NTA JOINT CSIR UGC NET December 2025 18th Dec 2025

Application No	[redacted]
Candidate Name	
Roll No.	
Test Date	
Test Time	
Subject	

Section : **PART-A**

Q.1

Five students graduated from a college, not all in the same year, after each has studied for four years. If batchmates Jiten and Anwar were between Ramesh and Prakash but senior to Sam while Ramesh had left the college before Jiten took admission, then it is certain that

1. Anwar was the most senior
2. Ramesh was the most senior
3. Sam was the most junior
4. Prakash was the most junior

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710139**
Option 1 ID : **916710553**
Option 2 ID : **916710554**
Option 3 ID : **916710555**
Option 4 ID : **916710556**
Status : **Answered**
Chosen Option : **2**

Q.2 Which among the following cities can be said most appropriately to bear the same relation to *Tamil Nadu* that **Pune** bears to *Maharashtra*; **Surat** to *Gujarat* and **Asansol** to *West Bengal*?

1. **Tirupati**
2. **Mysore**
3. **Chennai**
4. **Coimbatore**

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710129**
Option 1 ID : **916710513**
Option 2 ID : **916710514**
Option 3 ID : **916710515**
Option 4 ID : **916710516**
Status : **Answered**
Chosen Option : **3**

Q.3 The geometric mean of 100 observations is 25. If each observation is multiplied by 4, what will be the new geometric mean?

1. 100
2. 50
3. 25
4. $(25\times4)^{1/2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710128**
Option 1 ID : **916710509**
Option 2 ID : **916710510**
Option 3 ID : **916710511**
Option 4 ID : **916710512**
Status : **Answered**
Chosen Option : **1**

Q.4 Alloy A is formed by mixing iron (Fe) and nickel (Ni) in the ratio 3:4, while alloy B is formed by mixing Fe and Ni in the ratio 9:5. If equal quantities of alloys A and B are melted together to form a new alloy C, what will be the ratio of Fe to Ni in the alloy C?

1. 4:3
2. 5:3
3. 15:13
4. 13:9

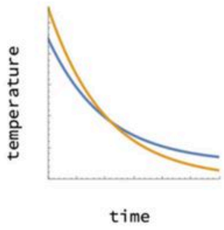
Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710135**
Option 1 ID : **916710537**
Option 2 ID : **916710538**
Option 3 ID : **916710539**
Option 4 ID : **916710540**
Status : **Answered**
Chosen Option : **1**

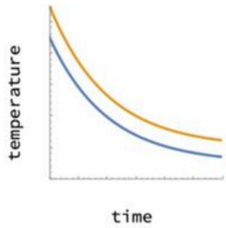
Q.5

Two identical metal bars are heated to different temperatures and allowed to cool in the same surroundings. Which one of the following figures correctly shows their cooling curves?

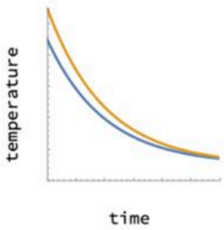
A



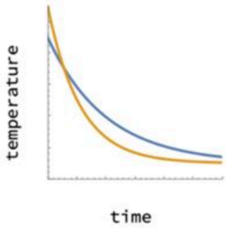
B



C



D



1. A
2. B
3. C
4. D

Options 1. 1

2. 2

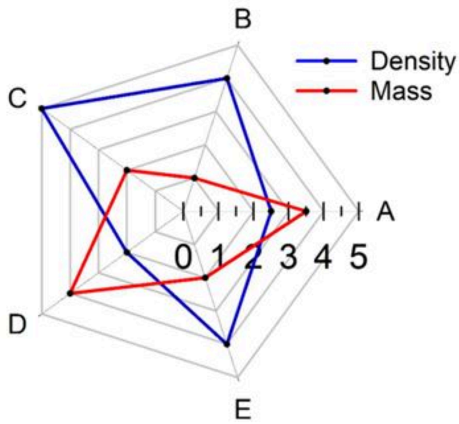
3. 3

4. 4

Question Type :	**MCQ**
Question ID :	**916710130**
Option 1 ID :	**916710517**
Option 2 ID :	**916710518**
Option 3 ID :	**916710519**
Option 4 ID :	**916710520**
Status :	**Not Answered**
Chosen Option :	**--**

Q.6

The following figure shows densities and masses of five objects (A to E).



The object with the largest volume is _____.

1. A
2. B
3. D
4. E

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710121**
Option 1 ID : **916710481**
Option 2 ID : **916710482**
Option 3 ID : **916710483**
Option 4 ID : **916710484**
Status : **Not Answered**
Chosen Option : **--**

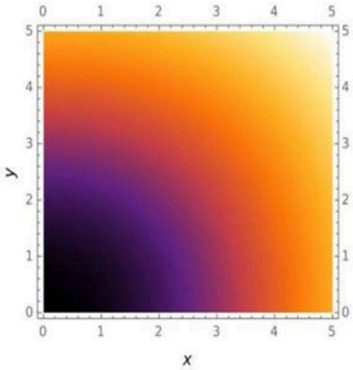
Q.7 In an exam, questions of three difficulty levels hard, medium, and easy fetch respectively 7, 3, and 2 marks if correct and 0 if incorrect. Three students got 30 marks each but in three different ways, though the total number of questions correctly answered by each student was the same. Then what could be the total number of questions correctly answered by each of these students?

1. 12
2. 10
3. 9
4. 6

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**916710126**
Option 1 ID :	**916710501**
Option 2 ID :	**916710502**
Option 3 ID :	**916710503**
Option 4 ID :	**916710504**
Status :	**Not Attempted and Marked For Review**
Chosen Option :	**--**

Q.8 The following plot shows temperature as a function of *x* and *y*. Along which path is the temperature change minimum?



1. x = constant or y = constant
2. $\frac{y}{x^2}$ = constant
3. $y^2 + x^2$= constant
4. y x = constant

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710133**
Option 1 ID : **916710529**
Option 2 ID : **916710530**
Option 3 ID : **916710531**
Option 4 ID : **916710532**
Status : **Not Answered**
Chosen Option : **--**

Q.9 The minimum height of a plane vertical mirror that will allow a 6-feet tall person to see himself fully in it

1. depends on the distance between the person and the mirror
2. is 3 feet
3. is 4.5 feet
4. is 6 feet

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710136**
Option 1 ID : **916710541**
Option 2 ID : **916710542**
Option 3 ID : **916710543**
Option 4 ID : **916710544**
Status : **Answered**
Chosen Option : **2**

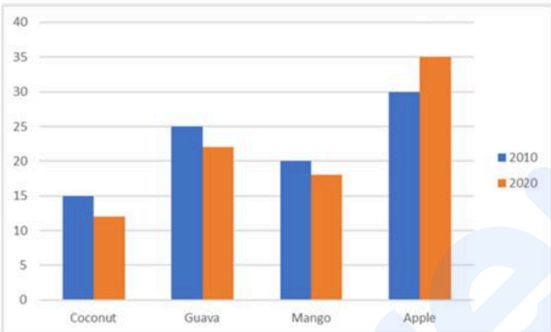
Q.10 Three periodic events repeat every 24 seconds, 54 seconds, and 56 seconds. If they coincide at 10:20:00, when will they next coincide?

1. 10:35:12
2. 10:45:20
3. 10:45:12
4. 10:35:20

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710140**
Option 1 ID : **916710557**
Option 2 ID : **916710558**
Option 3 ID : **916710559**
Option 4 ID : **916710560**
Status : **Answered**
Chosen Option : **3**

Q.11 The numbers (in millions) of coconut, guava, mango and apple trees in a region in 2010 and 2020 are shown in the following figure.



The maximum relative change in numbers was for

1. coconut trees
2. guava trees
3. mango trees
4. apple trees

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710138**
Option 1 ID : **916710549**
Option 2 ID : **916710550**
Option 3 ID : **916710551**
Option 4 ID : **916710552**
Status : **Answered**
Chosen Option : **4**

Q.12 Some, but not all, faces of a six-faced cubical fair die are painted red (R) and the remaining green (G); and the die is thrown until red faces come up on top 4 times. Consider the following sequences of colours listed left to right as they appear on the top.

A: **GRRRR**
B: **GRGRRR**
Which one of the following is true?

1. A is more probable than B
2. B is more probable than A
3. Both have the same probability
4. Whether A or B is more probable depends upon how many faces are painted green

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710137**
Option 1 ID : **916710545**
Option 2 ID : **916710546**
Option 3 ID : **916710547**
Option 4 ID : **916710548**
Status : **Answered**
Chosen Option : **1**

Q.13 12 लीटर क्षमता के एक पूर्ण भरे हुए पात्र से एक 8 लीटर क्षमता वाले खाली पात्र में 6 लीटर पानी को लाने के लिए न्यूनतम कितनी बार पानी उड़ेलना होगा जबकि एक 5 लीटर का खाली पात्र भी उपयोग के लिए उपलब्ध हो?

1. 4
2. 5
3. 6
4. 7

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710123**
Option 1 ID : **916710489**
Option 2 ID : **916710490**
Option 3 ID : **916710491**
Option 4 ID : **916710492**
Status : **Not Answered**
Chosen Option : **--**

Q.14 A lady bought some apples, each costing Rs. 25, and some bananas each costing Rs 6, for a total of Rs. 378. In how many ways could she have chosen the numbers of apples and bananas?

1. 1
2. 2
3. 3
4. 4

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710131**
Option 1 ID : **916710521**
Option 2 ID : **916710522**
Option 3 ID : **916710523**
Option 4 ID : **916710524**
Status : **Answered**
Chosen Option : **2**

Q.15 Suppose a_1, $a_2,\ldots,$ a_{300} are integers such that $a_{i-1} + a_i + a_{i+1}$ = 2025 for all *i* = 2,3, ..., 299.

If a_7 = – 5, a_9 = 37, then the value of a_{106} is

1. 1993
2. 37
3. – 5
4. 2030

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710124**
Option 1 ID : **916710493**
Option 2 ID : **916710494**
Option 3 ID : **916710495**
Option 4 ID : **916710496**
Status : **Not Answered**
Chosen Option : **--**

Q.16 How many 5-digit numbers can be formed from the digits 0, 2, 3, 4, 6, 7 and 9, using each at most once, which are divisible by 5?

1. 120
2. 240
3. 360
4. 720

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710134**
Option 1 ID : **916710533**
Option 2 ID : **916710534**
Option 3 ID : **916710535**
Option 4 ID : **916710536**
Status : **Answered**
Chosen Option : **3**

Q.17 A recent survey suggests that the total fertility rate in a country has fallen below 2.1, the population replacement ratio. This necessarily implies that the

1. infant mortality rate has increased reducing the net fertility ratio.
2. total population will decline.
3. population of young people is going to increase with a faster rate in the long run if the same status continues.
4. proportion of elderly people is going to decrease in the long run if the same status continues.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710125**
Option 1 ID : **916710497**
Option 2 ID : **916710498**
Option 3 ID : **916710499**
Option 4 ID : **916710500**
Status : **Not Answered**
Chosen Option : **--**

Q.18 In a class, 40% and 20% students passed in Mathematics and Physics, respectively, and 10% students passed in both subjects. What is the probability of a randomly selected student to have passed in Physics if the student already passed in Mathematics?

1. 1/2
2. 1/20
3. 1/4
4. 2/25

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710122**
Option 1 ID : **916710485**
Option 2 ID : **916710486**
Option 3 ID : **916710487**
Option 4 ID : **916710488**
Status : **Not Answered**
Chosen Option : **--**

Q.19 The value of $1 + (\frac{1}{2^1} + \frac{1}{3}) + (\frac{1}{2^2} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}) + \ldots + (\frac{1}{2^9} + \cdots + \frac{1}{1023})$ lies between

1. 2 and 10
2. 11 and 20
3. 21 and 30
4. 31 and 40

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710127**
Option 1 ID : **916710505**
Option 2 ID : **916710506**
Option 3 ID : **916710507**
Option 4 ID : **916710508**
Status : **Not Answered**
Chosen Option : **--**

Q.20 In a community, some artists are teachers, no teacher is a painter, all painters are artists, and all teachers are professionals. Then it can be definitely asserted that

1. no painter is a professional
2. all artists are professionals
3. no professionals are teachers
4. some artists are professionals

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710132**
Option 1 ID : **916710525**
Option 2 ID : **916710526**
Option 3 ID : **916710527**
Option 4 ID : **916710528**
Status : **Answered**
Chosen Option : **4**

Section : **PART-B**

Q.21 Let X be a Binomial(n, p) random variable, where $n \in \{5, 6\}$ and $p \in \{\frac{1}{4}, \frac{3}{4}\}$. If $X = 3$ is observed, then the maximum likelihood estimate of (n, p) is

1. $\left(5, \frac{1}{4}\right)$
2. $\left(5, \frac{3}{4}\right)$
3. $\left(6, \frac{3}{4}\right)$
4. $\left(6, \frac{1}{4}\right)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710174**
Option 1 ID : **916710693**
Option 2 ID : **916710694**
Option 3 ID : **916710695**
Option 4 ID : **916710696**
Status : **Not Answered**
Chosen Option : **--**

Q.22 Let $X = \{1, \ldots, 17\}$ and S_{17} be the group of permutations of X. For a subgroup G of S_{17} and $x \in X$, let

$$S_G(x) = \{\sigma \in G \mid \sigma(x) = x\}.$$

Which of the following statements is true for every subgroup G of S_{17}?

1. The number of pairs $(\sigma, x) \in G \times X$ such that $\sigma(x) = x$ is strictly greater than $\sum_{x \in X} |S_G(x)|$.
2. The number

$$\frac{1}{|G|} \sum_{x \in X} |S_G(x)|$$

is always an integer.

3. For all $x \in X$, $S_G(x)$ is a normal subgroup of G.
4. For all $x, y \in X$, $S_G(x)$ is isomorphic to $S_G(y)$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710159**
Option 1 ID : **916710633**
Option 2 ID : **916710634**
Option 3 ID : **916710635**
Option 4 ID : **916710636**
Status : **Not Answered**
Chosen Option : **--**

Q.23 Let I denote the 3×3 identity matrix. Given any three distinct matrices $A, B, C \in M_3(\mathbb{R})$, which of the following statements is necessarily true?

1. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = f(B) = f(C) = 0$ and $f(D) = I$.
2. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = f(B) = 0$, $f(C) = I$ and $f(D) = I$.
3. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = 0$, $f(B) = f(C) = I$ and $f(D) \neq 0$.
4. There exists $D \in M_3(\mathbb{R})$ and a polynomial $f \in \mathbb{R}[X]$ satisfying $f(A) = f(B) = 0$, $f(C) = I$ and $f(D) = 0$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710150**
Option 1 ID : **916710597**
Option 2 ID : **916710598**
Option 3 ID : **916710599**
Option 4 ID : **916710600**
Status : **Not Answered**
Chosen Option : **--**

Q.24 Let X and Y be topological spaces. Consider the following statement:

S: For every open subset $U \subseteq X \times Y$ and every $x \in X$ such that $\{x\} \times Y \subseteq U$, there is a neighbourhood W of x in X such that $W \times Y \subseteq U$.

Which of the following statements is true?

1. If $X \times Y$ is Hausdorff, then the statement **S** is true.
2. If Y is connected, then the statement **S** is true.
3. If $X \times Y$ is regular, then the statement **S** is true.
4. If Y is compact, then the statement **S** is true.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710160**
Option 1 ID : **916710637**
Option 2 ID : **916710638**
Option 3 ID : **916710639**
Option 4 ID : **916710640**
Status : **Answered**
Chosen Option : **1**

Q.25 Let $M, N \in M_3(\mathbb{C})$ be such that

$$\text{Trace}(M^k) = \text{Trace}(N^k), \qquad 1 \leq k \leq 3.$$

Which of the following statements is necessarily true?

1. $M^2 = N^2$
2. $M^3 = N^3$
3. The characteristic polynomials of M and N are the same.
4. The minimal polynomials of M and N are the same.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710149**
Option 1 ID : **916710593**
Option 2 ID : **916710594**
Option 3 ID : **916710595**
Option 4 ID : **916710596**
Status : **Answered**
Chosen Option : **4**

Q.26 Let $\{X_n : n \geq 0\}$ be any homogeneous Markov Chain on the state space $S = \{1, 2, 3, 4\}$ having the transition probability matrix $P = (p_{ij})_{i,j \in S}$ given by

$$P = \begin{pmatrix} \frac{1}{4} & 0 & \frac{2}{3} & \frac{1}{12} \\ 0 & 1 & 0 & 0 \\ \frac{1}{12} & 0 & \frac{1}{4} & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{12} & \frac{1}{4} \end{pmatrix}.$$

Which of the following statements about stationary distributions of any such Markov Chain is true?

1. There is no stationary distribution
2. Stationary distribution exists and is unique
3. There are exactly two stationary distributions
4. There are infinitely many stationary distributions

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710171**
Option 1 ID : **916710681**
Option 2 ID : **916710682**
Option 3 ID : **916710683**
Option 4 ID : **916710684**
Status : **Not Answered**
Chosen Option : **--**

Q.27 Let $\mathbb{F}_q$ be a finite field with q elements. For $n \geq 2$, let A be a $2n \times 2n$ matrix with entries in $\mathbb{F}_q$ such that $\text{rank}(A) = n$. Let $W = \{v \in \mathbb{F}_q^{2n} \mid Av = 0\}$. Which of the following is necessarily the number of $(n+2)$-dimensional subspaces of $\mathbb{F}_q^{2n}$ that contain W?

1. 1
2. $\dfrac{(q^{2n} - q^n)(q^{2n} - q^{n+1})}{q^n}$
3. $\dfrac{(q^{2n} - 1)(q^{2n} - q) \cdots (q^{2n} - q^{n-1})}{q^n}$
4. $\dfrac{(q^n - 1)(q^n - q)}{(q^2 - 1)(q^2 - q)}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710147**
Option 1 ID : **916710585**
Option 2 ID : **916710586**
Option 3 ID : **916710587**
Option 4 ID : **916710588**
Status : **Answered**
Chosen Option : **2**

Q.28 Let $\mathbb{D}$ denote the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$ and

$$X = \left\{ f : \mathbb{D} \to \mathbb{C} \mid f \text{ is holomorphic and satisfies } f(2z) = \frac{f(z)}{1 - (f(z))^2} \quad \text{for all} \quad |z| < \frac{1}{2} \right\}.$$

Which of the following statements is true?

1. X is uncountable.
2. X is infinite and countable.
3. Every element of X is an open map.
4. X has exactly one element.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710153**
Option 1 ID : **916710609**
Option 2 ID : **916710610**
Option 3 ID : **916710611**
Option 4 ID : **916710612**
Status : **Answered**
Chosen Option : **4**

Q.29 Let $u(x,t)$ be the solution of the partial differential equation

$$u_t = 16u_{xx} + 2,\ 0 < x < 7,\ t > 0$$

satisfying the conditions

$$u_x(0,t) = u(7,t) = 0,\ \ t > 0,$$
$$u(x,0) = 0,\ \ 0 < x < 7.$$

Then which of the following statements is true?

1. For each $x \in (0,7)$, $u(x,t) \to 0$ as $t \to \infty$.
2. For each $x \in (0,7)$, $u(x,t) \to \dfrac{x^2(7-x)^2}{16}$ as $t \to \infty$.
3. For each $x \in (0,7)$, $u(x,t) \to \dfrac{x^2}{16}(7-x)$ as $t \to \infty$.
4. For each $x \in (0,7)$, $u(x,t) \to \dfrac{1}{16}(49-x^2)$ as $t \to \infty$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710164**
Option 1 ID : **916710653**
Option 2 ID : **916710654**
Option 3 ID : **916710655**
Option 4 ID : **916710656**
Status : **Not Answered**
Chosen Option : **--**

Q.30 For a finite group G, let $S(G)$ denote the number of subgroups of G. Which of the following statements is necessarily true?

1. Let G and G' be finite groups such that $S(G) = S(G')$. Then G is isomorphic to G'.
2. If $S(G) = 4$, then $|G| = p^m$ for some prime number p and positive integer m.
3. If $S(G) = 5$, then G is a cyclic group.
4. For every positive integer n, there exists a finite group G such that $S(G) = n$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710158**
Option 1 ID : **916710629**
Option 2 ID : **916710630**
Option 3 ID : **916710631**
Option 4 ID : **916710632**
Status : **Answered**
Chosen Option : **2**

Q.31 Let X, Y, and Z be independent Normal random variables with means -1, 0, and 1, respectively, and variances 1, 1, and 3, respectively. Which of the following random variables has a Cauchy distribution with location parameter 0 and scale parameter 1?

1. $\dfrac{X-1}{|Y|}$
2. $\dfrac{X+2Y+Z}{X-2Y+Z}$
3. $\dfrac{Z-1}{Y}$
4. $\dfrac{X-Y}{X+Y}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710172**
Option 1 ID : **916710685**
Option 2 ID : **916710686**
Option 3 ID : **916710687**
Option 4 ID : **916710688**
Status : **Not Answered**
Chosen Option : **--**

Q.32 Suppose $y(x)$ is the extremal of the variational problem

$$J(y) = \int_0^4 \frac{(y')^2}{y^2}\,dx \quad \text{subject to} \quad y(0) = 1,\ y(4) = e^8.$$

Then which of the following statements is true?

1. $y(\log_e 2) = 2.$
2. $y(\log_e 3) = 9.$
3. $y(\log_e 4) = 4.$
4. $y(\log_e 5) = 5.$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MCQ**

Question ID : **916710166**

Option 1 ID : **916710661**

Option 2 ID : **916710662**

Option 3 ID : **916710663**

Option 4 ID : **916710664**

Status : **Not Answered**

Chosen Option : **--**

Q.33 For a non-negative real number a, let $\sqrt{a}$ denote its non-negative square-root. Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = x\left(\sqrt{x^2+3} - \sqrt{x^2+2}\right).$$

Which of the following statements is true?

1. $\lim_{x\to\infty} f(x) = \infty$
2. $\lim_{x\to\infty} f(x) = 0$
3. $\lim_{x\to\infty} f(x) = 1$
4. $\lim_{x\to\infty} f(x) = \frac{1}{2}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MCQ**

Question ID : **916710144**

Option 1 ID : **916710573**

Option 2 ID : **916710574**

Option 3 ID : **916710575**

Option 4 ID : **916710576**

Status : **Answered**

Chosen Option : **4**

Q.34 Consider the following statements:

(P) The initial value problem

$$y' = f(y), \text{ where } f(y) = \begin{cases} y\cos\frac{1}{y}, & y \neq 0 \\ 0, & y = 0 \end{cases}$$
$$y(0) = 0$$

has at most one solution.

(Q) If $z(x)$ is the solution of the initial value problem

$$z' = \frac{(1-z^4)^{100}}{1+z^2},$$
$$z(0) = 0,$$

then for each $\alpha \in (0,2)$, there exists $x_\alpha \in \mathbb{R}$ such that $z(x_\alpha) = 1 - \alpha$.

Then which of the following statements is true?

1. Both (P) and (Q) are true.
2. (P) is true, (Q) is FALSE.
3. (P) is FALSE, (Q) is true.
4. Both (P) and (Q) are FALSE.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710162**
Option 1 ID : **916710645**
Option 2 ID : **916710646**
Option 3 ID : **916710647**
Option 4 ID : **916710648**
Status : **Not Answered**
Chosen Option : **--**

Q.35 Consider the initial-boundary value problem (IBVP)

$$u_t + u_x = 0,\ 0 < x < \infty,\ t > 0,$$
$$u(x,0) = e^x,\ 0 < x < \infty,$$
$$u(0,t) = 1 - \sin t,\ t > 0.$$

Then which of the following statements is true?

1. There exists a unique solution $u(x,t)$ of IBVP such that $u(1,t) = e - \sin t$, for all $t < 1$.
2. There does NOT exist a solution $u(x,t)$ of IBVP such that $u(1,t) = e - \sin t$, for all $t > 1$.
3. There exists a solution $u(x,t)$ of IBVP such that $u(x,t) = e^{x-t}$, for all $t > x$.
4. There exists a solution $u(x,t)$ of IBVP such that $u(x,t) = e - \sin(t-x)$, for all $t < x$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710163**
Option 1 ID : **916710649**
Option 2 ID : **916710650**
Option 3 ID : **916710651**
Option 4 ID : **916710652**
Status : **Not Answered**
Chosen Option : **--**

Q.36 For a non-negative real number a, let $\sqrt{a}$ denote its non-negative square-root. Consider the sequence $\{x_n\}_{n\geq 1}$ defined by

$$x_1 = 1 \quad \text{and} \quad x_{n+1} = \sqrt{1 + \frac{x_n^2}{n}} \quad \text{for } n \geq 1.$$

Which of the following statements is true?

1. $\limsup_{n\to\infty} x_n = 1$
2. $\limsup_{n\to\infty} x_n = \sqrt{2}$
3. $\limsup_{n\to\infty} x_n = 2$
4. $\limsup_{n\to\infty} x_n = 1 + \sqrt{2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710143**
Option 1 ID : **916710569**
Option 2 ID : **916710570**
Option 3 ID : **916710571**
Option 4 ID : **916710572**
Status : **Not Answered**
Chosen Option : **--**

Q.37 Consider the following linear programming problem:

$$\text{Maximize} \quad 3x_1 + 4x_2$$

subject to

$$\begin{aligned} 3x_1 + 2x_2 &\leq 12, \\ 3x_1 + 5x_2 &\leq 15, \\ 2x_1 - x_2 &\geq 0, \\ x_2 &\leq 2, \\ \text{and } x_1, x_2 &\geq 0. \end{aligned}$$

Which of the following values is the optimum value of the objective function in the feasible region?

1. 11
2. 13
3. 14
4. 15

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710180**
Option 1 ID : **916710717**
Option 2 ID : **916710718**
Option 3 ID : **916710719**
Option 4 ID : **916710720**
Status : **Not Answered**
Chosen Option : **--**

Q.38 For $a \in \mathbb{R}$, let $y_1(x)$ and $y_2(x)$ be solutions of the differential equation

$$y'' + (e^{x^2} + \cos x)\, y = 0$$

such that

$$y_1(0) = 3,\ y_1'(0) = -1,\ y_2(0) = -5,\ y_2'(0) = a.$$

Suppose $W(y_1, y_2)(x)$ denotes the Wronskian of y_1 and y_2. If $W(y_1, y_2)\left(\frac{1}{2}\right) = 4$, then the value of a is

1. 2
2. 3
3. 4
4. -3

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710161**
Option 1 ID : **916710641**
Option 2 ID : **916710642**
Option 3 ID : **916710643**
Option 4 ID : **916710644**
Status : **Answered**
Chosen Option : **2**

Q.39 Let $\alpha, \beta \in (0, \infty)$. Consider the infinite series

$$\sum_{n \geq 4} \frac{1}{n(\log_e n)^{\alpha}(\log_e(\log_e n))^{\beta}}$$

Which of the following statements is true?

1. The series converges for all $\alpha \in (0, 1)$ and for all $\beta \in (1, \infty)$.
2. The series converges for $\alpha = 1$ and $\beta = 1$.
3. The series converges for $\alpha = 1$ and for all $\beta \in (1, \infty)$.
4. The series converges for all $\alpha \in (0, \infty)$ and for all $\beta \in (0, \infty)$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710146**
Option 1 ID : **916710581**
Option 2 ID : **916710582**
Option 3 ID : **916710583**
Option 4 ID : **916710584**
Status : **Not Answered**
Chosen Option : **--**

Q.40 Let $X = (X_1, X_2, X_3)^T$ be a 3×1 random vector with $E(X) = (3, 2, 1)^T$ and $\text{Cov}(X) = \Sigma = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & -2 \\ 0 & -2 & 3 \end{pmatrix}$. Suppose that $Y = (Y_1, Y_2, Y_3)^T = \frac{1}{\sqrt{3}} X$. Then the value of the multiple correlation coefficient between Y_1 and (Y_2, Y_3) equals

1. $\frac{2}{\sqrt{5}}$
2. $\sqrt{\frac{2}{5}}$
3. $\frac{2}{\sqrt{3}}$
4. $\frac{1}{\sqrt{3}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710178**
Option 1 ID : **916710709**
Option 2 ID : **916710710**
Option 3 ID : **916710711**
Option 4 ID : **916710712**
Status : **Not Answered**
Chosen Option : **--**

Q.41 Let S be a finite subset of $\mathbb{C}$ containing 0. Let $f : \mathbb{C} \setminus S \to \mathbb{C}$ be a holomorphic function which has a simple pole at 0. For $R > 0$, let γ_R denote the path $\gamma_R(t) = Re^{2\pi i t}$ for $t \in [0, 1]$. Which of the following statements is necessarily true?

1. The function $f(1/z)$ has a removable singularity at 0.
2. The function $f(1/z)$ has an essential singularity at 0.
3. If $\int_{\gamma_R} f(z) \mathrm{d}z = 0$ for some $R > 0$, then S is not a singleton set.
4. If S is not a singleton set, then $\int_{\gamma_R} f(z) \mathrm{d}z = 0$ for some $R > 0$ such that $S \cap \{z \in \mathbb{C} : |z| = R\} = \emptyset$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710154**
Option 1 ID : **916710613**
Option 2 ID : **916710614**
Option 3 ID : **916710615**
Option 4 ID : **916710616**
Status : **Answered**
Chosen Option : **4**

Q.42 Let h be a holomorphic function on $\mathbb{C} \setminus \{0\}$ such that

$$\lim_{|z|\to\infty} h(z) = 0.$$

For every $n \geq 1$, consider

$$f_n(z) = \sum_{k=1}^{n} h(z^k).$$

Let $\mathbb{D}$ denote the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$. Which of the following statements is necessarily true?

1. For all z with $|z| \geq 1$, the sequence $\{f_n(z)\}_{n\geq 1}$ converges.
2. For all $z \in \mathbb{D} \setminus \{0\}$, the sequence $\{f_n(z)\}_{n\geq 1}$ converges.
3. The sequence $\{f_n\}_{n\geq 1}$ converges pointwise to a holomorphic function on $\mathbb{D} \setminus [0, 1)$.
4. The sequence $\{f_n\}_{n\geq 1}$ converges pointwise to a holomorphic function on $\{z \in \mathbb{C} : |z| > 1\}$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710155**
Option 1 ID : **916710617**
Option 2 ID : **916710618**
Option 3 ID : **916710619**
Option 4 ID : **916710620**
Status : **Not Answered**
Chosen Option : **--**

Q.43 Consider a finite population of size $N = 100$. Let T_1 be the sample mean of a study variable based on a sample of size n $(1 < n < N)$ under simple random sampling with replacement scheme. Let T_2 be the sample mean of the same study variable based on a sample of size n under simple random sampling without replacement scheme. If $\text{Var}(T_1) = 9\text{Var}(T_2)$, then the sample size n equals

1. 33
2. 69
3. 89
4. 93

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710179**
Option 1 ID : **916710713**
Option 2 ID : **916710714**
Option 3 ID : **916710715**
Option 4 ID : **916710716**
Status : **Not Answered**
Chosen Option : **--**

Q.44 Let $\gamma_R(t) = 2 + i + Re^{2\pi it}$ for $t \in [0,1]$ and $R = 1, 2$. Which of the following statements is true?

1. $\int_{\gamma_1} \tan(z)\mathrm{d}z = 2\pi i$
2. $\int_{\gamma_1} \tan(z)\mathrm{d}z = -2\pi i$
3. $\int_{\gamma_2} \tan(z)\mathrm{d}z = 2\pi i$
4. $\int_{\gamma_2} \tan(z)\mathrm{d}z = -2\pi i$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710156**
Option 1 ID : **916710621**
Option 2 ID : **916710622**
Option 3 ID : **916710623**
Option 4 ID : **916710624**
Status : **Answered**
Chosen Option : **4**

Q.45 Let B be a 4×4 positive-definite real symmetric matrix which is not the identity matrix. Consider the inner product on $\mathbb{R}^4$ given by $\langle v, w \rangle = v^\top Bw$, where $v^\top$ denotes the transpose of v. Which of the following statements is **FALSE**?

1. If v and w are eigenvectors of B for distinct eigenvalues, then $\langle v, w \rangle = 0$.
2. If v is an eigenvector of B and $w \neq 0$ is such that $\langle w, v \rangle = 0$, then w is an eigenvector of B.
3. For every subspace $W \subseteq \mathbb{R}^4$, $W + W^\perp = \mathbb{R}^4$, where $W^\perp = \{v \in \mathbb{R}^4 \mid \langle v, w \rangle = 0 \text{ for all } w \in W\}$.
4. If $W \subseteq \mathbb{R}^4$ is an eigenspace of B, then W has an orthonormal basis.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710151**
Option 1 ID : **916710601**
Option 2 ID : **916710602**
Option 3 ID : **916710603**
Option 4 ID : **916710604**
Status : **Answered**
Chosen Option : **2**

Q.46 Let $u(x,t)$ be the solution of the initial value problem

$$u_{tt} - u_{xx} = e^{-t}, \; x \in \mathbb{R}, \, t > 0,$$
$$u(x,0) = \cos x, \; u_t(x,0) = 0, \; x \in \mathbb{R}.$$

Then which of the following statements is true?

1. For each $x_0 \in \mathbb{R}$, $e^t u(x_0,t) \to 0$ as $t \to \infty$.
2. For each $x_0 \in \mathbb{R}$, $|u(x_0,t)| \to 0$ as $t \to \infty$.
3. For each $x_0 \in \mathbb{R}$, there exists a $t_0 > 0$ such that $u(x_0,t_0) \geq 4$.
4. For each $x_0 \in \mathbb{R}$, there exists a $t_0 > 0$ such that $u(x_0,t_0) \leq -4$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710168**
Option 1 ID : **916710669**
Option 2 ID : **916710670**
Option 3 ID : **916710671**
Option 4 ID : **916710672**
Status : **Not Answered**
Chosen Option : **--**

Q.47 Let $f : (0,\infty) \to (0,\infty)$ be a uniformly continuous function. Define $g : (0,\infty) \to (0,\infty)$ by

$$g(x) = \begin{cases} x^2 & \text{if } 0 < x \leq 1, \\ x & \text{if } x > 1. \end{cases}$$

Which of the following statements is true?

1. $f \circ g$ is uniformly continuous, but $g \circ f$ is not uniformly continuous.
2. $g \circ f$ is uniformly continuous, but $f \circ g$ is not uniformly continuous.
3. Neither $f \circ g$ nor $g \circ f$ is uniformly continuous.
4. Both $f \circ g$ and $g \circ f$ are uniformly continuous.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710145**
Option 1 ID : **916710577**
Option 2 ID : **916710578**
Option 3 ID : **916710579**
Option 4 ID : **916710580**
Status : **Answered**
Chosen Option : **4**

Q.48

If $p(x)$ is the interpolating polynomial for the data

x	-2	-1	0	1	2
y	-1	3	1	-1	3

then the value of $p\left(\frac{1}{2}\right)$ is

1. $\frac{-3}{8}$
2. $\frac{-5}{8}$
3. $\frac{5}{8}$
4. $\frac{3}{8}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710165**
Option 1 ID : **916710657**
Option 2 ID : **916710658**
Option 3 ID : **916710659**
Option 4 ID : **916710660**
Status : **Not Answered**
Chosen Option : **--**

Q.49 Let S be a subset of the open interval $(0,1)$ that consists of all the real numbers $\alpha \in (0,1)$ whose infinite decimal expansion $\alpha = 0.a_1a_2a_3\cdots$ is such that $a_i \in \{0,2,4\}$ for all $i \geq 1$. Which of the following statements is true?

1. There is a bijective map from $\mathbb{N}$ to S.
2. There is a surjective map from S onto $(0,1)$.
3. There is a bijective map from $\mathbb{N}$ to $(0,1)\setminus S$.
4. S is a countable set and $(0,1)\setminus S$ is uncountable.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710142**
Option 1 ID : **916710565**
Option 2 ID : **916710566**
Option 3 ID : **916710567**
Option 4 ID : **916710568**
Status : **Answered**
Chosen Option : **2**

Q.50 Consider the quadratic form

$$f(x,y,z) = \begin{bmatrix} x & y & z \end{bmatrix} A \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where A is an invertible 3×3 symmetric matrix over $\mathbb{Q}$.
Assume that there exists $(\alpha,\beta,\gamma) \in \mathbb{C}^3 \setminus \{(0,0,0)\}$ such that $f(\alpha,\beta,\gamma)=0$.
Which of the following statements is necessarily true?

1. There exists $(a,b,c) \in \mathbb{R}^3 \setminus \{(0,0,0)\}$ such that $f(a,b,c)=0$.
2. If there exists $(a,b,c) \in \mathbb{R}^3 \setminus \{(0,0,0)\}$ such that $f(a,b,c)=0$, then $(a,b,c) \in \mathbb{Q}^3$.
3. $\{(a,b,c) \in \mathbb{Z}^3 \mid f(a,b,c)=0\}$ is a finite set.
4. If there exists $(a,b,c) \in \mathbb{Q}^3 \setminus \{(0,0,0)\}$ such that $f(a,b,c)=0$, then A has a positive eigenvalue and a negative eigenvalue.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710152**
Option 1 ID : **916710605**
Option 2 ID : **916710606**
Option 3 ID : **916710607**
Option 4 ID : **916710608**
Status : **Not Answered**
Chosen Option : **--**

Q.51 Let X_1 and X_2 be a random sample from Uniform$[0, \theta]$ distribution, where $\theta > 0$. For testing the hypothesis

$$H_0 : \theta = 1 \text{ against } H_1 : \theta = 2,$$

consider a test which rejects H_0 if $X_1 + X_2 > \frac{4}{5}$. Then, the probability of type-I error is

1. $\frac{8}{25}$
2. $\frac{13}{25}$
3. $\frac{17}{25}$
4. $\frac{22}{25}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**916710175**
Option 1 ID :	**916710697**
Option 2 ID :	**916710698**
Option 3 ID :	**916710699**
Option 4 ID :	**916710700**
Status :	**Not Answered**
Chosen Option :	**--**

Q.52 Let $\{Y_n : n \geq 1\}$ be a sequence of independent and identically distributed random variables, where $Y_1 \sim$ Bernoulli$(\frac{1}{2})$. Define $Z = \sum_{n=1}^{\infty} \frac{4Y_n}{5^n}$. Then, which of the following statements is true?

1. $P(Z \geq \frac{3}{5}) = 0.5, P(Z = \frac{4}{25}) = 0$
2. $P(Z \geq \frac{3}{5}) = 0.6, P(Z = \frac{4}{25}) = 0$
3. $P(Z \geq \frac{3}{5}) = 0.7, P(Z = \frac{4}{25}) = 0.16$
4. $P(Z \geq \frac{3}{5}) = 0.8, P(Z = \frac{4}{25}) = 0.16$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**916710170**
Option 1 ID :	**916710677**
Option 2 ID :	**916710678**
Option 3 ID :	**916710679**
Option 4 ID :	**916710680**
Status :	**Not Answered**
Chosen Option :	**--**

Q.53 Let A be a 3×3 matrix over complex numbers with trace 1 and determinant 1. Suppose, further, that one of the eigenvalues of A is 1. Which of the following statements is necessarily true?

1. The characteristic polynomial of A has repeated roots.
2. Every eigenvalue of A has absolute value 1.
3. A does not have any eigenvalue on the imaginary axis.
4. A^2 is the identity matrix.

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**916710148**
Option 1 ID :	**916710589**
Option 2 ID :	**916710590**
Option 3 ID :	**916710591**
Option 4 ID :	**916710592**
Status :	**Answered**
Chosen Option :	**2**

Q.54 Let $X|\theta \sim \text{Uniform}[0, \theta]$ and θ has an Exponential distribution with mean λ, where $\lambda > 3$ is known. If the realized value of X is 2025, then the posterior mode equals

1. 0
2. 2025
3. $\frac{2025}{\lambda}$
4. 2025λ

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**916710176**
Option 1 ID :	**916710701**
Option 2 ID :	**916710702**
Option 3 ID :	**916710703**
Option 4 ID :	**916710704**
Status :	**Not Answered**
Chosen Option :	**--**

Q.55 Let X be a single sample from an absolutely continuous distribution with probability density function

$$f(x|\theta) = \begin{cases} \frac{2}{\theta^2}(\theta - x), & \text{if } 0 < x < \theta \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is unknown. Which of the following intervals is a 95% confidence interval for θ?

1. $\left(X, \frac{X}{1-\sqrt{0.95}}\right)$
2. $\left(X, \frac{X}{0.95}\right)$
3. $\left(X, \frac{X}{\sqrt{0.95}}\right)$
4. $(0.95X, X)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710173**
Option 1 ID : **916710689**
Option 2 ID : **916710690**
Option 3 ID : **916710691**
Option 4 ID : **916710692**
Status : **Not Answered**
Chosen Option : **--**

Q.56 Which of the following statements is necessarily true?

1. The set of all finite subsets of $\mathbb{Z}$ is uncountable.
2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous and one-one function. If $S \subseteq \mathbb{R}$ is a countably infinite set, then $f(S)$ is a countably infinite set.
3. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Then there exists an uncountable subset $T \subseteq \mathbb{R}$ such that $f(T) \subseteq \mathbb{R}$ is uncountable.
4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. If $R \subseteq \text{image}(f)$ is a countable set, then $f^{-1}(R)$ is countable.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710141**
Option 1 ID : **916710561**
Option 2 ID : **916710562**
Option 3 ID : **916710563**
Option 4 ID : **916710564**
Status : **Answered**
Chosen Option : **2**

Q.57 Consider the linear model

$$Y_1 = \beta_1 + \beta_2 + \epsilon_1$$
$$Y_2 = \beta_1 + 2\beta_2 + \epsilon_2$$
$$Y_3 = \beta_1 + c\beta_2 + \epsilon_3,$$

where $\beta_1, \beta_2 \in \mathbb{R}$ are unknown parameters, and the uncorrelated errors $\epsilon_i, i = 1, 2, 3$ have zero mean and finite variance $\sigma^2(> 0)$. The constant c is such that $\hat{\beta}_1$ and $\hat{\beta}_2$ are uncorrelated, where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the best linear unbiased estimators of β_1 and β_2, respectively. Which of the following statements is the correct option for $(\text{Var}(\hat{\beta}_1), \text{Var}(\hat{\beta}_2))$?

1. $\left(\frac{\sigma^2}{3}, \frac{\sigma^2}{14}\right)$
2. $(3\sigma^2, 14\sigma^2)$
3. $\left(\frac{\sigma^2}{14}, \frac{\sigma^2}{3}\right)$
4. $(14\sigma^2, 3\sigma^2)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710177**
Option 1 ID : **916710705**
Option 2 ID : **916710706**
Option 3 ID : **916710707**
Option 4 ID : **916710708**
Status : **Not Answered**
Chosen Option : **--**

Q.58 Suppose $u(x)$ is the solution of the integral equation

$$u(x) = 3 + \int_0^x (x-t)u(t)\,dt.$$

Then which of the following statements is true?

1. $u(\pi) = 2e^{\pi}$.
2. $u'(\pi) = e^{\pi}$.
3. $u(\pi) + u'(\pi) = 3e^{\pi}$.
4. $u(\pi) - u'(\pi) = e^{\pi}$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**916710167**
Option 1 ID :	**916710665**
Option 2 ID :	**916710666**
Option 3 ID :	**916710667**
Option 4 ID :	**916710668**
Status :	**Not Answered**
Chosen Option :	**--**

Q.59 Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables having Binomial$(3, \frac{1}{4})$ distribution. For $j = 1, 2, \ldots$, define

$$Y_j = \begin{cases} 1, & \text{if } X_j \leq \sqrt{5} \\ 0, & \text{otherwise.} \end{cases}$$

If $\frac{1}{n}\sum_{j=1}^{n} Y_j^2$ converges almost surely to a constant c as $n \to \infty$, then the value of c equals

1. $\frac{1}{64}$
2. $\frac{37}{64}$
3. $\frac{63}{64}$
4. $\frac{27}{32}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710169**
Option 1 ID : **916710673**
Option 2 ID : **916710674**
Option 3 ID : **916710675**
Option 4 ID : **916710676**
Status : **Not Answered**
Chosen Option : **--**

Q.60 For a finite group G, let $H_G = \{g \in G \mid g^{15} = e\}$. Which of the following statements is necessarily true?

1. There exists a finite group G such that $|H_G|$ is even.
2. There exists a finite group G such that $|H_G| = 4n + 1$ for some $n \geq 3$.
3. For every finite group G, there exists a non-negative integer n such that $|H_G| = 4n+1$.
4. For every finite group G, there exists a non-negative integer n such that $|H_G| = 4n+3$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **916710157**
Option 1 ID : **916710625**
Option 2 ID : **916710626**
Option 3 ID : **916710627**
Option 4 ID : **916710628**
Status : **Answered**
Chosen Option : **2**

Section : **PART-C**

Q.61 Let $X_1, X_2, \ldots, X_{15}$ be a random sample from an Exponential distribution with the probability density function

$$f(x|\sigma) = \begin{cases} \frac{1}{\sigma}\exp\left(-\frac{x}{\sigma}\right), & \text{if } x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

where the unknown parameter σ is positive. Let $\bar{X} = \frac{1}{15}\sum_{i=1}^{15} X_i$. Suppose that ϕ denotes the likelihood ratio test for testing $H_0 : \sigma \leq 1$ against $H_1 : \sigma > 1$ at level $\alpha = 0.1$. It is given that $\chi^2_{15,0.1} = 22.307, \chi^2_{15,0.9} = 8.547, \chi^2_{30,0.1} = 40.256, \chi^2_{30,0.9} = 20.599$, where $P(W > \chi^2_{n,\alpha}) = \alpha$ and $W \sim \chi^2_n$. Then which of the following statements are true?

1. If the observed value of $\bar{X}$ is 0.6, then ϕ does not reject H_0
2. If the observed value of $\bar{X}$ is 1.6, then ϕ rejects H_0
3. If the observed value of $\bar{X}$ is 1.3, then ϕ rejects H_0
4. If the observed value of $\bar{X}$ is 1.2, then ϕ does not reject H_0

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710231**
Option 1 ID : **916710921**
Option 2 ID : **916710922**
Option 3 ID : **916710923**
Option 4 ID : **916710924**
Status : **Not Answered**
Chosen Option : **--**

Q.62 Which of the following statements are true?

1. $x^2 + xy^2 - x - y + 1$ is irreducible in $\mathbb{Q}[x, y]$.
2. $x^3 + 100x^2 + 25x + 10$ is irreducible in $\mathbb{Z}[x]$.
3. $2x^2 + 3xy + y^2 + 3x + 2y + 1$ is irreducible in $\mathbb{Q}[x, y]$.
4. $x^4 + 4x^2 + 3$ is irreducible in $\mathbb{Z}[x]$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710207**
Option 1 ID : **916710825**
Option 2 ID : **916710826**
Option 3 ID : **916710827**
Option 4 ID : **916710828**
Status : **Answered**
Chosen Option : **2,4**

Q.63 Let $X_1, X_2, \ldots, X_n$, $(n \geq 2)$ be a random sample from an Exponential distribution with the probability density function

$$f(x|\mu,\sigma) = \begin{cases} \frac{1}{\sigma}\exp\left(\frac{\mu - x}{\sigma}\right), & \text{if } x > \mu, \\ 0, & \text{otherwise,} \end{cases}$$

where parameters μ and σ are unknown and positive. Let $\bar{X}_n$, S_n^2 and $X_{1:n}$ denote the sample mean, the sample variance and the sample smallest order statistic, respectively and let $\theta = \frac{\sigma}{\mu}$. Then which of the following statements are true?

1. $S_n (X_{1:n})^{-1}$ is a consistent estimator of θ
2. $(\bar{X}_n - X_{1:n})(X_{1:n})^{-1}$ is a consistent estimator of θ
3. $(\bar{X}_n - X_{1:n})(\bar{X}_n - S_n)^{-1}$ is a consistent estimator of θ
4. $S_n (\bar{X}_n)^{-1}$ is a consistent estimator of θ

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710230**
Option 1 ID : **916710917**
Option 2 ID : **916710918**
Option 3 ID : **916710919**
Option 4 ID : **916710920**
Status : **Not Answered**
Chosen Option : **--**

Q.64 Consider a dataset of n observations given by $A = \{x_1, x_2, ..., x_n\}$. Create a new dataset, given by $B = \{x_1, x_2, ..., x_n, -x_1, -x_2, ..., -x_n\}$. Which of the following statements are always true?

1. Mean of the observations in dataset B is 0
2. Median of the observations in dataset B is 0
3. Variance of the observations in dataset $B \geq$ Variance of the observations in dataset A (Here variance is calculated with divisor as the number of observations in the corresponding dataset)
4. Range of the observations in dataset $B \geq$ Range of the observations in dataset A

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710223**
Option 1 ID : **916710889**
Option 2 ID : **916710890**
Option 3 ID : **916710891**
Option 4 ID : **916710892**
Status : **Not Answered**
Chosen Option : **--**

Q.65 Let G be a group of order 8. Which of the following statements are necessarily true?

1. If there are no elements of order 4 in G, then G is abelian.
2. If there are exactly two elements of order 4 in G, then G is abelian.
3. If there are exactly six elements of order 4 in G, then G is abelian.
4. There is an element of order 4 in G.

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710205**
Option 1 ID :	**916710817**
Option 2 ID :	**916710818**
Option 3 ID :	**916710819**
Option 4 ID :	**916710820**
Status :	**Answered**
Chosen Option :	**1,2**

Q.66 Let $\alpha = \sum_{\ell=1}^{2025} \frac{a_\ell}{10^\ell}$ with $a_\ell \in \{0, 1, \ldots, 9\}$ for all $1 \leq \ell \leq 2025$ and $a_{2025} \neq 0$. Let $(\beta_n)_{n\geq 1}$ be a strictly increasing sequence of positive real numbers in $(0, 1)$ that converges to α. For each $n \geq 1$, write

$$\beta_n = \sum_{\ell=1}^{\infty} \frac{b_{n,\ell}}{10^\ell} \quad \text{with} \quad b_{n,\ell} \in \{0, 1, \ldots, 9\},$$

for the infinite decimal expansion of β_n. Which of the following statements are necessarily true?

1. There exists a positive integer N such that for all $n \geq N$, $b_{n,\ell} = a_\ell$ for all $1 \leq \ell \leq 2023$.
2. There exists a positive integer N such that for all $n \geq N$, $b_{n,2025} = a_{2025} - 1$.
3. β_n is rational for infinitely many n.
4. There exists a positive integer N such that for all $n \geq N$, $b_{n,2024} = a_{2024} - 1$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710181**
Option 1 ID :	**916710721**
Option 2 ID :	**916710722**
Option 3 ID :	**916710723**
Option 4 ID :	**916710724**
Status :	**Not Answered**
Chosen Option :	**--**

Q.67 Let $\mathcal{F}$ be the set of all functions $f : [0,\infty) \to [0,\infty)$ that are Riemann integrable on $[0,t]$ for all $t \in [0,\infty)$. Which of the following statements are necessarily true?

1. If $f \in \mathcal{F}$ is a continuous function and $f(n) = 0$ for all positive integers n, then $\lim_{x\to\infty} \int_0^x f(t)\,dt$ exists in $\mathbb{R}$.
2. If $f \in \mathcal{F}$ is uniformly continuous and $\lim_{x\to\infty} \int_0^x f(t)\,dt$ exists in $\mathbb{R}$, then $\lim_{x\to\infty} f(x) = 0$.
3. There exists a function $f \in \mathcal{F}$ such that $\lim_{x\to\infty} \int_0^x f(t)\,dt$ exists in $\mathbb{R}$ but $\lim_{x\to\infty} f(x) \neq 0$.
4. If $f \in \mathcal{F}$ is Lebesgue measurable and $\lim_{x\to\infty} \int_0^x f(t)\,dt = 0$, then $f = 0$ almost everywhere on $[0,\infty)$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710187**
Option 1 ID : **916710745**
Option 2 ID : **916710746**
Option 3 ID : **916710747**
Option 4 ID : **916710748**
Status : **Not Answered**
Chosen Option : **--**

Q.68 Consider the ordinary differential equation (ODE)

$$y'' + r(x)y = 0,$$

where

$$r(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0. \end{cases}$$

Then which of the following statements are true?

1. There exists a non-trivial solution ϕ of ODE, and $\alpha, \beta \in \mathbb{R}$, $\alpha < \beta$ such that ϕ has infinitely many zeros in $[\alpha, \beta]$.
2. There exists a non-trivial solution ϕ of ODE, and a sequence $\{x_n\}$ such that $x_n \to \infty$ and $\phi(x_n) = 0$ for all $n \in \mathbb{N}$.
3. If ϕ_1, ϕ_2 are two linearly independent solutions of ODE, then their Wronskian is a constant function on $\mathbb{R}$.
4. There exists a non-trivial solution of ODE which vanishes at most at one point in $(0,\infty)$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710212**
Option 1 ID : **916710845**
Option 2 ID : **916710846**
Option 3 ID : **916710847**
Option 4 ID : **916710848**
Status : **Not Answered**
Chosen Option : **--**

Q.69 Consider $X = \prod_{c \in (0,\infty)} [0, c]$ together with the product topology, where $[0, c]$ is equipped with the euclidean topology. Which of the following statements are necessarily true?

1. X is metrizable.
2. X is compact.
3. X is Hausdorff.
4. X is connected.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710190**
Option 1 ID : **916710757**
Option 2 ID : **916710758**
Option 3 ID : **916710759**
Option 4 ID : **916710760**
Status : **Answered**
Chosen Option : **1,2,3,4**

Q.70 Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from Uniform$\left[-\frac{3\theta}{2}, \frac{\theta}{2}\right]$ distribution, where $\theta > 0$ is an unknown parameter. Let $\bar{X}, X_{(1)}$ and $X_{(n)}$ denote respectively the mean, the smallest order statistic and the largest order statistic of the sample. Then which of the following statements are true?

1. The method of moments estimator of θ is $-2\bar{X}$
2. The method of moments estimator of θ is $2\bar{X}$
3. The maximum likelihood estimator of θ is $\min\left\{-\frac{2X_{(1)}}{3}, 2X_{(n)}\right\}$
4. The maximum likelihood estimator of θ is $\max\left\{-\frac{2X_{(1)}}{3}, 2X_{(n)}\right\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710229**
Option 1 ID : **916710913**
Option 2 ID : **916710914**
Option 3 ID : **916710915**
Option 4 ID : **916710916**
Status : **Not Answered**
Chosen Option : **--**

Q.71 Let τ_{E} denote the euclidean topology on $\mathbb{R}$. Which of the following statements are necessarily true?

1. $(\mathbb{R}, \tau_{\mathrm{E}})$ is a normal space.
2. Let τ be a topology on $\mathbb{R}$. If the identity function of $\mathbb{R}$ is a continuous map from $(\mathbb{R}, \tau_{\mathrm{E}})$ to $(\mathbb{R}, \tau)$, then $(\mathbb{R}, \tau)$ is a regular space.
3. Let τ be a topology on $\mathbb{R}$. If the identity function of $\mathbb{R}$ is a continuous map from $(\mathbb{R}, \tau)$ to $(\mathbb{R}, \tau_{\mathrm{E}})$, then $(\mathbb{R}, \tau)$ is a Hausdorff space.
4. $\mathbb{R}$ is a regular space in the finite-complement topology.

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MSQ**

Question ID : **916710210**

Option 1 ID : **916710837**

Option 2 ID : **916710838**

Option 3 ID : **916710839**

Option 4 ID : **916710840**

Status : **Answered**

Chosen Option : **1,2,3,4**

Q.72 Let $y(x)$ be the solution of the integral equation (IE)

$$y(x) = e^x + e + 1 + \frac{1}{3}\int_0^1 y(t)\,dt,$$

and $R\left(x, t, \frac{1}{3}\right)$ be the resolvent kernel associated to IE.

Then which of the following statements are true?

1. $R\left(0, 1, \frac{1}{3}\right) = \frac{3}{2}$
2. $R\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}\right) = \frac{2}{3}$
3. $y(1) = 3e + 1$
4. $y(1) = e + 3$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710220**
Option 1 ID : **916710877**
Option 2 ID : **916710878**
Option 3 ID : **916710879**
Option 4 ID : **916710880**
Status : **Not Answered**
Chosen Option : **--**

Q.73 Suppose $X|\theta \sim \text{Binomial}(7, \theta)$, $0 < \theta < 1$, and the prior distribution of θ is $\text{Beta}(\alpha, \beta)$ where $\alpha > 0$ and $\beta > 0$ are known. Then which of the following statements MAY NOT be true?

1. Posterior mean of θ given $X = 2$ is less than the prior mean
2. Posterior mean of θ given $X = 3$ is greater than the prior mean
3. Posterior mean of θ given $X = 4$ is not equal to the prior mean
4. Posterior mean of θ given $X = 5$ is equal to the prior mean

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710234**
Option 1 ID : **916710933**
Option 2 ID : **916710934**
Option 3 ID : **916710935**
Option 4 ID : **916710936**
Status : **Not Answered**
Chosen Option : **--**

Q.74 Let $A \in M_4(\mathbb{C})$ be such that $A^2 = I$ and $\text{Trace}(A) = 0$. Which of the following statements are necessarily true?

1. The set $\{B \in M_4(\mathbb{C}) \mid AB + BA = 0\}$ is an 8-dimensional $\mathbb{C}$-vector space.
2. The set $\{B \in M_4(\mathbb{C}) \mid AB - BA = 0\}$ is an 8-dimensional $\mathbb{C}$-vector space.
3. There exists $B \in M_4(\mathbb{C})$ such that $AB + BA = 0$, $B^2 = I$ and $\text{Trace}(B) = 0$.
4. If $B \in M_4(\mathbb{C})$ is such that $AB + BA = 0$, then B is diagonalisable.

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710195**
Option 1 ID : **916710777**
Option 2 ID : **916710778**
Option 3 ID : **916710779**
Option 4 ID : **916710780**
Status : **Not Answered**
Chosen Option : **--**

Q.75 Consider an $M/M/2$ queuing system with the birth rate $\lambda = 4$ per minute, the death rate $\mu = 1$ per minute, and the total capacity of 3 customers (including the ones that are being served). Let p_0 and p_3 denote the long-run probabilities that the system will be empty (i.e. without customers) and will be blocked (i.e. full), respectively. Which of the following statements are true?

1. $p_0 + p_3 = \frac{17}{29}$
2. $p_0 > p_3$
3. $\frac{p_3}{p_0} = 2$
4. $p_3 - p_0 = \frac{15}{29}$

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710240**
Option 1 ID : **916710957**
Option 2 ID : **916710958**
Option 3 ID : **916710959**
Option 4 ID : **916710960**
Status : **Not Answered**
Chosen Option : **--**

Q.76 For $n \geq 3$, consider the space $M_n(\mathbb{C})$ of $n \times n$ complex matrices endowed with the inner product

$$\langle A, B \rangle = \text{Trace}(A^* B).$$

For $0 \leq k \leq n$, let

$$W_k = \left\{ A \in M_n(\mathbb{C}) : \langle A, B \rangle = 0 \text{ for all } B \in M_n(\mathbb{C}) \text{ with rank } k \right\}.$$

Which of the following statements are necessarily true?

1. $W_0 = \{0\}$
2. $W_1 = \{0\}$
3. $W_2 = \{0\}$
4. $W_n = \{0\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710198**
Option 1 ID : **916710789**
Option 2 ID : **916710790**
Option 3 ID : **916710791**
Option 4 ID : **916710792**
Status : **Not Answered**
Chosen Option : **--**

Q.77 There are six persons. On each day of the year 2024 at least one of them borrowed money from another. A person P is labeled *troublesome* in a calendar month M of 2024 if P borrowed from the same person at least twice during the month M. Which of the following statements are necessarily true?

1. In every calendar month of 2024, at least one person was troublesome.
2. In any two consecutive calendar months of 2024, at least one person was troublesome in at least one of the two calendar months.
3. At least one person was troublesome in two calendar months of 2024.
4. At least one person was troublesome in January 2024.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710204**
Option 1 ID : **916710813**
Option 2 ID : **916710814**
Option 3 ID : **916710815**
Option 4 ID : **916710816**
Status : **Not Answered**
Chosen Option : **--**

Q.78 Let V be a real vector space. Suppose that $\{u, v, w, x, y\} \subseteq V$ is a spanning set of V and that $\{u, v, x, y\}$ is linearly independent. Which of the following statements are necessarily true?

1. The dimension of V is 4 or 5.
2. If $2u - 3v + 5w = 0$, then $\{v, w, x, y\}$ is a basis of V.
3. If $u - 6v + 7w = 0$, then the span of $\{v, w, x\}$ is a 2-dimensional vector space.
4. If $u + w = v + x$, then $\{u, v, w, y\}$ is a basis of V.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710191**
Option 1 ID : **916710761**
Option 2 ID : **916710762**
Option 3 ID : **916710763**
Option 4 ID : **916710764**
Status : **Answered**
Chosen Option : **1,2,3,4**

Q.79 Suppose $y(x)$ is the extremal of the variational problem

$$J(y)=\int_0^1 x^2\,(y')^2\,dx$$

subject to

$$y(0)=0,\ y(1)=1,\ \int_0^1 y^2dx=\frac{1}{7}.$$

Then which of the following statements are true?

1. $y'\left(\frac{1}{2}\right)=\frac{3}{4}$
2. $y'\left(\frac{1}{\sqrt{3}}\right)=1$
3. $y'\left(\frac{1}{3}\right)=\frac{1}{3}$
4. $y'\left(\frac{1}{4}\right)=\frac{1}{2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710218**
Option 1 ID : **916710869**
Option 2 ID : **916710870**
Option 3 ID : **916710871**
Option 4 ID : **916710872**
Status : **Not Answered**
Chosen Option : **--**

Q.80 Let $X_1, X_2, \ldots, X_{20}$ be a random sample from $N_{12}(\mu, \Sigma)$, where $\mu \in \mathbb{R}^{12}$ and Σ is positive definite matrix. Suppose $\bar{X} = \frac{1}{20}\sum_{i=1}^{20} X_i$ and $S = \frac{1}{19}\sum_{i=1}^{20}(X_i - \bar{X})(X_i - \bar{X})^T$. Then which of the following statements are true?

1. $19(\bar{X}^T S \bar{X})(\bar{X}^T \Sigma \bar{X})^{-1} \sim \chi^2_{19}$
2. $E(S^{-1}) = \dfrac{19}{6}\Sigma^{-1}$
3. $S \sim W_{12}(19, 19\Sigma)$
4. $\text{trace}(19\Sigma^{-1}S) \sim \chi^2_{228}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710237**
Option 1 ID : **916710945**
Option 2 ID : **916710946**
Option 3 ID : **916710947**
Option 4 ID : **916710948**
Status : **Not Answered**
Chosen Option : **--**

Q.81 Let $u(x,t)$ be the solution of the initial-boundary value problem

$$u_{tt} - u_{xx} + u_t = 0,\ 0 < x < \pi,\ t > 0,$$
$$u(0,t) = 0,\ u(\pi,t) = 0,\ t > 0,$$
$$u(x,0) = \sin x,\ u_t(x,0) = 0,\ 0 < x < \pi.$$

For $t \geq 0$, define

$$E(t) = \int_0^\pi \left(u_t^2 + u_x^2\right)\, dx.$$

Then which of the following statements are true?

1. $E(t) \geq \pi$ for all $t > 0$.
2. $E(t) \leq \pi$ for all $t > 0$.
3. $E(t) \geq \frac{\pi}{2}$ for all $t > 0$.
4. $E(t) \leq \frac{\pi}{2}$ for all $t > 0$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710215**
Option 1 ID : **916710857**
Option 2 ID : **916710858**
Option 3 ID : **916710859**
Option 4 ID : **916710860**
Status : **Not Answered**
Chosen Option : **--**

Q.82 Let S be the set of all 2×2 matrices A such that the iterative sequence generated by the Gauss-Seidel method converges for every initial guess, when employed to solve the system of equations $A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Then which of the following statements are true?

1. $\begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix} \in S$
2. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \in S$
3. $\begin{pmatrix} 1 & 5 \\ 1 & 10 \end{pmatrix} \in S$
4. $\begin{pmatrix} -5 & 2 \\ 1 & -4 \end{pmatrix} \in S$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710216**
Option 1 ID : **916710861**
Option 2 ID : **916710862**
Option 3 ID : **916710863**
Option 4 ID : **916710864**
Status : **Not Answered**
Chosen Option : **--**

Q.83 Consider a commutative ring R with unity with at least five elements such that for any two elements $a, b \in R$, there exists $c \in R$ such that $a = bc$ or $b = ac$. Which of the following statements are necessarily true?

1. R has a unique maximal ideal.
2. Every finitely generated ideal of R is principal.
3. R is an integral domain.
4. R is a euclidean domain.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710206**
Option 1 ID : **916710821**
Option 2 ID : **916710822**
Option 3 ID : **916710823**
Option 4 ID : **916710824**
Status : **Answered**
Chosen Option : **1,2,3,4**

Q.84 Consider the following statements:

(P) The initial value problem

$$y' + y = e^{-x^2}, \ y(0) = 0$$

has a Taylor series solution about the point $x = 0$.

(Q) The initial value problem

$$y' + y = r(x),\ y(0) = 0, \text{ where } r(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

has a Taylor series solution about the point $x = 0$.

Then which of the following statements are true?

1. (P) is true.
2. (P) is FALSE.
3. (Q) is true.
4. (Q) is FALSE.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710213**
Option 1 ID : **916710849**
Option 2 ID : **916710850**
Option 3 ID : **916710851**
Option 4 ID : **916710852**
Status : **Not Answered**
Chosen Option : **--**

Q.85 Let $F : \mathbb{R}^3 \to \mathbb{R}$ be a continuously differentiable function such that

$$F(0,0,0) = -1, \quad \frac{\partial F}{\partial x}(0,0,0) = 2, \quad \frac{\partial F}{\partial y}(0,0,0) = 0, \text{ and } \frac{\partial F}{\partial z}(0,0,0) = 3.$$

For $\epsilon > 0$, define $\Omega_\epsilon = (-\epsilon, \epsilon) \times (-\epsilon, \epsilon) \subset \mathbb{R}^2$. Which of the following statements are necessarily true?

1. There exist $\epsilon > 0, \delta > 0$ and a continuously differentiable function $g : \Omega_\epsilon \to (-\delta, \delta)$ such that $g(0,0) = 0$ and $F(x, y, g(x,y)) = -1$ for all $(x,y) \in \Omega_\epsilon$.
2. There exist $\epsilon > 0, \delta > 0$ and a continuously differentiable function $h : \Omega_\epsilon \to (-\delta, \delta)$ such that $h(0,0) = 0$ and $F(x, h(x,z), z) = -1$ for all $(x,z) \in \Omega_\epsilon$.
3. There exist $\epsilon > 0, \delta > 0$ and continuously differentiable functions $k_1, k_2 : (-\epsilon, \epsilon) \to (-\delta, \delta)$ such that $k_1(0) = k_2(0) = 0$ and $F(x, k_1(x), k_2(x)) = -1$ for all $x \in (-\epsilon, \epsilon)$.
4. There exist $\epsilon > 0, \delta > 0$ and continuously differentiable functions $j_1, j_2 : (-\epsilon, \epsilon) \to (-\delta, \delta)$ such that $j_1(0) = j_2(0) = 0$ and $F(j_1(z), j_2(z), z) = -1$ for all $z \in (-\epsilon, \epsilon)$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710189**
Option 1 ID : **916710753**
Option 2 ID : **916710754**
Option 3 ID : **916710755**
Option 4 ID : **916710756**
Status : **Not Answered**
Chosen Option : **--**

Q.86 For each positive integer n, let $f_n : [-1, 1] \to \mathbb{R}$ be given by

$$f_n(x) = \begin{cases} -1, & \text{if } -1 \le x \le -\frac{1}{n}, \\ nx, & \text{if } -\frac{1}{n} < x < \frac{1}{n}, \\ 1, & \text{if } \frac{1}{n} \le x \le 1. \end{cases}$$

On which of the following intervals does the sequence $\{f_n\}_{n \ge 1}$ converge uniformly?

1. $[0, 1]$
2. $(0, 1]$
3. $(10^{-2025}, 1)$
4. $(-10^{-2025}, 10^{-2025}]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710185**
Option 1 ID : **916710737**
Option 2 ID : **916710738**
Option 3 ID : **916710739**
Option 4 ID : **916710740**
Status : **Answered**
Chosen Option : **3,4**

Q.87 Let (X, d) be a metric space. For a non-empty subset A of X, and $x \in X$, define

$$d(x, A) = \inf_{a \in A} d(x, a).$$

Which of the following statements are necessarily true?

1. For all $x, y \in X$ and every non-empty subset A of X, we have
$$d(x, A) - d(y, A) \leq d(x, y).$$
2. For every non-empty subset A of X, the function $x \mapsto d(x, A)$ is uniformly continuous on X.
3. A non-empty subset A of X is closed if and only if $d(x, A) > 0$ for all $x \in X \setminus A$.
4. If X is compact and $C_1, \ldots, C_k$ are non-empty closed sets of X such that $\bigcap_{1 \leq i \leq k} C_i = \emptyset$, then the minimum value of the function $x \mapsto \sum_{1 \leq i \leq k} d(x, C_i)$ on X is 0.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710209**
Option 1 ID : **916710833**
Option 2 ID : **916710834**
Option 3 ID : **916710835**
Option 4 ID : **916710836**
Status : **Answered**
Chosen Option : **1,2,3,4**

Q.88 Consider the following subset of real numbers

$$A = \left\{ (1 + (-1)^n)n - \frac{1}{n} \ : \ n \text{ is a positive integer} \right\}.$$

Which of the following statements are true?

1. A is bounded below but not bounded above.
2. A is bounded above but not bounded below.
3. $\inf A = -1$
4. $\sup A = 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710182**
Option 1 ID : **916710725**
Option 2 ID : **916710726**
Option 3 ID : **916710727**
Option 4 ID : **916710728**
Status : **Answered**
Chosen Option : **1,3**

Q.89 Let $f : (0, \infty) \to (0, \infty)$ be the function defined by

$$f(x) = \int_0^x \frac{\sqrt{t}}{1+t^2}\, dt,$$

where $\sqrt{t}$ denotes the positive square root for $t > 0$. Which of the following statements are true?

1. f is a uniformly continuous function.
2. f is a bounded function.
3. There exists $x \in (0, \infty)$ such that $f(x) = 0$.
4. The derivative of f is continuous.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710183**
Option 1 ID : **916710729**
Option 2 ID : **916710730**
Option 3 ID : **916710731**
Option 4 ID : **916710732**
Status : **Answered**
Chosen Option : **1,2,3,4**

Q.90 Consider the following real-valued functions F_1 and F_2 defined on $\mathbb{R}$, given by

$$F_1(x) = \begin{cases} 1, & \text{if } x \geq 1, \\ 0, & \text{otherwise}, \end{cases}$$

$$F_2(x) = \begin{cases} \int_0^x \exp(-t)\, dt, & \text{if } x \geq 0, \\ 0, & \text{otherwise}. \end{cases}$$

Define another function $F : \mathbb{R} \to [0,1]$ by $F(x) = \frac{2}{3}F_1(x) + \frac{1}{3}F_2(x)$ for all $x \in \mathbb{R}$. Which of the following statements are true?

1. F is non-decreasing on $\mathbb{R}$
2. $\lim_{x\to\infty} F(x) = 1$
3. F is left-continuous on $\mathbb{R}$
4. F is right-continuous on $\mathbb{R}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710224**
Option 1 ID : **916710893**
Option 2 ID : **916710894**
Option 3 ID : **916710895**
Option 4 ID : **916710896**
Status : **Not Answered**
Chosen Option : **--**

Q.91 Consider the following 2^5-factorial design with 8 blocks.

Block	Treatment combinations
1	$(1),\ acd,\ bce,\ abde$
2	$e,\ acde,\ bc,\ abd$
3	$a,\ cd,\ abce,\ bde$
4	$ab,\ bcd,\ ace,\ de$
5	$b,\ abcd,\ ce,\ ade$
6	$ac,\ d,\ abe,\ bcde$
7	$abc,\ bd,\ ae,\ cde$
8	$c,\ ad,\ be,\ abcde$

Which of the following statements are true?

1. ABC is confounded with blocks
2. BCD is confounded with blocks
3. CDE is confounded with blocks
4. $ABDE$ is confounded with blocks

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710238**
Option 1 ID : **916710949**
Option 2 ID : **916710950**
Option 3 ID : **916710951**
Option 4 ID : **916710952**
Status : **Not Answered**
Chosen Option : **--**

Q.92 Let $\mathbb{N}$ denote the set of all positive integers. For $n \in \mathbb{N}$, let $f_n : [0,1] \to \mathbb{R}$ be given by

$$f_n(x) = \begin{cases} n(1-nx), & \text{if } 0 \leq x \leq \frac{1}{n}, \\ 0, & \text{if } x > \frac{1}{n}. \end{cases}$$

Which of the following statements are necessarily true?

1. The sequence of functions $\{f_n\}_{n\geq 1}$ is uniformly bounded.
2. The sequence of functions $\{f_n\}_{n\geq 1}$ does not converge pointwise.
3. The sequence of functions $\{f_n\}_{n\geq 1}$ converges uniformly to the constant function 0.
4. The set $\{f_n : n \in \mathbb{N}\}$ is compact in $\mathcal{C}[0,1]$, where $\mathcal{C}[0,1]$ denotes the space of all real valued continuous functions on $[0,1]$ equipped with the supremum norm.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710184**
Option 1 ID : **916710733**
Option 2 ID : **916710734**
Option 3 ID : **916710735**
Option 4 ID : **916710736**
Status : **Answered**
Chosen Option : **1,3,4**

Q.93 Let f be an entire function. Consider the function g given by

$$g(z) = f(z) - \frac{1}{z}$$

for $z \in \mathbb{C} \setminus \{0\}$. Which of the following statements are necessarily true?

1. The function g has a pole at 0.
2. If $g(\alpha) = 0$, then $|\alpha| \neq 1$.
3. The function g has only finitely many zeros.
4. $\max_{|z|=1} |g(z)| \geq 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710201**
Option 1 ID : **916710801**
Option 2 ID : **916710802**
Option 3 ID : **916710803**
Option 4 ID : **916710804**
Status : **Answered**
Chosen Option : **1,2,3,4**

Q.94 Let ℓ^2 denote the vector space of all square summable sequences $\{a_n\}_{n\geq 1}$ of real numbers with the inner product given by

$$\langle \{a_n\}, \{b_n\} \rangle = \sum_{n=1}^{\infty} a_n b_n.$$

Let

$$H = \left\{ \{a_n\} \in \ell^2 : |a_n| \leq \frac{1}{n} \text{ for all positive integers } n \right\}.$$

Which of the following statements are true?

1. H contains an orthonormal basis of ℓ^2.
2. H is a linear subspace of ℓ^2.
3. H is a bounded subset of ℓ^2.
4. H is a convex subset of ℓ^2.

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710197**
Option 1 ID :	**916710785**
Option 2 ID :	**916710786**
Option 3 ID :	**916710787**
Option 4 ID :	**916710788**
Status :	**Not Answered**
Chosen Option :	**--**

Q.95 Let $\{X_n : n \geq 1\}$ be a sequence of independent and identically distributed random variables, where X_1 has an Exponential distribution with mean 1. Define

$$T_n = \max\{X_1, X_2, \ldots, X_n\} - \ln n, \quad n \geq 1.$$

Suppose $T_n \xrightarrow{d} Y$ as $n \to \infty$. Then which of the following statements are true?

1. Y has a Double Exponential distribution with location parameter $\ln(\ln 2)$ and scale parameter 1
2. Median of $Y = \ln(\ln 2)$
3. $P(Y \leq 0) = e^{-1}$
4. The derivative of the cumulative distribution function of Y at $\ln 3$ is $e^{-\frac{1}{3}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710227**
Option 1 ID :	**916710905**
Option 2 ID :	**916710906**
Option 3 ID :	**916710907**
Option 4 ID :	**916710908**
Status :	**Not Answered**
Chosen Option :	**--**

Q.96 Let $f : [0,1] \to [0,1]$ be a monotonically increasing function, that is, $a \le b$ implies that $f(a) \le f(b)$. For any $\alpha \in (0,1)$, let

$$L_\alpha^+ = \lim_{x\to\alpha^+} f(x) \quad \text{and} \quad L_\alpha^- = \lim_{x\to\alpha^-} f(x)$$

denote the right hand and the left hand limits respectively, provided they exist. For $\alpha \in (0,1)$, if L_α^+ and L_α^- exist, then define

$$U_\alpha = \begin{cases} (L_\alpha^-, L_\alpha^+) & \text{if } L_\alpha^- < L_\alpha^+, \\ \emptyset & \text{if } L_\alpha^+ \le L_\alpha^-. \end{cases}$$

Which of the following statements are true?

1. L_α^+ and L_α^- exist for every $\alpha \in (0,1)$.
2. If f is surjective, then f is continuous.
3. If f is Riemann integrable, then f is continuous.
4. If the left and right hand limits exist at $\alpha, \beta \in (0,1)$, $\alpha \ne \beta$, then $U_\alpha \cap U_\beta = \emptyset$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710186**
Option 1 ID : **916710741**
Option 2 ID : **916710742**
Option 3 ID : **916710743**
Option 4 ID : **916710744**
Status : **Answered**
Chosen Option : **1,4**

Q.97 Suppose that the transition probability matrix of a homogeneous Markov chain with state space $\{1,2,3,4\}$ is given by

$$P = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{8} & 0 & \frac{7}{8} & 0 \\ 0 & 0 & \frac{1}{9} & \frac{8}{9} \end{pmatrix}.$$

Which of the following statements are true?

1. State 2 is a positive recurrent state
2. Mean recurrence time of state 1 is $\frac{7}{4}$
3. State 4 is a transient state
4. State 3 is aperiodic and ergodic

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710225**
Option 1 ID : **916710897**
Option 2 ID : **916710898**
Option 3 ID : **916710899**
Option 4 ID : **916710900**
Status : **Not Answered**
Chosen Option : **--**

Q.98 Let X and Y be two independent random variables such that the moment generating functions of X and Y are

$$M_X(t) = e^{3(e^t - 1)}, \quad t \in \mathbb{R},$$

and

$$M_Y(t) = \left(\tfrac{1}{2}e^{-3t} + \tfrac{1}{2}e^{3t}\right)^2, \quad t \in \mathbb{R},$$

respectively. Then which of the following statements are true?

1. $P(XY = 0) = \dfrac{1 + e^{-3}}{2}$
2. $E(X + Y) = 3$
3. $\text{Var}(X + Y) = 21$
4. $\text{Cov}(X + Y, X - Y) = 0$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710228**
Option 1 ID : **916710909**
Option 2 ID : **916710910**
Option 3 ID : **916710911**
Option 4 ID : **916710912**
Status : **Not Answered**
Chosen Option : **--**

Q.99 Let A be a non-zero 3×3 matrix with integer entries. Let $\lambda_i \in \mathbb{C}, 1 \le i \le 3$ be all the eigenvalues of A (not necessarily distinct). Which of the following statements are necessarily true?

1. There exists a cubic polynomial $f(X) \in \mathbb{Q}[X]$ such that $f(\lambda_i) = 0$ for all $1 \le i \le 3$.
2. There exists a quadratic polynomial $f(X) \in \mathbb{Q}[X]$ such that $f(\lambda_i) = 0$ for all $1 \le i \le 3$.
3. If $f(X) \in \mathbb{Q}[X]$ is such that $f(\lambda_i) = 0$ for all $1 \le i \le 3$, then $f(A) = 0$.
4. If $f(X) \in \mathbb{Q}[X]$ is a cubic polynomial such that $f(\lambda_i) = 0$ for all $1 \le i \le 3$, then $f(A) = 0$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710194**
Option 1 ID : **916710773**
Option 2 ID : **916710774**
Option 3 ID : **916710775**
Option 4 ID : **916710776**
Status : **Answered**
Chosen Option : **3,4**

Q.100 A mechanical system is described using generalized position q and generalized momentum p. Let Q and P denote new generalized position and generalized momentum variables respectively, generated by the generating function $F(q,P) = q^2e^P$, and Q,P are canonical coordinates. Let $G(p,Q)$ be a function such that $G(2,e) = 0$, and it generates the same canonical coordinates Q,P.

Then which of the following statements are true?

1. $G(p,Q) = -Q\left[1 + \log_e\left(\frac{p^2}{4Q}\right)\right]$
2. $G(p,Q) = -pQ\left[1 + \log_e\left(\frac{Q^2}{4p}\right)\right]$
3. $p = 2qe^P,\ Q = q^2e^P$
4. $p = -2qe^P,\ Q = -q^2e^P$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710222**
Option 1 ID : **916710885**
Option 2 ID : **916710886**
Option 3 ID : **916710887**
Option 4 ID : **916710888**
Status : **Not Answered**
Chosen Option : **--**

Q.101 Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and $f : \mathbb{D} \to \mathbb{D}$ be a holomorphic function which satisfies $f(-1/2) = 0$. Which of the following statements are necessarily true?

1. $|f(-1/5)| \leq 1/5$
2. $|f(-1/5)| \leq 1/3$
3. $|f'(-1/2)| \leq 1/2$
4. $|f'(-1/2)| \leq 4/3$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710200**
Option 1 ID : **916710797**
Option 2 ID : **916710798**
Option 3 ID : **916710799**
Option 4 ID : **916710800**
Status : **Not Answered**
Chosen Option : **--**

Q.102 For $t \in [0, 2\pi]$, let $\gamma_1(t) = e^{it}$ and $\gamma_2(t) = 1 + i + e^{it}$. Which of the following statements are true?

1. $\int_{\gamma_1} \frac{dz}{z \sin z} = 0$
2. $\int_{\gamma_1} \frac{dz}{z \sin z} = 2\pi i$
3. $\int_{\gamma_2} \frac{dz}{z \sin z} = 2\pi i$
4. $\int_{\gamma_2} \frac{dz}{z \sin z} = 0$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710202**
Option 1 ID : **916710805**
Option 2 ID : **916710806**
Option 3 ID : **916710807**
Option 4 ID : **916710808**
Status : **Answered**
Chosen Option : **1,4**

Q.103 Let V be a finite-dimensional $\mathbb{R}$-vector space and $T : V \to V$ a linear operator such that T^2 is diagonalizable over $\mathbb{R}$. Which of the following statements are necessarily true?

1. If T is not diagonalizable over $\mathbb{R}$, then T^2 has an eigenvalue ≤ 0.
2. If T^2 has only negative eigenvalues, then $\dim V$ is an even integer.
3. If T^2 has only non-negative eigenvalues, then T is diagonalizable over $\mathbb{R}$.
4. For each non-zero $v \in V$, $\{v, Tv, T^2v\}$ is linearly dependent.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710193**
Option 1 ID : **916710769**
Option 2 ID : **916710770**
Option 3 ID : **916710771**
Option 4 ID : **916710772**
Status : **Not Answered**
Chosen Option : **--**

Q.104 Let $X_1, X_2, \ldots, X_7$ be a random sample drawn from a continuous distribution with unknown unique median M. The null hypothesis $H_0 : M = 2$ is tested against the alternative $H_1 : M > 2$ at level of significance 0.05 using the right-tailed test based on the Sign test statistic K, which is the number of observations in the sample greater than 2. If the observed sample is $-3, -6, 1, 9, 4, 10, 12,$ which of the following statements are true?

1. H_0 is rejected
2. The p-value of the test is greater than 0.01
3. Under H_0, $E(K) = 3.5$
4. If $X_{(i)}$ denotes the i-th smallest observation of the sample, then $[X_{(2)}, X_{(6)}]$ is a confidence interval for M with confidence coefficient at least 0.95

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710233**
Option 1 ID : **916710929**
Option 2 ID : **916710930**
Option 3 ID : **916710931**
Option 4 ID : **916710932**
Status : **Not Answered**
Chosen Option : **--**

Q.105 Suppose $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the differential equation

$$x^2y'' + \frac{(1+\sin x)x}{2}y' - \frac{3}{2}(\cos x)y = 0, \ x > 0$$

satisfying $y_2(0) = 0$.

Then which of the following statements are true?

1. $\lim_{x \to 0^+} \frac{y_2(x)}{x^2 y_1(x)}$ exists.
2. $\lim_{x \to 0^+} \frac{y_2(x)}{x y_1(x)}$ exists.
3. $\lim_{x \to 0^+} \frac{x y_1(x)}{y_2(x)}$ does NOT exist.
4. $\lim_{x \to 0^+} \frac{x^2 y_1(x)}{y_2(x)}$ exists.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710211**
Option 1 ID : **916710841**
Option 2 ID : **916710842**
Option 3 ID : **916710843**
Option 4 ID : **916710844**
Status : **Not Answered**
Chosen Option : **--**

Q.106 Suppose two fair dice are thrown independently at random. Let X and Y be the numbers on the upper face of the first die and that of the second die, respectively. Then which of the following statements are true?

1. $P(X-Y=0 \mid X+Y=2) = P(X-Y=0 \mid X+Y=12)$
2. $E\left(\frac{X-Y}{X+Y}\right) = 0$
3. $\text{Cov}(X+Y, X-Y) = 0$
4. $(X+Y)$ and $(X-Y)$ are independent

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710226**
Option 1 ID :	**916710901**
Option 2 ID :	**916710902**
Option 3 ID :	**916710903**
Option 4 ID :	**916710904**
Status :	**Not Answered**
Chosen Option :	**--**

Q.107 Let V be a 4-dimensional complex vector space and A a linear operator on V. Which of the following statements are necessarily true?

1. There exist $\lambda \in \mathbb{C}$ and a non-zero $v \in V$ such that $Av = \lambda v$.
2. There exist $\lambda, \mu \in \mathbb{C}$ and linearly independent vectors $v, w \in V$ such that $Av = \lambda v$ and $Aw = \mu w$.
3. There exist $\lambda, \mu, \delta \in \mathbb{C}$ and linearly independent vectors $v, w \in V$ such that $Av = \lambda v$ and $Aw = \mu v + \delta w$.
4. There exists a three-dimensional subspace W such that $Aw \in W$ for all $w \in W$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710196**
Option 1 ID :	**916710781**
Option 2 ID :	**916710782**
Option 3 ID :	**916710783**
Option 4 ID :	**916710784**
Status :	**Answered**
Chosen Option :	**1,4**

Q.108 Let $p \geq 3$ be a prime number and $f(x) \in \mathbb{Q}[x]$ an irreducible polynomial of degree p. Suppose that $a_1, \ldots, a_p \in \mathbb{C}$ are the roots of f and that

$$a_1 \notin \mathbb{R},\ a_2 \notin \mathbb{R},\ \text{and } a_i \in \mathbb{R} \text{ for all } 3 \leq i \leq p.$$

Let $K = \mathbb{Q}(a_1, \ldots, a_p)$ be the subfield of $\mathbb{C}$ generated by the roots of f. Consider the Galois group G of K over $\mathbb{Q}$ as a subgroup of S_p, the group of permutations of $\{a_1, \ldots, a_p\}$. Which of the following statements are true?

1. The transposition $(1\ 2)$ belongs to G.
2. $|G|$ is divisible by p.
3. A p-cycle belongs to G.
4. $G = S_p$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710208**
Option 1 ID : **916710829**
Option 2 ID : **916710830**
Option 3 ID : **916710831**
Option 4 ID : **916710832**
Status : **Not Answered**
Chosen Option : **--**

Q.109 Let V be a real vector space and $L(V)$ denote the space of linear operators on V. Let $T \in L(V)$ be a non-zero operator such that $T^2 = T$. Consider the subspace W of $L(V)$ spanned by

$$\{I, T^n : n \text{ is a positive integer}\}.$$

Which of the following statements are necessarily true?

1. The set $\{S \in W : S^2 = S\}$ contains exactly 2 elements.
2. $\dim_{\mathbb{R}}(W) = 2$
3. If $U \in L(V)$ is such that $U^2 = U$ and $(T+U)^2 = T+U$, then $TU = 0$.
4. If $U \in L(V)$ is such that $U^2 = U$ and $(T-U)^2 = T-U$, then $(TU)^2 = TU$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710192**
Option 1 ID : **916710765**
Option 2 ID : **916710766**
Option 3 ID : **916710767**
Option 4 ID : **916710768**
Status : **Answered**
Chosen Option : **2,4**

Q.110 Let G be a finite non-abelian group. Which of the following statements are necessarily true?

1. If d is a positive integer that divides $|G|$, then G has a subgroup of order d.
2. The map $f : G \times G \to G$ given by $f(a, b) = ab$ is not a group homomorphism.
3. Suppose that for every positive integer d that divides $|G|$, there exists a subgroup of G of order d. Then G has at least three normal subgroups.
4. $|G| \neq 16$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710203**
Option 1 ID :	**916710809**
Option 2 ID :	**916710810**
Option 3 ID :	**916710811**
Option 4 ID :	**916710812**
Status :	**Answered**
Chosen Option :	**2,3**

Q.111 Consider a series system comprising four components, having independent and identically distributed lifetimes with hazard rate $\lambda(t) = \frac{1}{1+t}, t > 0$, and survival function $S(t), t > 0$. If Y denotes the lifetime of the series system, then which of the following statements are true?

1. Cumulative hazard function of each component is $H(t) = 2\ln(1 + t), t > 0$
2. $S(t) = \dfrac{1}{(1+t)^2}, t > 0$
3. $P(Y < \frac{1}{2}) = \{S(\frac{1}{2})\}^4$
4. $P(Y < \frac{1}{2}) = \dfrac{65}{81}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**916710239**
Option 1 ID :	**916710953**
Option 2 ID :	**916710954**
Option 3 ID :	**916710955**
Option 4 ID :	**916710956**
Status :	**Not Answered**
Chosen Option :	**--**

Q.112 Let a, b be distinct positive real numbers. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x,y) = \begin{cases} \frac{(ax+by)^2}{ax^2+by^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Which of the following statements are necessarily true?

1. $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.
2. The partial derivatives of f at $(0,0)$ do not exist.
3. $\lim_{x\to 0} f(x,0) = \lim_{y\to 0} f(0,y)$
4. f is differentiable at $(0,0)$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710188**
Option 1 ID : **916710749**
Option 2 ID : **916710750**
Option 3 ID : **916710751**
Option 4 ID : **916710752**
Status : **Answered**
Chosen Option : **1**

Q.113 Let $X_1, X_2, \ldots, X_n$ $(n > 5)$ be independent random variables such that

$$X_t = \alpha + \alpha^2 t + \epsilon_t, \quad \text{for } t = 1, \ldots, n,$$

where, $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ are independent and identically distributed $N(0, \sigma^2)$ random variables. Here $\alpha \in \mathbb{R}$ and $\sigma > 0$ are unknown parameters. Which of the following statements are true?

1. $(X_1, X_2, \ldots, X_n)$ is a sufficient statistic for (α, σ)
2. $(\sum_{t=1}^n X_t, \sum_{t=1}^n tX_t, \sum_{t=1}^n t^2 X_t^2)$ is a jointly minimal sufficient statistic for (α, σ)
3. $X_4 - X_3 - X_2 + X_1$ is an ancillary statistic
4. $\dfrac{X_4 + X_1 - X_2 - X_3}{X_5 + X_2 - X_3 - X_4}$ is an ancillary statistic

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710232**
Option 1 ID : **916710925**
Option 2 ID : **916710926**
Option 3 ID : **916710927**
Option 4 ID : **916710928**
Status : **Not Answered**
Chosen Option : **--**

Q.114 Suppose $y(x)$ is the extremal of the variational problem

$$J(y) = \int_0^1 \left((y')^2 \sin x + (2\cos x)y\right)\, dx$$

subject to $y(0) = 0,\ y(1) = 1.$

Then which of the following statements are true?

1. $y\left(\frac{1}{2}\right) = 1$
2. $y'(0) = 1$
3. $y\left(\frac{1}{4}\right) = 2$
4. $y'\left(\frac{1}{2}\right) = 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710219**
Option 1 ID : **916710873**
Option 2 ID : **916710874**
Option 3 ID : **916710875**
Option 4 ID : **916710876**
Status : **Not Answered**
Chosen Option : **--**

Q.115 Let $Y = X\beta + \epsilon$ be a multiple linear regression model with p regressors and an intercept, where $\beta = (\beta_0, \beta_1, \ldots, \beta_p)^T$ and the random error $\epsilon \sim N_n(0, \sigma^2 I), \sigma > 0$ and $n > p + 1$. Here, the least squares estimation method provides unique estimator of β, say $\hat{\beta}$. Let the total sum of squares (corrected), sum of squares due to regression/model and sum of squares due to error/residual, based on $\hat{\beta}$, be denoted by Y^TAY, Y^TBY and Y^TCY, respectively, so that $Y^TAY = Y^TBY + Y^TCY$. Which of the following statements are always true?

1. $\dfrac{Y^TAY}{\sigma^2}$ follows a central χ^2 distribution with $(n-1)$ degrees of freedom.
2. $\dfrac{Y^TBY}{\sigma^2}$ follows a central χ^2 distribution with p degrees of freedom if $\beta_1 = \beta_2 = \cdots = \beta_p = 0$.
3. $\dfrac{Y^TBY}{Y^TCY}$ follows a central F distribution with $(p, n-p)$ degrees of freedom.
4. Y^TBY and Y^TCY are independently distributed if and only if $\beta_1 = \beta_2 = \cdots = \beta_p = 0$.

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710236**
Option 1 ID : **916710941**
Option 2 ID : **916710942**
Option 3 ID : **916710943**
Option 4 ID : **916710944**
Status : **Not Answered**
Chosen Option : **--**

Q.116 Let $\lambda \in \mathbb{R}$ be such that the integral equation

$$y(x) = \lambda \int_{-1}^{1} \left(5xt^3 + 4x^2t + 3xt\right) y(t)\, dt$$

admits a non-trivial solution $y(x)$ such that $y(1) = \frac{5}{2}$.

Then which of the following statements are true?

1. $y(0) + y'(0) = \frac{3}{2}$
2. $y\left(\frac{1}{2}\right) + y'\left(\frac{1}{2}\right) = \frac{7}{2}$
3. $y(-1) + y'(-1) = -1$
4. $y\left(\frac{1}{3}\right) + y'\left(\frac{1}{3}\right) = \frac{14}{9}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MSQ**
Question ID : **916710221**
Option 1 ID : **916710881**
Option 2 ID : **916710882**
Option 3 ID : **916710883**
Option 4 ID : **916710884**
Status : **Not Answered**
Chosen Option : **--**

Q.117 Let $\{x_n\}$ be a convergent iterative sequence generated by Newton-Raphson method for solving the equation $\sin x - 1 = 0$ such that $x_n \to \frac{\pi}{2}$ as $n \to \infty$. For $n \in \mathbb{N}$, let $e_n = x_n - \frac{\pi}{2}$.

Let $p > 0$ be such that $\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^p}$ exists and is non-zero.

Then which of the following statements are true?

1. $p = 1$
2. $\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^p} = \frac{1}{2}$
3. $p = 2$
4. $\lim_{n\to\infty} \frac{|e_{n+1}|}{|e_n|^p} = 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710217**
Option 1 ID : **916710865**
Option 2 ID : **916710866**
Option 3 ID : **916710867**
Option 4 ID : **916710868**
Status : **Not Answered**
Chosen Option : **--**

Q.118 Consider the boundary value problem (BVP)

$$u_{xx} + u_{yy} = 0 \text{ in } \Omega = \{(x, y) \in \mathbb{R}^2 \ : \ x^2 + y^2 < 1\},$$
$$u(x, y) = e^{x+y} \text{ on } \partial\Omega = \{(x, y) \in \mathbb{R}^2 \ : \ x^2 + y^2 = 1\}.$$

Then which of the following statements are true?

1. There exists a unique solution to BVP.
2. The BVP does NOT have a solution.
3. There exists a solution u to BVP such that $u(x, y) = \frac{1+e}{2}$ for some $(x, y) \in \Omega \cup \partial\Omega$.
4. There exists a solution u to BVP such that $u(x, y) = \frac{1+e^3}{2}$ for some $(x, y) \in \Omega \cup \partial\Omega$.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710214**
Option 1 ID : **916710853**
Option 2 ID : **916710854**
Option 3 ID : **916710855**
Option 4 ID : **916710856**
Status : **Not Answered**
Chosen Option : **--**

Q.119 Consider the simple linear regression model $Y_i = \beta x_i + \epsilon_i, i = 1, 2, \ldots, n$, where $\beta > 0$, $\sum_{i=1}^{n} x_i^2 > 0$, and the uncorrelated errors ϵ_i, $i = 1, 2, \ldots, n$, have zero mean and finite variance $\sigma^2 (> 0)$. Let $\tilde{\beta}_1 = \sum_{i=1}^{n} a_i^* Y_i$, where a_i^*'s minimize $E\left(\sum_{i=1}^{n} a_i Y_i - \beta\right)^2$ with respect to scalars $a_1, a_2, \ldots, a_n$. Let $\tilde{\beta}_2$ be the ordinary least squares estimator of β. Which of the following statements are true?

1. $\text{Var}(\tilde{\beta}_1) > \text{Var}(\tilde{\beta}_2), E(\tilde{\beta}_1 - \beta)^2 > E(\tilde{\beta}_2 - \beta)^2$
2. $\text{Var}(\tilde{\beta}_1) < \text{Var}(\tilde{\beta}_2), E(\tilde{\beta}_1 - \beta)^2 < E(\tilde{\beta}_2 - \beta)^2$
3. $\text{Var}(\tilde{\beta}_1) > \text{Var}(\tilde{\beta}_2), E(\tilde{\beta}_1) < E(\tilde{\beta}_2)$
4. $\text{Var}(\tilde{\beta}_1) < \text{Var}(\tilde{\beta}_2), E(\tilde{\beta}_1) > E(\tilde{\beta}_2)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710235**
Option 1 ID : **916710937**
Option 2 ID : **916710938**
Option 3 ID : **916710939**
Option 4 ID : **916710940**
Status : **Not Answered**
Chosen Option : **--**

Q.120 Let $f : \mathbb{C} \to \mathbb{C}$ be defined by

$$f(z) = \sin^2 z + \cos^2 |z|.$$

Which of the following statements are true?

1. f is a real valued function.
2. $f(z) = 1$ for all $z \in \mathbb{C}$.
3. f is not an entire function.
4. f has finitely many zeros on the imaginary axis.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **916710199**
Option 1 ID : **916710793**
Option 2 ID : **916710794**
Option 3 ID : **916710795**
Option 4 ID : **916710796**
Status : **Answered**
Chosen Option : **3,4**

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