{Total Work Done} = 59.60\,\text{kJ/kg}$$

Final Answer:

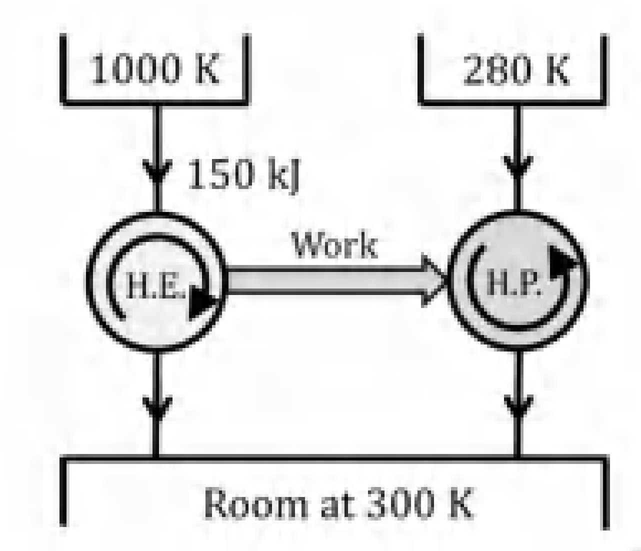
$$\text{Total Work Done} = 59.60\,\text{kJ/kg}$$

Quick Tip

For piston-cylinder systems, determine the polytropic index and use the ideal gas law for simplifications.

---

47. A heat pump (H.P.) is driven by the work output of a heat engine (H.E.) as shown in the figure. The heat engine extracts 150 kJ of heat from the source at 1000 K. The heat pump absorbs heat from the ambient at 280 K and delivers heat to the room maintained at 300 K. Considering the combined system to be ideal, the total amount of heat delivered to the room together by the heat engine and heat pump is _____ kJ (answer in integer).



Correct Answer: $1620\,\text{kJ}$

Solution: Step 1: Given:

$$Q_1 = 150\,\text{kJ}, \quad Q_2 + Q_4 = ?$$

For the Heat Engine:

$$\eta = 1 - \frac{T_L}{T_H}$$

Substitute the values:

$$\eta = 1 - \frac{300}{1000} = 1 - \frac{Q_2}{Q_1}$$

Rearranging:

$$Q_2 = 0.3 Q_1$$

Substitute $Q_1 = 150\,\text{kJ}$:

$$Q_2 = 0.3 \times 150 = 45\,\text{kJ}$$

The net work output is given by:

$$W_{\text{net}} = Q_1 - Q_2$$

Substitute the values:

$$W_{\text{net}} = 150 - 45 = 105\,\text{kJ}$$

For the Heat Pump:

The coefficient of performance (COP) of the heat pump is given by:

$$(COP)_{\text{HP}} = \frac{Q_4}{W_{\text{net}}} = \frac{T_H}{T_H - T_L}$$

Substitute the values:

$$(COP)_{\text{HP}} = \frac{300}{300-280} = \frac{300}{20} = 15$$

From the COP equation:

$$Q_4 = (COP)_{\text{HP}} \times W_{\text{net}}$$

Substitute the values:

$$Q_4 = 15 \times 105 = 1575\,\text{kJ}$$

Total Heat Provided:

$$Q_2 + Q_4 = 45 + 1575 = 1620\,\text{kJ}$$

Conclusion: The total heat provided to the room is:

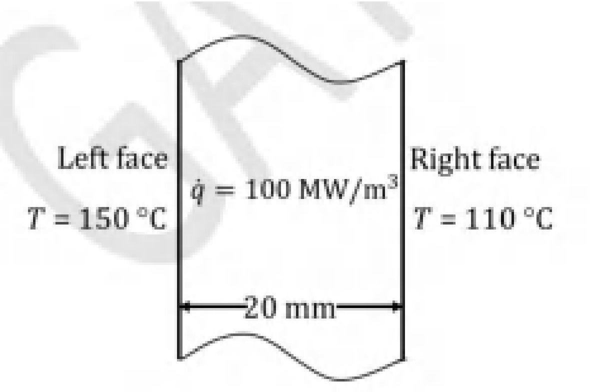
$$Q_2 + Q_4 = 1620\,\text{kJ}$$

Quick Tip

Use energy balance for heat engine and heat pump combined cycles.

---

48. Consider a slab of $20\,$mm thickness. There is a uniform heat generation of $q = 100\,$MW/m^3 inside the slab. The left and right faces of the slab are maintained at $150°$C and $110°$C, respectively. The plate has a constant thermal conductivity of $200\,$W/(m·K). Considering a 1-D steady-state heat conduction, the location of the maximum temperature from the left face will be at _____ mm (answer in integer).



Correct Answer: $6\,$mm

Solution: Step 1: From the General Heat Conduction Equation (GHCE):

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_g}{k} = 0$$

Solving this differential equation, the temperature distribution is:

$$T(x) = T_1 + (T_2 - T_1)\frac{x}{\delta} + \frac{\dot{q}_g}{2k}\left[\delta x - x^2\right]$$

To determine the location of the maximum temperature, we set the first derivative of $T(x)$ with respect to x to zero:

$$\frac{dT}{dx} = 0$$

The derivative of $T(x)$ is:

$$\frac{dT}{dx} = (T_2 - T_1)\frac{1}{\delta} + \frac{\dot{q}_g}{2k}\left[\delta - 2x\right] = 0$$

Simplify to solve for x:

$$(T_2 - T_1)\frac{1}{\delta} + \frac{\dot{q}_g}{2k}\left[\delta - 2x\right] = 0$$

Substitute the given values:

$$T_1 = 110°\text{C},\ T_2 = 150°\text{C},\ \delta = 0.02\,\text{m},\ \dot{q}_g = 100 \times 10^6\,\text{W/m}^3,\ k = 200\,\text{W/mK}$$

Substitute into the equation:

$$(110 - 150)\frac{1}{0.02} + \frac{100 \times 10^6}{2 \times 200}\left[0.02 - 2x\right] = 0$$

Simplify step-by-step:

$$-2000 + \frac{100 \times 10^6}{400}\left[0.02 - 2x\right] = 0$$

$$-2000 + 250000\left[0.02 - 2x\right] = 0$$

$$-2000 + 250000 \times 0.02 - 500000x = 0$$

$$-2000 + 5000 - 500000x = 0$$

$$3000 = 500000x$$

Solve for x:

$$x = \frac{3000}{500000} = 0.006\,\text{m} = 6\,\text{mm}$$

Conclusion: The location of the maximum temperature is at $x = 6\,\text{mm}$.

Quick Tip

Use steady-state heat conduction equations and differentiate for maxima.

49. A condenser is used as a heat exchanger in a large steam power plant in which steam is condensed to liquid water. The condenser is a shell and tube heat exchanger which consists of 1 shell and 20,000 tubes. Water flows through each of the tubes at a rate of 1 kg/s with an inlet temperature of $30°$C. The steam in the condenser shell condenses at the rate of 430 kg/s at a temperature of $50°$C. If the heat of vaporization is 2.326 MJ/kg and specific heat of water is 4 kJ/(kg·K), the effectiveness of the heat exchanger is _____ *(rounded off to 3 decimal places)*.

Correct Answer: 0.624

Solution: Step 1: Heat transfer in the condenser. The heat transferred by the steam is:

$$Q_{\text{steam}} = m_{\text{steam}} \cdot h_{\text{fg}},$$

where $m_{\text{steam}} = 430\,\text{kg/s}$ and $h_{\text{fg}} = 2.326\,\text{MJ/kg}$:

$$Q_{\text{steam}} = 430 \cdot 2.326 = 1000.18\,\text{MW}.$$

Step 2: Heat absorbed by water. The heat absorbed by water is:

$$Q_{\text{water}} = m_{\text{water}} \cdot c_p \cdot \Delta T,$$

where $m_{\text{water}} = 20,000\,\text{kg/s}$, $c_p = 4\,\text{kJ/(kg·K)}$, and $\Delta T = 20\,\text{K}$:

$$Q_{\text{water}} = 20,000 \cdot 4 \cdot 20 = 1600\,\text{MW}.$$

Step 3: Effectiveness of the heat exchanger. The effectiveness is:

$$\epsilon = \frac{Q_{\text{actual}}}{Q_{\text{max}}},$$

where $Q_{\text{actual}} = Q_{\text{steam}}$:

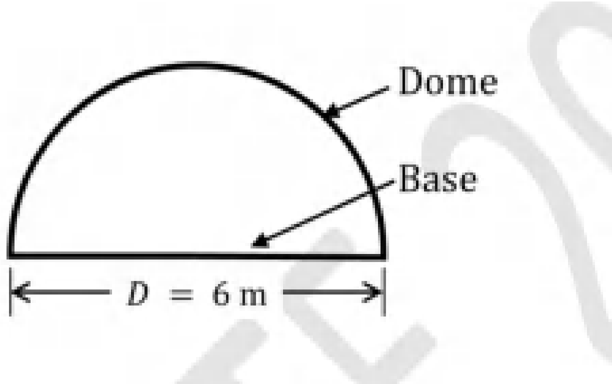
$$\epsilon = \frac{1000.18}{1600} = 0.624.$$

Conclusion: The effectiveness of the heat exchanger is 0.624.

Quick Tip

For heat exchangers, always compare actual heat transfer with the maximum possible heat transfer.

50. Consider a hemispherical furnace of diameter $D = 6\,\text{m}$ with a flat base. The dome of the furnace has an emissivity of 0.7, and the flat base is a blackbody. The base and the dome are maintained at uniform temperatures of $300\,\text{K}$ and $1200\,\text{K}$, respectively. Under steady-state conditions, the rate of radiation heat transfer from the dome to the base is ____ kW (rounded off to the nearest integer). Use Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\,\text{W/(m}^2\cdot\text{K}^4)$.



Correct Answer: $2720\,\text{kW}$

Solution: Step 1: Surface area of the dome. The surface area of a hemisphere is:

$$A = 2\pi r^2,$$

where $r = \frac{D}{2} = 3\,\text{m}$:

$$A = 2\pi(3)^2 = 56.55\,\text{m}^2.$$

Step 2: Net heat transfer. The net radiation heat transfer is given by:

$$Q = \sigma \cdot \epsilon \cdot A \cdot (T_1^4 - T_2^4),$$

where $T_1 = 1200\,\text{K}$, $T_2 = 300\,\text{K}$, and $\epsilon = 0.7$:

$$Q = 5.67 \times 10^{-8} \cdot 0.7 \cdot 56.55 \cdot [(1200)^4 - (300)^4].$$

Step 3: Simplify the calculation.

$$Q = 5.67 \times 10^{-8} \cdot 0.7 \cdot 56.55 \cdot (2.0736 \times 10^{12} - 8.1 \times 10^8),$$

$$Q = 2720\,\text{kW}.$$

Conclusion: The rate of radiation heat transfer is $2720\,\text{kW}$.

Quick Tip

For radiation problems, always check the emissivity and use the Stefan-Boltzmann law.

51. A liquid fills a horizontal capillary tube whose one end is dipped in a large pool of the liquid. Experiments show that the distance L travelled by the liquid meniscus inside the capillary in time t is given by

$$L = k\gamma^a R^b \mu^c \sqrt{t},$$

where γ is the surface tension, R is the inner radius of the capillary, and μ is the dynamic viscosity of the liquid. If k is a dimensionless constant, then the exponent a is ______ (rounded off to 1 decimal place).

Solution: Step 1: Dimensional analysis of L. From the given equation:

$$[L] = [k][\gamma]^a[R]^b[\mu]^c[t]^{0.5}.$$

The dimensions are as follows:

$$[L] = L, \quad [\gamma] = MT^{-2}, \quad [R] = L, \quad [\mu] = ML^{-1}T^{-1}, \quad [t] = T.$$

Step 2: Equating dimensions. Equating the dimensions on both sides:

$$L = (MT^{-2})^a(L)^b(ML^{-1}T^{-1})^c(T^{0.5}).$$

Simplifying:

$$L = M^{a+c}L^{b-c}T^{-2a-c+0.5}.$$

Step 3: Solving for a. To balance L, M, and T: - M: $a + c = 0 \Rightarrow c = -a$. - L: $b - c = 1 \Rightarrow b = 1 - c = 1 + a$. - T: $-2a - c + 0.5 = 0 \Rightarrow -2a - (-a) + 0.5 = 0 \Rightarrow -a + 0.5 = 0 \Rightarrow a = 0.5$.

Conclusion: The value of a is **0.5**.

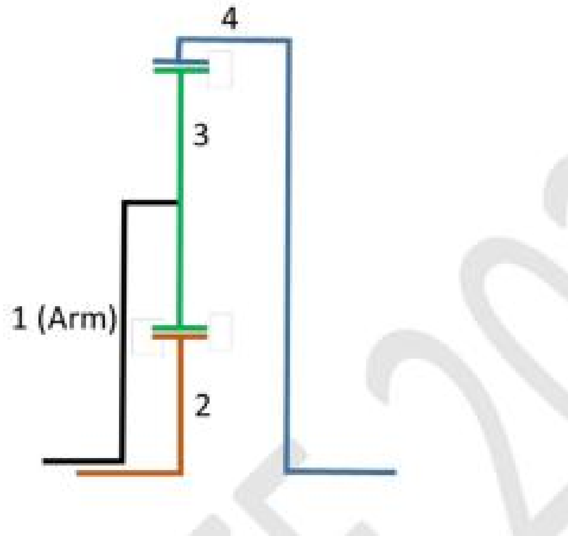
Quick Tip

For problems involving dimensional analysis, systematically balance the dimensions of mass, length, and time.

—

52. The Levi type-A train illustrated in the figure has gears with module $m = 8$ mm/tooth. Gears 2 and 3 have 19 and 24 teeth, respectively. Gear 2 is fixed and

internal gear 4 rotates at 20 rev/min counter-clockwise. The magnitude of angular velocity of the arm is ______ rev/min. (rounded off to 2 decimal places)



Correct Answer: 15.58

Solution: Given Data:

$$m = 8\,\text{mm}, \quad T_2 = 19, \quad T_3 = 24, \quad N_2 = 0, \quad N_4 = 20\,\text{rpm}$$

Step 1: Calculate Total Motion for Gear 4 (T_4)

The total motion for Gear 4 is given by:

$$mT_4 = mT_2 + 2mT_3$$

Substituting the given values $T_2 = 19$ and $T_3 = 24$:

$$T_4 = T_2 + 2T_3$$

$$T_4 = 19 + 2 \times 24 = 67$$

Step 2: Equations for Total Motion

$$x + y = 0 \quad (\text{since } N_2 = 0) \quad \ldots(1)$$

$$N_4 = 20 = y - x \cdot \frac{T_2}{T_4}$$

$$20 = y - x \cdot \frac{19}{67} \quad \ldots(2)$$

From equation (1), $y = -x$. Substituting this into equation (2):

$$20 = -x - x \cdot \frac{19}{67}$$

$$20 = -x\left(1 + \frac{19}{67}\right)$$

Simplifying:

$$20 = -x \cdot \frac{67+19}{67}$$

$$20 = -x \cdot \frac{86}{67}$$

$$x = -20 \cdot \frac{67}{86}$$

$$x \approx -15.58$$

Substituting $x = -15.58$ into equation (1):

$$y = -x = 15.58$$

Step 3: Determine Direction of Motion

- $x = -15.58\,\text{rpm}$: Clockwise
- $y = 15.58\,\text{rpm}$: Counterclockwise

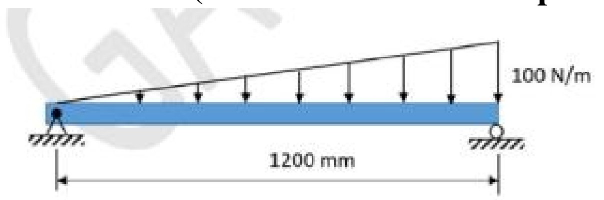
Final Answer:

$$x \approx 15.58\,\text{clockwise}, \quad y \approx 15.58\,\text{counterclockwise}.$$

Quick Tip

Use gear ratios and relations systematically to solve problems involving gear trains.

53. A horizontal beam of length $1200\,$mm is pinned at the left end and is resting on a roller at the other end as shown in the figure. A linearly varying distributed load is applied on the beam. The magnitude of maximum bending moment acting on the beam is ______ N.m. (round off to 1 decimal place)



Correct Answer: 9.23

Solution: Step 1: Calculate the Net Load, W

The net load is given as:

$$W = \frac{1}{2} \times 100 \times 1.2$$

Simplifying:

$$W = 60\,\text{kN}$$

Step 2: Moment about point A ($\sum M_A = 0$)

$$R_B \cdot 1.2 - 60 \cdot 0.8 = 0$$

Rearranging to find R_B:

$$R_B = \frac{60 \cdot 0.8}{1.2} = \frac{60 \cdot 2}{3} = 40\,\text{kN}$$

Step 3: Vertical Force Equilibrium ($\sum F_V = 0$)

$$R_A + R_B = 60$$

Substituting $R_B = 40\,\text{kN}$:

$$R_A = 60 - 40 = 20\,\text{kN}$$

Step 4: Bending Moment at a Distance x from A

The bending moment M_x at a distance x from A is given by:

$$M_x = \left[R_A \cdot x - \frac{1}{2}\left(\frac{100}{1.2} \cdot x\right) \cdot x\right] \cdot \frac{x}{3}$$

Substituting $R_A = 20$:

$$M_x = 20x - \frac{100}{7.2}x^3$$

Step 5: Maximizing the Bending Moment (M_x)

To find the maximum M_x, take the derivative of M_x with respect to x and equate it to 0:

$$\frac{dM_x}{dx} = 20 - \frac{3 \cdot 100}{7.2}x^2 = 0$$

Rearranging to find x:

$$20 = \frac{300}{7.2}x^2$$

$$x^2 = \frac{20 \cdot 7.2}{300} = \frac{144}{300} = 0.48$$

$$x = \sqrt{0.48} \approx 0.6928\,\text{m}$$

Step 6: Maximum Bending Moment ($M_{\textbf{max}}$)

Substitute $x = 0.6928$ into the equation for M_x:

$$M_{\text{max}} = 20(0.6928) - \frac{100}{7.2}(0.6928)^3$$

Simplify:

$$M_{\text{max}} = 13.856 - \frac{100}{7.2} \cdot 0.3326$$

$$M_{\text{max}} = 13.856 - 4.618 \approx 9.2376\,\text{Nm}$$

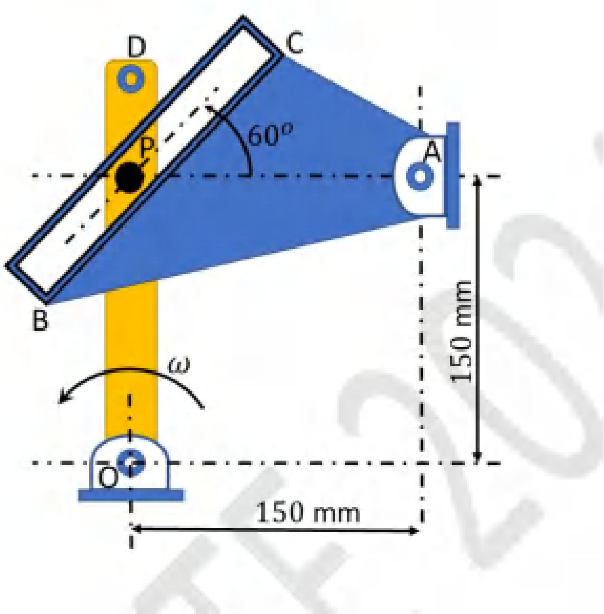
Final Answer: The maximum bending moment is approximately $9.2376\,\text{Nm}$.

Quick Tip

For linearly varying loads, always locate the position of the maximum bending moment before calculating.

—

54. At the instant when OP is vertical and AP is horizontal, the link OD is rotating counter-clockwise at a constant rate $\omega = 7$ rad/s. Pin P on link OD slides in the slot BC of link ABC which is hinged at A, and causes a clockwise rotation of the link ABC. The magnitude of angular velocity of link ABC for this instant is ______ rad/s. (rounded off to 2 decimal places)



Correct Answer: 12.12

Solution: The given relationship is:

$$V_{123} = \omega_2(I_{12}I_{23}) = \omega_3(I_{13}I_{23})$$

Step 1: Calculate I_{12P}

Using the sine rule in $\triangle I_{12P}I_{23}$:

$$\frac{I_{12P}}{\sin 60^\circ} = \frac{150}{\sin 75^\circ}$$

Rearranging to find I_{12P}:

$$I_{12P} = \frac{150 \cdot \sin 60^\circ}{\sin 75^\circ}$$

Substituting the values $\sin 60^\circ = \frac{\sqrt{3}}{2} \approx 0.866$ and $\sin 75^\circ \approx 0.965$:

$$I_{12P} = \frac{150 \cdot 0.866}{0.965} \approx 134.49\,\text{mm}$$

Step 2: Calculate I_{13P}

Using the sine rule in $\triangle I_{13P}I_{23}$:

$$\frac{I_{13P}}{\sin 30^\circ} = \frac{150}{\sin 105^\circ}$$

Rearranging to find I_{13P}:

$$I_{13P} = \frac{150 \cdot \sin 30^\circ}{\sin 105^\circ}$$

Substituting the values $\sin 30^\circ = 0.5$ and $\sin 105^\circ \approx 0.966$:

$$I_{13P} = \frac{150 \cdot 0.5}{0.966} \approx 77.65\,\text{mm}$$

Step 3: Equate Angular Velocities

From the given relationship:

$$\omega_2(I_{12} \cdot I_{23}) = \omega_3(I_{13} \cdot I_{23})$$

Substituting $I_{12P} = 134.49$ mm and $I_{13P} = 77.65$ mm:

$$7 \cdot 134.49 = \omega_3 \cdot 77.65$$

Simplifying to find ω_3:

$$\omega_3 = \frac{7 \cdot 134.49}{77.65} \approx 12.124\,\text{rad/sec}$$

Rounding to two decimal places:

$$\omega_3 \approx 12.12\,\text{rad/sec}$$

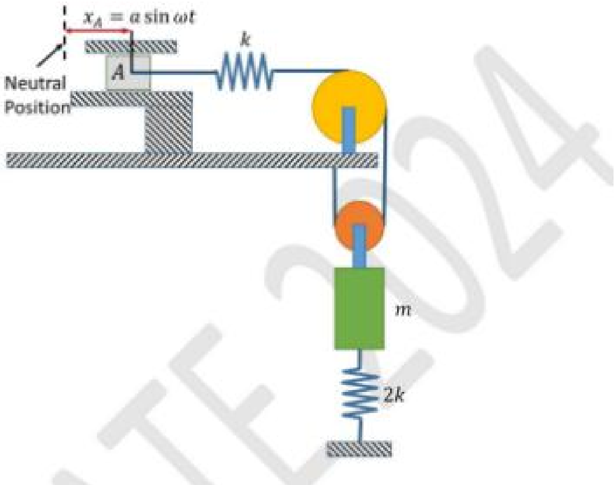
Final Answer: $\omega_3 = 12.12\,\text{rad/sec}$

Quick Tip

Use relative velocity and geometry for solving angular velocity problems in mechanisms.

—

55. A vibratory system consists of mass m, a vertical spring of stiffness $2k$, and a horizontal spring of stiffness k. The end A of the horizontal spring is given a horizontal motion $x_A = a \sin \omega t$. The other end of the spring is connected to an inextensible rope that passes over two massless pulleys as shown. Assume $m = 10$ kg, $k = 1.5$ kN/m, and neglect friction. The magnitude of critical driving frequency for which the oscillations of mass m tend to become excessively large is ______ rad/s. (answer in integer)



Solution: Step 1:

For excessively large oscillation of mass M, the driving frequency (ω) should be equal to natural undamped frequency (ω_n).

To calculate natural undamped frequency, we are assuming block A is fixed.

Total kinetic energy $(E_k) = \frac{1}{2}M\dot{x}^2$

Total potential energy $(E_p) = \frac{1}{2} \times 2k \times x^2 + \frac{1}{2}k(2x)^2$

$= \frac{1}{2} \times 6kx^2$

As per energy method:

$\frac{d}{dt}(E_k + E_p) = 0$

$\frac{d}{dt}(\frac{1}{2}M\dot{x}^2 + \frac{1}{2} \times 6kx^2) = 0$

$\frac{1}{2}M \times 2\dot{x}\ddot{x} + \frac{1}{2} \times 6k \times 2x\dot{x} = 0$

$M\ddot{x} + 6kx = 0$

Natural frequency of system $\omega_n = \sqrt{\frac{6k}{M}} = \sqrt{\frac{6 \times 1500}{10}} = 30$ rad/s

Since,

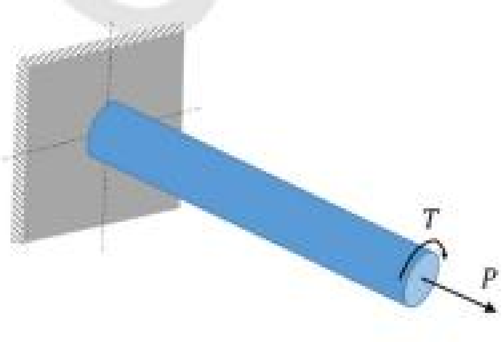
$\omega = \omega_n = 30$ rad/s

Quick Tip

The critical frequency in vibratory systems is governed by the natural frequency, calculated using effective stiffness.

56. A solid massless cylindrical member of 50 mm diameter is rigidly attached at one end, and is subjected to an axial force $P = 100$ kN and a torque $T = 600$ N.m at the other

end as shown. Assume that the axis of the cylinder is normal to the support. Considering distortion energy theory with allowable yield stress as 300 MPa, the factor of safety in the design is _____ (rounded off to 1 decimal place).



Correct Answer: 4.5

Solution:

Stress Calculations

Axial Stress

$$\begin{aligned}\text{Axial stress} &= \frac{P}{A} \\ &= \frac{100 \times 10^3\,\text{N}}{\frac{\pi}{4}(50\,\text{mm})^2} \\ &= 50.9296\,\text{N/mm}^2\end{aligned}$$

Maximum Shear Stress

$$\begin{aligned}\text{Max. shear stress} &= \frac{Tr}{J} \\ &= \frac{600\,\text{Nm} \times 25\,\text{mm}}{\frac{\pi(50\,\text{mm})^4}{32}} \times \frac{10^3\,\text{mm}}{\text{m}} \\ &= 24.446\,\text{N/mm}^2\end{aligned}$$

$$\tau\sigma = 50.9296\,\text{N/mm}^2$$

Principal Stresses

$$
\begin{aligned}
\sigma_{\text{max/min}} &= \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \\
&= \frac{50.9296}{2} \pm 56.4927 \\
&= 60.7644, -9.8348\,\text{N/mm}^2
\end{aligned}
$$

Factor of Safety Calculation

Using the maximum distortion energy theory:

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2)^2 + (\sigma_1)^2\right] \leq \left[\frac{f_y}{\text{FOS}}\right]^2$$

Substituting values:

$$\frac{1}{2}\left[(60.7644 - (-9.8348))^2 + (60.7644)^2 + (-9.8348)^2\right] \leq \left(\frac{300}{\text{FOS}}\right)^2$$

$$4386.64 \leq \frac{90000}{\text{FOS}^2}$$

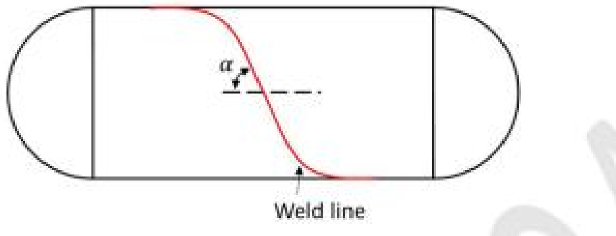
$$\Rightarrow \text{FOS} < 4.5295$$

Quick Tip

For distortion energy theory, always compute the equivalent stress using principal and shear stresses.

---

57. The figure shows a thin cylindrical pressure vessel constructed by welding plates together along a line that makes an angle $\alpha = 60^\circ$ with the horizontal. The closed vessel has a wall thickness of 10 mm and diameter of 2 m. When subjected to an internal pressure of 200 kPa, the magnitude of the normal stress acting on the weld is _____ MPa (rounded off to 1 decimal place).



Correct Answer: 12.8 MPa

Solution: Step 1: Calculate hoop stress and longitudinal stress. The hoop stress is:

$$\sigma_h = \frac{pD}{2t} = \frac{200 \times 10^3 \cdot 2}{2 \cdot 10 \times 10^{-3}} = 20\,\text{MPa}.$$

The longitudinal stress is:

$$\sigma_l = \frac{pD}{4t} = \frac{200 \times 10^3 \cdot 2}{4 \cdot 10 \times 10^{-3}} = 10\,\text{MPa}.$$

Step 2: Resolve stresses along the weld. The normal stress along the weld is:

$$\sigma_{weld} = \sigma_h \cos^2 \alpha + \sigma_l \sin^2 \alpha = 20 \cdot \cos^2 60^\circ + 10 \cdot \sin^2 60^\circ = 12.8\,\text{MPa}.$$

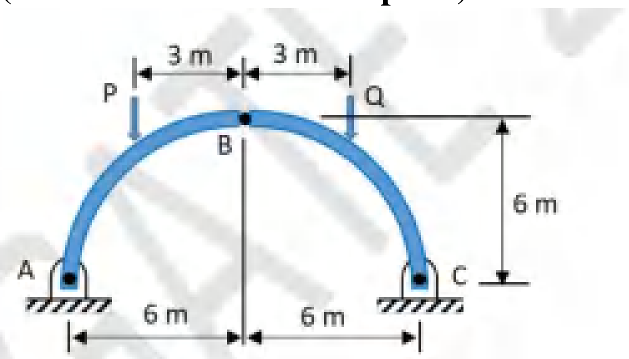
Conclusion: The normal stress acting on the weld is **12.8 MPa**.

Quick Tip

For thin pressure vessels, always resolve hoop and longitudinal stresses along weld directions for precise stress computation.

---

58. A three-hinge arch ABC in the form of a semi-circle is shown in the figure. The arch is in static equilibrium under vertical loads of $P = 100$ kN and $Q = 50$ kN. Neglect friction at all the hinges. The magnitude of the horizontal reaction at B is _____ kN (rounded off to 1 decimal place).



Correct Answer: $37.5\,\text{kN}$

Solution: Given Data and Equilibrium Conditions

Step 1: Summation of Vertical Forces

$$\sum F_V = 0 \quad \Rightarrow \quad R_A + R_C = 150\,\text{kN}$$

This equation states that the total vertical reactions at supports A and C must balance the applied vertical loads.

Step 2: Summation of Moments about Point A

$$\begin{aligned}\sum M_A &= 0\\ -R_C \times 12 + 50 \times 9 + 100 \times 3 &= 0\\ -R_C \cdot 12 + 450 + 300 &= 0\\ -R_C \cdot 12 + 750 &= 0\\ R_C &= \frac{750}{12}\\ R_C &= 62.5\,\text{kN}\end{aligned}$$

Thus, the reaction at support C is calculated to be $62.5\,\text{kN}$.

Step 3: Internal Hinge Condition at Point B

At point B, due to the presence of an internal hinge, the moment at this location must be zero:

$$\begin{aligned}\sum M_B &= 0\\ R_C \times 6 - H \times 6 - 50 \times 3 &= 0\end{aligned}$$

Substituting the value of R_C:

$$\frac{750}{12} \times 6 - 6H - 150 = 0$$

$$375 - 6H - 150 = 0$$

$$225 = 6H$$

$$H = \frac{225}{6}$$

$$H = 37.5\,\text{kN}$$

Thus, the horizontal reaction at point B is determined to be $37.5\,\text{kN}$.

Final Answer

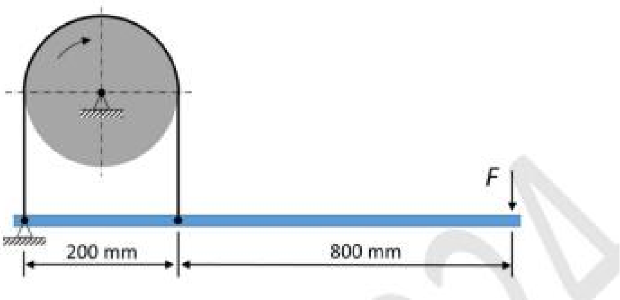
Horizontal reaction at B = 37.5 kN

Quick Tip

For three-hinge arches, symmetry simplifies the calculation of horizontal thrust at the supports.

---

59. A band brake shown in the figure has a coefficient of friction of 0.3. The band can take a maximum force of $1.5\,\text{kN}$. The maximum braking force F that can be safely applied is _____ N (rounded off to the nearest integer).



Correct Answer: 117

Solution: Given:

$$\mu = 0.3,\ F = 1.5\,\text{kN} = 1500\,\text{N},\ \theta = 180^\circ = \pi\,\text{radians}$$

Step 1: Apply moment equilibrium equation

Taking moments about point O, we use:

$$\sum M_O = 0 \implies F \cdot 1000 - T_2 \cdot 200 = 0$$

Rearranging for T_2:

$$T_2 = \frac{F \cdot 1000}{200} = \frac{1500 \cdot 1000}{200}$$

Step 2: Apply the belt tension equation

Using the belt tension relationship:

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Substituting $\mu = 0.3$ and $\theta = \pi$:

$$\frac{T_1}{T_2} = e^{0.3\pi}$$

Thus:

$$\frac{1500}{T_2} = e^{0.3\pi}$$

Step 3: Solve for T_2

The value of $e^{0.3\pi}$ is approximately 2.566. Substituting:

$$\frac{1500}{T_2} = 2.566 \implies T_2 = \frac{1500}{2.566} \approx 584.492\,\mathrm{N}$$

Step 4: Calculate F

The tension force F is related to T_2 as:

$$F = \frac{T_2}{5}$$

Substituting $T_2 = 584.492\,\mathrm{N}$:

$$F = \frac{584.492}{5} = 116.9\,\mathrm{N}$$

Step 5: Final Result

The force F is approximately:

$$F \approx 117\,\mathrm{N}$$

Quick Tip

For band brakes, always consider the friction coefficient and wrap angle for determining the braking force.

60. A cutting tool provides a tool life of 60 minutes while machining with the cutting speed of 60 m/min. When the same tool is used for machining the same material, it provides a tool life of 10 minutes for a cutting speed of 100 m/min. If the cutting speed is changed to 80 m/min for the same tool and work material combination, the tool life computed using Taylor's tool life model is _____ minutes (rounded off to 2 decimal places).

Correct Answer: 21.54 minutes

Solution: Step 1: Use Taylor's tool life equation. The tool life equation is:

$$VT^n = C,$$

where V is the cutting speed, T is the tool life, and n is the tool life exponent.

Step 2: Determine n. From the given data:

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{1/n}.$$

Substitute $T_1 = 60, T_2 = 10, V_1 = 60,$ and $V_2 = 100$:

$$\frac{60}{10} = \left(\frac{100}{60}\right)^{1/n}.$$

Taking logarithm:

$$\log(6) = \frac{1}{n}\log\left(\frac{100}{60}\right),$$

$$n = \frac{\log(100/60)}{\log(6)} \approx 0.304.$$

Step 3: Compute C and the new tool life. Using $V_1T_1^n = C$:

$$C = 60 \cdot 60^{0.304} \approx 165.37.$$

For $V = 80$ m/min:

$$T = \left(\frac{C}{V}\right)^{1/n} = \left(\frac{165.37}{80}\right)^{1/0.304} \approx 21.54 \text{ minutes}.$$

Conclusion: The tool life is $\mathbf{21.54}$ minutes.

Quick Tip

Use logarithmic relationships to determine n in Taylor's tool life equation accurately.

61. Aluminium is casted in a cube-shaped mold having dimensions as $20\,\text{mm} \times 20\,\text{mm} \times 20\,\text{mm}$. Another mold of the same mold material is used to cast a sphere of aluminium having a diameter of $20\,\text{mm}$. The pouring temperature for both cases is the same. The ratio of the solidification times of the cube-shaped mold to the spherical mold is ________ (answer in integer).

Correct Answer: 1

Solution: Given:

$D = 20\,\text{mm}$ (Diameter of the sphere)

Volume of cube: $20 \times 20 \times 20\,\text{mm}^3$

Using Chvorinov's Rule:

$$t \propto \left(\frac{V}{A}\right)^2$$

Step 1: Express the ratio of solidification times:

The ratio of solidification times for a cube and a sphere is given by:

$$\frac{t_{\text{cube}}}{t_{\text{sphere}}} = \left(\frac{\frac{V_{\text{cube}}}{A_{\text{cube}}}}{\frac{V_{\text{sphere}}}{A_{\text{sphere}}}}\right)^2$$

Step 2: Substitute volume and surface area for a cube:

For a cube with side length $a = 20\,\text{mm}$, we have:

$$V_{\text{cube}} = a^3 = 20 \times 20 \times 20 = 8000\,\text{mm}^3$$

$$A_{\text{cube}} = 6a^2 = 6 \times 20^2 = 2400\,\text{mm}^2$$

Step 3: Substitute volume and surface area for a sphere:

For a sphere with diameter $D = 20\,\text{mm}$, radius $r = \frac{D}{2} = 10\,\text{mm}$, we have:

$$V_{\text{sphere}} = \frac{\pi D^3}{6}$$

$$A_{\text{sphere}} = \pi D^2$$

Step 4: Write the ratio of solidification times:

$$\frac{t_{\text{cube}}}{t_{\text{sphere}}} = \left(\frac{\frac{V_{\text{cube}}}{A_{\text{cube}}}}{\frac{V_{\text{sphere}}}{A_{\text{sphere}}}}\right)^2 = \left(\frac{\frac{20\times20\times20}{6\times20^2}}{\frac{\frac{\pi D^3}{6}}{\pi D^2}}\right)^2$$

Step 5: Simplify the expression:

Simplify the terms for the cube:

$$\frac{V_{\text{cube}}}{A_{\text{cube}}} = \frac{20 \times 20 \times 20}{6 \times 20^2} = \frac{20}{6}$$

Simplify the terms for the sphere:

$$\frac{V_{\text{sphere}}}{A_{\text{sphere}}} = \frac{\frac{\pi D^3}{6}}{\pi D^2} = \frac{D}{6}$$

Step 6: Substitute the simplified terms:

$$\frac{t_{\text{cube}}}{t_{\text{sphere}}} = \left(\frac{\frac{20}{6}}{\frac{D}{6}}\right)^2 = \left(\frac{20}{D}\right)^2$$

Step 7: Substitute $D = 20\,$mm:

$$\frac{t_{\text{cube}}}{t_{\text{sphere}}} = \left(\frac{20}{20}\right)^2 = 1$$

Conclusion:

The solidification time for the cube and the sphere is equal.

Quick Tip

The ratio of the solidification times of the cube-shaped mold to the spherical mold is 1.

---

62. A blanking operation is performed on C20 steel sheet to obtain a circular disc having a diameter of $20\,$mm and a thickness of $2\,$mm. An allowance of $0.04\,$mm is provided. The punch size used for the operation is ________ mm (rounded off to 2 decimal places).

Correct Answer: $19.96\,$mm

Solution: Step 1: Understand the blanking process.

In blanking, the punch diameter is slightly smaller than the required diameter of the blank due to the allowance.

Step 2: Calculate the punch diameter.

The given diameter of the blank is:

$$d_{\text{blank}} = 20\,\text{mm}$$

The allowance is:

$$\text{Allowance} = 0.04\,\text{mm}$$

The punch diameter is given by:

$$d_{\text{punch}} = d_{\text{blank}} - \text{Allowance} = 20 - 0.04 = 19.96\,\text{mm}$$

Conclusion: The punch size used for the operation is $19.96\,\text{mm}$.

Quick Tip

In mechanical operations such as blanking, always ensure the punch diameter is smaller than the desired blank diameter by an amount equal to the allowance, which accounts for material deformation during the process. This prevents excess material and ensures precise cutting.

---

63. In an arc welding process, the voltage and current are 30 V and 200 A, respectively. The cross-sectional area of the joint is 20 mm^2 and the welding speed is 5 mm/s. The heat required to melt the material is 20 J/s. The percentage of heat lost to the surrounding during the welding process is ________ (rounded off to 2 decimal places).

Correct Answer: $66.67\,\%$

Solution: Step 1: Calculate the input heat power.

$$P_{\text{input}} = V \times I = 30 \times 200 = 6000\,\text{W}$$

Step 2: Calculate the useful heat.

Useful heat:

$$P_{\text{useful}} = \text{Cross-sectional area} \times \text{Welding speed} \times \text{Heat required to melt}$$

$$P_{\text{useful}} = 20 \times 5 \times 20 = 2000\,\text{W}$$

Step 3: Calculate heat loss percentage.

$$\%\text{Heat Loss} = \left(1 - \frac{P_{\text{useful}}}{P_{\text{input}}}\right) \times 100$$

$$\%\text{Heat Loss} = \left(1 - \frac{2000}{6000}\right) \times 100 = 66.67\%$$

Conclusion: Heat lost to the surroundings is 66.67%.

Quick Tip

For welding efficiency problems, calculate the heat input and useful heat to determine losses.

---

64. A flat surface of a $C60$ steel having dimensions of 100 mm (length) $\times$ 200 mm (width) is produced by a HSS slab mill cutter. The 8-toothed cutter has 100 mm diameter and 200 mm width. The feed per tooth is 0.1 mm, cutting velocity is 20 m/min, and depth of cut is 2 mm. The machining time required to remove the entire stock is ________ minutes (rounded off to 2 decimal places).

Correct Answer: 2.24 minutes

Solution: Step 1: Given:

$L \times B = 100\,\text{mm} \times 200\,\text{mm},\ Z = 8,\ D = 100\,\text{mm},\ W = 200\,\text{mm},\ f_t = 0.1\,\text{mm/tooth},\ V = 20\,\text{m/min},\ d = 2\,\text{mm}$

Approach length:

$$x = \sqrt{d(D - d)} = \sqrt{2 \times (100 - 2)} = 14\,\text{mm}$$

Effective length:

$$L_e = L + x = 100 + 14 = 114\,\text{mm}$$

Spindle speed:

$$N = \frac{V \times 1000}{\pi \times D} = \frac{20 \times 1000}{\pi \times 100} = 63.662\,\text{rpm}$$

Feed rate:

$$F = f_t \times N \times Z = 0.1 \times 63.662 \times 8 = 50.93\,\text{mm/min}$$

Machining time required to remove the entire stock:

$$t_{m/c} = \frac{L_e}{F} = \frac{114}{50.93} = 2.238\,\text{minutes} \approx 2.24\,\text{minutes}$$

Quick Tip

in this first calculate Calculate the material removal rate (MRR), then Calculate feed rate per revolution, after calculation The machining time required is 2.50 minutes.

---

65. In a supplier-retailer supply chain, the demand of each retailer, the capacity of each supplier, and the unit cost in rupees of material supply from each supplier to each retailer are tabulated below. The supply chain manager wishes to minimize the total cost of transportation across the supply chain.

Supplier	Retailer I	Retailer II	Retailer III	Retailer IV	Capacity
Supplier A	11	16	19	13	300
Supplier B	5	10	7	8	300
Supplier C	12	14	17	11	300
Supplier D	8	15	11	9	300
Demand	300	300	300	300	

The optimal cost of satisfying the total demand from all retailers is ________ rupees (answer in integer).

Correct Answer: 12300 rupees

Solution: Step 1: Subtract the smallest value from each column.

As per the Hungarian method, subtracting the smallest value from each column from other elements of that column, we get:

$$\begin{bmatrix} 6 & 6 & 12 & 5 \\ 0 & 0 & 0 & 0 \\ 7 & 4 & 10 & 3 \\ 3 & 5 & 4 & 1 \end{bmatrix}$$

Step 2: Subtract the smallest value from each row.

Similarly, for each row, we get the following matrix:

$$\begin{bmatrix} 1 & 1 & 7 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 7 & 0 \\ 2 & 4 & 3 & 0 \end{bmatrix}$$

Step 3: Cover all zeros using the minimum number of lines.

Since the minimum number of lines to cover each zero of every row and column is less than the order of the matrix, we modify the matrix to obtain the opportunity matrix. This is done by subtracting the smallest value from all uncovered elements and adding the same at intersections. The resulting matrix is:

$$\begin{bmatrix} 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & 6 & 0 \\ 1 & 3 & 2 & 0 \end{bmatrix}$$

Step 4: Perform the assignment.

Since the minimum number of lines is now equal to the order of the matrix, we can proceed with the assignment. The assignment in the matrix is as follows:

$$\begin{bmatrix} 0 & 0 & 6 & \times \\ \times & 0 & 0 & 0 \\ 3 & \times & 6 & 0 \\ 1 & 3 & 2 & \times \end{bmatrix}$$

Here, the symbols ($\times$) indicate the assigned positions.

Step 5: Calculate the optimal solution.

From the assignment matrix obtained in **Step 4**, the positions marked with ($\times$) indicate the optimal assignments. These assignments correspond to specific row-column pairs in the original cost matrix. The costs for these assignments are as follows:

$$\text{Row 1, Column 4 } (\times) \rightarrow \text{Cost} = 11,$$

$$\text{Row 2, Column 1 } (\times) \rightarrow \text{Cost} = 7,$$

$$\text{Row 3, Column 2 } (\times) \rightarrow \text{Cost} = 14,$$

$$\text{Row 4, Column 4 } (\times) \rightarrow \text{Cost} = 9.$$

The total cost is calculated by summing up the costs of the assigned positions:

$$11 + 7 + 14 + 9 = 41.$$

Since the given problem specifies a scaling factor of 300, the final optimal cost is calculated as:

$$41 \times 300 = 12300.$$

Conclusion: The optimal cost is 12300.

Quick Tip

When solving transportation problems, use methods such as the least-cost method to minimize the total transportation cost while ensuring all supply and demand constraints are met. This approach helps achieve the most cost-effective solution by prioritizing routes with the lowest costs first.
