

PERMUTATION AND COMBINATION

In our day-to-day life, we have various things to do. We can find all the possible ways to do those works by using permutation and combination.

Permutation

Factorial

It can be defined as the product of all-natural numbers up to that number i.e., $n!$

$$= n \times (n - 1) \times (n - 2) \times (n - 3) \times \dots \times 1.$$

Example:

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$5! = 5 \times 4! = 5 \times 4 \times 3! = 5 \times 4 \times 3 \times 2! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Permutation

It is defined as the possible number of different arrangements which can be made by taking some or all given things at a time. In permutation order of things matters i.e., if two numbers A and B are arranged then AB and BA are counted as two different permutations.

Formula of Permutation:

$${}^n P_r = \frac{n!}{(n - r)!}$$

Points to Remember

- ${}^n P_n = n!$
- ${}^n P_0 = 1$

Combination

It is defined as the possible number in which given things can be selected. In combination order of things does not matter i.e., if two numbers A and B are to be selected then AB and BA are counted as one combination.

Formula of Combination:

$${}^n C_r = \frac{n!}{r!(n - r)!}$$

Points to Remember

- ${}^nC_n = 1$
- ${}^nC_0 = 1$
- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n = 2^n$

Permutation vs Combination

Points to Remember

- Whenever we want to arrange n things at n places, we can arrange it in $n!$ ways.
- Whenever we want to select r things out of n things, we can select it in nC_r ways.
- Whenever we want to arrange r things out of n things, we can arrange it in nP_r ways.
- Number of ways in which r objects can be arranged out of n objects if q things are similar $= {}^nP_r / q! = n! / q! (n-r)!$
- If, ${}^nC_x = {}^nC_y$, then either $x = y$ or $(x + y) = n$
- Number of circular permutations of n different objects $= (n - 1)!$
- Number of circular permutations of n different objects if clockwise or anti-clockwise arrangement are not considered $= (n - 1)! / 2$

Example 1: 5 letters A, B, C, D and E are given, then find

A. In how many ways 5 letters can be arranged?

B. In how many ways 3 of 5 letters can be selected.

C. In how many ways 3 of 5 letters can be arranged

Solution:

We can find the number of possible arrangements by using factorial $= 5! = 120$

We can find a number of ways of selection by using combination, ${}^5C_3 = 5! / (2! \times 3!) = (5 \times 4 \times 3!) / (2 \times 3!) = 10$

We can find the required arrangement by using permutation, ${}^5P_3 = 5! / 3! = (5 \times 4 \times 3!) / 3! = 20$

Example 2: In how many different ways can the letters of the word 'HAPPY' be arranged?

Solution:

HAPPY = 5 Letter word with letter "P" repeating two times

Hence, No. of ways 5 letters word can be arranged with a letter repeating itself 2 times = $5! / 2! = 120 / 2 = 60$

Example 3: Find the number of different ways of forming a committee consisting of 3 men and 3 women from 6 men and 5 women.

Solution:

No. of ways of selecting 3 men out of 6 men = 6C_3

And, No. of ways of selecting 3 women out of 5 women = 5C_3

So, the no. of ways of forming a committee = ${}^6C_3 \times {}^5C_3 = 200$

Example 4: Find the number of ways in which 5 beads can be strung into a necklace.

Solution:

Total number of beads, $n = 5$

Required number of ways = $(5 - 1)! / 2 = 12$

Example 5: In a meeting 5 persons are present, find the number of handshakes if each person shakes his hand with every other.

Solution:

Number of ways in which 5 people can shake hand with each other once = ${}^5C_2 = 10$

Example 6: Find the number of straight lines that can be formed by 10 non-collinear points.

Solution: Number of straight lines formed by 10 non-collinear points = ${}^{10}C_2 = 45$

Example 7: Find the number of triangles that can be made by using 10 points in a plane, out of which 3 are collinear.

Solution: Number of triangles formed by 10 points out of which 3 are collinear points = ${}^{10}C_3 - {}^3C_3 = 120 - 1 = 119$