

64th C. C. Exam., 2019**MATHEMATICS****Time Allowed : 3 Hours.]****[Maximum Marks : 300**

*Candidates are required to answer six questions in all,
selecting three from each Section.*

SECTION – I

1. (a) State Cayley-Hamilton theorem and using it find the inverse of the matrix

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

- (b) Let $T: V_4 \rightarrow V_3$ be a linear map defined by
 $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$. Find the rank and nullity of T .

2. (a) Transform the equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ to polar coordinate system.

- (b) Evaluate the Euler-Poisson integral $\int_0^\infty e^{-x^2} dx$

3. (a) Find the centre and radius of the circle

$$x^2 + y^2 + z^2 - 2y - 4z = 11, x + 2y + 2z = 15$$

- (b) Find the curvature and torsion of the curve

→

$$\vec{r} = (a(u - \sin u), a(1 - \cos u), bu)$$

4. (a) Solve: $(y+1)\frac{dy}{dx} + x(y^2 + 2y) = x$

- (a) Solve: $(D^4 - 16)y = x^2 - e^x, D \equiv \frac{d}{dx}$

5. (a) Prove that: $\nabla \times \left(\frac{\vec{a} \times \vec{r}}{r^n} \right) = \frac{2-n}{r^n} \vec{a} + \frac{n(\vec{a} \cdot \vec{r})}{r^{n+2}} \vec{r}$

- (b) A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular coordinates. Find its covariant components in spherical coordinates.

6. (a) Find the intrinsic equation of a common catenary. Find also its equation in cartesian coordinate system.
(b) Define projectile. Find the maximum height and time of flight of a projectile.

SECTION - II

7. (a) Define a group and prove that the set of n th root of unit is a group with respect to multiplication.
(b) If U is an ideal of the ring R , then show that R/U is a ring and is a homomorphic image of R .
8. (a) Define a compact set and show that the continuous image of a compact set is compact.
(b) If $xyz = abc$, then show that the minimum value of $bcx + cay + abz$ is $3abc$

9. (a) Find the expansion of $f(z) = \frac{7z-2}{(z+1)(z-2)}$ in the region $1 < z+1 < 3$.
- (b) Using Charpit's method, solve the following partial differential equation : $z^2 = pqxy$
10. (a) Using regula-falsi method, solve the polynomial equation $x^3 - x + 1 = 0$.
- (b) Obtain the cubic spline approximation of the function defined by the data
- | | | | | | |
|--------|---|---|---|----|-----|
| x | : | 0 | 1 | 2 | 3 |
| $f(x)$ | : | 1 | 2 | 33 | 144 |
- with $M(0) = 0, M(3) = 0$.
11. (a) The following data gives the marks obtained by two students A and B in a certain examination :
- A : 85 68 64 91 86 82
- B : 78 73 76 81 81 83
- Find who is more consistent and who is more intelligent.
- (b) If X and Y are independent random variables, then show that $E(XY) = E(X) E(Y)$
12. (a) Solve the following transportation problem :

		Distribution centre				
		1	2	3	4	
Source	1	2	3	11	7	6
	2	1	0	6	1	1
	3	5	8	15	9	10
		7	5	3	2	

- (b) Solve the dual of the following linear programming problem by simplex method :

$$\begin{aligned}
 &\text{Maximize} && 45x_1 + 80x_2 \\
 &\text{Subject to} && 5x_1 + 20x_2 \leq 400 \\
 &&& 10x_1 + 15x_2 \leq 450 \\
 &&& x_1, x_2 \geq 0
 \end{aligned}$$