NTA Joint CSIR UGC NET JUNE 2026_ 17th and 18th July

Application No	
Test Date	17/07/2026
Test Time	3:00 PM - 6:00 PM
Subject	Mathematical Sciences

Section : **PART A**

Q.1

There are four towns P, Q, R and T. Q is to the South-West of P, R is to the East of Q and South-East of P and T is to the North of R in line with Q, P. In which direction of P is T located?

1. South-East
2. North
3. North-East
4. East

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908734**
Option 1 ID :	**4589082933**
Option 2 ID :	**4589082934**
Option 3 ID :	**4589082935**
Option 4 ID :	**4589082936**
Status :	**Not Answered**
Chosen Option :	**--**

Q.2

A certain sum of money becomes double of itself in 5 years at simple rate of interest. In how many years will it become 16 times of itself at the same rate of simple interest?

1. 80
2. 70
3. 75
4. 90

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908748**
Option 1 ID :	**4589082989**
Option 2 ID :	**4589082990**
Option 3 ID :	**4589082991**
Option 4 ID :	**4589082992**
Status :	**Answered**
Chosen Option :	**3**

Q.3 In which of the following, entropy will decrease?

1. Water is boiled
2. Sugar is dissolved in water
3. Ice is heated to melt
4. Egg is boiled in water

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908741**
Option 1 ID :	**4589082961**
Option 2 ID :	**4589082962**
Option 3 ID :	**4589082963**
Option 4 ID :	**4589082964**
Status :	**Answered**
Chosen Option :	**2**

Q.4 There are two numbers such that the difference between them is 10 and their ratio is 5:3. Find the product of these two numbers

1. 150
2. 375
3. 275
4. 80

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908744**
Option 1 ID :	**4589082973**
Option 2 ID :	**4589082974**
Option 3 ID :	**4589082975**
Option 4 ID :	**4589082976**
Status :	**Answered**
Chosen Option :	**2**

Q.5

Suppose there are '9' identical gold-coins out of which 8 coins have same weight. One of the coin is counterfeit and weighs slightly less than other coins. You have a classical balance scale for weighing.

What is the minimum number of weighings required to definitively isolate and identify the counterfeit coin?

1. 3
2. 4
3. 2
4. 8

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908736**
Option 1 ID :	**4589082941**
Option 2 ID :	**4589082942**
Option 3 ID :	**4589082943**
Option 4 ID :	**4589082944**
Status :	**Marked For Review**
Chosen Option :	**4**

Q.6

The largest possible triangle is inscribed in a semi-circle of diameter 14 cm. What would be the area inside the semi-circle which is not occupied by the triangle?

1. 35 sq. cm
2. 30 sq. cm
3. 28 sq. cm
4. 48 sq. cm

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908747**
Option 1 ID :	**4589082985**
Option 2 ID :	**4589082986**
Option 3 ID :	**4589082987**
Option 4 ID :	**4589082988**
Status :	**Not Answered**
Chosen Option :	**--**

Q.7 During the interaction of population of two different species identify the type where only one species is benefited and is detrimental to the other species.

1. Commensalism and Amensalism
2. Commensalism and Competition
3. Parasitism and Predation
4. Parasitism and Amenalism

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908742**
Option 1 ID :	**4589082965**
Option 2 ID :	**4589082966**
Option 3 ID :	**4589082967**
Option 4 ID :	**4589082968**
Status :	**Not Answered**
Chosen Option :	**--**

Q.8 XUR, TQN, PMJ, ____?_____, HEB.

Which of the following options will complete the above series by replacing the question mark?

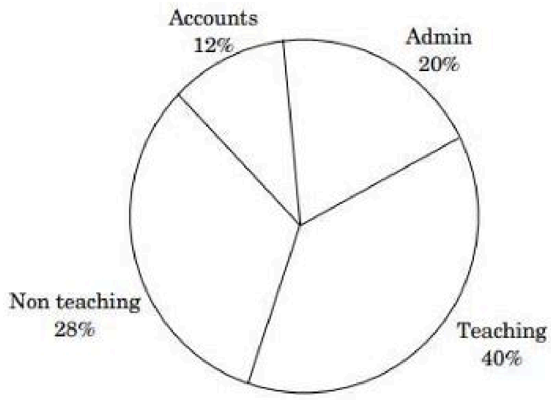
1. KHE
2. LIF
3. LJH
4. MJG

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908732**
Option 1 ID :	**4589082925**
Option 2 ID :	**4589082926**
Option 3 ID :	**4589082927**
Option 4 ID :	**4589082928**
Status :	**Answered**
Chosen Option :	**2**

Q.9 The pie chart shows the percentage of employees working in different departments of a University. The table shows the ratio of males to females in each of these departments.

If the total number of employees = 2000, what is the ratio between female employees in Admin and non teaching departments?



Category	Male : Female
Accounts	7 : 5
Admin	2 : 3
Teaching	5 : 3
Non teaching	3 : 4

1. 3 : 4
2. 5 : 3
3. 2 : 3
4. 4 : 3

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908737**
Option 1 ID : **4589082945**
Option 2 ID : **4589082946**
Option 3 ID : **4589082947**
Option 4 ID : **4589082948**
Status : **Answered**
Chosen Option : **1**

Q.10 The present population of a city is 6000. If the number of males increases by 14% and the number of females increases by 22%, then the population will become 6900. What is the present population of females in the city?

1. 900
2. 700
3. 750
4. 850

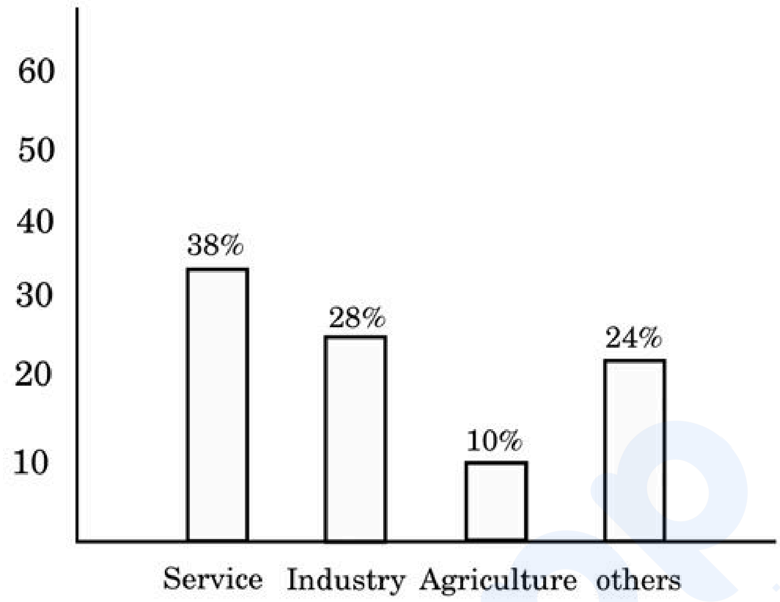
Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908743**
Option 1 ID :	**4589082969**
Option 2 ID :	**4589082970**
Option 3 ID :	**4589082971**
Option 4 ID :	**4589082972**
Status :	**Answered**
Chosen Option :	**3**

Q.11 The following chart shows the sector wise percentage break up of Rs. 650 crore, which is the GDP of a small country with a population of 1 billion people. 80% of the population constitutes its working population engaged in various sector and contributing to GDP.

Which of the following has lowest contribution to GDP?

% share of various sectors in GDP



1. Service
2. Industry
3. Agriculture
4. Non working population

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908739**
Option 1 ID :	**4589082953**
Option 2 ID :	**4589082954**
Option 3 ID :	**4589082955**
Option 4 ID :	**4589082956**
Status :	**Not Answered**
Chosen Option :	**--**

Q.12 Candidate in a competitive examination consisted of 60% men and 40% woman. 70% of men and 75% of women cleared the qualifying test and entered the final test where 80% men and 70% women were successful. Which of the following statement is correct?

1. Overall success rate is below 50%
2. Success rate is higher for women
3. More men cleared the examination than women
4. Both 1 and 2

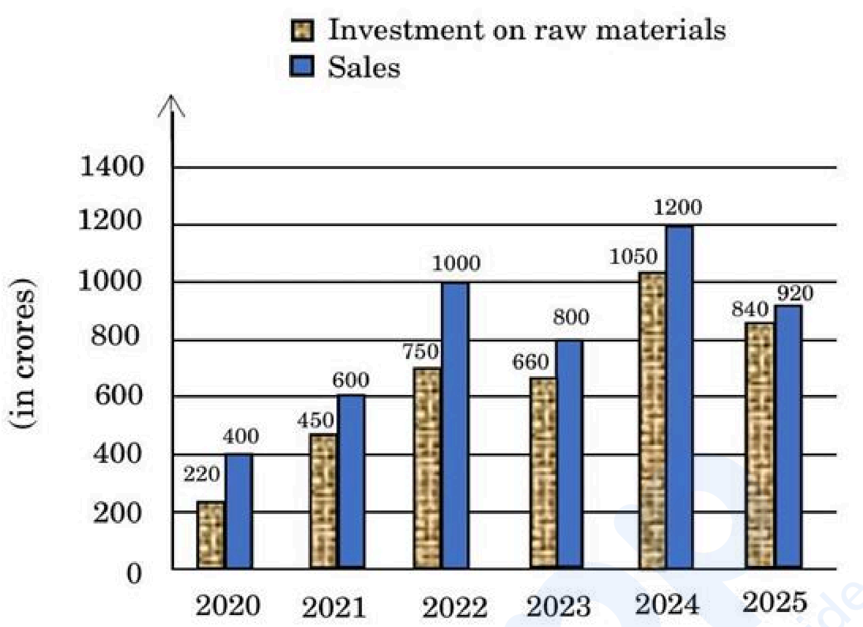
Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908750**
Option 1 ID :	**4589082997**
Option 2 ID :	**4589082998**
Option 3 ID :	**4589082999**
Option 4 ID :	**4589083000**
Status :	**Answered**
Chosen Option :	**3**

Q.13 The graph represents investment on raw material and sales of an organization from 2020 to 2025 (calendar year)

Study the bar graph and answer the question.

In which year was there a maximum percentage increase in the sales of products as compared the previous year?



1. 2021
2. 2023
3. 2022
4. 2024

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908738**
Option 1 ID :	**4589082949**
Option 2 ID :	**4589082950**
Option 3 ID :	**4589082951**
Option 4 ID :	**4589082952**
Status :	**Not Answered**
Chosen Option :	**--**

Q.14 Consider the relationship statement between two natural numbers :

A. the difference between them is 6.

B. the difference between their squares is 60.

Find the sum of the cubes of these two numbers.

1. 620
2. 520
3. 720
4. 820

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908749**
Option 1 ID : **4589082993**
Option 2 ID : **4589082994**
Option 3 ID : **4589082995**
Option 4 ID : **4589082996**
Status : **Answered**
Chosen Option : **2**

Q.15 Three alarm bells ring at intervals 30 sec, 40 sec and 45 sec respectively. They started ringing simultaneously at 4:00 am in the morning.

At what time will they all ring simultaneously:

1. 4:06 am
2. 4:36 am
3. 4:40 am
4. 5:00 am

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908745**
Option 1 ID : **4589082977**
Option 2 ID : **4589082978**
Option 3 ID : **4589082979**
Option 4 ID : **4589082980**
Status : **Answered**
Chosen Option : **1**

Q.16 A process is used to separate substances in chemical mixtures. What is this process known as?

1. Distillation
2. Filtration
3. Chromatography
4. Crystallisation

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908740**
Option 1 ID :	**4589082957**
Option 2 ID :	**4589082958**
Option 3 ID :	**4589082959**
Option 4 ID :	**4589082960**
Status :	**Not Answered**
Chosen Option :	**--**

Q.17 In a certain code if '1000' is written as '1728' and '125' is written as '343'. And '343' is written as '729', in the same code '512' will be written as:

1. 1000
2. 1331
3. 990
4. 1010

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908733**
Option 1 ID :	**4589082929**
Option 2 ID :	**4589082930**
Option 3 ID :	**4589082931**
Option 4 ID :	**4589082932**
Status :	**Not Answered**
Chosen Option :	**--**

Q.18 There are four positive integers say A, B, C and D such that $A \leq B \leq C \leq D$. Consider the following statements :

A. The unique mode of the data is 6.

B. The median score of the data is 7.

What is the minimum possible value of the largest number D?

1. 6
2. 7
3. 8
4. 9

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908746**
Option 1 ID : **4589082981**
Option 2 ID : **4589082982**
Option 3 ID : **4589082983**
Option 4 ID : **4589082984**
Status : **Not Answered**
Chosen Option : **--**

Q.19 Read the following information to answer the question

$P + Q$ means P is the brother of Q;

$P \times Q$ mean P is the father of Q;

$P - Q$ means P is the sister of Q.

Which of the following relation shows that I is the niece of K?

1. $K + Y + Z - I$
2. $K + Y \times I - Z$
3. $Z - I \times Y + K$
4. $K \times Y + I - Z$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908735**
Option 1 ID : **4589082937**
Option 2 ID : **4589082938**
Option 3 ID : **4589082939**
Option 4 ID : **4589082940**
Status : **Answered**
Chosen Option : **2**

Q.20 Select a figure from amongst the four alternatives which when placed in the blank space of fig (X) would complete the pattern?

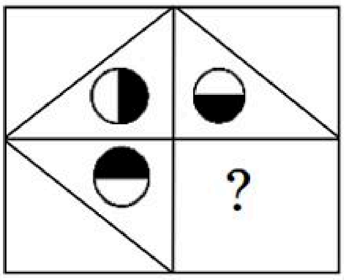
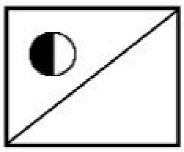
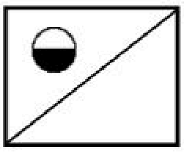
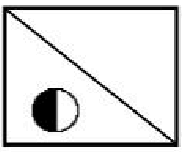
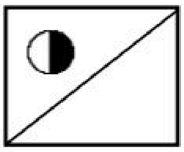


Fig (X)

1. 

2. 

3. 

4. 

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908731**
Option 1 ID : **4589082921**
Option 2 ID : **4589082922**
Option 3 ID : **4589082923**
Option 4 ID : **4589082924**
Status : **Answered**
Chosen Option : **1**

Section : **PART B**

Q.21

Let X_i be independent random variables following exponential distribution with mean $\frac{1}{i\theta}$, $i = 1, 2, 3$ and $\theta > 0$ is unknown. Which of the following statements is true?

(1) $3X_1 + 3X_2 + 3X_3$ is sufficient estimator for θ

(2) $X_1 + 2X_2 + 3X_3$ is sufficient estimator for θ

(3) $3X_1 + 2X_2 + X_3$ is sufficient estimator for θ

(4) $X_1 + \frac{1}{2}X_2 + \frac{1}{3}X_3$ is sufficient estimator for θ

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908786**
Option 1 ID :	**4589083141**
Option 2 ID :	**4589083142**
Option 3 ID :	**4589083143**
Option 4 ID :	**4589083144**
Status :	**Not Answered**
Chosen Option :	**--**

Q.22

Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2 - 4xy^2 + y^3$. Let the directional derivatives of f at the points $(1, 1)$ and $(1, 0)$ in the direction of $\vec{u} = \alpha\hat{\imath} + \beta\hat{\jmath}$ be $14/5$ and $6/5$, respectively. Then, the value of $25(\alpha - \beta)^2$ is equal to

(1) 1

(2) 9

(3) 16

(4) 49

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908753**
Option 1 ID :	**4589083009**
Option 2 ID :	**4589083010**
Option 3 ID :	**4589083011**
Option 4 ID :	**4589083012**
Status :	**Not Attempted and Marked For Review**
Chosen Option :	**--**

Q.23 Let $P_{30}(\mathbb{R})$ be the vector space of all real polynomials of degree less than or equal to 30. Let $T : P_{30}(\mathbb{R}) \to P_{30}(\mathbb{R})$ be a linear transformation such that $T^2 = 0$. Then, the maximum possible value of $\text{rank}(T)$ is

(1) 8

(2) 15

(3) 24

(4) 31

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908760**
Option 1 ID :	**4589083037**
Option 2 ID :	**4589083038**
Option 3 ID :	**4589083039**
Option 4 ID :	**4589083040**
Status :	**Answered**
Chosen Option :	**2**

Q.24 Let x, b and c be 3×1 real matrices and

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 1 & 5 & 3 \\ 2 & 2 & 5 \end{bmatrix}.$$

Let the Gauss-Seidel iterative scheme for solving the system of linear equations $Ax = b$ be $x^{(k+1)} = Gx^{(k)} + c,\ k = 0, 1, 2, ...$ Then the trace of the iteration matrix G is

(1) $\frac{33}{50}$

(2) $\frac{27}{50}$

(3) $\frac{23}{50}$

(4) $\frac{13}{50}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908767**
Option 1 ID :	**4589083065**
Option 2 ID :	**4589083066**
Option 3 ID :	**4589083067**
Option 4 ID :	**4589083068**
Status :	**Not Attempted and Marked For Review**
Chosen Option :	**--**

Q.25

Let $M_3(\mathbb{R})$ denote the vector space of all 3×3 matrices with real entries. Let $T: M_3(\mathbb{R}) \to M_3(\mathbb{R})$ be a linear transformation defined by $T(A) = A^T$. Consider the following two statements:

S1: T is diagonalizable

S2: Trace $(T) = 6$

Then,

(1) only S1 is true

(2) only S2 is true

(3) both S1 and S2 are true

(4) neither S1 nor S2 is true

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908761**
Option 1 ID :	**4589083041**
Option 2 ID :	**4589083042**
Option 3 ID :	**4589083043**
Option 4 ID :	**4589083044**
Status :	**Answered**
Chosen Option :	**1**

Q.26

The number of solutions of $x_1 + x_2 + x_3 = 17$, where $0 \leq x_1 \leq 7$, $0 \leq x_2 \leq 8$, $0 \leq x_3 \leq 12$ and $x_1, x_2, x_3 \in \mathbb{Z}$, is

(1) 57

(2) 59

(3) 60

(4) 148

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908771**
Option 1 ID :	**4589083081**
Option 2 ID :	**4589083082**
Option 3 ID :	**4589083083**
Option 4 ID :	**4589083084**
Status :	**Not Attempted and Marked For Review**
Chosen Option :	**--**

Q.27 Consider the primal problem (**P**):

$$\text{minimize } z = 3x_1 + x_2 - 4x_3$$
$$\text{s.t.} \quad 5x_1 + x_2 + 2x_3 \geq 10,$$
$$x_1 + 2x_2 - x_3 \leq 1,$$
$$x_1, x_2, x_3 \geq 0.$$

Then the dual of the problem (**P**)

(1) is infeasible

(2) is unbounded

(3) has finite optimal solutions

(4) is either infeasible or unbounded

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908784**
Option 1 ID :	**4589083133**
Option 2 ID :	**4589083134**
Option 3 ID :	**4589083135**
Option 4 ID :	**4589083136**
Status :	**Not Answered**
Chosen Option :	**--**

Q.28 Let $f : [0,1] \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} 0, & x = 0, \\ (-1)^n (1+n), & \dfrac{1}{n+1} < x \leq \dfrac{1}{n}, \quad n \in \mathbb{N}. \end{cases}$$

Consider the following two statements:

S1: $\int_0^1 f(x)dx$ is convergent.

S2: $\int_0^1 |f(x)|dx$ is divergent.

Then,

(1) only S1 is true

(2) only S2 is true

(3) both S1 and S2 are true

(4) neither S1 nor S2 is true

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908756**
Option 1 ID : **4589083021**
Option 2 ID : **4589083022**
Option 3 ID : **4589083023**
Option 4 ID : **4589083024**
Status : **Answered**
Chosen Option : **2**

Q.29

If the partial differential equation

$$u_{xx} + (\cos x + \sin x)u_{xy} + \left(y \sin x + \frac{1}{4}\right)u_{yy} + e^x u_y + x^2y^2u_x + 3u = 0,$$

where $u = u(x, y)$, is parabolic in $S = \{(x, y) \in \mathbb{R}^2 : y = \phi(x)\}$, then the value of $\lim_{x \to 0} \phi(x)$ is

(1) 1

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) $\frac{1}{8}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908765**
Option 1 ID :	**4589083057**
Option 2 ID :	**4589083058**
Option 3 ID :	**4589083059**
Option 4 ID :	**4589083060**
Status :	**Answered**
Chosen Option :	**2**

Q.30

For two square matrices A and B, let $A \sim B$ and $A \nsim B$ denote A is similar to B, and A is NOT similar to B, respectively. Let

$$P = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}, Q = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } R = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Which of the following statements is true?

(1) $P \sim Q$ and $Q \nsim R$

(2) $P \nsim Q$ and $Q \sim R$

(3) $P \nsim Q$ and $Q \nsim R$

(4) $P \sim Q$ and $Q \sim R$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908758**
Option 1 ID :	**4589083029**
Option 2 ID :	**4589083030**
Option 3 ID :	**4589083031**
Option 4 ID :	**4589083032**
Status :	**Answered**
Chosen Option :	**1**

Q.31 Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x|\theta) = \theta x^{\theta-1}, 0 < x < 1$ and $\theta > 0$ is an unknown parameter. Which of the following statements is FALSE?

(1) $\frac{\sum_{i=1}^{n} X_i}{n-\sum_{i=1}^{n} X_i}$ is method of moments estimator for θ

(2) $\frac{-n}{\sum_{i=1}^{n} \log_e X_i}$ is maximum likelihood estimator for θ

(3) $\frac{n}{n-\sum_{i=1}^{n} \log_e X_i}$ is maximum likelihood estimator for $\frac{\theta}{1+\theta}$

(4) $\frac{2n}{n+\sum_{i=1}^{n} \log_e X_i}$ is maximum likelihood estimator for $\frac{2\theta}{1+\theta}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908787**
Option 1 ID :	**4589083145**
Option 2 ID :	**4589083146**
Option 3 ID :	**4589083147**
Option 4 ID :	**4589083148**
Status :	**Not Answered**
Chosen Option :	**--**

Q.32 Let A be a 2×2 positive definite matrix with $\text{trace}(A - 2I) = 2$ and $\det(A^2 - 2I) = 28$. Then, the determinant of the adjoint of the matrix A is

(1) 4

(2) 8

(3) 16

(4) 64

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908762**
Option 1 ID :	**4589083045**
Option 2 ID :	**4589083046**
Option 3 ID :	**4589083047**
Option 4 ID :	**4589083048**
Status :	**Not Attempted and Marked For Review**
Chosen Option :	**--**

Q.33 Consider the transition probability matrix of a homogeneous Markov chain with state space $S = \{1, 2, 3, 4\}$ as

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1/4 & 1/4 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/4 & 3/4 \end{bmatrix}.$$

Which of the following statements is true?

(1) Only state 1 is transient

(2) Only state 2 is transient

(3) Both states 1 and 2 are transient

(4) All states are recurrent

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908785**
Option 1 ID :	**4589083137**
Option 2 ID :	**4589083138**
Option 3 ID :	**4589083139**
Option 4 ID :	**4589083140**
Status :	**Not Answered**
Chosen Option :	**--**

Q.34 Let X be a real-valued random variable having the probability density function $f_X(x)$ such that $f_X(\mu + x) = f_X(\mu - x)$ for a constant $\mu \in \mathbb{R}$. Let $\mathrm{N}(\mu, \sigma^2)$ denote the normal distribution with mean μ and variance σ^2. For $Y = 2\mu - X$, consider the following two statements:

S1: $f_X(\cdot)$ is the density function of Y

S2: $Y \sim \mathrm{N}(\mu, \sigma^2)$ if $X \sim \mathrm{N}(\mu, \sigma^2)$

Then,

(1) only S1 is true

(2) only S2 is true

(3) both S1 and S2 are true

(4) neither S1 nor S2 is true

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908782**
Option 1 ID :	**4589083125**
Option 2 ID :	**4589083126**
Option 3 ID :	**4589083127**
Option 4 ID :	**4589083128**
Status :	**Not Answered**
Chosen Option :	**--**

Q.35 Let $\boldsymbol{X}$ be Wishart $W_p(n, \boldsymbol{I}_p)$ distributed with $n > p$ and $\boldsymbol{A}$ be a $p \times p$ idempotent matrix of rank r $(r > 1)$. Let χ_n^2 denote a central Chi-Squared distribution with n degrees of freedom. Which of the following statements is true?

(1) trace $(\boldsymbol{AX}) \sim \chi_r^2$

(2) trace $(\boldsymbol{AX}) \sim \chi_n^2$

(3) trace $(\boldsymbol{AX}) \sim \chi_{pr}^2$

(4) trace $(\boldsymbol{AX}) \sim \chi_{nr}^2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908780**
Option 1 ID : **4589083117**
Option 2 ID : **4589083118**
Option 3 ID : **4589083119**
Option 4 ID : **4589083120**
Status : **Not Answered**
Chosen Option : **--**

Q.36 The Lebesgue outer measure of the set $\{x \in [-1,1]: f(x) \geq 0\}$, where

$$f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0, \end{cases}$$

is

(1) $\frac{2}{\pi}(1 - \log_e 2)$

(2) $\frac{2}{\pi}(\pi - \log_e 2)$

(3) $\frac{1}{\pi}(1 + 2\log_e 2)$

(4) $\frac{2}{\pi}(\pi + \log_e 2)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908751**
Option 1 ID : **4589083001**
Option 2 ID : **4589083002**
Option 3 ID : **4589083003**
Option 4 ID : **4589083004**
Status : **Not Answered**
Chosen Option : **--**

Q.37

Consider the partial differential equation (PDE)

$$z^2 e^{2y}\cos^2(x)\,(z_x)^2 + z^2\sin^2(x)\,(z_y)^2 - 3ze^{2y}\sin(2x)\,z_x = 0;\ \ x \in \left(0, \frac{\pi}{2}\right), y \in \mathbb{R}.$$

Then, for the PDE,

(1) $z = 1 + y$ is a singular solution

(2) $z = 3$ is a singular solution

(3) $z = y - 3$ is a singular solution

(4) singular solution does NOT exist

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908763**
Option 1 ID :	**4589083049**
Option 2 ID :	**4589083050**
Option 3 ID :	**4589083051**
Option 4 ID :	**4589083052**
Status :	**Answered**
Chosen Option :	**2**

Q.38

Let $z = z(x, y)$ be the solution of the partial differential equation

$$x(2z + y)z_x + y(2x^2 - z)z_y + (y + 4x^2)z = 0;\ \ z(x, x) = 1,\ \ \forall\, x \in \mathbb{R}.$$

If $z(1, \sqrt{8}) = \alpha^3$, then the value of $\alpha^3 - 4\alpha^2 + 2\alpha$ is

(1) $\sqrt{2}$

(2) $2\sqrt{2}$

(3) $3\sqrt{2}$

(4) $4\sqrt{2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908764**
Option 1 ID :	**4589083053**
Option 2 ID :	**4589083054**
Option 3 ID :	**4589083055**
Option 4 ID :	**4589083056**
Status :	**Not Answered**
Chosen Option :	**--**

Q.39

The sum of all the incongruent solutions of $90x \equiv 12 \pmod{102}$, which are positive integers less than or equal to 102, is

(1) 334

(2) 351

(3) 367

(4) 391

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908772**
Option 1 ID :	**4589083085**
Option 2 ID :	**4589083086**
Option 3 ID :	**4589083087**
Option 4 ID :	**4589083088**
Status :	**Not Answered**
Chosen Option :	**--**

Q.40

The minimum number of subsets of the set $\Omega = [0,1]$ to be added to the collection

$$\mathcal{F} = \left\{\phi, \left[0, \frac{1}{2}\right), \{1\}\right\},$$

so that it will become a field on Ω, is

(1) 2

(2) 3

(3) 4

(4) 5

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908783**
Option 1 ID :	**4589083129**
Option 2 ID :	**4589083130**
Option 3 ID :	**4589083131**
Option 4 ID :	**4589083132**
Status :	**Not Answered**
Chosen Option :	**--**

Q.41 If a particular integral of the partial differential equation

$$yz_{yy} - 3z_y = xy^3, \qquad y > 0$$

is of the form

$$z(x,y) = \frac{1}{4}\, x^a y^b \log_e y^c, \qquad a,b,c \in \mathbb{R},$$

then the value of $a + b + c$ is

(1) 2

(2) 4

(3) 6

(4) 8

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908768**
Option 1 ID : **4589083069**
Option 2 ID : **4589083070**
Option 3 ID : **4589083071**
Option 4 ID : **4589083072**
Status : **Answered**
Chosen Option : **3**

Q.42 Let $z = z(x,y)$ be the solution of the partial differential equation

$$z_{xx} - 2z_{xy} + z_{yy} + 2z_x - 2z_y + z = e^{x+2y}$$

defined on $\mathbb{R}^2$. If $z(0,y) = e^y, z(1,y) = 0,\ y \in \mathbb{R}$, then the value of $z(2,0)$ is

(1) $e^2 + 1$

(2) $e^2 - 1$

(3) 1

(4) 2

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908766**
Option 1 ID : **4589083061**
Option 2 ID : **4589083062**
Option 3 ID : **4589083063**
Option 4 ID : **4589083064**
Status : **Not Answered**
Chosen Option : **--**

Q.43

For $x \in (-1,1)$, the power series $\sum_{n=1}^{\infty} n^2 x^n$ converges to

(1) $\dfrac{x(1+x)}{(1-x)^3}$

(2) $\dfrac{x^2(1+x)}{(1-x)^3}$

(3) $\dfrac{x(1+x)}{(1-x)^2}$

(4) $\dfrac{x^2(1+x)}{(1-x)^2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908752**
Option 1 ID :	**4589083005**
Option 2 ID :	**4589083006**
Option 3 ID :	**4589083007**
Option 4 ID :	**4589083008**
Status :	**Answered**
Chosen Option :	**1**

Q.44

Let G be the group under usual multiplication of 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $ad - bc \neq 0$, and a, b, c, d are integers modulo 11. Then the order of a 3-Sylow subgroup of G is

(1) 3

(2) 9

(3) 27

(4) 81

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908773**
Option 1 ID :	**4589083089**
Option 2 ID :	**4589083090**
Option 3 ID :	**4589083091**
Option 4 ID :	**4589083092**
Status :	**Answered**
Chosen Option :	**1**

Q.45 Let $y = y(x)$ be the solution of the differential equation $y'' - 5y' + 4y = 0$ with initial conditions $y(0) = 0$ and $y'(0) = -1$. If $u(x) = y''(x)$, $\forall\, x \in \mathbb{R}$, then the corresponding integral equation is given by

(1) $u(x) = 4x + 5 + \int_0^x (5 + 4x + 4t)\, u(t)\, dt$

(2) $u(x) = 4x - 5 + \int_0^x (5 - 4x + 4t)\, u(t)\, dt$

(3) $u(x) = -4x + 5 + \int_0^x (5 + 4x + 4t)\, u(t)\, dt$

(4) $u(x) = -4x + 5 + \int_0^x (5 + 4x - 4t)\, u(t)\, dt$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908769**
Option 1 ID :	**4589083073**
Option 2 ID :	**4589083074**
Option 3 ID :	**4589083075**
Option 4 ID :	**4589083076**
Status :	**Answered**
Chosen Option :	**4**

Q.46

Consider the following ANOVA table for a balanced one-way ANOVA model:

Source of variation	Sum of squares	Degrees of freedom	Mean sum of squares	F_0
Treatment	a	2	d	1
Error	72	b	e	
Total	84	c		

Then the size of each experimental or treatment group is

(1) 5

(2) 6

(3) 7

(4) 8

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908789**
Option 1 ID : **4589083153**
Option 2 ID : **4589083154**
Option 3 ID : **4589083155**
Option 4 ID : **4589083156**
Status : **Not Answered**
Chosen Option : **--**

Q.47

Let X and Y be two topological spaces. Consider the cartesian product $X \times Y$ with the product topology. Which of the following options ensures that the projection map $f: X \times Y \to X$ is a closed map?

(1) connectedness of X

(2) connectedness of Y

(3) compactness of X

(4) compactness of Y

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908778**
Option 1 ID : **4589083109**
Option 2 ID : **4589083110**
Option 3 ID : **4589083111**
Option 4 ID : **4589083112**
Status : **Not Answered**
Chosen Option : **--**

Q.48

Let $\hat{\bar{Y}}_R$ denote the ratio estimator of the population mean of the study variable y, defined by

$$\hat{\bar{Y}}_R = \frac{\bar{y}_3}{\bar{x}_3}\bar{x}_4.$$

Here $\bar{y}_3$ and $\bar{x}_3$ are the sample means of the study variable y and the auxiliary variable x, respectively. Note that samples of size 3 for y and x are taken from the populations $\{c, 0, 1, 1\}$ and $\{0, 1, 0, 1\}$, respectively (without replacement). Further, $\bar{y}_4$ denotes the population mean of the study variable. If the expected value of $\hat{\bar{Y}}_R$ is $7/8$, then the value of c is

(1) 0

(2) 3

(3) 2

(4) 1

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908790**
Option 1 ID : **4589083157**
Option 2 ID : **4589083158**
Option 3 ID : **4589083159**
Option 4 ID : **4589083160**
Status : **Not Answered**
Chosen Option : **--**

Q.49

Let $\sum_{n\geq 0} a_n z^n$ converge at $z_1 = \frac{1+i}{\sqrt{2}}$ and diverge at $z_2 = \frac{1-i}{\sqrt{2}}$. If R is the radius of convergence of the series, then which of the following statements is true?

(1) $R < 1$

(2) $R = 1$

(3) $R > 1$

(4) $R = \infty$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908774**
Option 1 ID : **4589083093**
Option 2 ID : **4589083094**
Option 3 ID : **4589083095**
Option 4 ID : **4589083096**
Status : **Not Answered**
Chosen Option : **--**

Q.50

Consider the following two statements:

S1: Let f be twice continuously differentiable function on $[a,b]$, such that $f(a) = f(b) = 0$. If $f(c) > 0$ for some $c \in (a,b)$, then there exists a $\xi \in (a,b)$ such that $f''(\xi) < 0$.

S2: If $g(x) = x^2 \sin x$, $x \in \mathbb{R}$, and $g(h) = g(0) + hg'(\xi_h)$ for some $\xi_h \in (0,h)$, then $\lim_{h \to 0+} \frac{\xi_h}{h} = \frac{1}{2}$.

Then,

(1) only S1 is true

(2) only S2 is true

(3) both S1 and S2 are true

(4) neither S1 nor S2 is true

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908754**
Option 1 ID : **4589083013**
Option 2 ID : **4589083014**
Option 3 ID : **4589083015**
Option 4 ID : **4589083016**
Status : **Not Answered**
Chosen Option : **--**

Q.51

Consider the sequence of functions $\{f_n\}$ on $[0,1]$ given by

$$f_n(x) = \lim_{k \to \infty} (\cos(n!\,\pi x))^{2k}.$$

Let the limit function of $\{f_n\}$ be f. Which of the following statements is true?

(1) f is Lebesgue integrable on $[0,1]$

(2) $\{f_n\}$ is uniformly convergent on $[0,1]$

(3) All discontinuities of f on $[0,1]$ are removable

(4) f is differentiable at all the rational points on $[0,1]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908755**
Option 1 ID : **4589083017**
Option 2 ID : **4589083018**
Option 3 ID : **4589083019**
Option 4 ID : **4589083020**
Status : **Not Answered**
Chosen Option : **--**

Q.52 Let $f_\rho(\boldsymbol{x})$ be the probability density function (pdf) of a bivariate normal distributed random vector $\boldsymbol{X} = (X_1, X_2)^T$ with mean vector $(0, 0)^T$ and covariance matrix

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}, \qquad \rho \in (-1, 1).$$

Let

$$g(\boldsymbol{x}) = \frac{1}{2} f_{\rho_1}(\boldsymbol{x}) + \frac{1}{2} f_{\rho_2}(\boldsymbol{x}), \boldsymbol{x} \in \mathbb{R}^2;\ \rho_1, \rho_2 \in (-1, 1), \rho_1 \neq \rho_2$$

and

$$h(\boldsymbol{x}) = f_\rho(\boldsymbol{x}) f_{-\rho}(\boldsymbol{x}), \boldsymbol{x} \in \mathbb{R}^2.$$

Let $\mathrm{N}(\mu, \sigma^2)$ denote the normal distribution with mean μ and variance σ^2. Which of the following statements is FALSE?

(1) The marginal pdfs of X_1, X_2 under g are $\mathrm{N}(0,1)$

(2) The conditional distribution of X_1 given $X_2 = x_2$ under f_ρ is $\mathrm{N}(\rho x_2, 1 - \rho^2)$

(3) g is a pdf of a bivariate normal distribution for all ρ_1, ρ_2

(4) The function $h(\boldsymbol{x})$ is NOT a pdf

Options 1. 1

2. 2

3. 3

4. 4

Question Type :	**MCQ**
Question ID :	**458908779**
Option 1 ID :	**4589083113**
Option 2 ID :	**4589083114**
Option 3 ID :	**4589083115**
Option 4 ID :	**4589083116**
Status :	**Not Answered**
Chosen Option :	**--**

Q.53

Let $M_4(\mathbb{R})$ denote the set of all 4×4 matrices with real entries. Let

$$\mathfrak{B} = \{A \in M_4(\mathbb{R}) : \text{trace}(2A + 3I) \neq -5\}.$$

Then $\mathfrak{B}$ is

(1) a compact set

(2) a connected set

(3) a closed set

(4) an open set

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908757**
Option 1 ID :	**4589083025**
Option 2 ID :	**4589083026**
Option 3 ID :	**4589083027**
Option 4 ID :	**4589083028**
Status :	**Not Answered**
Chosen Option :	**--**

Q.54

Let $D = \{z \in \mathbb{C} : |z| < 1\}$. For each $\alpha \in \{z \in \mathbb{C} : |z| \leq 1\}$, define

$$X_\alpha = \{f : D \to D : f(\alpha z) = f(z), z \in D, f \text{ is holomorphic}\}.$$

Which of the following statements is true?

(1) For all $\alpha \in D$, X_α consists of all constant functions only

(2) For some $\alpha \in D$, X_α contains a non-constant function

(3) For each $\alpha \in D$, X_α contains a non-constant function

(4) X_α consists of all constant functions for some $\alpha \in D$ but NOT for all $\alpha \in D$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908776**
Option 1 ID :	**4589083101**
Option 2 ID :	**4589083102**
Option 3 ID :	**4589083103**
Option 4 ID :	**4589083104**
Status :	**Not Answered**
Chosen Option :	**--**

Q.55 Consider the following logistic regression model with the even number of observations n:

$$\mathbb{E}(y_i) = \begin{cases} \pi_{\text{ev}} = \dfrac{e^{\beta_0-\beta_1}}{1+e^{\beta_0-\beta_1}}, & \text{if } i \text{ is even,} \\ \pi_{\text{od}} = \dfrac{e^{\beta_0+\beta_1}}{1+e^{\beta_0+\beta_1}}, & \text{if } i \text{ is odd.} \end{cases}$$

Suppose the sum of $y_i's$ for even and odd $i's$ are $\frac{n}{4}$ and $\frac{n}{8}$, respectively. Then the odd ratio of the model, based on the maximum likelihood estimation of π_{ev} and π_{od} is

(1) $\dfrac{1}{\sqrt{2}}$

(2) $\sqrt{2}$

(3) $\sqrt{3}$

(4) $\dfrac{1}{\sqrt{3}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908788**
Option 1 ID :	**4589083149**
Option 2 ID :	**4589083150**
Option 3 ID :	**4589083151**
Option 4 ID :	**4589083152**
Status :	**Not Answered**
Chosen Option :	**--**

Q.56

Consider the sets

$$B_1 = \{\, (a,b) : a < b \text{ and } a,b \in \mathbb{R}\},$$
$$B_2 = \{\, [a,b) :\ a < b \text{ and } a,b \in \mathbb{R}\} \text{ and}$$
$$B_3 = \{\, [a,b) :\ a < b \text{ and } a,b \in \mathbb{Q}\}$$

of some intervals in $\mathbb{R}$. Let τ_1, τ_2 and τ_3 be the smallest topologies on $\mathbb{R}$ containing B_1, B_2 and B_3, respectively. Which of the following statements is true?

(1) $\mathbb{R}$ is NOT a Hausdorff space in τ_3

(2) $\tau_2 = \tau_3$

(3) $\tau_1 \subseteq \tau_3$

(4) $\mathbb{R}$ is connected in τ_3

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908777**
Option 1 ID : **4589083105**
Option 2 ID : **4589083106**
Option 3 ID : **4589083107**
Option 4 ID : **4589083108**
Status : **Not Answered**
Chosen Option : **--**

Q.57

Let $D = \{z \in \mathbb{C} : |z| < 1\}$. Let $\phi : D \to D$ be a holomorphic function such that $\phi\left(\frac{1}{2}\right) = 0$ and $\phi\left(-\frac{1}{2}\right) = \frac{4}{5}$. Then, the value of $\phi\left(\frac{1}{3}\right)$ is equal to

(1) $\frac{2}{7}$

(2) $\frac{1}{5}$

(3) $\frac{2}{5}$

(4) $\frac{3}{5}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908775**
Option 1 ID : **4589083097**
Option 2 ID : **4589083098**
Option 3 ID : **4589083099**
Option 4 ID : **4589083100**
Status : **Not Answered**
Chosen Option : **--**

Q.58 Let $P_4(\mathbb{R})$ denote the real vector space of all polynomials of degree less than or equal to 4. Let V and W be two subspaces of $P_4(\mathbb{R})$ given by

$$V = \{p \in P_4(\mathbb{R}) \mid p'(2) = 0 \text{ and } p(1) = p(3)\} \text{ and}$$
$$W = \{p \in P_4(\mathbb{R}) \mid p'(0) = p(0) = 0\}.$$

Then the dimension of the subspace $V + W$ is

(1) 5

(2) 6

(3) 4

(4) 3

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908759**
Option 1 ID :	**4589083033**
Option 2 ID :	**4589083034**
Option 3 ID :	**4589083035**
Option 4 ID :	**4589083036**
Status :	**Not Answered**
Chosen Option :	**--**

Q.59 Let X be a random variable with probability density function f. The uniformly most powerful test of size 0.1 in testing

$$H_0: f(x) = 6x^5, 0 < x < 1 \text{ against } H_1: f(x) = 10x^9, 0 < x < 1$$

has the power

(1) $1 - \left(\frac{9}{10}\right)^{3/5}$

(2) $\left(\frac{9}{10}\right)^{5/3}$

(3) $1 - \left(\frac{9}{10}\right)^{5/3}$

(4) $\left(\frac{9}{10}\right)^{3/5}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**458908781**
Option 1 ID :	**4589083121**
Option 2 ID :	**4589083122**
Option 3 ID :	**4589083123**
Option 4 ID :	**4589083124**
Status :	**Not Answered**
Chosen Option :	**--**

Q.60 List-I contains a term and List-II supports a related example/fact of List I.

	List-I		List-II
A.	Generalized coordinates	I.	Vibrators
B.	Rheonomic system	II.	Forces are derivable by potential energy
C.	Non-holonomic system	III.	Need NOT be associated with three mutually perpendicular directions
D.	Conservative system	IV.	Sphere rolling on a plane

Which of the following options matches List-I with the correct option of List-II?

(1) A-I, B-II, C-IV, D-III

(2) A-III, B-IV, C-I, D-II

(3) A-III, B-I, C-IV, D-II

(4) A-II, B-IV, C-III, D-I

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **458908770**
Option 1 ID : **4589083077**
Option 2 ID : **4589083078**
Option 3 ID : **4589083079**
Option 4 ID : **4589083080**
Status : **Not Answered**
Chosen Option : **--**

Section : **PART C**

Q.61 Let A be an $n \times n$ matrix with real entries and $\mu(x)$ be the minimal polynomial of A. Which of the following statements is/are true?

(1) If $\mu(x) = x^2 + 1$, then $A^4 = I$

(2) If $\mu(x) = x^2 + x$, then $A^8 = I$

(3) If $\mu(x) = x^2 - x + 1$, then $A^6 = I$

(4) If $\mu(x) = x^2 + x + 1$, then $A^3 = I$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **458908804**
Option 1 ID : **4589083213**
Option 2 ID : **4589083214**
Option 3 ID : **4589083215**
Option 4 ID : **4589083216**
Status : **Answered**
Chosen Option : **1,4**

Q.62 Consider two populations π_1 and π_2 with associated densities

$$f_1(x) = 1,\ 0 < x < 1 \text{ and } f_2(x) = 6x(1-x),\ 0 < x < 1,$$

respectively. The prior probabilities of π_1 and π_2 are equal. For the Bayes classifier under 0-1 loss function, which of the following statements is/are true?

(1) Classify x to π_2 if and only if $x \in \left(\frac{3-\sqrt{3}}{6}, \frac{3+\sqrt{3}}{6}\right)$

(2) Classify x to π_2 if and only if $x \in \left(\frac{3+\sqrt{3}}{6}, 1\right)$

(3) Classify x to π_1 if and only if $x \in \left(0, \frac{3+\sqrt{3}}{6}\right)$

(4) The probability of misclassifying x to π_2 is $\frac{1}{\sqrt{3}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908838**
Option 1 ID :	**4589083349**
Option 2 ID :	**4589083350**
Option 3 ID :	**4589083351**
Option 4 ID :	**4589083352**
Status :	**Not Answered**
Chosen Option :	**--**

Q.63 Consider the linear programming problem (LPP)

$$\text{Maximize/Minimize } z = x_1 + 2x_2 + x_3$$

$$\begin{aligned} \text{s.t.} \quad & 3x_1 + x_2 + x_3 \le 4, \\ & 2x_2 - x_3 \ge 2, \\ & x_1 + 5x_3 \le 0, \\ & x_1, x_2, x_3 \ge 0. \end{aligned}$$

Which of the following statements is/are true?

(1) $(0, 1, 0)$ is a vertex of the feasible region of the LPP

(2) $(0, 3, 0)$ is a point in the feasible region of the LPP

(3) The minimum value of the LPP is 2

(4) The maximum value of the LPP is 8

Options 1. 1

2. 2

3. 3

4. 4

Question Type :	**MSQ**
Question ID :	**458908843**
Option 1 ID :	**4589083369**
Option 2 ID :	**4589083370**
Option 3 ID :	**4589083371**
Option 4 ID :	**4589083372**
Status :	**Answered**
Chosen Option :	**1,2,4**

Q.64

Let $\mathcal{T}$ be the smallest topology on $\mathbb{Z}$ containing the collection

$$\mathbb{B} = \{\{0\}, \{-n, n\} : n \in \mathbb{N}\}.$$

Which of the following statements is/are true?

(1) $\mathbb{N} \cup \{0\}$ is closed in $(\mathbb{Z}, \mathcal{T})$

(2) $\mathbb{N} \cup \{0\}$ is dense in $(\mathbb{Z}, \mathcal{T})$

(3) The number of elements in finite closed subsets of $\mathbb{Z} \backslash \{0\}$ in $(\mathbb{Z}, \mathcal{T})$ is an even number

(4) There are infinitely many closed subsets of $(\mathbb{Z}, \mathcal{T})$ of order 5

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908832**
Option 1 ID :	**4589083325**
Option 2 ID :	**4589083326**
Option 3 ID :	**4589083327**
Option 4 ID :	**4589083328**
Status :	**Not Answered**
Chosen Option :	**--**

Q.65

Which of the following statements is/are true?

(1) $\sqrt{-7}$ is a prime element of $\mathbb{Z}[\sqrt{-7}]$

(2) 4 divides $3 + \sqrt{-7}$ in $\mathbb{Z}[\sqrt{-7}]$

(3) 4 is a prime element of $\mathbb{Z}[\sqrt{-7}]$

(4) Units of $\mathbb{Z}[\sqrt{-7}]$ are 1 and -1

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908822**
Option 1 ID :	**4589083285**
Option 2 ID :	**4589083286**
Option 3 ID :	**4589083287**
Option 4 ID :	**4589083288**
Status :	**Not Answered**
Chosen Option :	**--**

Q.66 Consider the problem of predicting Y using the linear model $\beta_0 + \beta_1 Z_1 + \beta_2 Z_2$, where Y, Z_1 and Z_2 are random variables and have a joint distribution with mean vector $\boldsymbol{\mu} = \begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}$ and covariance matrix

$$\Sigma = \begin{pmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/3 \\ 1/2 & 1/3 & 1 \end{pmatrix}.$$

Which of the following statements is/are true?

(1) The multiple correlation coefficient of Y on (Z_1, Z_2) is $\frac{1}{2}$

(2) The multiple correlation coefficient of Y on (Z_1, Z_2) is $\sqrt{\frac{3}{8}}$

(3) The best linear predictor of Y is $\frac{7}{4} + \frac{1}{2} Z_1 + \frac{1}{2} Z_2$

(4) The best linear predictor of Y is $\frac{17}{4} + \frac{3}{8} Z_1 + \frac{3}{8} Z_2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908839**
Option 1 ID :	**4589083353**
Option 2 ID :	**4589083354**
Option 3 ID :	**4589083355**
Option 4 ID :	**4589083356**
Status :	**Not Answered**
Chosen Option :	**--**

Q.67 Which of the following statements is/are true?

(1) Every uncountable set has non-zero Lebesgue measure

(2) Let G be the set of real numbers which can be presented in the form,

$$\frac{c_1}{5}+\frac{c_2}{5^2}+\cdots+\frac{c_n}{5^n}+\cdots, \text{where } c_n \in \{0,4\}, n \in \mathbb{N}.$$

Then, G has Lebesgue measure $2/5$

(3) The set of real numbers in $[0,1]$ whose decimal expansion does NOT contain the digit 7, has Lebesgue measure zero

(4) Let $\mathbb{Q}$ be the set of all rationals, enumerated as $a_1, a_2, a_3, \ldots$ and

$$S=\bigcup_{k=1}^{\infty}\left(a_k-\frac{1}{k^3}, a_k+\frac{1}{k^3}\right).$$

Then, for any closed subset T of $\mathbb{R}$, the set $S \setminus T$ has positive Lebesgue measure

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908800**
Option 1 ID :	**4589083197**
Option 2 ID :	**4589083198**
Option 3 ID :	**4589083199**
Option 4 ID :	**4589083200**
Status :	**Not Answered**
Chosen Option :	**--**

Q.68 Which of the following statements is/are true?

(1) If α is real root of $x^3-3x-1=0$, then $\sqrt{2} \notin \mathbb{Q}(\alpha)$

(2) The polynomial $x^4-\sqrt{2}$ is irreducible over $\mathbb{Q}(\sqrt{2})$

(3) $[\mathbb{Q}(\sqrt[8]{2}):\mathbb{Q}]=8$

(4) $[\mathbb{Q}(\sqrt[8]{2}):\mathbb{Q}(\sqrt{2})]=2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908826**
Option 1 ID :	**4589083301**
Option 2 ID :	**4589083302**
Option 3 ID :	**4589083303**
Option 4 ID :	**4589083304**
Status :	**Answered**
Chosen Option :	**1,2,3**

Q.69 Let $\mathrm{BV}[a,b]$ be the class of functions that are of bounded variation over $[a,b]$. Which of the following statements is/are FALSE?

(1) If $f \in \mathrm{BV}[a,b]$, then f is measurable on $[a,b]$

(2) If $f \in \mathrm{BV}[a,b]$, then there exist two monotonic functions f_1 and f_2 on $[a,b]$ such that $\lim_{x \to a+} f(x) = \inf_{x \in (a,b)} f_1(x) - \inf_{x \in (a,b)} f_2(x)$

(3) If $f \in \mathrm{BV}[a,b]$, then $\lim_{x \to b-} f(x)$ exists

(4) There exists an element in $\mathrm{BV}[a,b]$ which is NOT Riemann integrable on $[a,b]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908793**
Option 1 ID :	**4589083169**
Option 2 ID :	**4589083170**
Option 3 ID :	**4589083171**
Option 4 ID :	**4589083172**
Status :	**Answered**
Chosen Option :	**4**

Q.70 For $n \in \mathbb{N}$, let y_n be the eigenfunction corresponding to the eigenvalue λ_n for the Sturm–Liouville problem

$$x(xy')' + \lambda y = 0; \qquad y'(1) = 0, \qquad y(e) = 0,$$

where e is the base of natural logarithm. Then which of the following statements is/are true?

(1) $\lambda_n = \dfrac{(2n-1)^2\pi^2}{e^2}$

(2) $\lambda_n = \dfrac{(2n-1)^2\pi^2}{4}$

(3) $y_n = \cos\left(\dfrac{(2n-1)\pi x}{e}\right)$

(4) $y_n = \cos\left(\dfrac{(2n-1)\pi \log_e x}{2}\right)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908809**
Option 1 ID :	**4589083233**
Option 2 ID :	**4589083234**
Option 3 ID :	**4589083235**
Option 4 ID :	**4589083236**
Status :	**Answered**
Chosen Option :	**2,4**

Q.71

Let $y = y(x)$ be the solution of the integral equation

$$y(x) = \frac{ax}{2} + \int_0^1 e^{(x+t)}\, y(t)dt, \; a \in \mathbb{R}.$$

Then which of the following statements is/are true?

(1) For $a \neq 0$, the integral equation has a unique solution

(2) For $a = 0$, the integral equation has a trivial solution

(3) For $a = 2$, $y(2) = \dfrac{6}{3 - e^2}$

(4) For $a = 1$, $y(2) = \dfrac{4}{3 - e^2}$

Options
1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908819**
Option 1 ID :	**4589083273**
Option 2 ID :	**4589083274**
Option 3 ID :	**4589083275**
Option 4 ID :	**4589083276**
Status :	**Answered**
Chosen Option :	**1,2,3,4**

Q.72 Consider the following three statements:

S1: The initial value problem: $2(y')^2 + 7y^2 + 10 = 0,\ y(0) = 1$

has a unique solution in $\mathbb{R}$.

S2: Let $D = \{(x, y) \in \mathbb{R}^2 : |x - 1| \le 1, |y - 2| \le 2\}$. Then the initial value problem $y' = x^2 + y^4,\ y(1) = 2$ defined on D has a unique solution for $x \in \left(\frac{129}{130}, \frac{131}{130}\right)$.

S3: On $\mathbb{R}$, the functions x^3 and $x^2|x|$ are linearly dependent solutions of

$$x^2y'' - 4xy' + 6y = 0.$$

Then

(1) S1 is true but S2 is FALSE

(2) Both S1 and S3 are FALSE

(3) S2 is true but S3 is FALSE

(4) S3 is true but S2 is FALSE

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **458908810**
Option 1 ID : **4589083237**
Option 2 ID : **4589083238**
Option 3 ID : **4589083239**
Option 4 ID : **4589083240**
Status : **Answered**
Chosen Option : **2,3**

Q.73 Let $y(x) = xe^x$ be a solution of the differential equation $y'' + \alpha y' + \beta y = 0$, where α and β are real numbers. If $p = p(x)$ is the solution of

$$p'' + \alpha p' + \beta p = \frac{e^x}{x};\ x > 0,\ \ p(1) = p'(1) = e,$$

then which of the following statements is/are true?

(1) $(\alpha + \beta)^2 = 1$

(2) $(\alpha + \beta)^2 = 2$

(3) $p(e) = 2e^e$

(4) $p(e) = e^{(e+1)}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **458908813**
Option 1 ID : **4589083249**
Option 2 ID : **4589083250**
Option 3 ID : **4589083251**
Option 4 ID : **4589083252**
Status : **Not Answered**
Chosen Option : **--**

Q.74 Let $c(x)$ and $m(x)$ denote the characteristic and minimal polynomials of a real square matrix A, respectively. Let J_A be the Jordan form of the matrix A, that is, $A = SJ_AS^{-1}$ for some invertible matrix S. If $c(x) = m(x)(x^2 - 3x + 2)$ and $\left(m(x)\right)^2 = c(x)(x - 2)$, then which of the following statements is/are true?

(1) $c(4) - m(4) = 60$

(2) Total number of Jordan blocks in J_A is 3

(3) Sum of all the elements in J_A is 9

(4) Sum of all the elements in $(J_A)^4$ is 84

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **458908801**
Option 1 ID : **4589083201**
Option 2 ID : **4589083202**
Option 3 ID : **4589083203**
Option 4 ID : **4589083204**
Status : **Answered**
Chosen Option : **1,2,3**

Q.75 Consider an $M/M/1$ queue system. The customers arrival rate is $\lambda = 8$ customers per hour and the service rate is 10 customers per hour in this system. If the system is in steady-state, then which of the following statements is/are true?

(1) The mean waiting time is 24 minutes

(2) The mean service time is 6 minutes

(3) The mean queue length is 3.20

(4) The mean number of the customers in the system is 4

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908841**
Option 1 ID :	**4589083361**
Option 2 ID :	**4589083362**
Option 3 ID :	**4589083363**
Option 4 ID :	**4589083364**
Status :	**Not Answered**
Chosen Option :	**--**

Q.76 Let S be a subset of $\mathbb{R}^2$ defined by

$$S = \left\{\left(\frac{1}{n}, \frac{1}{n}\right) \in \mathbb{R}^2 : n \in \mathbb{N}\right\}.$$

Which of the following statements is/are true?

(1) Every point of S is an isolated point of S

(2) $(0, 0)$ is a limit point of S

(3) S is NOT a compact set of $\mathbb{R}^2$

(4) Every open cover of S has a finite subcover

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908791**
Option 1 ID :	**4589083161**
Option 2 ID :	**4589083162**
Option 3 ID :	**4589083163**
Option 4 ID :	**4589083164**
Status :	**Answered**
Chosen Option :	**1,2,3**

Q.77 Let f be an entire function such that $f(z) = f\left(\frac{1}{z}\right)$ for all $z \in \mathbb{C}\setminus\{0\}$. Which of the following statements is/are true?

(1) $\oint_{|z|=1} \frac{f'(z)}{z^2}\, dz = 0$

(2) The image of $\mathbb{C}$ under f is an open subset of $\mathbb{C}$

(3) The image of $\mathbb{C}$ under f' is an open subset of $\mathbb{C}$

(4) If $\sum_{n\geq 0} a_n z^n$ is the power series expansion of f at 0, then $\lim_{n\to\infty} n!\, a_n = 0$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908828**
Option 1 ID :	**4589083309**
Option 2 ID :	**4589083310**
Option 3 ID :	**4589083311**
Option 4 ID :	**4589083312**
Status :	**Answered**
Chosen Option :	**1,4**

Q.78 Consider a design with the following arrangements:

Blocks	Treatments
1	A, B, C
2	A, B, D
3	A, C, D
4	B, C, D

Which of the following statements is/are true?

(1) The design is connected

(2) The design is binary

(3) The design is symmetric

(4) The design is proper

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MSQ**
Question ID : **458908834**
Option 1 ID : **4589083333**
Option 2 ID : **4589083334**
Option 3 ID : **4589083335**
Option 4 ID : **4589083336**
Status : **Not Answered**
Chosen Option : **--**

Q.79

Let

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \quad \text{and} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3.$$

Which of the following statements is/are true?

(1) $\max\limits_{x \neq 0} \dfrac{x^T A^2 x}{x^T x} = 2(3 + \sqrt{2})$

(2) $\min\limits_{x \neq 0} \dfrac{x^T A^2 x}{x^T x} = 2(3 - 2\sqrt{2})$

(3) $\max\limits_{x \neq 0} \dfrac{x^T A^{-1} x}{x^T x} = \dfrac{2 + \sqrt{2}}{2}$

(4) $\min\limits_{x \neq 0} \dfrac{x^T A^{-1} x}{x^T x} = \dfrac{1 + \sqrt{2}}{2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908805**
Option 1 ID :	**4589083217**
Option 2 ID :	**4589083218**
Option 3 ID :	**4589083219**
Option 4 ID :	**4589083220**
Status :	**Answered**
Chosen Option :	**1,2,3**

Q.80 Suppose $y = y(x)$ is the extremal for the variational problem

$$J[y(x)] = \int_0^1 (x - y')^2 \, dx$$

subject to $y(0) = 0, y(1) = 2$. Which of the following statements is/are true?

(1) $\max_{x \in [0,1]} y(x) = 3$

(2) $\min_{x \in [0,1]} y(x) = 0$

(3) $y\left(\frac{1}{2}\right) = \frac{7}{8}$

(4) $y'\left(\frac{1}{2}\right) = 2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908818**
Option 1 ID :	**4589083269**
Option 2 ID :	**4589083270**
Option 3 ID :	**4589083271**
Option 4 ID :	**4589083272**
Status :	**Answered**
Chosen Option :	**2,3,4**

Q.81 Let X and Y be two random variables having the joint density function

$$f_{X,Y}(x, y) = x(y - x)e^{-y}, \qquad 0 \le x \le y < \infty.$$

Which of the following statements is/are true?

(1) $f_{X\mid Y}(x \mid y) = 6x(y - x)y^{-3}, \qquad 0 \le x \le y < \infty$

(2) $f_{Y\mid X}(x \mid y) = (y - x)e^{x-y}, \qquad 0 \le x \le y < \infty$

(3) $\mathbb{E}[Y \mid X] = X$

(4) $\mathbb{E}[X \mid Y] = \frac{Y}{2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908849**
Option 1 ID :	**4589083393**
Option 2 ID :	**4589083394**
Option 3 ID :	**4589083395**
Option 4 ID :	**4589083396**
Status :	**Not Answered**
Chosen Option :	**--**

Q.82 Let $y_1 = y_1(x)$ and $y_2 = y_2(x)$ be two linearly independent solutions of the differential equation $y'' - 4xy' + (4x^2 - 1)y = 0, x \in \mathbb{R}$. Which of the following statements is/are true for all $x \in \mathbb{R}$?

(1) Both $e^{-x^2}y_1$ and $e^{-x^2}y_2$ are periodic

(2) Both y_1 and y_2 are bounded

(3) Both y_1 and y_2 are unbounded

(4) $e^{-x^2}y_1$ is NOT periodic and $e^{-x^2}y_2$ is periodic

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908811**
Option 1 ID :	**4589083241**
Option 2 ID :	**4589083242**
Option 3 ID :	**4589083243**
Option 4 ID :	**4589083244**
Status :	**Not Answered**
Chosen Option :	**--**

Q.83 Consider the following linear regression models:

$$\mathbb{E}(y_1) = \beta_0 + \beta_2,$$
$$\mathbb{E}(y_2) = \beta_0 + \beta_1 + 2\beta_2,$$
$$\mathbb{E}(y_3) = \beta_0 + 2\beta_1 + 3\beta_2,$$

where β_0, β_1 and β_2 are unknown real constants. Which of the following linear functions of β_i's is/are estimable?

(1) β_0

(2) β_2

(3) $-\beta_1 - \beta_2$

(4) $2\beta_1 + 2\beta_2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908835**
Option 1 ID :	**4589083337**
Option 2 ID :	**4589083338**
Option 3 ID :	**4589083339**
Option 4 ID :	**4589083340**
Status :	**Not Answered**
Chosen Option :	**--**

Q.84

For $n \geq 3$, let $f_n : [0, 1] \to \mathbb{R}$ be defined as

$$f_n(x) = \begin{cases} n^2 x, & 0 \leq x \leq \dfrac{3}{2n} \\ -n^2\left(x - \dfrac{3}{n}\right), & \dfrac{3}{2n} \leq x \leq \dfrac{3}{n} \\ 0, & \text{otherwise.} \end{cases}$$

Which of the following statements is/are true?

(1) f_n is Riemann integrable, $\forall\, n \geq 3$

(2) $\displaystyle\int_0^1 f_n(x)\, dx = 2, \ \ \forall\, n \geq 3$

(3) $\displaystyle\lim_{n\to\infty} f_n(x) = 0, \ \ \forall\, x \in [0, 1]$

(4) If $f_n \to f, \forall\, x \in [0, 1]$, then $\displaystyle\int_0^1 f(x)\, dx = \lim_{n\to\infty} \int_0^1 f_n(x)\, dx$

Options 1. 1

2. 2

3. 3

4. 4

Question Type :	**MSQ**
Question ID :	**458908795**
Option 1 ID :	**4589083177**
Option 2 ID :	**4589083178**
Option 3 ID :	**4589083179**
Option 4 ID :	**4589083180**
Status :	**Not Answered**
Chosen Option :	**--**

Q.85 Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space. For $n \geq 1$, consider a sequence of random variables defined on $(\Omega, \mathcal{F}, \mathbb{P})$ as

$$X_n(\omega) = \begin{cases} 2^n, & \omega \in \left(0, \frac{1}{n}\right), \\ 0, & \text{otherwise.} \end{cases}$$

For $n \to \infty$, which of the following statements is/are true?

(1) $X_n \to 0$ almost surely

(2) $\mathbb{P}(|X_n| > \epsilon \text{ infinitely often}) = 0, \ \forall \, \epsilon > 0$

(3) $X_n \to 0$ in probability

(4) $X_n \to 0$ in L^p, $p > 0$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MSQ**
Question ID : **458908847**
Option 1 ID : **4589083385**
Option 2 ID : **4589083386**
Option 3 ID : **4589083387**
Option 4 ID : **4589083388**
Status : **Not Answered**
Chosen Option : **--**

Q.86

Let V be the vector space of all 3×3 matrices over the real field. Let

$$w_1 = \left\{ \begin{pmatrix} a & a & c \\ 0 & b & -b \\ c & 0 & d \end{pmatrix} \in V \mid a, b, c, d \in \mathbb{R} \right\}$$

and

$$w_2 = \left\{ \begin{pmatrix} 0 & x & y \\ x & y & z \\ -z & 0 & 0 \end{pmatrix} \in V \mid x, y, z \in \mathbb{R} \right\}$$

be two subspaces of V. Which of the following statements is/are true?

(1) $\dim(w_1) = 4$

(2) $\dim(w_2) = 3$

(3) $\dim(w_1 \cap w_2) \geq 1$

(4) $\dim(w_1 + w_2) = 7$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908806**
Option 1 ID :	**4589083221**
Option 2 ID :	**4589083222**
Option 3 ID :	**4589083223**
Option 4 ID :	**4589083224**
Status :	**Answered**
Chosen Option :	**1,2,3**

Q.87

Let $\boldsymbol{X} = (X_1, X_2, X_3)^T$ be a random vector with covariance matrix

$$\Sigma = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

and Y_1 be the first principal component of Σ. Which of the following statements is/are true?

(1) $Y_1 = \frac{1}{\sqrt{3}}(X_1 + X_2 + X_3)$

(2) The correlation coefficient between Y_1 and the variable X_1 is $\frac{1}{\sqrt{3}}$

(3) Y_1 explains more than 75% of total variation

(4) $\mathrm{Var}(Y_1) = 4$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908837**
Option 1 ID :	**4589083345**
Option 2 ID :	**4589083346**
Option 3 ID :	**4589083347**
Option 4 ID :	**4589083348**
Status :	**Not Answered**
Chosen Option :	**--**

Q.88

Which of the following statements is/are true?

(1) If G is a non-abelian group of order 243, then the group of automorphism of G has at least 9 elements

(2) The order of the group of automorphisms of the symmetric group S_3 is 6

(3) The center of S_3 is NOT equal to $\{I\}$, where I is the identity element of S_3

(4) Let U_{18} be the group of integers relatively prime to 18 under multiplication modulo 18. Then the order of the element 13 in U_{18} is 6

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908821**
Option 1 ID :	**4589083281**
Option 2 ID :	**4589083282**
Option 3 ID :	**4589083283**
Option 4 ID :	**4589083284**
Status :	**Not Answered**
Chosen Option :	**--**

Q.89

Consider the partial differential equation

$$\frac{\partial u}{\partial x} - \sqrt{1-y^2}\,\frac{\partial u}{\partial y} + 2u = 0, \quad u(\phi(y), y) = \psi(y), \quad x \in \mathbb{R}, |y| < 1,$$

where $u = u(x, y)$. Which of the following statements is/are true?

(1) If $\phi(y) = \cos^{-1} y$ and $\psi(y) = y$, then the solution $u = u(x, y)$ does NOT exist

(2) If $\phi(y) = 1$ and $\psi(y) = y$, then the solution $u = u(x, y)$ is bounded for all $x >\ 1$ and $|y| < 1$

(3) If $\phi(y) = 1$ and $\psi(y) = y$, then the solution $u = u(x, y)$ exists and $u(0, 0) = \sin 1$

(4) If $\phi(y) = 2$ and $\psi(y) = y^2$, then the solution $u = u(x, y)$ exists

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908812**
Option 1 ID :	**4589083245**
Option 2 ID :	**4589083246**
Option 3 ID :	**4589083247**
Option 4 ID :	**4589083248**
Status :	**Not Answered**
Chosen Option :	**--**

Q.90

Let X and Y be random variables on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Which of the following statements is/are true?

(1) $\mathbb{E}\big[\mathbb{E}[X|Y]\big] = \mathbb{E}[X]$

(2) $\mathbb{E}[\mathbb{E}[X|Y]|Y] = \mathbb{E}[X|Y]$

(3) $\mathrm{Var}[X|Y] = \mathbb{E}[X^2|Y] - (\mathbb{E}[X|Y])^2$

(4) $\mathrm{Var}[X] = \mathrm{Var}\big[\mathbb{E}[X|Y]\big] + \mathbb{E}\big[\mathrm{Var}[X|Y]\big]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908846**
Option 1 ID :	**4589083381**
Option 2 ID :	**4589083382**
Option 3 ID :	**4589083383**
Option 4 ID :	**4589083384**
Status :	**Not Answered**
Chosen Option :	**--**

Q.91 Let X_1, X_2, X_3, X_4 be a random sample of size 4 from an absolutely continuous distribution symmetric about 0. For $i = 1, 2, 3, 4$, let R_i denote the rank of $|X_i|$ among $|X_1|, |X_2|, |X_3|$ and $|X_4|$. If

$$T^+ = \sum_{i=1,\, X_i>0}^{4} R_i$$

is the Wilcoxon Signed-Ranks statistic, then which of the following statements is/are true?

(1) $\mathbb{P}(T^+ \text{ is taking all possible even values}) = \frac{1}{2}$

(2) $\mathbb{E}(T^+) = 5$

(3) $\mathbb{P}(T^+ \text{ is taking all possible odd values}) = \frac{7}{16}$

(4) $\mathbb{P}(T^+ = 0) = \frac{1}{8}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type :	**MSQ**
Question ID :	**458908836**
Option 1 ID :	**4589083341**
Option 2 ID :	**4589083342**
Option 3 ID :	**4589083343**
Option 4 ID :	**4589083344**
Status :	**Not Answered**
Chosen Option :	**--**

Q.92

Consider an $M/M/3$ queueing system with the arrival rate $\lambda = 10$ per hour, the service rate $\mu = 5$ per hour and the total number of servers is 3. In the steady state, for $n = 0, 1, 2, \ldots,$ let p_n be the probability that there are n customers in the system. Which of the following statements is/are true?

(1) $p_0 = \frac{1}{9}$

(2) $p_0 + p_1 = \frac{1}{3}$

(3) $2p_0 = p_3$

(4) $p_3 + p_4 + \cdots = \frac{4}{9}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type :	**MSQ**
Question ID :	**458908844**
Option 1 ID :	**4589083373**
Option 2 ID :	**4589083374**
Option 3 ID :	**4589083375**
Option 4 ID :	**4589083376**
Status :	**Not Answered**
Chosen Option :	**--**

Q.93

Consider the partial differential equation (PDE)

$$u_{tt} = 4u_{xx}; \quad 0 < x < \pi, \ \ t > 0$$
$$u_x(0,t) = u_x(\pi,t) = 0, \ \ t > 0$$
$$u(x,0) = \sin x, \ \ u_t(x,0) = 0, \ \ 0 < x < \pi.$$

If $u(x,t) = \frac{A}{\pi} + \frac{B}{\pi}\sum_{k=1}^{\infty}\frac{\cos(2kx)\cos(4kt)}{(1-4k^2)}$ is the solution of the PDE, then which of the following statements is/are true?

(1) $A + B = 6$

(2) $A^2 + B^2 = 20$

(3) $A^2 - B^2 = 12$

(4) $A - B = 10$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908814**
Option 1 ID :	**4589083253**
Option 2 ID :	**4589083254**
Option 3 ID :	**4589083255**
Option 4 ID :	**4589083256**
Status :	**Not Answered**
Chosen Option :	**--**

Q.94 Let $T: \mathbb{R}^3(\mathbb{R}) \to \mathbb{R}^3(\mathbb{R})$ be a linear transformation such that

$$T(1,1,1) = (1,-1,1), T(1,-1,1) = (1,1,-1) \text{ and } T(1,1,-1) = (1,1,1).$$

Let M be the matrix representation of T with respect to the standard basis of $\mathbb{R}^3$.

Which of the following statements is/are true?

(1) $T(2,-1,5) = (2,2,-1)$

(2) If $T^{-1}(2,1,-3) = (\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma = 3$

(3) $\text{Trace}(M) = 0$

(4) The matrix $(M + I)$ has only one real eigenvalue

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908807**
Option 1 ID :	**4589083225**
Option 2 ID :	**4589083226**
Option 3 ID :	**4589083227**
Option 4 ID :	**4589083228**
Status :	**Not Answered**
Chosen Option :	**--**

Q.95 If the quadrature formula

$$\int_0^2 f(x)\,dx = \frac{1}{2}\left[a\,f(0) + b\,f\left(\frac{2}{3}\right) + c\,f(2)\right]; \quad a,b,c \in \mathbb{R}$$

is exact for polynomials up to degree n and its truncation error (for sufficiently differentiable function f) is denoted by T.E., then which of the following statements is/are true?

(1) $a^2 + b^2 + c^2 = 10$

(2) $\text{T.E.} = -\frac{4}{9} f'''(\xi)$, for some $\xi \in (0,2)$

(3) $n = 2$

(4) $\text{T.E.} = -\frac{2}{27} f'''(\xi)$, for some $\xi \in (0,2)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908816**
Option 1 ID :	**4589083261**
Option 2 ID :	**4589083262**
Option 3 ID :	**4589083263**
Option 4 ID :	**4589083264**
Status :	**Not Answered**
Chosen Option :	**--**

Q.96 Let the function $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} \dfrac{x^6 - y^3}{|x| + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0). \end{cases}$$

Which of the following statements is/are true?

(1) f is differentiable at $(0,0)$

(2) f is unbounded in a neighborhood of $(0,0)$

(3) $f_x(1,1) + f_y(1,1) > 0$

(4) At $(1,1)$, f increases along the direction $\dfrac{(\hat{\imath} + \hat{\jmath})}{\sqrt{2}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908797**
Option 1 ID :	**4589083185**
Option 2 ID :	**4589083186**
Option 3 ID :	**4589083187**
Option 4 ID :	**4589083188**
Status :	**Not Answered**
Chosen Option :	**--**

Q.97 Which of the following statements is/are true?

(1) $$\lim_{n\to\infty}\left(\int_0^{\frac{\pi}{4}} \frac{t^n \sin t}{1+\cos^2 t}\, dt\right) = 0$$

(2) For any continuous function $f\colon [0,1] \to [0,1)$,

$$\lim_{n\to\infty}\left(n\int_0^1 \big(f(t)\big)^n\, dt\right) \le 1$$

(3) For any continuously differentiable function $f\colon [0,1] \to \mathbb{R}$,

$$\lim_{n\to\infty}\left(\int_0^1 f(t)\sin(nt)\ dt\right) \le 1$$

(4) For any continuously differentiable function $f\colon [0,1] \to (1,\infty)$,

$$\left(\int_0^1 f(t)\cos^2(nt)\ dt\right) \to \infty \text{ as } n \to \infty$$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908794**
Option 1 ID :	**4589083173**
Option 2 ID :	**4589083174**
Option 3 ID :	**4589083175**
Option 4 ID :	**4589083176**
Status :	**Not Answered**
Chosen Option :	**--**

Q.98 Consider the sequence $\{a_n\}$, where

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}, \qquad n \in \mathbb{N}.$$

Which of the following statements is/are true?

(1) $a_{2026} > 6$

(2) $a_{2052} < \frac{13}{2}$

(3) $a_{52} < \frac{7}{2}$

(4) $a_{153} < \frac{9}{2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **458908798**
Option 1 ID : **4589083189**
Option 2 ID : **4589083190**
Option 3 ID : **4589083191**
Option 4 ID : **4589083192**
Status : **Not Answered**
Chosen Option : **--**

Q.99 Let $u = \left(\frac{3}{2}, \ \frac{i}{2}, \ \ i\right)^T \in \mathbb{C}^3$ and $A = I - 2uu^*$, where u^* denotes the conjugate transpose of u. Which of the following statements is/are true?

(1) A is a skew-Hermitian matrix

(2) $\langle Ax, Ax \rangle = \langle x, x \rangle$ for any vector $x \in \mathbb{C}^3$, where $\langle x, y \rangle = x^* y; \ \ x, y \in \mathbb{C}^3$

(3) $AA^* = A^*A$

(4) If for some 3×3 real matrices B and C, $A = B + iC$, then B is a symmetric matrix

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **458908808**
Option 1 ID : **4589083229**
Option 2 ID : **4589083230**
Option 3 ID : **4589083231**
Option 4 ID : **4589083232**
Status : **Not Answered**
Chosen Option : **--**

Q.100 Let X be a random variable that takes values $-2, -1, 0, 1$ and 2 only with probability distribution $\mathbb{P}(X = 0) = 1 - \theta$, $\mathbb{P}(X = i) = \frac{\theta}{4}$ for $i = -2, -1, 1, 2$ and $\theta \in [0, 1]$ is unknown. Let $I(\cdot)$ denote the indicator function. Consider a random sample $X_1, X_2, \ldots, X_n$ of size n from the distribution of X. Which of the following statements is/are true?

(1) $\frac{1}{n}\sum_{i=1}^{n} I(X_i \neq 0)$ is a maximum likelihood estimator of θ

(2) $1 - \frac{1}{n}\sum_{i=1}^{n} I(X_i = 0)$ is a maximum likelihood estimator of θ

(3) $\frac{1}{n}\sum_{i=1}^{n} I(X_i = 0)$ is a maximum likelihood estimator of θ

(4) $1 - \frac{1}{n}\sum_{i=1}^{n} I(X_i \neq 0)$ is a maximum likelihood estimator of θ

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908845**
Option 1 ID :	**4589083377**
Option 2 ID :	**4589083378**
Option 3 ID :	**4589083379**
Option 4 ID :	**4589083380**
Status :	**Not Answered**
Chosen Option :	**--**

Q.101 Consider an initial value problem (IVP)

$$\frac{dy}{dx} = f(y) \quad \text{with} \quad y(x_0) = y_0.$$

For $n = 1, 2, \ldots$, let y_n denote approximate value of $y(x)$ at $x_n = x_0 + nh,\ h > 0$.
A scheme

$$y_{n+1} = y_n + \frac{1}{4}k_1^{(n)} + \frac{3}{4}k_2^{(n)};\ n = 0, 1, 2, \ldots$$

where $k_1^{(n)} = hf(y_n)$ and $k_2^{(n)} = hf\left(y_n + \frac{2}{3}k_1^{(n)}\right)$ is applied to solve the IVP.
Which of the following statements is/are true?

(1) The scheme has local truncation error $O(h^3)$

(2) The order of convergence of the scheme is 2

(3) If $f(y) = y$, $(x_0, y_0) = (0, 3)$ and $h = 0.1$, then $k_2^{(0)} = 0.32$

(4) If $f(y) = y^2$, $(x_0, y_0) = (0, 0)$ and $h = 0.1$, then $y_{10} = 0.85$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908817**
Option 1 ID :	**4589083265**
Option 2 ID :	**4589083266**
Option 3 ID :	**4589083267**
Option 4 ID :	**4589083268**
Status :	**Not Answered**
Chosen Option :	**--**

Q.102 Let $K = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{B}$ be a collection of all open intervals (a, b) along with all sets of the form $(a, b)\setminus K$, where $a, b \in \mathbb{R}$. Suppose $\mathbb{R}_K$ denotes the topology on $\mathbb{R}$ generated by the collection $\mathbb{B}$. Which of the following statements is/are true?

(1) $\mathbb{R}_K$ is Hausdorff

(2) $\mathbb{R}_K$ is regular

(3) $\mathbb{R}_K$ is normal

(4) 0 is NOT a limit point of K in $\mathbb{R}_K$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908831**
Option 1 ID :	**4589083321**
Option 2 ID :	**4589083322**
Option 3 ID :	**4589083323**
Option 4 ID :	**4589083324**
Status :	**Not Answered**
Chosen Option :	**--**

Q.103

Let m be the number of ring homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{20}$ and n be the number of ring homomorphisms from $\mathbb{Z}$ to $\mathbb{Z}$. Which of the following statements is/are true?

(1) $m = 2$

(2) $m = 3$

(3) $n = 3$

(4) $n = 2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908823**
Option 1 ID :	**4589083289**
Option 2 ID :	**4589083290**
Option 3 ID :	**4589083291**
Option 4 ID :	**4589083292**
Status :	**Not Answered**
Chosen Option :	**--**

Q.104

Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from Uniform$(\theta, 2\theta)$ distribution, where $\theta > 0$ is an unknown. Let Z_1 and Z_2 be the smallest and largest order statistic of the X_i's, respectively. Consider the estimators:

$$T_1 = \frac{n+1}{2n+1} Z_2, \quad T_2 = \frac{n+1}{5n+4}(Z_1 + 2Z_2) \text{ and } T_3 = \frac{1}{n+1} Z_1.$$

Which of the following statements is/are true?

(1) T_1 is an unbiased estimator for θ

(2) T_2 is an unbiased estimator for θ

(3) $\frac{1}{3} T_2 + \frac{2}{3} T_3$ is an unbiased estimator for θ

(4) $\frac{5}{6} T_1 + \frac{1}{6} T_2$ is an unbiased estimator for θ

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908850**
Option 1 ID :	**4589083397**
Option 2 ID :	**4589083398**
Option 3 ID :	**4589083399**
Option 4 ID :	**4589083400**
Status :	**Not Answered**
Chosen Option :	**--**

Q.105 Let $n(P)$ denote the number of elements of a set P.

Let $S = \{m \in \mathbb{Z} : -10 \leq m \leq 10\}$. Let

$$A = \begin{pmatrix} a & 1 \\ 1 & a \end{pmatrix} \text{ and } B = \begin{pmatrix} b & -1 \\ 1 & b \end{pmatrix}$$

where $a \in S_1 \subseteq S$ and $b \in S_2 \subseteq S$. If all the solutions of

$$\frac{du}{dt} = Au \quad \text{and} \quad \frac{dv}{dt} = Bv$$

approach to zero as $t \to \infty$, then which of the following statements is/are true?

(1) $S_1 \subset S_2$

(2) $S_2 \subset S_1$

(3) $n(S_1 \cup S_2) = 10$

(4) $n(S_1 \cap S_2) = 7$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908802**
Option 1 ID :	**4589083205**
Option 2 ID :	**4589083206**
Option 3 ID :	**4589083207**
Option 4 ID :	**4589083208**
Status :	**Not Answered**
Chosen Option :	**--**

Q.106 Which of the following statements is/are true?

(1) $\dfrac{\mathbb{Z}_{11}[x]}{\langle x^3 + 8x^2 + 4x + 1 \rangle}$ is a field

(2) In the ring $\mathbb{Z}_{11}[x]$, the ideal $I = \langle x^3 + 8x^2 + 4x + 1 \rangle$ is NOT contained in the ideal $J = \langle x^2 + x + 8 \rangle$

(3) $\dfrac{\mathbb{Z}_3[x]}{\langle x^3 + x^2 + x + 2 \rangle}$ is a field

(4) The field $\dfrac{\mathbb{Z}_3[x]}{\langle x^3 + x^2 + x + 2 \rangle}$ has 27 elements

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908824**
Option 1 ID :	**4589083293**
Option 2 ID :	**4589083294**
Option 3 ID :	**4589083295**
Option 4 ID :	**4589083296**
Status :	**Not Answered**
Chosen Option :	**--**

Q.107

Let $\{X_n\}_{n\geq 0}$ be a homogeneous Markov chain with state space $S = \{1, 2, 3, 4\}$ and the transition probability matrix

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix}.$$

Which of the following statements is/are true?

(1) All the states are recurrent

(2) State 4 is aperiodic

(3) The mean recurrence time from state 4 to state 4 is $\frac{5}{2}$

(4) $\mathbb{P}(X_1 = 4, X_2 = 3, X_3 = 2,\ X_4 = 1 \mid X_0 = 4) = \dfrac{1}{16}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908842**
Option 1 ID :	**4589083365**
Option 2 ID :	**4589083366**
Option 3 ID :	**4589083367**
Option 4 ID :	**4589083368**
Status :	**Not Answered**
Chosen Option :	**--**

Q.108

Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T \neq 0, T^2 \neq 0$ but $T^3 = 0$. If $T(1,2,3) = (0,1,3)$ and $T(2,1,2) = (0,2,3)$, then which of the following statements is/are true?

(1) T is NOT invertible

(2) If $T(3,0,1) = (\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma = 6$

(3) $\mathrm{Rank}(T) = 2$

(4) $\mathrm{Rank}(T^2) = 2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908803**
Option 1 ID :	**4589083209**
Option 2 ID :	**4589083210**
Option 3 ID :	**4589083211**
Option 4 ID :	**4589083212**
Status :	**Not Answered**
Chosen Option :	**--**

Q.109 Let R be the radius of convergence of non-zero power series $\sum_{n\geq 0} a_n z^n$. Define $c_k = \max\{|a_n| : 0 \leq n \leq k\}$, $k \geq 0$. If R' is the radius of convergence of power series $\sum_{k\geq 0} c_k z^k$, then which of the following statements is/are true?

(1) $R' > R$

(2) $R' \leq R$

(3) $R' \leq 1$

(4) $R' \leq \min\{1, R\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908829**
Option 1 ID :	**4589083313**
Option 2 ID :	**4589083314**
Option 3 ID :	**4589083315**
Option 4 ID :	**4589083316**
Status :	**Not Answered**
Chosen Option :	**--**

Q.110 Suppose $\boldsymbol{X} = (X_1, X_2)^T$ follows bivariate normal distribution

$$N_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & \rho\\ \rho & 1\end{pmatrix}\right), \quad \rho \in (-1, 1).$$

Which of the following statements is/are true?

(1) $X_1 + X_2$ and $X_1 - X_2$ are uncorrelated

(2) $X_1 + X_2$ and $X_1 - X_2$ are independent

(3) $X_1 + X_2$ and $X_1 X_2$ are uncorrelated if X_1 and X_2 are independent

(4) $X_1 + X_2$ and $X_1 X_2$ are independent if X_1 and X_2 are independent

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908840**
Option 1 ID :	**4589083357**
Option 2 ID :	**4589083358**
Option 3 ID :	**4589083359**
Option 4 ID :	**4589083360**
Status :	**Not Answered**
Chosen Option :	**--**

Q.111 Which of the following statements is/are true?

(1) $[\mathbb{Q}(i+1):\mathbb{Q}] = 2$

(2) $\left[\mathbb{Q}\left(\sqrt{3+2\sqrt{2}}\right):\mathbb{Q}\right] = 4$

(3) $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2},\sqrt{3})$

(4) $[\mathbb{Q}(\sqrt{2}+3i):\mathbb{Q}] = 4$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908825**
Option 1 ID :	**4589083297**
Option 2 ID :	**4589083298**
Option 3 ID :	**4589083299**
Option 4 ID :	**4589083300**
Status :	**Not Answered**
Chosen Option :	**--**

Q.112 Let f be an entire function such that $f \not\equiv 1$. For all $n \in \mathbb{N}$, let $C_n := ne^{i\theta}$, $\theta \in [0, 2\pi]$. Define for all $n \in \mathbb{N}$,

$$I_n = \frac{1}{2\pi i}\int_{C_n} \frac{f'(z)}{f(z)-1}\,dz,$$

whenever the integral exists. Then which of the following statements is/are true?

(1) f has pole at ∞ if and only if there exists $N \in \mathbb{N}$ such that $I_n = I_N, \forall\, n \geq N$

(2) If f has pole at ∞, then there exists $N \in \mathbb{N}$ such that $I_n = I_N, \forall\, n \geq N$

(3) If f has removable singularity at ∞, then $I_n = 0, \forall\, n \in \mathbb{N}$

(4) There exists f with essential singularity at ∞ such that $I_n = 0, \forall\, n \in \mathbb{N}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908830**
Option 1 ID :	**4589083317**
Option 2 ID :	**4589083318**
Option 3 ID :	**4589083319**
Option 4 ID :	**4589083320**
Status :	**Not Answered**
Chosen Option :	**--**

Q.113 Let $D = \{z \in \mathbb{C}: |z| < 1\}$. Let $2 \leq m \in \mathbb{N}$ and

$$\mathbb{F}_m = \left\{f: D \to D: f \text{ is holomorphic}, f\left(e^{\frac{2\pi i}{m}} z\right) = f(z), \forall\, z \in D\right\}.$$

Which of the following statements is/are true?

(1) $\mathbb{F}_m$ is the set of all constant functions

(2) $\mathbb{F}_m$ is the set of all polynomials of degree less than or equal to m

(3) $z^n \in \mathbb{F}_m, 1 \leq n \in \mathbb{N}$ if and only if m divides n

(4) $h \in \mathbb{F}_m$ if and only if for some holomorphic function $g: D \to D$,

$$h(z) = \frac{1}{m} \sum_{k=0}^{m-1} g\left(e^{\frac{2k\pi i}{m}} z\right), z \in D$$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908827**
Option 1 ID :	**4589083305**
Option 2 ID :	**4589083306**
Option 3 ID :	**4589083307**
Option 4 ID :	**4589083308**
Status :	**Not Answered**
Chosen Option :	**--**

Q.114 Consider the following simple linear regression model

$$y_i = \beta_0 + ic\beta_1 + \epsilon_i, \quad i = 1, 2, \ldots, n, (n > 2)$$

where β_0 and β_1 are unknown real parameters, c is a real constant and the errors ϵ_i are such that

$$\mathbb{E}(\epsilon_i) = 0, \ \mathrm{Var}(\epsilon_i) = 1 \ \text{and} \ \mathrm{Cov}(\epsilon_i, \epsilon_j) = 0 \ \text{for all} \ i \neq j.$$

Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the best linear unbiased estimators of β_0 and β_1, respectively. If $\mathrm{Cov}(\hat{\beta}_0, \hat{\beta}_1) = 1$, then which of the following statements is/are true?

(1) $\mathrm{Var}(\hat{\beta}_0) = \dfrac{2(2n+1)}{n(n-1)}$

(2) $\mathrm{Var}(\hat{\beta}_0) = \dfrac{2(n+2)}{n(n+1)}$

(3) $c = \dfrac{6}{n(1-n)}$

(4) $c = \dfrac{6}{n(n+1)}$

Options 1. 1

2. 2

3. 3

4. 4

Question Type : **MSQ**
Question ID : **458908833**
Option 1 ID : **4589083329**
Option 2 ID : **4589083330**
Option 3 ID : **4589083331**
Option 4 ID : **4589083332**
Status : **Not Answered**
Chosen Option : **--**

Q.115

Let $\alpha \in \mathbb{N}$ and let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function given by

$$f(x,y) = \begin{cases} \dfrac{xy^\alpha}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Which of the following statements is/are true?

(1) f is continuous on $\mathbb{R}^2$, $\forall\, \alpha \geq 1$

(2) f is differentiable on $\mathbb{R}^2$, $\forall\, \alpha \geq 3$

(3) $f_x(0,0) = f_y(0,0) = 0, \forall\, \alpha \in \mathbb{N}$

(4) $\dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right)_{(0,0)} = 0, \forall\, \alpha \geq 3$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908792**
Option 1 ID :	**4589083165**
Option 2 ID :	**4589083166**
Option 3 ID :	**4589083167**
Option 4 ID :	**4589083168**
Status :	**Not Answered**
Chosen Option :	**--**

Q.116

Which of the following functions is/are uniformly continuous on the indicated intervals?

(1) $f(x) = \dfrac{\cos^2 x}{1+x^2+\sin^2 x}$ on $[0,\infty)$

(2) $f(x) = \dfrac{1}{x}\sin\left(\dfrac{1}{x}\right)$ on $[1,\infty)$

(3) $f(x) = x\sin\left(\dfrac{2}{\pi x}\right)$ on $\left(0,\dfrac{\pi}{2}\right)$

(4) $f(x) = \inf\{|x-a| : a \in [1,2]\}$ on $\mathbb{R}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908796**
Option 1 ID :	**4589083181**
Option 2 ID :	**4589083182**
Option 3 ID :	**4589083183**
Option 4 ID :	**4589083184**
Status :	**Not Answered**
Chosen Option :	**--**

Q.117 Let $C[0,1]$ denote the set of all continuous functions defined on $[0,1]$. Let $(C[0,1],d)$ be a metric space with

$$d(f,g) = \int_0^1 |f(t) - g(t)|\, dt, \quad \forall f,g \in C[0,1].$$

For $n \geq 2$, let $f_n: [0,1] \to \mathbb{R}$ be given by

$$f_n(t) = \begin{cases} 0, & 0 \leq t \leq \frac{1}{2} \\ n\left(t - \frac{1}{2}\right), & \frac{1}{2} \leq t \leq \frac{1}{2} + \frac{1}{n} \\ 1, & \frac{1}{2} + \frac{1}{n} \leq t \leq 1. \end{cases}$$

Which of the following statements is/are true?

(1) $\{f_n\}$ is a Cauchy sequence in $(C[0,1],d)$

(2) $(C[0,1],d)$ is NOT a complete metric space

(3) $\lim_{n\to\infty} f_n(t)$ exists $\forall\, t \in [0,1]$

(4) $\lim_{n\to\infty} f_n \in C[0,1]$, provided the limit exists

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **458908799**
Option 1 ID : **4589083193**
Option 2 ID : **4589083194**
Option 3 ID : **4589083195**
Option 4 ID : **4589083196**
Status : **Not Answered**
Chosen Option : **--**

Q.118

If the sequence $\{x_n\}$:

$$x_{n+1} = x_n + \alpha f(x_n), \quad \alpha \in \mathbb{R}, \quad n = 0, 1, 2, \ldots$$

converges to the root of $f(x) = e^{-x} - \sin x = 0$ in $\left[0, \frac{\pi}{2}\right]$ for any $x_0 \in \left[0, \frac{\pi}{2}\right]$, then which of the following statements is/are true?

(1) $\alpha \in (0, 0.2)$

(2) $\alpha \in (0.2, 0.6)$

(3) $\alpha \in (0.5, 0.8)$

(4) $\alpha \in (0.9, 1)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908815**
Option 1 ID :	**4589083257**
Option 2 ID :	**4589083258**
Option 3 ID :	**4589083259**
Option 4 ID :	**4589083260**
Status :	**Not Answered**
Chosen Option :	**--**

Q.119

A bead of unit mass slides over a smooth wire of the form $y = x^2$. Let g denote the acceleration due to gravity. If $\big(x(t), y(t)\big)$ is the position of the bead at any time t, and $\dot{x}(t) = \frac{dx}{dt}$, $\ddot{x}(t) = \frac{d^2x}{dt^2}$, then which of the following statements is/are true?

(1) The Lagrangian function for the bead's motion is $\frac{1}{2}(1 + 3x^2)\dot{x}^2 - 2gx^2$

(2) The potential function for the bead's motion is gx^2

(3) The kinetic energy function for the bead's motion is $\frac{1}{2}(1 + 4x^2)\dot{x}^2$

(4) Equation of motion of the bead is $(1 + 4x^2)\ddot{x} + 4x\dot{x}^2 + 2gx = 0$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908820**
Option 1 ID :	**4589083277**
Option 2 ID :	**4589083278**
Option 3 ID :	**4589083279**
Option 4 ID :	**4589083280**
Status :	**Not Answered**
Chosen Option :	**--**

Q.120 Consider a two-component system such that the failure of each component occurs in accordance with a Poisson process of intensity $\lambda > 0$. Which of the following statements is/are true?

(1) In series system, the reliability function is $R(t) = e^{-2\lambda t}$

(2) In series system, the hazard function is $r(t) = 2\lambda$

(3) In parallel system, the reliability function is $R(t) = 1 - (1 - e^{-\lambda t})^2$

(4) In parallel system, the hazard function is $r(t) = 2\lambda(1 - e^{-\lambda t})$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**458908848**
Option 1 ID :	**4589083389**
Option 2 ID :	**4589083390**
Option 3 ID :	**4589083391**
Option 4 ID :	**4589083392**
Status :	**Not Answered**
Chosen Option :	**--**

Exam Summary

MATHEMATICAL SCIENCES

Section Name	No. of Questions	Answered	Not Answered	Marked for Review	Answered & Marked for Review	Not Visited
PART A	20	11	8	0	1	0
PART B	40	10	12	4	0	14
PART C	60	13	23	0	0	24

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