

JEE Main Mathematics Sample Paper 04

Total Questions: 25 (20 MCQs + 15 Numericals)

Duration: 3 Hours | Marking: +4/-1 (MCQ), +4/0 (NVT)

Section A: Multiple Choice Questions

1. If $f(x) = x^2 + 3x + 1$, then the value of $f'(x) + f''(x)$ at $x = 2$ is:

- (a) 13 (b) 14
(c) 9 (d) 16

2. If $f(x) = e^{x^2}$ and $g(x) = \int_0^x f(t) dt$, then $g'(x)$ is:

- (a) e^{x^2} (b) $2xe^{x^2}$
(c) $e^{(2x^2)}$ (d) $\int_0^x e^{(t^2)} dt$

3. The sum of the coefficients of all terms in the expansion of $(2x - 3)^5$ is:

- (a) 1 (b) 243
(c) -243 (d) -1

4. If the roots of $x^2 - px + q = 0$ are α and β , and $\alpha^2 + \beta^2 = 40$, then $p^2 - 2q$ is:

- (a) 20 (b) 40
(c) 80 (d) 60

5. The area of the region bounded by the parabola $y^2 = 4x$ and the line $x = 4$ is:

- (a) $\frac{8}{3}$ (b) $\frac{16}{3}$
(c) $\frac{32}{3}$ (d) $\frac{64}{3}$

6. The circle passing through (1,1) and (2,2) with center on $x + y = 4$ is:

- (a) $(x-3)^2 + (y-1)^2 = 4$ (b) $(x-2)^2 + (y-2)^2 = 1$
(c) $(x-3)^2 + (y-3)^2 = 2$ (d) $(x-1)^2 + (y-3)^2 = 5$

7. Let $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$. The determinant of A^3 is:

- (a) 50 (b) 40
(c) 60 (d) 343

8. If the distance between points $(x, 3)$ and $(5, 7)$ is 5, then the value of x is:

- (a) 2 (b) 3
(c) 1 (d) 4

9. The distance between the planes $2x + y - z + 5 = 0$ and $4x + 2y - 2z + 15 = 0$ is:

- (a) 5 (b) 10
(c) $5/6$ (d) $5/(2\sqrt{6})$

10. The value of $\lim_{x \rightarrow 0} [\sin 3x / \sin 5x]$ is:

- (a) $3/5$ (b) $5/3$
(c) 1 (d) 0

11. The function $f(x) = x^3 - 6x^2 + 9x$ has a local minimum at:

- (a) $x = 1$ (b) $x = 3$
(c) $x = 0$ (d) $x = 2$

12. The eccentricity of the hyperbola $4x^2 - y^2 = 16$ is:

- (a) $\sqrt{2}$ (b) 2
(c) $\sqrt{3}$ (d) $\sqrt{5}$

13. If infinite GP sum is 6 and sum of squares is 12, the first term is:

- (a) 2 (b) 3
(c) 4 (d) 1

14. If roots of $ax^2 + bx + c = 0$ are in ratio 2:3, then:

- (a) $6b^2 = 25ac$ (b) $2b^2 = 9ac$
(c) $b^2 = 6ac$ (d) $25b^2 = 6ac$

15. The value of $\int_0^{\pi/4} \tan x \, dx$ is:

- (a) $\ln 2$ (b) $\pi/4$
(c) $1/2 \ln 2$ (d) 1

16. Number of solutions of $\sin x + \cos x = 1$ in $[0, 2\pi]$ is:

- (a) 1 (b) 2
(c) 3 (d) 4

17. Vector $a = 2i - 3j + 4k$ is perpendicular to $b = i + \lambda j - k$. λ is:

- (a) $-2/3$ (b) 10
(c) -5 (d) $2/3$

18. The order of the differential equation $y'' + 3y' + y = 0$ is:

- (a) 1 (b) 2
(c) 3 (d) 0

19. Volume of tetrahedron with coord axes and plane $x/2 + y/3 + z/4 = 1$ is:

- (a) 8 (b) 4
(c) 12 (d) 6

20. The number of ways of arranging the letters of the word "MATHS" is:

- (a) 60 (b) 120
(c) 240 (d) 720

Section B: Numerical Value Questions

21. Find the number of subsets of a set containing 10 elements. _____

22. Find the number of divisors of 360. _____

23. A triangle has sides $a=6$, $b=8$, and $c=10$. Find its area. _____

24. Find the value of $\sum_{k=1}^{10} k^2$. _____

25. If $\int_0^a x^3 dx = 16$, find the value of a . _____

Answer Key & Detailed Explanation

1. [c] $f(x) = 2x + 3$; $f'(x) = 2$. $f'(2) = 7$. Sum = $7 + 2 = 9$.

2. [a] By Leibniz rule/FTC, derivative of an integral is the function itself: $g'(x) = f(x) = e^{(x^2)}$.

3. [d] Put $x=1$: $(2-3)^5 = (-1)^5 = -1$.

4. [b] $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = p^2 - 2q$. Given as 40.

5. [d] Area = $2 * \int_0^4 2\sqrt{x} dx = 4 * [x^{3/2}/(3/2)]_0^4 = 64/3$.

6. [d] Center must be (1,3) to satisfy $x+y=4$ and be equidistant from (1,1) and (2,2). Radius²=5.

7. [d] $\det(A) = 6 - (-1) = 7$. $\det(A^3) = 7^3 = 343$.

8. [a] $(x-5)^2 + 16 = 25 \Rightarrow (x-5)^2=9 \Rightarrow x-5=\pm 3$. $x=2$ or 8 .

9. [d] Distance = $|d_2-d_1|/\sqrt{(a^2+b^2+c^2)} = |7.5-5|/\sqrt{6} = 2.5/\sqrt{6} = 5/(2\sqrt{6})$.

10. [a] Standard limit formula: $(\sin ax / \sin bx)$ as $x \rightarrow 0$ is $a/b = 3/5$.

11. [b] $f(x)=3(x-1)(x-3)$. $f''(3)=6 > 0$, so $x=3$ is local minimum.

12. [d] $a^2=4$, $b^2=16$. $e = \sqrt{(1 + b^2/a^2)} = \sqrt{(1+4)} = \sqrt{5}$.

13. [b] $a/(1-r)=6$ and $a^2/(1-r^2)=12$. Solving gives $a=3$, $r=1/2$.

14. [a] Standard result for root ratio $m:n$ is $(m+n)^2/mn = b^2/ac$. $(5)^2/6 = b^2/ac \Rightarrow 25/6 = b^2/ac$.

15. [c] $[\ln|\sec x|]_0^{\pi/4} = \ln\sqrt{2} = 1/2 \ln 2$.

16. [c] Solutions are $x=0$, $\pi/2$, 2π . Total 3 solutions.

17. [a] Dot product: $2(1) - 3\lambda - 4 = 0 \Rightarrow -3\lambda = 2 \Rightarrow \lambda = -2/3$.

18. [b] Highest derivative is y'' , so order is 2.

19. [b] Volume = $1/6 * |abc| = 1/6 * (2*3*4) = 4$.

20. [b] Permutations of 5 distinct letters = $5! = 120$.

21. [1024] $2^{10} = 1024$.

22. [24] $360 = 2^3 \cdot 3^2 \cdot 5^1$. Divisors = $4*3*2 = 24$.

23. [24] Right triangle: $1/2 * 6 * 8 = 24$.

24. [385] $n(n+1)(2n+1)/6$ for $n=10$ is 385.

25. [2.82] $a^4/4 = 16 \Rightarrow a^4=64 \Rightarrow a = 2\sqrt{2} \approx 2.82$.