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Your Personal Exams Guide

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GATE EC 2011 Question Paper (13-Feb-2011) (Shift 2)

Total Time: 3 Hour **Total Marks:** 100

Instructions

Sl No.	Section Name	No. of Question	Maximum Marks
1	General Aptitude	10	15
2	Electronics and Communication Engineering	55	85

1.) A total of 180 minutes is allotted for the examination.

2.) The server will set your clock for you. In the top right corner of your screen, a countdown timer will display the remaining time for you to complete the exam. Once the timer reaches zero, the examination will end automatically. The paper need not be submitted when your timer reaches zero.

3.) There will, however, be sectional timing for this exam. You will have to complete each section within the specified time limit. Before moving on to the next section, you must complete the current one within the time limits.

Answers

1. **Answer: d**

Explanation:

Understanding the Original Pair: Gladiator : Arena

The original pair given is "Gladiator : Arena". To understand the relationship, let's define each term:

- **Gladiator:** A professional combatant or performer in ancient Rome who engaged in fights, often to the death, for the entertainment of the public.
- **Arena:** The central, open area in a Roman amphitheater where gladiatorial contests, public spectacles, and other events took place. It was the specific venue for the gladiator's activity.

Therefore, the relationship between "Gladiator" and "Arena" is that of a **specialized performer or participant** and the **specific, dedicated venue or place where their primary activity or performance takes place**. This activity is often public, formal, and sometimes high-stakes.

Analyzing the Relationship Type for Word Analogy

When analyzing word analogies, it's important to precisely identify the connection. For "Gladiator : Arena", the key aspects of the relationship include:

- **Agent/Professional:** A gladiator is a professional engaged in a specific line of work.
- **Dedicated Venue:** An arena is a place specifically designed and used for that profession's activity.
- **Formal/Public Activity:** The activity performed by the gladiator in the arena is typically public and follows certain formal rules or customs.

We are looking for an option pair that exhibits this 'Professional/Agent : Dedicated Venue for Primary, Formal Activity' relationship most closely.

Evaluating the Given Options

Let's examine each of the provided options to see how their relationships compare to "Gladiator : Arena":

- **Dancer : Stage:**
  - **Dancer:** A person whose profession is dancing or performing.
  - **Stage:** A raised platform in a theater or other venue where artistic performances take place.
  - **Relationship:** A Dancer performs on a Stage. This is a 'Performer : Place of Performance' relationship, which is quite similar to the original pair.
- **Commuter : Train:**
  - **Commuter:** A person who travels regularly between two places, typically from home to work.
  - **Train:** A form of public transport.
  - **Relationship:** A Commuter travels by (or on) a Train. This is a 'Person : Mode of Transport' relationship, which is distinctly different from the 'Agent : Place of Primary Activity' relationship we are looking for.
- **Teacher : Classroom:**
  - **Teacher:** A professional who instructs or educates students.
  - **Classroom:** A room in a school or educational institution where teaching and learning take place.
  - **Relationship:** A Teacher teaches in a Classroom. This represents a 'Professional : Place of Work/Primary Activity' relationship. It is a strong candidate because it involves a specific professional and their dedicated workspace.
- **Lawyer : Courtroom:**
  - **Lawyer:** A professional who practices law, representing clients in legal matters.
  - **Courtroom:** A room in a courthouse where legal proceedings, trials, and hearings are conducted.
  - **Relationship:** A Lawyer conducts their professional legal arguments and activities in a Courtroom. This is a 'Professional : Dedicated Venue for Formal/High-Stakes Activity' relationship.

Identifying the Best Analogy

Upon evaluating the options, "Dancer : Stage", "Teacher : Classroom", and "Lawyer : Courtroom" all present a 'person and their activity place' relationship. However, we need to find the pair that best captures the specific nuances of "Gladiator : Arena".

- The gladiator's activity in the arena is often adversarial, public, and involves high stakes (life or death). It's a specialized skill performed in a formal, designated setting for an audience.
- While a dancer performs on a stage, the activity is typically artistic performance rather than an adversarial contest.
- A teacher works in a classroom, which is a dedicated place, but the activity of teaching is instructional and collaborative, not typically adversarial or a 'contest'.
- A **Lawyer** in a **Courtroom** provides the closest parallel:
  - **Lawyer:** A highly specialized professional.
  - **Courtroom:** A formal, dedicated venue specifically designed for legal proceedings.
  - **Activity:** A lawyer's work in a courtroom (presenting cases, making arguments, cross-examining) is often adversarial, highly skilled, and conducted in a public, formal setting with significant outcomes (high stakes), much like a gladiator's contest.

The relationship between a **Lawyer** and a **Courtroom** most accurately reflects the complex relationship of a **Gladiator** and an **Arena**, emphasizing a specialized individual performing a crucial, often adversarial, and public activity within a designated, formal environment.

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2. **Answer: a**

   Explanation:

Understanding the Election Scenario

This problem involves calculating the total number of voters in an election based on initial promises made by voters to candidates P and Q, and subsequent changes in their voting intentions. We are given that candidate P lost the election by 2 votes.

Step-by-Step Calculation of Total Voters

Let's denote the total number of voters as N.

Initial Voter Promises:

- Percentage of voters promised to vote for P: 40%
- Percentage of voters promised to vote for Q: The rest, which is $100\% - 40\% = 60\%$

Changes in Voting Intentions:

- Voters who promised P but switched to Q: 15% of those who promised P.
- Voters who promised Q but switched to P: 25% of those who promised Q.

Calculating Actual Votes for P and Q:

Let's calculate the number of voters switching:

- Number of voters switching from P to Q = 15% of 40% of $N = 0.15 \times (0.40 \times N) = 0.06 \times N$
- Number of voters switching from Q to P = 25% of 60% of $N = 0.25 \times (0.60 \times N) = 0.15 \times N$

Now, let's find the final actual votes for each candidate:

- **Actual Votes for P** = (Voters initially promised to P) - (Voters who switched from P to Q) + (Voters who switched from Q to P) Actual Votes for P = $(0.40 \times N) - (0.06 \times N) + (0.15 \times N)$ Actual Votes for P = $(0.40 - 0.06 + 0.15) \times N$ Actual Votes for P = $0.49 \times N$
- **Actual Votes for Q** = (Voters initially promised to Q) - (Voters who switched from Q to P) + (Voters who switched from P to Q) Actual Votes for Q = $(0.60 \times N) - (0.15 \times N) + (0.06 \times N)$ Actual Votes for Q = $(0.60 - 0.15 + 0.06) \times N$ Actual Votes for Q = $0.51 \times N$

Using the Election Outcome to Find Total Voters:

We are given that P lost the election by 2 votes. This means the number of votes for P is 2 less than the number of votes for Q.

Mathematically, this can be written as:

Actual Votes for P = Actual Votes for Q - 2

Substituting the expressions we found:

$0.49 \times N = (0.51 \times N) - 2$

Now, we solve for N:

$2 = (0.51 \times N) - (0.49 \times N)$

$2 = (0.51 - 0.49) \times N$

$2 = 0.02 \times N$

$N = \frac{2}{0.02}$

$N = \frac{200}{2}$

$N = 100$

Summary of Votes:

Candidate	Initial Promise	Switched Away	Switched To	Actual Votes
P	$0.40N$	$0.06N$ (to Q)	$0.15N$ (from Q)	$0.49N$
Q	$0.60N$	$0.15N$ (to P)	$0.06N$ (from P)	$0.51N$

With $N = 100$:

- Actual Votes for P = $0.49 \times 100 = 49$
- Actual Votes for Q = $0.51 \times 100 = 51$

The difference is $51 - 49 = 2$ votes, which matches the condition that P lost by 2 votes.

Conclusion

The total number of voters was 100.

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3. **Answer: c**

Explanation:

Problems: Understanding the Sentence Context

The key to correctly completing this sentence lies in understanding the implication of the phrase "**counter-productive**." The sentence states, "It was her view that the country's problems had been _______ by foreign technocrats, so that to invite them to come back would be **counter-productive**."

- The term "counter-productive" means that something has the opposite effect of what is desired, or makes a situation worse rather than better.
- If inviting the foreign technocrats back would be counter-productive, it logically follows that their previous involvement must have had a negative impact on the country's problems. They didn't solve or merely identify the problems; instead, they made them worse.

Options: Evaluating Word Choices

Let's analyze each given option in the context of the sentence:

- **identified:** This word means to recognize or pinpoint something. If the technocrats simply "identified" the problems, inviting them back might be helpful for implementing solutions, not "counter-productive." This option does not fit the negative consequence implied.
- **ascertained:** Similar to "identified," this means to discover or confirm something for certain. It implies gaining knowledge, not worsening a situation. Therefore, it does not align with the "counter-productive" outcome mentioned in the sentence.

- **exacerbated:** This verb means to make a problem, a bad situation, or a negative feeling worse. If the foreign technocrats "exacerbated" the country's problems, it means they intensified them or made them more severe. In this case, inviting them back would indeed be "counter-productive" because their past actions led to a worsening of the problems. This word perfectly fits the logical flow of the sentence.
- **analysed:** This means to examine something in detail to understand its nature or structure. While technocrats might analyze problems, this word itself does not suggest they made the problems worse. Analysis is often a prerequisite for solving problems and doesn't inherently imply a negative impact that would make their return "counter-productive."

Exacerbated: The Appropriate Word Choice

Considering the meaning of "counter-productive," the only word among the options that suggests the foreign technocrats' previous involvement made the country's problems worse is **exacerbated**. Therefore, "exacerbated" is the most appropriate word to complete the given sentence, creating a coherent and logical statement.

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4. **Answer: b**

Explanation:

Frequency: Understanding Its Opposite Meaning

The question asks us to identify the word that is most nearly opposite in meaning to the word "Frequency." To answer this, we need to understand the core meaning of "Frequency" and then analyze each given option.

Defining Frequency

The word **Frequency** refers to the rate at which something occurs or is repeated over a particular period of time. It indicates how often an event or occurrence happens.

For example, if a bus comes every 10 minutes, its frequency is high. If it comes once a day, its frequency is low.

- **High frequency** implies something happens often or is common.
- **Low frequency** implies something happens seldom or is uncommon.

Analyzing the Options for Opposite Meaning

Let's examine each option to see which one stands as the most direct opposite to "Frequency."

- **1. periodicity**

  Periodicity refers to the quality or state of being periodic, meaning something recurs at regular intervals. This concept is closely related to frequency. For instance, a periodic event has a certain frequency. Therefore, "periodicity" is not an opposite; it is more of a related concept or even a synonym in certain contexts, particularly when discussing recurring events.

- **2. rarity**

  Rarity refers to the state or quality of being rare; something that does not occur often or is uncommon. If something occurs with high **frequency**, it is common. If something occurs with low **frequency**, it is rare. Thus, "rarity" directly expresses the idea of infrequency or uncommonness, making it the most suitable opposite of "Frequency."

- **3. gradualness**

  Gradualness refers to the quality of being gradual, meaning something happens or changes slowly, by small degrees. This word describes the pace or manner of change, not how often something occurs. There is no direct opposite relationship between "gradualness" and "Frequency."

- **4. persistency**

  Persistency refers to the quality of persisting; continuing firmly or existing over a prolonged period. While something that persists might happen repeatedly, "persistency" itself doesn't quantify how often something happens (its frequency). An event can persist rarely or frequently. Therefore, "persistency" is not the opposite of "Frequency."

Conclusion: The Opposite of Frequency

Based on our analysis, "rarity" clearly stands out as the most nearly opposite in meaning to "Frequency." Where "Frequency" denotes how often something occurs, "rarity" denotes how seldom it occurs.

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5. **Answer: d**

Explanation:

Indian Medical Association's Ethical Guidelines for Gene Manipulation

Understanding the ethical implications of advanced medical procedures like gene manipulation is crucial. The question asks about specific conditions under which the Indian Medical Association (IMA) permits human gene manipulation, emphasizing that it should only occur when conventional treatments are inadequate.

Understanding Gene Manipulation and Ethical Principles

Gene manipulation, often referred to as gene therapy or genetic engineering, involves altering an individual's genes to treat or prevent disease. Due to its profound implications for human health and identity, strict ethical guidelines are essential. These guidelines typically ensure that such powerful interventions are considered only as a last resort, when existing, less invasive, or less risky treatments have proven ineffective.

Analyzing the Options for 'Unsatisfactory Treatments'

Let's carefully examine each option provided to determine which word best completes the sentence, aligning with the ethical principles of medical intervention:

- **Option 1: similar**

  If "similar" treatments are unsatisfactory, it does not necessarily imply that all other possible treatments, even those dissimilar, have been exhausted. Ethical guidelines typically demand a broader failure of conventional care before resorting to gene manipulation. This option is too restrictive in its scope.

- **Option 2: most**

  Stating that "most" treatments are unsatisfactory is vague. It doesn't specify which majority of treatments have failed, nor does it guarantee that all viable alternatives have been considered. Ethical considerations demand a more comprehensive assessment of treatment failure.

- **Option 3: uncommon**

  Whether treatments are "uncommon" or common is not the primary ethical criterion for deciding on gene manipulation. The key is their efficacy and whether they offer a satisfactory solution to the disease. An uncommon but effective treatment might still be preferable to gene manipulation. This option is irrelevant to the ethical justification.

- **Option 4: available**

  The word "available" is the most appropriate choice. It implies that all treatments that are currently known, accessible, and typically used for the specific disease have been tried or thoroughly considered and found to be unsatisfactory. This aligns with the principle that gene manipulation, being a complex and potentially high-risk intervention, should be a treatment of last resort when all conventional, "available" options have failed to provide adequate relief or cure.

Conclusion on Ethical Treatment Selection

The Indian Medical Association's ethical guidelines underscore a cautious approach to gene manipulation. Human genes should only be manipulated to correct diseases when all **available** treatments have been deemed unsatisfactory. This ensures that gene therapy is pursued only when conventional medical science cannot adequately address the patient's condition, highlighting its role as a significant, carefully considered intervention in the medical field.

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6. **Answer: b**

Explanation:

Horses' Contribution to Medicine

The passage highlights a significant, though often overlooked, role that horses have played in the field of medicine. This process involves utilizing the natural immune response of horses to create valuable medical treatments.

Understanding the Immunity Building Process in Horses

The passage describes a specific process where horses were used to develop serums. Let's break down this process:

- **Toxin Injection:** Horses were injected with "toxins of diseases." This means they were exposed to substances that cause illness.
- **Immunity Development:** Following the injection of toxins, the horses' bodies naturally "built up immunities" in their blood. This implies their immune systems responded by producing antibodies to fight off these toxins.
- **Serum Creation:** Once sufficient immunity was built, a "serum was made from their blood." This serum, rich in antibodies, could then be used to treat diseases in humans.
- **Disease Treatment:** Specifically, serums to combat diphtheria and tetanus were developed using this method, demonstrating the effectiveness of the process.

Analyzing the Inference about Horses' Immunity

The question asks what can be inferred about horses based on this passage. We need to look for a conclusion that logically follows from the described medical process.

Let's evaluate each option:

- **Option 1: <p>given immunity to diseases</p>**
  This option is incorrect. The passage states that horses "built up immunities" after being injected with toxins. This means they developed immunity themselves in response to exposure, rather than being "given" pre-existing immunity from an external source. The purpose of the process was to *induce* immunity in them, not to confer it upon them.
- **Option 2: <p>generally quite immune to diseases</p>**
  This is the most accurate inference. For horses to be successfully injected with disease toxins, survive the exposure, and then produce a strong immune response (antibodies) without succumbing to severe illness, they must possess a robust immune system. If horses were easily susceptible or fragile, this method would not be viable. Their capacity to withstand the toxins and effectively "build up immunities" suggests a general physiological resilience or strong immune response mechanism, which can be interpreted as being "generally quite immune" in the sense of being able to robustly fight off or adapt to disease challenges. This inherent strength makes them suitable for such medical procedures.
- **Option 3: <p>given medicines to fight toxins</p>**
  This option is incorrect. The passage clearly states horses were "injected with toxins," not given medicines to fight those toxins. The horses' own bodies produced the "fight" in the form of immunity.
- **Option 4: <p>given diphtheria and tetanus serums</p>**
  This option is incorrect. The passage explains that serums for diphtheria and tetanus were "made from their blood" (the horses' blood). This means the horses were the *source* of the serums, not the recipients of them.

Key Takeaway: Horses' Role in Medical Advancements

The success of developing serums from horses' blood, specifically for serious diseases like diphtheria and tetanus, implies that horses have a naturally strong immune system. This allows them to withstand controlled exposure to toxins and produce powerful antibodies, making them invaluable contributors to early medical advancements.

7. **Answer: b**

Explanation:

Calculation:

Consumption (km/liter) means distance covered by the car in 1 liter of fuel

⇒ Fuel consumption per liter = $\frac{1}{Consumption\ (km/liter)}$

	P	Q	R	S
Distance	15	75	40	10
Speed	15	45	75	10
Consumption (km/liter)	60	90	75	30
Fuel consumption per km	$\frac{1}{60}$ = 0.016	$\frac{1}{90}$ = 0.011	$\frac{1}{75}$ = 0.013	$\frac{1}{30}$ = 0.033

From the above table, fuel consumption per km was least during the lap Q.

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8. **Answer: c**

Explanation:

Solving the Toffee Sharing Problem

This problem involves a sequence of actions where toffees are taken and returned. To find the original number of toffees, we need to work backward from the final known quantity.

Working Backwards Step-by-Step

Let's denote the number of toffees at different stages:

- N = Original number of toffees.
- N_R = Number of toffees after R's actions.
- N_S = Number of toffees after S's actions.
- N_T = Number of toffees after T's actions (final amount).

We are given that the final amount, N_T, is 17 toffees.

Reversing T's Actions

T took half of the remaining toffees and returned two. Let the number of toffees just before T acted be X.

- Amount T took = $\frac{1}{2}X$
- Amount left after T took his share = $X - \frac{1}{2}X = \frac{1}{2}X$
- After T returned 2 toffees, the amount became $\frac{1}{2}X + 2$.

We know this final amount is 17:

$$\frac{1}{2}X + 2 = 17$$

Subtract 2 from both sides:

$$\frac{1}{2}X = 17 - 2$$

$$\frac{1}{2}X = 15$$

Multiply by 2 to find X:

$$X = 15 \times 2 = 30$$

So, there were 30 toffees before T took his share ($N\_S = 30$).

Reversing S's Actions

S took 1/4th of the remaining toffees and returned three. Let the number of toffees just before S acted be Y.

- Amount S took = $\frac{1}{4}Y$
- Amount left after S took his share = $Y - \frac{1}{4}Y = \frac{3}{4}Y$

- After S returned 3 toffees, the amount became $\frac{3}{4}Y + 3$.

This amount equals the toffees before T acted, which is *X = 30*:

$$\frac{3}{4}Y + 3 = 30$$

Subtract 3 from both sides:

$$\frac{3}{4}Y = 30 - 3$$

$$\frac{3}{4}Y = 27$$

Multiply by $\frac{4}{3}$ to find *Y*:

$$Y = 27 \times \frac{4}{3}$$

$$Y = 9 \times 4 = 36$$

So, there were 36 toffees before S took his share (*N_R = 36*).

Reversing R's Actions

R took 1/3rd of the original toffees and returned four. Let the original number of toffees be *N*.

- Amount R took = $\frac{1}{3}N$
- Amount left after R took his share = $N - \frac{1}{3}N = \frac{2}{3}N$
- After R returned 4 toffees, the amount became $\frac{2}{3}N + 4$.

This amount equals the toffees before S acted, which is *Y = 36*:

$$\frac{2}{3}N + 4 = 36$$

Subtract 4 from both sides:

$$\frac{2}{3}N = 36 - 4$$

$$\frac{2}{3}N = 32$$

Multiply by $\frac{3}{2}$ to find N:

$$N = 32 \times \frac{3}{2}$$

$$N = 16 \times 3 = 48$$

Therefore, the original number of toffees was 48.

Verification of the Solution

Let's check if starting with 48 toffees leads to the final count of 17:

1. **Start:** 48 toffees.
2. **R's turn:** R takes $\frac{1}{3} \times 48 = 16$. Bowl has $48 - 16 = 32$. R returns 4. Bowl has $32 + 4 = 36$.
3. **S's turn:** S takes $\frac{1}{4} \times 36 = 9$. Bowl has $36 - 9 = 27$. S returns 3. Bowl has $27 + 3 = 30$.
4. **T's turn:** T takes $\frac{1}{2} \times 30 = 15$. Bowl has $30 - 15 = 15$. T returns 2. Bowl has $15 + 2 = 17$.

The final count of 17 matches the problem statement, confirming our calculation.

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9. Answer: d

Explanation:

Function Understanding: Absolute Value Definition

The given function is $f(y) = \frac{|y|}{y}$. To understand this function, we need to consider the definition of the absolute value function, $|y|$.

- If y is a positive real number $(y > 0)$, then $|y| = y$.
- If y is a negative real number $(y < 0)$, then $|y| = -y$.
- The function is undefined when $y = 0$, as division by zero is not allowed.

Based on this, we can define $f(y)$ in two cases:

- **Case 1:** $y > 0$
  If $y > 0$, then $f(y) = \frac{y}{y} = 1$.

- **Case 2:** $y < 0$
  If $y < 0$, then $f(y) = \frac{-y}{y} = -1$.

Evaluating f(q) for Non-Zero Real Number q

The problem states that q is any non-zero real number. We need to evaluate $f(q)$ based on whether q is positive or negative.

- **When $q > 0$ (q is positive):**
  According to our function definition for $y > 0$, $f(q) = 1$.
- **When $q < 0$ (q is negative):**
  According to our function definition for $y < 0$, $f(q) = -1$.

Evaluating f(-q) for Non-Zero Real Number q

Next, we need to evaluate $f(-q)$. The sign of $-q$ depends on the sign of q.

- **When $q > 0$ (q is positive):**
  If q is positive, then $-q$ will be negative $(-q < 0)$.
  According to our function definition for $y < 0$, $f(-q) = -1$.
- **When $q < 0$ (q is negative):**
  If q is negative, then $-q$ will be positive $(-q > 0)$.
  According to our function definition for $y > 0$, $f(-q) = 1$.

Calculating the Difference f(q) - f(-q)

Now we will calculate the difference $f(q) - f(-q)$ for both cases of q.

- **Case 1: When $q > 0$**

  We found $f(q) = 1$ and $f(-q) = -1$.

  So, $f(q) - f(-q) = 1 - (-1) = 1 + 1 = 2$.

- **Case 2: When $q < 0$**

  We found $f(q) = -1$ and $f(-q) = 1$.

So, $\mathrm{f(q) - f(-q) = -1 - 1 = -2}$.

Condition	$\mathrm{f(q)}$	$\mathrm{f(-q)}$	$\mathrm{f(q) - f(-q)}$
$\mathrm{q > 0}$	1	-1	$1 - (-1) = 2$
$\mathrm{q < 0}$	-1	1	$-1 - 1 = -2$

Final Absolute Value Calculation: $|f(q) - f(-q)|$

Finally, we need to find the absolute value of the difference, $\mathrm{|f(q) - f(-q)|}$.

- **From Case 1 ($\mathrm{q > 0}$):**
  $\mathrm{|f(q) - f(-q)| = |2| = 2}$.
- **From Case 2 ($\mathrm{q < 0}$):**
  $\mathrm{|f(q) - f(-q)| = |-2| = 2}$.

In both possible scenarios for q (positive or negative), the value of $\mathrm{|f(q) - f(-q)|}$ is 2.

The final answer is 2.

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10. **Answer: c**

Explanation:

Sum of Series 4 + 44 + 444 + ...

To find the sum of 'n' terms of the given series $S_n = 4 + 44 + 444 + \cdots + \mathrm{n\ terms}$, we can use a systematic approach by transforming each term into a form involving powers of 10.

Series Transformation for Summation

The given series is:

$$S_n = 4 + 44 + 444 + \cdots + \mathrm{n\ terms}$$

Step 1: Factor out 4 from each term.

$$S_n = 4(1 + 11 + 111 + \cdots + \text{n terms})$$

Step 2: Multiply and divide the expression inside the parenthesis by 9. This step is crucial as it helps convert the repeating digit numbers into a difference of powers of 10 and 1.

$$S_n = 4\left(\frac{1}{9}\right)(9 + 99 + 999 + \cdots + \text{n terms})$$

$$S_n = \frac{4}{9}(9 + 99 + 999 + \cdots + \text{n terms})$$

Step 3: Express each term inside the parenthesis using powers of 10.

- The first term, 9, can be written as $10 - 1$.
- The second term, 99, can be written as $10^2 - 1$.
- The third term, 999, can be written as $10^3 - 1$.
- Following this pattern, the n-th term will be $10^n - 1$.

So, the series becomes:

$$S_n = \frac{4}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \cdots + (10^n - 1)]$$

Grouping Terms and Applying Formulas

Step 4: Group the positive powers of 10 together and the negative ones together.

$$S_n = \frac{4}{9}[(10 + 10^2 + 10^3 + \cdots + 10^n) - (1 + 1 + 1 + \cdots + \text{n times})]$$

Step 5: Calculate the sum of the first part, which is a Geometric Progression (GP).

The first part is $10 + 10^2 + 10^3 + \cdots + 10^n$.

This is a geometric progression with:

- First term $a = 10$
- Common ratio $r = \frac{10^2}{10} = 10$
- Number of terms n

The sum of n terms of a GP is given by the formula $S_n = \frac{a(r^n-1)}{r-1}$.

Substituting the values:

$$\text{Sum of GP} = \frac{10(10^n - 1)}{10 - 1} = \frac{10(10^n - 1)}{9}$$

$$\text{Sum of GP} = \frac{10^{n+1} - 10}{9}$$

Step 6: Calculate the sum of the second part.

The second part is $1 + 1 + 1 + \cdots + \text{n times}$.

The sum of 'n' ones is simply n.

Final Summation of Series

Step 7: Substitute the sums of both parts back into the main equation for S_n.

$$S_n = \frac{4}{9}\left[\left(\frac{10^{n+1} - 10}{9}\right) - n\right]$$

Step 8: Simplify the expression.

To combine the terms inside the square brackets, find a common denominator, which is 9:

$$S_n = \frac{4}{9}\left[\frac{10^{n+1} - 10 - 9n}{9}\right]$$

Multiply the fractions:

$$S_n = \frac{4}{9 \times 9}[10^{n+1} - 9n - 10]$$

$$S_n = \frac{4}{81}[10^{n+1} - 9n - 10]$$

Conclusion

The sum of n terms of the series $4 + 44 + 444 + \ldots$ is $\frac{4}{81}[10^{n+1} - 9n - 10]$.

This result matches option 3 provided in the question.

11. **Answer: d**

Explanation:

Poynting Vector Fundamentals

The complex Poynting vector, denoted as $\vec{P}$, is a fundamental concept in electromagnetism that describes the directional energy flux density (power per unit area) of an electromagnetic field. For a time-harmonic field, it is typically defined as:

$$\vec{P} = \frac{1}{2}\left(\vec{E} \times \vec{H}^*\right)$$

where $\vec{E}$ is the complex electric field and $\vec{H}^*$ is the complex conjugate of the magnetic field. The real part of the complex Poynting vector, $Re\left(\vec{P}\right)$, represents the time-averaged power density, which is the actual power flow per unit area.

Power Density Behavior of a Point Source

When considering a point source radiating power in an infinite, homogeneous, and lossless medium, the electromagnetic energy spreads out. As the waves propagate outwards, the energy is distributed over an increasingly larger spherical surface. Consequently, the power density, represented by $Re\left(\vec{P}\right)$, must decrease with increasing radial distance from the source.

- For an ideal isotropic point source, the time-averaged power density $Re\left(\vec{P}\right)$ is inversely proportional to the square of the radial distance r from the source. This relationship can be expressed as $Re\left(\vec{P}\right) \propto 1/r^2$.
- Therefore, statement 1, which suggests that $Re\left(\vec{P}\right)$ remains constant at any radial distance, is **false**.
- Similarly, statement 2, which claims that $Re\left(\vec{P}\right)$ increases with increasing radial distance, is also **false**.

Radiated Power Through a Spherical Surface

The total time-averaged power radiated by the point source and flowing through a closed spherical surface S of radius r, centered at the source, is calculated by integrating the real part of the Poynting vector over the entire surface. This is given by the surface integral:

\[P_{total} = \DoubleContourIntegral;_s Re\left({\vec P } \right)\cdot \hat n\ d\vec s \]

Here, $\hat{n}$ denotes the unit outward normal vector to the surface element $d\vec{s}$ on the spherical surface S.

In an ideal, perfectly lossless medium, the principle of energy conservation states that the total power radiated by a source remains constant as it propagates. This implies that the total average power flowing through any closed surface enclosing the source should be the same, regardless of the size or radial distance of the surface. If $Re\left(\vec{P}\right)$ decreases as $1/r^2$, then integrating it over a spherical surface of area $4\pi r^2$ would result in a constant value.

Analysis of Statements

Let's evaluate the remaining statements based on the question and the provided correct answer:

- Statement 3 suggests that \(\DoubleContourIntegral;_s Re\left({\vec P } \right)\cdot \hat n\ d\vec s \) remains constant at any radial distance from the source. This is consistent with the conservation of power in an ideal lossless medium.
- Statement 4 suggests that \(\DoubleContourIntegral;_s Re\left({\vec P } \right)\cdot \hat n\ d\vec s \) decreases with increasing radial distance from the source.

According to the information provided, the correct statement is that the integral \(\DoubleContourIntegral;_s Re\left({\vec P } \right)\cdot \hat n\ d\vec s \) decreases with increasing radial distance from the source. This indicates that, within the specific context of this problem, the total effective radiated power diminishes as it propagates outwards to greater distances, despite the medium being described as lossless. This behavior implies that there might be some form of attenuation or dispersion of the total integrated power not accounted for by simple inverse-square spreading in this particular scenario.

---

12. **Answer: c**

Explanation:

The problem asks us to determine the phase velocity of a wave along a transmission line, given its excitation frequency, the phase difference between two points, and the distance separating these points. This is a fundamental concept in wave propagation and transmission line theory.

Phase Velocity Understanding

Phase velocity (v_p) is the speed at which the phase of a wave propagates in a medium. It is a crucial parameter in understanding how signals travel along transmission lines. For a sinusoidal wave, the phase velocity is related to the angular frequency (ω) and the phase constant (β) by the formula:

$$v_p = \frac{\omega}{\beta}$$

Where:

- ω is the angular frequency (in radians per second).
- β is the phase constant (in radians per meter), representing the change in phase per unit length.

Given Parameters Analysis

Let's list the information provided in the question:

- **Characteristic impedance** (Z_0): 50 Ω.
- **Load impedance** (Z_L): 50 Ω. Since $Z_0 = Z_L$, the transmission line is perfectly matched, meaning there are no reflections. This information, while important for transmission line analysis, is not directly used in the calculation of phase velocity based on phase difference and distance.

- **Excitation frequency** (f): 10 GHz. We need to convert this to angular frequency ω.
- **Phase difference** ($\Delta\phi$) between two points: $\frac{\pi}{4}$ radians.
- **Distance** (Δz) between the two points: 2 mm. We need to convert this to meters.

Step-by-Step Calculation

1. Frequency Conversion

First, convert the given frequency f from GHz to Hz and then calculate the angular frequency ω:

Given $f = 10 \text{ GHz} = 10 \times 10^9 \text{ Hz}$

The formula for angular frequency is $\omega = 2\pi f$.

$$\omega = 2\pi \times (10 \times 10^9) \text{ rad/s}$$

$$\omega = 2\pi \times 10^{10} \text{ rad/s}$$

2. Distance Conversion

Next, convert the given distance Δz from millimeters to meters:

Given $\Delta z = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$

3. Phase Constant Calculation

The phase difference $\Delta\phi$ between two points separated by a distance Δz on a transmission line is given by the formula:

$$\Delta\phi = \beta \times \Delta z$$

From this, we can find the phase constant β:

$$\beta = \frac{\Delta\phi}{\Delta z}$$

Substitute the given values for $\Delta\phi$ and Δz:

$$\beta = \frac{\frac{\pi}{4} \text{ rad}}{2 \times 10^{-3} \text{ m}}$$

$$\beta = \frac{\pi}{4 \times 2 \times 10^{-3}} \text{ rad/m}$$

$$\beta = \frac{\pi}{8 \times 10^{-3}} \text{ rad/m}$$

$$\beta = \frac{\pi}{8} \times 10^{3} \text{ rad/m}$$

4. Phase Velocity Determination

Finally, we can calculate the phase velocity v_p using the formula $v_p = \frac{\omega}{\beta}$:

Substitute the calculated values for ω and β:

$$v_p = \frac{2\pi \times 10^{10} \text{ rad/s}}{\frac{\pi}{8} \times 10^{3} \text{ rad/m}}$$

$$v_p = \frac{2\pi \times 10^{10}}{\pi \times 10^{3}/8} \text{ m/s}$$

$$v_p = \frac{2\pi \times 10^{10} \times 8}{\pi \times 10^{3}} \text{ m/s}$$

Cancel out π from the numerator and denominator:

$$v_p = \frac{2 \times 10^{10} \times 8}{10^{3}} \text{ m/s}$$

$$v_p = 16 \times 10^{(10-3)} \text{ m/s}$$

$$v_p = 16 \times 10^{7} \text{ m/s}$$

To express this in standard scientific notation:

$$v_p = 1.6 \times 10^{8} \text{ m/s}$$

Comparison with Options

Let's compare our calculated phase velocity with the given options:

Option	Phase Velocity (v_p)
1	$0.8 \times 10^8 \ \mathrm{m/s}$
2	$1.2 \times 10^8 \ \mathrm{m/s}$
3	$1.6 \times 10^8 \ \mathrm{m/s}$
4	$3 \times 10^8 \ \mathrm{m/s}$

The calculated phase velocity $1.6 \times 10^8 \ \mathrm{m/s}$ matches Option 3.

This problem demonstrates the application of fundamental wave equations to determine the phase velocity on a transmission line, using measurable parameters like frequency, phase difference, and distance.

---

13. **Answer: d**

Explanation:

Information Rate Basics

The question asks us to determine the information rate of an analog signal under specific conditions. We are given the signal's bandwidth, the sampling method (Nyquist rate), the number of quantization levels, and the transmission rate of the quantized samples. The information rate is essentially the total number of bits transmitted per second. To find this, we need to understand how many bits each sample represents and how many such samples are transmitted per second.

Analog Signal Bandwidth and Sampling

An **analog signal** is specified as being **bandlimited** to $4 \ \mathrm{KHz}$. This means its maximum frequency component (B) is $4 \ \mathrm{KHz}$.

The signal is **sampled at Nyquist rate**. The Nyquist rate (f_s) is the minimum sampling rate required to perfectly reconstruct a bandlimited analog signal from its samples without aliasing. It is given by:

$$f_s = 2B$$

Where B is the bandwidth of the signal.

- Given bandwidth, $B = 4\ \text{KHz}$
- Nyquist rate, $f_s = 2 \times 4\ \text{KHz} = 8\ \text{KHz} = 8000\ \text{samples/second}$.

While the Nyquist rate tells us how frequently the signal should be sampled, the problem specifies a different transmission rate for the **quantized samples**. This distinction is crucial for calculating the actual information rate.

Quantized Levels and Bits per Sample

After sampling, the samples are **quantized into 4 levels**. Quantization is the process of mapping continuous-amplitude samples into a finite number of discrete amplitude levels.

We are told that the quantized levels are independent and equally probable. When there are M equally probable levels, the number of bits (n) required to represent each sample is given by the formula:

$$M = 2^n$$

Or equivalently:

$$n = \log_2(M)$$

- Number of quantization levels, $M = 4$
- Bits per sample, $n = \log_2(4) = 2\ \text{bits/sample}$.

This means each sample, after being quantized, carries 2 bits of information.

Information Rate Computation

The problem states: "If we transmit two **quantized samples** per sec". This is the rate at which the quantized data is being sent over a communication channel. This rate, let's call it R_s, is 2 samples/sec.

The information rate (R) is calculated by multiplying the number of bits per sample by the rate at which these samples are transmitted.

$$\text{Information Rate } (R) = (\text{Transmission rate of samples}) \times (\text{Bits per sample})$$

Using the values we've found:

- Transmission rate of samples (R_s) = 2 samples/sec
- Bits per sample (n) = 2 bits/sample

Therefore, the information rate is:

$$R = 2 \text{ samples/sec} \times 2 \text{ bits/sample} = 4 \text{ bits/sec}$$

Final Information Rate

Based on the analysis, transmitting two quantized samples per second, where each sample is represented by 2 bits, results in an information rate of 4 bits per second.

Parameter	Value
Analog Signal Bandwidth (B)	4 KHz
Nyquist Rate (f_s)	$8000 \text{ samples/second}$
Number of Quantization Levels (M)	4
Bits per Sample $(n = \log_2 M)$	2 bits/sample
Transmission Rate of Quantized Samples (R_s)	2 samples/sec
Information Rate $(R = R_s \times n)$	4 bits/sec

The calculated information rate is 4 bits/s.

14. **Answer: b**

Explanation:

Concept:

Every branch of a root locus diagram starts at a pole (K = 0) and terminates at a zero (K = ∞) of the open-loop transfer function.

Application:

Zero = -1

Notice that there are two-locus lines going out from the pole at -3, which implies that there are two poles at s = -3,

Pole = 0, -2, -3, -3

On the real axis to the right side of any section, if the sum of total number of poles and zeros are odd, root locus diagram exists in that section.

∴

$G(s)\,H(s) = \frac{k(s+1)}{s(s+2)(s+3)^2}$

⭐ **Important Points**

1. Root locus diagram is symmetrical with respect to the real axis.

2. Number of branches of the root locus diagram are:

N = P if P ≥ Z

= Z, if P ≤ Z

3. Number of asymptotes in a root locus diagram = |P − Z|

4. Centroid: It is the intersection of the asymptotes and always lies on the real axis. It is denoted by σ.

$\sigma = \frac{\sum P_i - \sum Z_i}{|P-Z|}$

ΣPi is the sum of real parts of finite poles of G(s)H(s)

ΣZi is the sum of real parts of finite zeros of G(s)H(s)

5. Angle of asymptotes: $\theta_l = \frac{(2l+1)\pi}{P-Z}$

l = 0, 1, 2, ... |P − Z| − 1

6. Break-in/away points: These exist when there are multiple roots on the root locus diagram.

At the breakpoints gain K is either maximum and/or minimum.

So, the roots of $\frac{dK}{ds}$ are the break points.

---

15. **Answer: b**

Explanation:

To determine if a system is stable and causal, we need to carefully analyze its impulse response, $h(n)$. The given impulse response for the system is $h(n) = 2^n u(n-2)$.

System Impulse Response Details

The impulse response $h(n) = 2^n u(n-2)$ describes how the system reacts to an impulse. The term $u(n-2)$ is a unit step function, which has the following properties:

- $u(n-2) = 1$ when the argument $(n-2)$ is greater than or equal to zero, i.e., for $n-2 \geq 0$, which simplifies to $n \geq 2$.
- $u(n-2) = 0$ when the argument $(n-2)$ is less than zero, i.e., for $n-2 < 0$, which simplifies to $n < 2$.

Based on this, the impulse response $h(n)$ can be explicitly written as:

- $h(n) = 2^n \cdot 1 = 2^n$ for $n \geq 2$

- $h(n) = 2^n \cdot 0 = 0$ for $n < 2$

This means that $h(n)$ has non-zero values only when n is 2 or greater.

Causality of the System

A discrete-time Linear Time-Invariant (LTI) system is considered **causal** if its impulse response $h(n)$ is zero for all negative values of n. In other words, for a system to be causal, $h(n)$ must be equal to 0 for all $n < 0$.

- From our detailed analysis of $h(n) = 2^n u(n-2)$, we found that $h(n) = 0$ for all $n < 2$.
- Since the condition $n < 0$ falls completely within the range $n < 2$, it naturally follows that $h(n) = 0$ for all $n < 0$.

Therefore, the system defined by $h(n) = 2^n u(n-2)$ is **causal**.

Stability of the System

A discrete-time LTI system is **Bounded-Input Bounded-Output (BIBO) stable** if its impulse response $h(n)$ is absolutely summable. This critical condition for stability is expressed mathematically as:

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

Let's evaluate this sum for the given impulse response $h(n) = 2^n u(n-2)$:

$$\sum_{n=-\infty}^{\infty} |2^n u(n-2)|$$

Since $u(n-2)$ is zero for $n < 2$, the summation effectively starts from $n = 2$:

$$\sum_{n=2}^{\infty} |2^n|$$

Because 2^n is always a positive value for real n, $|2^n|$ is simply 2^n. So the sum becomes:

$$\sum_{n=2}^{\infty} 2^n = 2^2 + 2^3 + 2^4 + \dots$$

This expression represents a geometric series. For a geometric series $\sum_{k=N}^{\infty} ar^k$ to converge to a finite value, the absolute value of the common ratio r must be strictly less than 1 $(|r| < 1)$. In this specific series:

- The first term (for $n = 2$) is $a = 2^2 = 4$.
- The common ratio $r = 2$.

Since the common ratio $|r| = 2$, which is clearly not less than 1 $(|r| \geq 1)$, this geometric series diverges. This means the sum is infinitely large:

$$\sum_{n=2}^{\infty} 2^n = \infty$$

Because the sum of the absolute values of the impulse response is infinite, the system is **not stable**.

System Classification Summary

Based on our thorough analysis of the system's impulse response $h(n) = 2^n u(n-2)$:

- The system is determined to be **causal** because $h(n) = 0$ for all $n < 0$.
- The system is determined to be **not stable** because the sum of the absolute values of its impulse response diverges, meaning $\sum_{n=-\infty}^{\infty} |h(n)| = \infty$.

Therefore, the system is causal but not stable.

---

16. **Answer: d**

Explanation:

Network Impulse Response Derivation

The relationship between the unit step response and the unit impulse response of a linear, time-invariant (LTI) network is a fundamental concept in signals and systems.

The unit impulse response of a network is obtained by differentiating its unit step response with respect to time.

Given the unit step response of the network, denoted as $s(t)$, is:

$s(t) = (1 - e^{-\alpha t})$

Impulse Response Calculation

To determine the unit impulse response, denoted as $h(t)$, we must differentiate the given unit step response $s(t)$ with respect to time t.

The mathematical relationship between the unit impulse response and the unit step response is:

$h(t) = \frac{d}{dt} s(t)$

Let us apply this differentiation to the given unit step response $s(t) = 1 - e^{-\alpha t}$:

- The derivative of a constant term (which is '1' in this case) with respect to t is always 0.
- For the exponential term, recall the general differentiation rule for e^{ax}, which is $\frac{d}{dx}(e^{ax}) = ae^{ax}$.
- Applying this rule to $-e^{-\alpha t}$, where $a = -\alpha$, the derivative will be $-(-\alpha e^{-\alpha t})$.

Combining these differentiation rules, we perform the step-by-step calculation:

$h(t) = \frac{d}{dt}(1 - e^{-\alpha t})$

$h(t) = \frac{d}{dt}(1) - \frac{d}{dt}(e^{-\alpha t})$

$h(t) = 0 - (-\alpha e^{-\alpha t})$

$h(t) = \alpha e^{-\alpha t}$

Final Network Response

Therefore, the unit impulse response of the given network is $\alpha e^{-\alpha t}$. This result indicates how the network responds to a very short duration, high-magnitude input, which is represented by the unit impulse function.

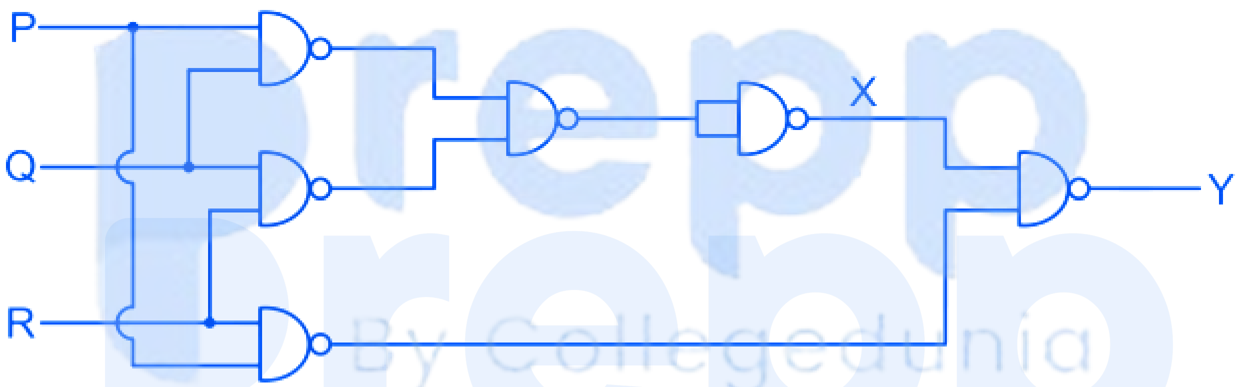
Comparing our calculated unit impulse response with the provided options, we find a match with option 4.

---

17. **Answer: b**

Explanation:

From the figure,



$X = \overline{PQ}.\overline{QR} = \left(\overline{PQ + QR}\right)$

$\therefore Y = \overline{\left(\overline{PQ + QR}\right)\,\overline{PR}} = PQ + QR + PR$

if two (or) more inputs are zero,

$\rightarrow$ Y = 0

if two (or) more inputs are one,

$\rightarrow$ Y = 1

---

18. **Answer: a**

Explanation:

For the parallel RLC circuit, the resonant frequency is

$w_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times10^{-6}\times1\times10^{-9}}} = 10\ M\ rad/sec$

∴ given circuit is in resonance

∴ impedance of parallel RLC = Z_{max} = R = 2KΩ

∴ gain = - g_m (Z_C || R_L)

= - g_m (2K || 2K)

∴ gain is maximum at resonance

∴ option (1) is correct

---

19. **Answer: c**

Explanation:

Drift Current in Semiconductors Explained

Drift current is a fundamental concept in semiconductor physics, describing the flow of charge carriers (electrons and holes) under the influence of an external electric field. This type of current is directly responsible for the operation of many electronic devices.

Drift Current Dependence

The magnitude of the drift current in a semiconductor depends primarily on two key factors:

- **Electric Field (E):** The electric field provides the force that accelerates the charge carriers. A stronger electric field will result in a larger drift velocity of the carriers, and thus a larger drift current.
- **Carrier Concentration (n or p):** This refers to the number of free electrons (n) or holes (p) available in the semiconductor material. More carriers mean more

charges available to move under the influence of the electric field, leading to a higher drift current.

Mathematical Representation of Drift Current

The drift current density ($\vec{J}_{\text{drift}}$) is mathematically expressed as the sum of electron drift current density ($\vec{J}_{n,\text{drift}}$) and hole drift current density ($\vec{J}_{p,\text{drift}}$).

For electrons, the drift current density is:

$$\vec{J}_{n,\text{drift}} = qn\mu_n\vec{E}$$

For holes, the drift current density is:

$$\vec{J}_{p,\text{drift}} = qp\mu_p\vec{E}$$

Where:

- q is the elementary charge (magnitude of charge of an electron).
- n is the electron concentration.
- p is the hole concentration.
- μ_n is the electron mobility.
- μ_p is the hole mobility.
- $\vec{E}$ is the applied electric field.

The total drift current density is the sum of these two components:

$$\vec{J}_{\text{drift}} = \vec{J}_{n,\text{drift}} + \vec{J}_{p,\text{drift}} = q(n\mu_n + p\mu_p)\vec{E}$$

From these equations, it is clear that drift current directly depends on both the carrier concentration (n and p) and the electric field ($\vec{E}$).

Drift Current vs. Diffusion Current

It is important to distinguish drift current from diffusion current, another major current mechanism in semiconductors.

- **Drift Current:** Caused by an applied **electric field**, resulting in the movement of carriers. Depends on electric field and carrier concentration.

- **Diffusion Current:** Caused by a **carrier concentration gradient** (difference in carrier concentration from one region to another). Carriers move from a region of higher concentration to a region of lower concentration. This current does NOT directly depend on the electric field.

Therefore, options that include "carrier concentration gradient" as a dependency for drift current are incorrect because that describes diffusion current. Similarly, "only the electric field" is incorrect because the number of available carriers (concentration) is also crucial.

Conclusion on Drift Current Factors

Based on the principles of semiconductor physics, drift current in semiconductors is a direct function of both the applied electric field and the concentration of charge carriers (electrons and holes) present in the material.

---

20. **Answer: b**

Explanation:

Zener Diode for Voltage Stabilization

A **Zener diode** is a specialized type of semiconductor diode that is designed to operate reliably in the reverse-biased direction. Unlike conventional diodes, which are typically used to block current in reverse bias, a Zener diode allows current to flow when the reverse voltage reaches a specific value, known as the **Zener breakdown voltage**. This unique characteristic makes it highly effective for applications requiring **voltage stabilization** or regulation.

Zener Diode Biasing Regions Explained

To understand why a Zener diode is used in a particular bias mode for voltage stabilization, it's important to differentiate its operating regions:

- **Forward Bias Region:** In this mode, the Zener diode behaves much like a standard p-n junction diode. When the forward voltage exceeds the diode's cut-in voltage (approximately 0.7V for silicon), it conducts current with a relatively small and constant voltage drop across it. However, this region is not suitable for voltage regulation as its primary purpose is forward conduction, not voltage stabilization against fluctuations.
- **Reverse Bias Region (Below Breakdown):** When a reverse voltage is applied across the Zener diode but it is less than its Zener breakdown voltage (V_Z), only a very small leakage current flows. The diode acts like an open circuit with very high resistance. In this region, it does not provide any voltage regulation.
- **Reverse Breakdown Region:** This is the critical operating region for **voltage stabilization**. When the reverse voltage across the Zener diode reaches or slightly exceeds its specified Zener breakdown voltage (V_Z), a controlled breakdown occurs. In this region, even if the current through the diode changes significantly, the voltage across its terminals remains remarkably constant, very close to V_Z. This constant voltage characteristic is the fundamental property utilized for regulating voltage in circuits.

Operating Principle for Voltage Stabilization

When a **Zener diode** is used in a **voltage stabilization circuit**, it is always biased in the **reverse breakdown region**. Here’s how it works:

- The Zener diode is typically connected in parallel with the load that requires a stable voltage.
- A series resistor is used to limit the current through the Zener diode and protect it.
- If the input voltage supplied to the circuit fluctuates, or if the current drawn by the load changes, the Zener diode adjusts the current flowing through itself to maintain a constant voltage across the load. For instance, if the input voltage increases, the Zener diode draws more current, keeping the voltage across the load stable at V_Z. Conversely, if the input voltage decreases, it draws less current.
- This ability to maintain a nearly constant voltage across its terminals, despite variations in input voltage or load current, is why the **reverse breakdown region**

is essential for its role as a **voltage regulator**.

Zener Diode Biasing Modes for Voltage Regulation

Bias Mode	Behavior	Application for Voltage Stabilization
Forward Bias	Conducts like a normal diode ($\approx$ 0.7V drop).	Not suitable.
Reverse Bias (Below Breakdown)	Blocks current, high resistance.	Not suitable.
Reverse Breakdown Region	Maintains nearly constant voltage (V_Z) despite varying current.	**Ideal and necessary for voltage stabilization.**

Conclusion on Zener Diode Biasing for Stabilization

In summary, the specific property that makes a **Zener diode** useful for **voltage stabilization circuits** is its ability to maintain a constant voltage across its terminals once it enters the breakdown state. Therefore, it must be operated in the **reverse breakdown region** to achieve effective voltage regulation.

---

21. **Answer: a**

Explanation:

We can see from the circuit,

$$\frac{V_{out}}{V_{in}} = \frac{(R||X_c)}{(R||X_c)+R+X_c}$$

Solving the above by putting $X_c = \frac{1}{j\omega C}$ we get,

$$\frac{V_{out}}{V_{in}} = \frac{j\omega RC}{j\omega RC+(1+j\omega RC)^2}$$

Given:

$V_{in} = \cos\left(\frac{t}{RC}\right)$ ---(1)

By comparing (1) with cos(ωt), we get

$\omega = \frac{1}{RC}$

$\therefore \frac{V_{out}}{V_{in}} = \frac{j}{j+(1+j)^2} = \frac{1}{3}$

$\therefore V_{out} = \frac{1}{3}V_i$

$= \frac{V_p}{3}\cos(\frac{t}{RC})$

---

22. **Answer: d**

Explanation:

Integral Evaluation using Divergence Theorem

To evaluate the given integral \(\smallint \mathop \smallint \limits_{\rm{s}}^{\rm{\;}} 5{\rm{\vec r}} \cdot {\rm{\vec nds\;}}\) over a closed surface S surrounding a volume V, we can use the powerful tool of the **Divergence Theorem**, also known as Gauss's Theorem. This theorem provides a relationship between a surface integral of a vector field and the volume integral of its divergence.

Understanding the Divergence Theorem

The Divergence Theorem states that for a vector field $\vec{\mathrm{F}}$ and a closed surface S enclosing a volume V, the flux of $\vec{\mathrm{F}}$ through S is equal to the volume integral of the divergence of $\vec{\mathrm{F}}$ over V. Mathematically, it is expressed as:

\[\smallint \mathop \smallint \limits_{\rm{S}}^{\rm{\;}} {\rm{\vec F}} \cdot {\rm{\vec ndS\;}} = \smallint \mathop \smallint \limits_{\rm{V}}^{\rm{\;}} \smallint \left({\nabla \cdot {\rm{\vec F}}} \right){\rm{dV\;}} \]

Here:

- $\vec{F}$ is the vector field.
- $\vec{n}$ is the outward unit normal vector to the closed surface S.
- dS is the differential surface area element.
- $\nabla \cdot \vec{F}$ is the divergence of the vector field $\vec{F}$.
- dV is the differential volume element.

Applying the Divergence Theorem to the Problem

Let's break down the solution step-by-step:

1. **Identify the Vector Field $\vec{F}$:**

   The given integral is of the form \(\smallint \mathop \smallint \limits_{\rm{s}}^{\rm{\;}} 5{\rm{\vec r}} \cdot {\rm{\vec nds\;}}\). By comparing it with the Divergence Theorem's left side \(\smallint \mathop \smallint \limits_{\rm{S}}^{\rm{\;}} {\rm{\vec F}} \cdot {\rm{\vec ndS\;}}\), we can clearly identify our vector field $\vec{F}$ as $5\vec{r}$.

   We know that the **position vector** $\vec{r}$ in Cartesian coordinates is given by $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

   Therefore, the vector field $\vec{F}$ is:

   $$\vec{F} = 5\vec{r} = 5(x\hat{i} + y\hat{j} + z\hat{k}) = 5x\hat{i} + 5y\hat{j} + 5z\hat{k}$$

2. **Calculate the Divergence of $\vec{F}$ ($\nabla \cdot \vec{F}$):**

   The divergence of a vector field $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ is calculated as:

   $$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

   For our vector field $\vec{F} = 5x\hat{i} + 5y\hat{j} + 5z\hat{k}$:

   $$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(5x) + \frac{\partial}{\partial y}(5y) + \frac{\partial}{\partial z}(5z)$$

   Performing the partial derivatives:

   $$\nabla \cdot \vec{F} = 5 + 5 + 5 = 15$$

3. **Convert the Surface Integral to a Volume Integral:**

   Now, substitute the divergence value into the Divergence Theorem formula:

   \[\smallint \mathop \smallint \limits_{\rm{s}}^{\rm{\;}} 5{\rm{\vec r}} \cdot {\rm{\vec nds\;}} = \smallint \mathop \smallint \limits_{\rm{V}}^{\rm{\;}} \smallint \left({15} \right){\rm{dV\;}} \]

4. **Evaluate the Volume Integral:**

   The constant 15 can be taken outside the integral:

   \[\smallint \mathop \smallint \limits_{\rm{V}}^{\rm{\;}} \smallint 15{\rm{dV\;}} = 15 \smallint \mathop \smallint \limits_{\rm{V}}^{\rm{\;}} \smallint {\rm{dV\;}} \]

   The integral \(\smallint \mathop \smallint \limits_{\rm{V}}^{\rm{\;}} \smallint {\rm{dV\;}} \) represents the total **volume V** enclosed by the surface S.

   Therefore, the value of the integral is:

   \[\smallint \mathop \smallint \limits_{\rm{s}}^{\rm{\;}} 5{\rm{\vec r}} \cdot {\rm{\vec nds\;}} = 15V \]

Final Result

The value of the integral \(\smallint \mathop \smallint \limits_{\rm{s}}^{\rm{\;}} 5{\rm{\vec r}} \cdot {\rm{\vec nds\;}}\) for a closed surface S surrounding volume V, with $\vec{r}$ as the **position vector** and $\vec{n}$ as the unit normal on S, is $15V$.

---

23. Answer: a

Explanation:

Rectangular Waveguide Modes: An Introduction

A rectangular waveguide is a hollow metallic conductor used to guide electromagnetic waves. These waves propagate in specific patterns called modes. In a rectangular waveguide, modes are generally classified into Transverse Electric (TE) modes and Transverse Magnetic (TM) modes.

The modes are denoted by TE_{mn} or TM_{mn}, where 'm' and 'n' are integers representing the half-wavelength variations of the electric or magnetic field across the larger and smaller dimensions of the waveguide, respectively. If 'a' is the larger dimension and 'b' is the smaller dimension of the waveguide, then 'm' relates to variations along 'a' and 'n' relates to variations along 'b'.

Waveguide Mode Conditions for Existence

The existence of specific TE_{mn} and TM_{mn} modes in a rectangular waveguide is governed by the boundary conditions and the nature of the electromagnetic fields within the waveguide. For a mode to propagate, it must satisfy Maxwell's equations and the boundary conditions at the waveguide walls.

- **Transverse Magnetic (TM) Modes (TM_{mn}):** For TM_{mn} modes, the electric field component along the direction of propagation (E_z) must be non-zero within the waveguide and zero at the conducting walls. For E_z to exist in a non-trivial form and satisfy the boundary conditions, both eigen numbers 'm' and 'n' must be positive integers. That is, $\mathrm{m} \geq 1$ and $\mathrm{n} \geq 1$. If either 'm' or 'n' is zero, the E_z component becomes zero everywhere, meaning no TM wave can propagate.
- **Transverse Electric (TE) Modes (TE_{mn}):** For TE_{mn} modes, the magnetic field component along the direction of propagation (H_z) must be non-zero within the waveguide, and its normal derivative must be zero at the conducting walls. For H_z to exist, at least one of the eigen numbers 'm' or 'n' must be non-zero. That is, $\mathrm{m} \geq 0$, $\mathrm{n} \geq 0$, but not both $\mathrm{m} = 0$ and $\mathrm{n} = 0$ simultaneously ($\mathrm{m} + \mathrm{n} \neq 0$). For example, TE_{10} and TE_{01} modes can exist. The TE_{00} mode, where both 'm' and 'n' are zero, does not exist because it would imply a uniform H_z field with no variation, which cannot propagate as a wave in a waveguide.

Cut-off Frequency of Waveguide Modes

Each propagating mode has a specific cut-off frequency (f_c), below which the wave cannot propagate and becomes evanescent (attenuates rapidly). The general formula for the cut-off frequency in a rectangular waveguide with dimensions 'a' (larger) and 'b' (smaller) is:

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Or, using the speed of light $c = \frac{1}{\sqrt{\mu\epsilon}}$:

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Here, μ is the permeability and ϵ is the permittivity of the dielectric material filling the waveguide (usually air or vacuum).

Analyzing Rectangular Waveguide Statements

Let's evaluate each given statement based on the mode existence conditions and cut-off frequency formula:

Statement	Analysis	Conclusion
The TM_{10} mode of the waveguide does not exist.	For TM_{mn} modes, both 'm' and 'n' must be ≥ 1. In TM_{10}, $\mathrm{n}=0$, which violates this condition. Therefore, the TM_{10} mode cannot exist.	**TRUE**
The TE_{10} mode of the waveguide does not exist.	For TE_{mn} modes, at least one of 'm' or 'n' must be non-zero. In TE_{10}, $\mathrm{m}=1$ (non-zero) and $\mathrm{n}=0$. This satisfies the condition for TE mode existence. The TE_{10} mode is actually the dominant mode in a rectangular waveguide, meaning it has the lowest cut-off frequency.	**FALSE**
The TM_{10} and the TE_{10} modes both exist and have the same cut-off frequencies.	As established, the TM_{10} mode does not exist. While the cut-off frequency formula $\mathrm{f}_c=\frac{\mathrm{c}}{2a}$ would be the same if both existed (as $\mathrm{n}=0$ for both), the non-existence of TM_{10} makes this statement false.	**FALSE**
The TM_{10} and the TM_{01} modes both exist and have the same cut-off frequencies.	For TM_{mn} modes, both 'm' and 'n' must be ≥ 1. In TM_{10}, $\mathrm{n}=0$, and in TM_{01}, $\mathrm{m}=0$. Therefore, neither of these TM modes can exist.	**FALSE**

Based on the analysis, the statement that the TM_{10} mode of the waveguide does not exist is accurate according to the fundamental principles of waveguide mode theory.

---

24. Answer: c

Explanation:

Differential Equation Solution Explained

This problem asks us to find the solution to a given first-order differential equation with an initial condition. Understanding how to solve such equations is fundamental in calculus and its applications.

Understanding the Given Differential Equation

The differential equation provided is:

$$\frac{dy}{dx} = ky$$

This equation describes a relationship where the rate of change of y with respect to x is directly proportional to y itself, with k as the constant of proportionality. This type of equation often models exponential growth or decay.

We are also given an initial condition:

$$y(0) = c$$

This condition tells us the value of y at a specific point $x = 0$, which helps us find the unique particular solution from the general solution.

Solving the Differential Equation by Separation of Variables

The given differential equation $\frac{dy}{dx} = ky$ is a separable differential equation. This means we can rearrange the terms so that all y terms are on one side with dy, and all x terms (or constants) are on the other side with dx.

Step 1: Separate the variables.

Divide both sides by y (assuming $y \neq 0$) and multiply both sides by dx:

$$\frac{dy}{y} = k\,dx$$

Step 2: Integrate both sides.

Now, we integrate both sides of the separated equation:

$$\int \frac{1}{y}\,dy = \int k\,dx$$

Performing the integration:

$$\ln|y| = kx + C_1$$

where C_1 is the constant of integration.

Step 3: Solve for y.

To eliminate the natural logarithm, we exponentiate both sides (use e as the base):

$$e^{\ln|y|} = e^{kx+C_1}$$

Using the property $e^{a+b} = e^a \cdot e^b$, we get:

$$|y| = e^{kx} \cdot e^{C_1}$$

Let $A = e^{C_1}$. Since C_1 is an arbitrary constant, A will be an arbitrary positive constant. Thus:

$$|y| = Ae^{kx}$$

This implies $y = \pm Ae^{kx}$. We can combine $\pm A$ into a single constant C, where C can be any non-zero real number. Note that if $y = 0$ is a trivial solution (which it is if $c = 0$), then C can also be zero. So, the general solution is:

$$y = Ce^{kx}$$

Applying the Initial Condition $y(0) = c$

We use the given initial condition $y(0) = c$ to find the specific value of the constant C .

Substitute $x = 0$ and $y = c$ into the general solution $y = Ce^{kx}$:

$$c = Ce^{k \cdot 0}$$

Since $e^0 = 1$:

$$c = C \cdot 1$$

$$C = c$$

Final Solution for the Differential Equation

Now, substitute the value of C back into the general solution:

$$y = ce^{kx}$$

This is the particular solution that satisfies both the differential equation and the given initial condition.

Comparing with the Options

Let's compare our derived solution with the provided options:

- Option 1: $x = ce^{-ky}$ - Does not match.
- Option 2: $x = ke^{cy}$ - Does not match.
- Option 3: $y = ce^{kx}$ - Matches our solution.
- Option 4: $y = ce^{-kx}$ - Does not match (note the negative sign in the exponent).

Therefore, the correct solution is $y = ce^{kx}$.

---

25. **Answer: b**

Explanation:

Understanding the characteristics of different modulation systems is crucial in communication engineering. This question asks us to match specific attributes related to signal transmission to their corresponding modulation techniques. Let's analyze each pair given in List I and List II to find the best matches.

Modulation System Attributes Explained

We will examine each attribute from List I and determine which modulation system from List II best fits the description.

P. Power efficient transmission of signal – FM (Frequency Modulation)

- **FM Efficiency:** Frequency Modulation (FM) is known for its superior noise performance and power efficiency compared to Amplitude Modulation (AM) at high signal-to-noise ratios. In FM, the information is encoded in the frequency variations of the carrier, while the amplitude remains constant. This constant amplitude allows the transmitter to operate at peak power efficiency. Additionally, FM systems can employ limiters at the receiver to remove amplitude noise, leading to a significant improvement in signal quality and effective power utilization for reliable signal transmission.

Q. Most bandwidth efficient transmission of voice signals – SSB (Single Sideband)

- **SSB Bandwidth:** Single Sideband (SSB) modulation is highly efficient in terms of bandwidth usage. It transmits only one of the two sidebands (either upper or lower) and suppresses the carrier signal. This significantly reduces the required bandwidth to approximately half of what is needed for conventional AM or DSB-SC (Double Sideband Suppressed Carrier) modulation. For voice signals, which typically occupy a relatively narrow frequency range, SSB's ability to conserve bandwidth makes it ideal for applications where spectrum efficiency is critical, such as long-distance radio communication.

R. Simplest receiver structure – Conventional AM (Amplitude Modulation)

- **AM Receiver Simplicity:** Conventional Amplitude Modulation (AM) receivers, especially those using an envelope detector, are remarkably simple in design and construction. An envelope detector typically consists of a diode, a resistor, and a capacitor, making it very cost-effective and easy to implement. This simplicity is one of the primary reasons why conventional AM has been widely used for broadcasting despite its lower power and bandwidth efficiency compared to other modulation schemes.

S. B.W efficient transmission of signals with significant DC component – VSB (Vestigial Sideband)

- **VSB for DC Component:** Vestigial Sideband (VSB) modulation is a technique that transmits one full sideband and a small portion (vestige) of the other sideband, along with the carrier signal. This method is a compromise between the bandwidth efficiency of SSB and the simplicity of AM. VSB is particularly well-suited for transmitting signals that contain a significant DC (direct current) component or very low-frequency components, such as video signals in analog television broadcasting. The presence of the vestigial sideband helps in preserving these low-frequency components during demodulation, preventing phase distortion that might occur with pure SSB. It offers better bandwidth efficiency than conventional AM while still allowing for simpler demodulation than SSB for signals with DC content.

Matching Summary

Based on the analysis above, we can create the following matches:

Attribute (List I)	Modulation System (List II)
P. Power efficient transmission of signal.	2. FM
Q. Most bandwidth efficient transmission of voice signals.	4. SSB
R. Simplest receiver structure.	1. Conventional AM
S. B.W efficient transmission of signals with significant DC component.	3. VSB

Therefore, the correct mapping is P – 2, Q – 4, R – 1, S – 3.

---

26. Answer: a

Explanation:

$$\frac{100d^2y}{dt^2} - \frac{20dy}{dt} + y = x(t)$$

Apply L.T. both sides

$(100s^2 - 20s + 1)\ Y(s) = \frac{1}{s} [\because x(t) \times (s) = \frac{1}{3}]$

$Y(s) = \frac{1}{s(100s^2 - 20s + 1)}$

So we have poles with positive real part ⇒ system is unstable.

---

27. **Answer: a**

Explanation:

G (jω) = 5 + jω

ω = 0

G (jω) = 5 + j 0

ω = 10

G (jω) = 5 + j 10

ω = ∞

G (jω) = 5 + j ∞

∴ G (jω) is a straight line parallel to jω axis

---

28. **Answer: b**

Explanation:

Trigonometric Fourier Series of Even Functions Explained

The trigonometric Fourier series is a powerful mathematical tool used to represent any periodic function as a sum of sines and cosines. Understanding its properties for specific types of functions, like even functions, is fundamental in signal processing and many engineering disciplines.

Understanding the General Trigonometric Fourier Series

A general trigonometric Fourier series for a periodic function $f(t)$ with period T is expressed as:

$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t))$$

Where $\omega_0 = \frac{2\pi}{T}$ is the fundamental angular frequency. The coefficients a_0, a_n, and b_n are calculated using the following integrals over one period $[0, T]$ or $[-T/2, T/2]$:

- **DC Term (Average Value):** $a_0 = \frac{1}{T}\int_0^T f(t)dt$
- **Cosine Coefficients:** $a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt$
- **Sine Coefficients:** $b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_0 t)dt$

Even Function Properties and Fourier Series

An **even function** $f(t)$ is defined by the property that $f(-t) = f(t)$. Examples include $\cos(t)$, t^2, or any function symmetric about the vertical axis. The key to understanding its Fourier series lies in the properties of integrals involving even and odd functions.

When integrating over a symmetric interval, such as $[-L, L]$:

- The integral of an **even function** $g(t)$ is twice the integral over half the interval: $\int_{-L}^{L} g(t)dt = 2\int_0^L g(t)dt$.
- The integral of an **odd function** $h(t)$ is zero: $\int_{-L}^{L} h(t)dt = 0$.

Analyzing Coefficients for an Even Function

Let's apply these properties to the Fourier series coefficients for an even function $f(t)$:

- **DC Term (a_0):**

  The DC term a_0 represents the average value of the function. Since $f(t)$ is an even function, its average value can be non-zero. For example, a constant function $f(t) = C$ is an even function, and its a_0 would be C. Therefore, an even function **can have DC terms**.

- **Cosine Terms (a_n):**

  The term inside the integral for a_n is $f(t)\cos(n\omega_0 t)$. Since $f(t)$ is an even function and $\cos(n\omega_0 t)$ is also an even function, their product $f(t)\cos(n\omega_0 t)$ will be an **even function** (even × even = even). The integral of an even function over a symmetric interval is generally non-zero. Thus, for an even function, **cosine terms (a_n) are present** in its Fourier series.

- **Sine Terms (b_n):**

  The term inside the integral for b_n is $f(t)\sin(n\omega_0 t)$. Since $f(t)$ is an even function and $\sin(n\omega_0 t)$ is an odd function, their product $f(t)\sin(n\omega_0 t)$ will be an **odd function** (even × odd = odd). The integral of an odd function over a symmetric interval $[-T/2, T/2]$ is always zero. Therefore, $b_n = 0$ for all n. This means that the trigonometric Fourier series of an even function **does not have sine terms**.

- **Odd Harmonic Terms:**

  Harmonic terms refer to components where n is an integer. Odd harmonic terms are those where $n = 1, 3, 5, \dots$. Since cosine terms ($a_n \cos(n\omega_0 t)$) are present for an even function, and n can be an odd integer, an even function's Fourier series **can have odd harmonic cosine terms**. For example, a symmetric square wave (an even function) only consists of odd harmonic cosine terms.

Conclusion on Even Function Fourier Series Components

Based on the analysis of the Fourier coefficients for an even function:

Component Type	Presence in Even Function Fourier Series
DC terms (a_0)	Can be present
Cosine terms ($a_n \cos(n\omega_0 t)$)	Are present
Sine terms ($b_n \sin(n\omega_0 t)$)	Are **not** present (coefficients $b_n = 0$)
Odd harmonic terms (e.g., $a_1 \cos(\omega_0 t), a_3 \cos(3\omega_0 t)$)	Can be present (as cosine terms)

Therefore, the trigonometric Fourier series of an even function does not have the **Sine terms** because their coefficients b_n are always zero due to the integral of an odd function over a symmetric interval being zero.

---

29. **Answer: a**

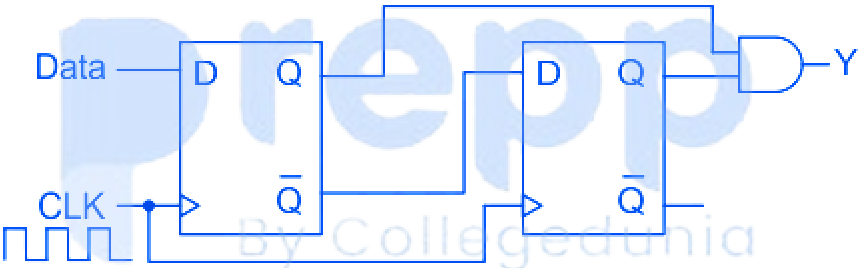
Explanation:

Concept:

D flip-flop can be built using NAND gate or with NOR gate. Whenever the clock signal is LOW, the input is never going to affect the output state. The clock has to be high for the inputs to get active. Thus, D flip-flop is a controlled Bi-stable latch where the clock signal is the control signal. Again, this gets divided into positive edge triggered D flip flop and negative edge triggered D flip-flop

Truth table of D Flip-Flop:

Clock	INPUT	OUTPUT	
	D	Q	Q'
LOW	x	0	1
HIGH	0	0	1
HIGH	1	1	0



New,

$$y_n = Q_{1_n} Q_{2_n}$$

$$= Q_{1_n} D_{2_n}$$

$$= Q_{1_n} \overline{Q_{1_{(n-1)}}}$$

$$= D_{1_n} \overline{D_{1_{(n-1)}}}$$

The output will be high when,

$$D_{1_{(n-1)}} = 0 \; and \; D_{1_n} = 1$$

i.e when the data changes from '0' to '1'

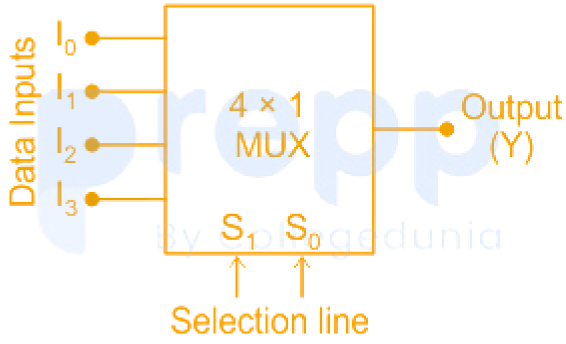
---

30. Answer: d

Explanation:

Concept:

In a 4 × 1 MUX



Truth-Table

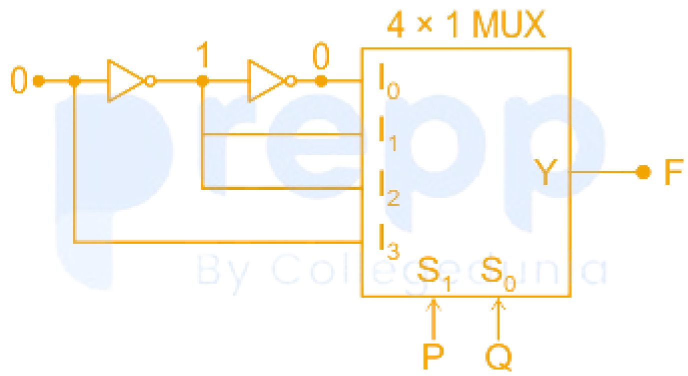
S_1	S_0	V
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

Y = Output = $\bar{S}_1\bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1\bar{S}_0 I_2 + S_1 S_0 I_3$

MUX contains AND gate followed by OR gate

Calculation:

By re-drawing circuit diagram



$\therefore I_0 = 0, I_1 = 1, I_2 = 1, I_3 = 0$ & $(P = S_1, Q = S_0)$

Now output of 4 × 1 MUX is

$Y = F = (P'Q')0 + (P'Q)1 + (PQ')1 + (PQ)0$

$\therefore F = PQ' + P'Q = P \oplus Q$

$\therefore F = XOR\,(P, Q)$

---

31. Answer: d

Explanation:

At low frequencies,

$i.e.\ w = 0 :\ L \rightarrow\ S.C$

$\therefore\ V_0\ = 0V$

similarly at higher frequencies,

$i.e.w = \infty : L \rightarrow\ O.C$

$\therefore\ V_0 = i_i R_1$

So, the given circuit passes only high frequencies so it acts like a high pass filter.

---

32. **Answer: d**

Explanation:

Silicon PN Junction Overview

A **silicon PN junction** is a fundamental semiconductor device formed by joining P-type and N-type silicon materials. When this junction is **forward biased**, an external voltage is applied in a way that allows current to flow easily through the device. This occurs when the positive terminal of the voltage source is connected to the P-type material and the negative terminal to the N-type material. For a silicon diode, a significant current starts flowing when the forward bias voltage reaches approximately 0.7 V at room temperature, which is known as the cut-in voltage or knee voltage.

Temperature Dependence of Forward Bias Voltage

The electrical characteristics of a **PN junction** diode are significantly influenced by temperature. When a diode is **forward biased** with a **constant current** flowing through it, its **forward bias voltage** changes as the temperature varies. Specifically, as the temperature of the diode increases, the intrinsic carrier concentration within the semiconductor material also increases. This leads to an increase in the reverse saturation current (I_S) of the diode.

The relationship between the diode current (I_D) and forward voltage (V_D) is given by the Shockley diode equation:

$I_D = I_S \left(e^{\frac{V_D}{\eta V_T}} - 1 \right)$

Where:

- I_D is the diode current
- I_S is the reverse saturation current (highly temperature-dependent)
- V_D is the diode voltage
- η is the ideality factor (1 for germanium, 2 for silicon)

- V_T is the thermal voltage, given by $V_T = \frac{kT}{q}$, where k is Boltzmann's constant, T is the absolute temperature, and q is the electron charge.

When the **forward current** (I_D) is kept **constant**, an increase in I_S (due to increased temperature) must be compensated by a decrease in V_D to maintain the equality. Therefore, for a fixed forward current, the **forward bias voltage** across a **silicon PN junction** decreases as the temperature rises.

Calculation of Voltage Change

For a **silicon PN junction** diode, the **forward bias voltage** typically decreases by approximately 2 mV to 2.5 mV for every 1°C increase in temperature, assuming the forward current is held constant. This value is known as the temperature coefficient of the forward voltage.

In this question:

- The temperature is **increased by 10°C**.
- The diode is a **silicon PN junction**.
- The diode is **forward biased** with a **constant current**.

Using the widely accepted temperature coefficient for silicon diodes, which is approximately $-2.5 \text{ mV/}^\circ\text{C}$ (the negative sign indicates a decrease in voltage with increasing temperature), we can calculate the total change in forward bias voltage:

$\text{Change in Voltage} = (\text{Temperature Coefficient}) \times (\text{Change in Temperature})$

$\text{Change in Voltage} = (-2.5 \text{ mV/}^\circ\text{C}) \times (10^\circ \text{C})$

$\text{Change in Voltage} = -25 \text{ mV}$

The result of -25 mV means that the **forward bias voltage** across the **PN junction** decreases by 25 mV.

Summary Table of Diode Temperature Effects

Understanding these temperature effects is crucial for designing stable electronic circuits.

Parameter	Effect with Increasing Temperature (Constant Current)
Forward Bias Voltage (V_F)	Decreases
Reverse Saturation Current (I_S)	Increases (approximately doubles for every 10°C rise)
Temperature Coefficient ($\Delta V_F / \Delta T$ for Silicon)	Approximately $-2.0 \text{ mV/}^\circ\text{C}$ to $-2.5 \text{ mV/}^\circ\text{C}$

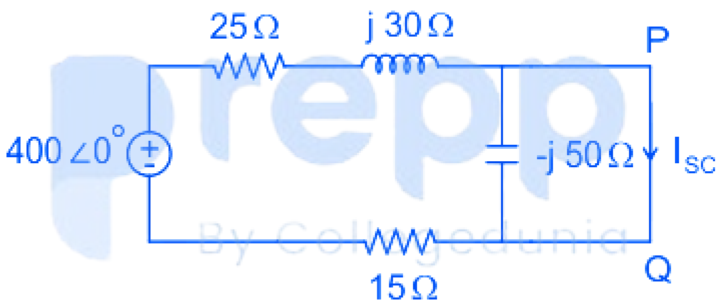
Therefore, when the temperature of a **silicon PN junction**, forward biased with a **constant current**, is **increased by 10°C**, its **forward bias voltage** decreases by 25 mV.

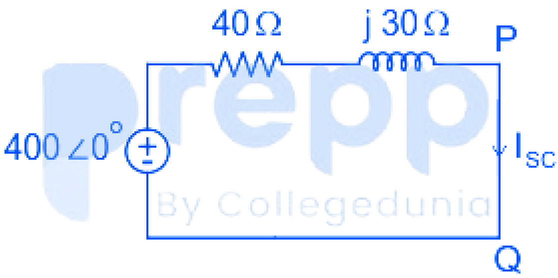
---

33. **Answer: a**

Explanation:

Converting current source into voltage source form, we get





The capacitor is connected across the short circuited branch hence it can be removed.

$$\therefore I_{SC} = \frac{400\angle 0^\circ}{40+j30}$$
$$= 6.4 - j4.8\ A$$

(1) is correct.

---

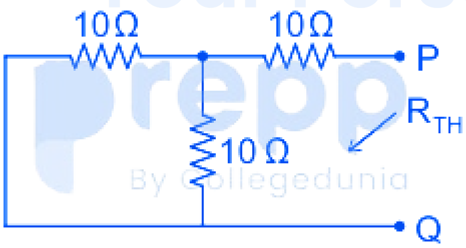
34. Answer: c

Explanation:

Concept:

From LMS

Calculation:



R_{TH} = (10 || 10) + 10 = 15 Ω

---

35. Answer: a

Explanation:

To evaluate the given complex integral, we need to understand the concept of singularities and their location relative to the contour of integration.

The integral we need to evaluate is:

$$\mathop \oint \limits_c^\; \frac{{ - 3z + 5}}{{{z^2} + 4z + 5}}dz$$

The contour C is given by the circle $|z| = 1$.

Integral Singularity Analysis

First, we need to find the singularities of the integrand. Singularities occur where the denominator is zero. Let's set the denominator equal to zero and solve for z:

$$z^2 + 4z + 5 = 0$$

This is a quadratic equation of the form $az^2 + bz + c = 0$, where $a=1$, $b=4$, and $c=5$.

We can use the quadratic formula to find the roots:

$$z = \frac{{ - b \pm \sqrt {{b^2} - 4ac} }}{{2a}}$$

Substitute the values of a, b, and c into the formula:

$$z = \frac{{ - 4 \pm \sqrt {{4^2} - 4(1)(5)} }}{{2(1)}}$$

$$z = \frac{{ - 4 \pm \sqrt {16 - 20} }}{2}$$

$$z = \frac{{ - 4 \pm \sqrt { - 4} }}{2}$$

Since $\sqrt{-4} = \sqrt{4 \times -1} = 2i$, we have:

$$z = \frac{{ - 4 \pm 2i}}{2}$$

This gives us two distinct roots (singularities):

- $z_1 = \frac{{ - 4 + 2i}}{2} = -2 + i$
- $z_2 = \frac{{ - 4 - 2i}}{2} = -2 - i$

Contour Boundary Check

Next, we need to determine if these singularities lie inside, on, or outside the given contour $C: |z| = 1$. The contour $|z|=1$ represents a circle centered at the origin with a radius of 1.

Location of $z_1 = -2 + i$

To check if z_1 is inside the circle, we calculate its modulus (distance from the origin):

$$|z_1| = |-2 + i| = \sqrt{{(-2)^2} + {(1)^2}}$$

$$|z_1| = \sqrt{4 + 1} = \sqrt{5}$$

Since $\sqrt{5} \approx 2.236$ and $2.236 > 1$, the singularity z_1 is outside the contour $|z|=1$.

Location of $z_2 = -2 - i$

Similarly, for z_2:

$$|z_2| = |-2 - i| = \sqrt{{(-2)^2} + {(-1)^2}}$$

$$|z_2| = \sqrt{4 + 1} = \sqrt{5}$$

Since $\sqrt{5} \approx 2.236$ and $2.236 > 1$, the singularity z_2 is also outside the contour $|z|=1$.

Integral Value Determination

Since both singularities of the integrand $f(z) = \frac{{ - 3z + 5}}{{{z^2} + 4z + 5}}$ are located outside the given contour $C: |z| = 1$, the function $f(z)$ is analytic everywhere inside and on the contour C.

According to **Cauchy's Integral Theorem**:

If a function \(f(z)\) is analytic at all points inside and on a simple closed contour \(C\), then the integral of \(f(z)\) over \(C\) is zero.

In this case, because there are no singularities inside the contour, the conditions for Cauchy's Integral Theorem are met.

Therefore, the value of the integral is 0.

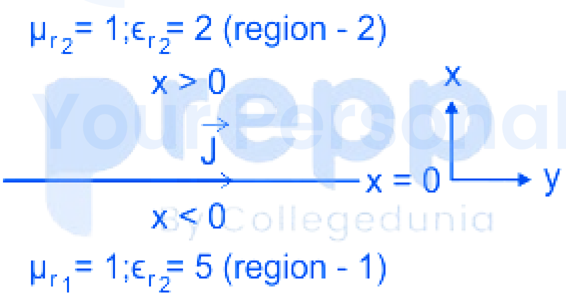
Summary of Singularity Analysis

Singularity	Value	Modulus \(|z|\)	Location relative to \(|z|=1\)
\(z_1\)	\(-2 + i\)	\(\sqrt{5} \approx 2.236\)	Outside
\(z_2\)	\(-2 - i\)	\(\sqrt{5} \approx 2.236\)	Outside

---

36. Answer: a

Explanation:



Boundary conditions for magnetic field are

$B_{n_1} = B_{n_2}$ and $H_{t_1} - H_{t_2} = -\vec{J}_s \times \hat{a}_n$

$\Rightarrow B_{x_1} = B_{x_2}$ ....(x is normal direction)

$\Rightarrow H_{x2} = \frac{1 \times 3\hat{u}_x}{2}$

$H_{x2} = 1.5\hat{u}_x$

Now,

$(H_{t1} - H_{t2}) = -(10\hat{u}_y) \times (\hat{u}_x)$

$= 10\ \hat{u}_z$

$H_{t_2} = H_{t_1} - 10\hat{u}_z = 30\hat{u}_y - 10\hat{u}_z$

$\Rightarrow H_{t2} = 30\hat{u}_y - 10\hat{u}_z$

$\Rightarrow \vec{H}_2 = \vec{H}_{x2} + \vec{H}_{t_2} = 1.5\hat{u}_x + 30\hat{u}_y - 10\hat{u}_z$

---

37. Answer: a

Explanation:

Calculating Load Impedance (Z_L) from VSWR and Voltage Maximum Location

This problem involves analyzing a terminated transmission line. We are given the **characteristic impedance** (Z_0), the Voltage Standing Wave Ratio (VSWR), and the position of the first voltage maximum relative to the load. Our goal is to determine the **load impedance** (Z_L) that causes these conditions.

Understanding VSWR and Reflection Coefficient

The Voltage Standing Wave Ratio (VSWR) on a transmission line is a measure of the standing wave pattern formed due to reflections from the load. It is defined as the ratio of the maximum voltage amplitude to the minimum voltage amplitude along the line.

VSWR is directly related to the magnitude of the voltage reflection coefficient ($|\Gamma|$) at the load by the following formula:

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Calculating the Magnitude of the Reflection Coefficient ($|\Gamma|$)

We are given that the VSWR is 5. We can use this value to find the magnitude of the reflection coefficient ($|\Gamma|$):

$$5 = \frac{1+|\Gamma|}{1-|\Gamma|}$$

Rearranging the formula to solve for $|\Gamma|$:

$$5(1-|\Gamma|) = 1+|\Gamma|$$

$$5-5|\Gamma| = 1+|\Gamma|$$

$$5-1 = |\Gamma|+5|\Gamma|$$

$$4 = 6|\Gamma|$$

$$|\Gamma| = \frac{4}{6} = \frac{2}{3}$$

So, the magnitude of the reflection coefficient is $\frac{2}{3}$.

Determining the Phase of the Reflection Coefficient (θ_Γ)

The locations of voltage maxima and minima on the transmission line depend on the phase of the reflection coefficient (θ_Γ). The voltage along the line (measured from the load at $z=0$) can be represented such that maxima occur at distances z_{max} where $2\beta z_{max} + \theta_\Gamma = 2n\pi$ (where n is an integer, β is the phase constant, and λ is the wavelength).

We are told that the first voltage maximum occurs at a distance $z = \lambda/4$ from the load.

Substituting $z = \lambda/4$ into the condition for voltage maxima:

$$2\beta\left(\frac{\lambda}{4}\right) + \theta_\Gamma = 2n\pi$$

Since $\beta = \frac{2\pi}{\lambda}$, we have:

$$2\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{4}\right)+\theta_\Gamma=2n\pi$$

$$\pi+\theta_\Gamma=2n\pi$$

$$\theta_\Gamma=(2n-1)\pi$$

The simplest value for θ_Γ that satisfies this condition is when $n=1$, which gives $\theta_\Gamma=\pi$.

If $\theta_\Gamma=\pi$, the voltage minima occur when $2\beta z+\pi=(2m+1)\pi$, which simplifies to $2\beta z=2m\pi$, or $z=m\frac{\pi}{\beta}=m\frac{\lambda}{2}$. This means minima occur at $z=0,\lambda/2,\lambda,\ldots$. Correspondingly, maxima occur midway between minima, at $z=\lambda/4,3\lambda/4,\ldots$. The condition that the *first* voltage maximum is at $\lambda/4$ is consistent with $\theta_\Gamma=\pi$.

Calculating the Load Impedance (Z_L)

Now that we have the magnitude ($|\Gamma|=\frac{2}{3}$) and phase ($\theta_\Gamma=\pi$), we can find the complex reflection coefficient:

$$\Gamma=|\Gamma|e^{j\theta_\Gamma}=\frac{2}{3}e^{j\pi}=\frac{2}{3}(-1)=-\frac{2}{3}$$

The load impedance (Z_L) can be calculated using the characteristic impedance (Z_0) and the reflection coefficient (Γ) with the following formula:

$$Z_L=Z_0\frac{1+\Gamma}{1-\Gamma}$$

Substituting the known values ($Z_0=50\,\Omega$ and $\Gamma=-2/3$):

$$Z_L=50\,\Omega\times\frac{1+(-\frac{2}{3})}{1-(-\frac{2}{3})}$$

$$Z_L=50\,\Omega\times\frac{1-\frac{2}{3}}{1+\frac{2}{3}}$$

$$Z_L=50\,\Omega\times\frac{\frac{3}{3}-\frac{2}{3}}{\frac{3}{3}+\frac{2}{3}}$$

$$Z_L=50\,\Omega\times\frac{\frac{1}{3}}{\frac{5}{3}}$$

$$Z_L = 50\,\Omega \times \frac{1}{5}$$

$$Z_L = 10\,\Omega$$

Conclusion

Based on the provided VSWR of 5 and the location of the first voltage maximum at $\lambda/4$ from the load, the calculated load impedance (Z_L) for the transmission line with a characteristic impedance of $50\,\Omega$ is $10\,\Omega$. This indicates a purely resistive load that is significantly mismatched.

---

38. Answer: a

Explanation:

The total transfer function H(f) = (j2πf - 1)

$S_x(f) = |H(f)|^2\, S_x(f)\; R_x(\tau) \overset{F}{\leftrightarrow} S_x(f)$

$= (4\pi^2 f^2 + 1)e^{-\pi t2}\; (\because e^{-\pi t2} \overset{F}{\leftrightarrow} e^{-\pi t2})$

---

39. Answer: a

Explanation:

The initial state of the Johnson counter is as follows –

	D_2	D_1	D_0
State -1	0	0	0
State-2	1	0	0
State-3	1	1	0

State-4	1	1	1	
State-5	0	1	1	
State-6	0	0	1	
State-7	0	0	0	(it is equivalent to state 1)

So the corresponding digital output assuming LSB bit contribute unit Analog output is –

State 1	0
State 2	4
State 3	6
State 4	7
State 5	3
State 6	1

So, option (1) is correct.

---

40. **Answer: d**

Explanation:

Synchronous Counter Design with D Flip-Flops

A **synchronous counter** is a type of sequential circuit where all flip-flops are clocked simultaneously by a single clock signal. This ensures that all flip-flop outputs change state at the same time, unlike asynchronous counters. D flip-flops are common building blocks for such counters because their next state is simply equal to their D input (i.e., $Q_{next} = D$).

Understanding the Counter Sequence

The question specifies a two-D flip-flop synchronous counter that follows the sequence: $00 \to 11 \to 01 \to 10 \to 00 \to \dots$ Here, Q_B represents the Most Significant Bit (MSB) and Q_A represents the Least Significant Bit (LSB).

Deriving the State Table

To determine the required inputs D_A and D_B for the D flip-flops, we first need to construct a state table. This table lists the current state ($Q_B Q_A$), the next state ($Q_B^+ Q_A^+$) as per the given sequence, and the corresponding D inputs. For a D flip-flop, the D input is simply the next state of its output (e.g., $D_A = Q_A^+$ and $D_B = Q_B^+$).

Current State ($Q_B Q_A$)	Next State ($Q_B^+ Q_A^+$)	Required D Inputs ($D_B D_A$)
00	11	11
11	01	01
01	10	10
10	00	00

Boolean Expressions for D Inputs

From the "Required D Inputs ($D_B D_A$)" column in the state table, we can write down the Boolean expressions for D_A and D_B in terms of the current states Q_A and Q_B.

Input D_A Derivation

Let's look at the values of D_A:

- When $Q_B Q_A = 00$, $D_A = 1$
- When $Q_B Q_A = 01$, $D_A = 0$
- When $Q_B Q_A = 10$, $D_A = 0$
- When $Q_B Q_A = 11$, $D_A = 1$

From this, we can see that D_A is 1 when Q_B and Q_A are both 0 or both 1. This corresponds to the XNOR (Exclusive-NOR) operation between Q_A and Q_B. The Boolean expression for D_A is:

$$D_A = (\overline{Q_A Q_B}) + (Q_A Q_B)$$

Input D_B Derivation

Now, let's look at the values of D_B:

- When $Q_B Q_A = 00$, $D_B = 1$
- When $Q_B Q_A = 01$, $D_B = 1$
- When $Q_B Q_A = 10$, $D_B = 0$
- When $Q_B Q_A = 11$, $D_B = 0$

From this, we observe that D_B is 1 when Q_B is 0, and D_B is 0 when Q_B is 1. This means D_B is the complement of Q_B.

The Boolean expression for D_B is:

$$D_B = \overline{Q_B}$$

Matching with Options

Comparing our derived Boolean expressions for D_A and D_B with the given options:

- Our derived $D_A = (\overline{Q_A Q_B}) + (Q_A Q_B)$
- Our derived $D_B = \overline{Q_B}$

This exact combination matches option 4:

$D_A = (\overline{Q_A Q_B} + Q_A Q_B)$; $D_B = \overline{Q_B}$

41. **Answer: c**

Explanation:

Since the Gate and Drain of pull-down transistor is shorted. So it has operated in the saturation region, let pull up transistor operated in the saturation region.

Then $I_{D1} = I_{D2}$

$\frac{1}{2}\mu_n C_{ox}\left(\frac{\omega}{L}\right)_1 (V_{GS1} - V_{T1})^2$

$= \frac{1}{2}\mu_n C_{ox}\left(\frac{\omega}{L}\right)_2 (V_{Gs2} - V_{T2})^2$

$4\,[5 - V_x - 1]^2 = 1[V_x - 1]^2$

$2(4 - V_x) = V_x - 1$

$9 = 3\,V_x \rightarrow V_x = 3V$

$V_{GS1} = 5 - 3 = 2$, $V_{DS1} = 6 - 3 = 3V$

$\therefore V_{DS1} > (V_{GS1} - VT_1)$

So our assumption is correct

$\therefore V_x = 3\,V$

So option (3) is correct.

---

42. **Answer: d**

Explanation:

System Output for LTI Systems

To find the output $y(t)$ of a Linear Time-Invariant (LTI) system, we use the convolution operation between the input signal $x(t)$ and the system's impulse response $h(t)$. The relationship is given by:

$$y(t) = x(t) * h(t)$$

Where $*$ denotes convolution.

Input and Impulse Response Analysis

We are given the following:

- **Input signal:** $x(t) = e^{-2t}u(t) + \delta(t-6)$
- **Impulse response:** $h(t) = u(t)$

The input signal $x(t)$ can be broken down into two components due to the linearity property of LTI systems:

1. The exponential component: $x_1(t) = e^{-2t}u(t)$
2. The impulse component: $x_2(t) = \delta(t-6)$

Therefore, the total output $y(t)$ will be the sum of the outputs produced by each component:

$$y(t) = y_1(t) + y_2(t)$$

Where $y_1(t) = x_1(t) * h(t)$ and $y_2(t) = x_2(t) * h(t)$.

Convolution for the Exponential Component ($y_1(t)$)

We need to calculate $y_1(t) = (e^{-2t}u(t)) * u(t)$. The convolution integral is defined as:

$$y_1(t) = \int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau$$

Substituting $x_1(\tau) = e^{-2\tau}u(\tau)$ and $h(t-\tau) = u(t-\tau)$:

$$y_1(t) = \int_{-\infty}^{\infty} e^{-2\tau}u(\tau)u(t-\tau)d\tau$$

The unit step function $u(\tau)$ is 1 for $\tau \geq 0$ and 0 otherwise. The unit step function $u(t-\tau)$ is 1 for $t - \tau \geq 0 \implies \tau \leq t$ and 0 otherwise. For the product $u(\tau)u(t-\tau)$ to be non-zero, both conditions must be met, meaning $0 \leq \tau \leq t$. This holds for $t \geq 0$. If $t < 0$, the product is 0, and thus $y_1(t) = 0$. So, for $t \geq 0$, the integral becomes:

$$y_1(t) = \int_0^t e^{-2\tau} d\tau$$

Now, we evaluate the integral:

$$y_1(t) = \left[\frac{e^{-2\tau}}{-2}\right]_0^t$$

$$y_1(t) = -\frac{1}{2}(e^{-2t} - e^{-2\cdot 0})$$

$$y_1(t) = -\frac{1}{2}(e^{-2t} - 1)$$

$$y_1(t) = \frac{1}{2}(1 - e^{-2t})$$

Since this result is valid for $t \geq 0$, we multiply it by the unit step function $u(t)$:

$$y_1(t) = 0.5[1 - e^{-2t}]u(t)$$

Convolution for the Impulse Component ($y_2(t)$)

Next, we calculate $y_2(t) = \delta(t-6) * u(t)$. A key property of convolution with an impulse function is:

$$f(t) * \delta(t - t_0) = f(t - t_0)$$

Applying this property, where $f(t) = u(t)$ and $t_0 = 6$:

$$y_2(t) = u(t-6)$$

Total System Output

Finally, we sum the two components to get the total output $y(t)$:

$$y(t) = y_1(t) + y_2(t)$$

$$y(t) = 0.5[1 - e^{-2t}]u(t) + u(t-6)$$

Comparison with Options

Comparing our derived output with the given options:

Option	Expression
1	$[1 - \exp(-2t)u(t)] + u(t+6)$
2	$[1 - \exp(-2t)]u(t) + u(t+6)$
3	$0.5[1 - \exp(-2t)]u(t) + u(t+6)$
4	$0.5[1 - \exp(-2t)]u(t) + u(t-6)$

Our calculated output $y(t) = 0.5[1 - e^{-2t}]u(t) + u(t-6)$ perfectly matches Option 4.

---

43. **Answer: d**

Explanation:

BJT Collector Current Calculation in Common-Emitter Mode

This explanation details the calculation of the collector current (I_C) for a Bipolar Junction Transistor (BJT) operating in the common-emitter (CE) configuration. We are given the common-base current gain (α), the collector base junction reverse bias saturation current (I_{CO}), and the base current (I_B) when the BJT is in the active region.

Understanding BJT Parameters (α, β, I_{CO})

Key parameters for BJT analysis include:

- **Common-Base Current Gain (α)**: This represents the ratio of collector current change to emitter current change in common-base configuration. It's typically close to 1. Formula: $\alpha = \frac{\Delta I_C}{\Delta I_E}$.
- **Common-Emitter Current Gain (β)**: This represents the ratio of collector current change to base current change in common-emitter configuration. It is related to α. Formula: $\beta = \frac{\alpha}{1-\alpha}$.
- **Collector Base Junction Reverse Bias Saturation Current (I_{CO})**: This is the small leakage current flowing from collector to base when the collector-base junction is reverse-biased and the emitter circuit is open. It contributes to the total collector current.
- **Base Current (I_B)**: The current flowing into the base terminal.
- **Collector Current (I_C)**: The current flowing out of the collector terminal.

Calculating Common-Emitter Gain (β)

First, we need to calculate the common-emitter current gain (β) using the provided common-base current gain ($\alpha = 0.98$). The relationship is:

$$\beta = \frac{\alpha}{1 - \alpha}$$

Substituting the value of α:

$$\beta = \frac{0.98}{1 - 0.98} = \frac{0.98}{0.02} = 49$$

So, the common-emitter current gain (β) is 49.

Determining Collector Current (I_C) using the Active Region Formula

For a BJT operating in the active region in common-emitter mode, the collector current (I_C) is primarily determined by the base current (I_B) and the current gain (β). However, it also includes a component related to the saturation current (I_{CO}). A precise formula relating these quantities is:

$$I_C = \beta I_B + (1 + \beta) I_{CO}$$

This formula accounts for both the amplified base current and the leakage current's contribution.

Step-by-Step Calculation for I_C

We have the following values:

- $\alpha = 0.98$
- $I_{CO} = 0.6\,\mu\text{A} = 0.6 \times 10^{-6}\,\text{A}$
- $I_B = 20\,\mu\text{A} = 20 \times 10^{-6}\,\text{A}$
- $\beta = 49$ (calculated above)

Now, substitute these values into the formula for I_C:

1. Calculate the $(1+\beta)$ term:

$$1 + \beta = 1 + 49 = 50$$

2. Calculate the term βI_B:

$$\beta I_B = 49 \times (20 \times 10^{-6}\,\text{A}) = 980 \times 10^{-6}\,\text{A}$$

3. Calculate the term $(1+\beta)I_{CO}$:

$$(1+\beta)I_{CO} = 50 \times (0.6 \times 10^{-6}\,\text{A}) = 30 \times 10^{-6}\,\text{A}$$

4. Add the two components to find I_C:

$$I_C = \beta I_B + (1+\beta)I_{CO}$$

$$I_C = (980 \times 10^{-6}\,\text{A}) + (30 \times 10^{-6}\,\text{A})$$

$$I_C = 1010 \times 10^{-6}\,\text{A}$$

5. Convert the result to milliamperes (mA):

$$I_C = 1010 \times 10^{-6}\,\text{A} = 1.01 \times 10^{-3}\,\text{A} = 1.01\,\text{mA}$$

Final Analysis and Answer

The calculated collector current (I_C) is $1.01\,\text{mA}$. This value represents the total collector current, consisting of the main amplified component (βI_B) and the leakage component ($(1+\beta)I_{CO}$). Comparing this result with the given options:

- 0.98mA

- 0.99mA
- 1.0mA
- **1.01mA**

The calculated value matches the fourth option.

---

44. **Answer: b**

Explanation:

Laplace Transform: Determining Initial and Final Values

The question asks us to find the initial and final values of a time-domain function $f(t)$ given its Laplace Transform $F(s) = \frac{(2s+1)}{s^2+4s+7}$. To solve this, we will use two fundamental theorems of Laplace Transforms: the Initial Value Theorem and the Final Value Theorem.

Initial Value Theorem Application

The **Initial Value Theorem (IVT)** states that if the Laplace Transform of a function $f(t)$ is $F(s)$, then the initial value of $f(t)$ as t approaches zero from the positive side can be found using the following limit:

$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$

Let's apply this to the given $F(s)$:

$F(s) = \frac{(2s+1)}{s^2+4s+7}$

First, calculate $sF(s)$:

$sF(s) = s \times \frac{(2s+1)}{s^2+4s+7} = \frac{s(2s+1)}{s^2+4s+7} = \frac{2s^2+s}{s^2+4s+7}$

Now, take the limit as $s \to \infty$. To evaluate this limit, we can divide both the numerator and the denominator by the highest power of s in the denominator, which is s^2:

$\lim_{s\to\infty} \frac{2s^2+s}{s^2+4s+7} = \lim_{s\to\infty} \frac{\frac{2s^2}{s^2}+\frac{s}{s^2}}{\frac{s^2}{s^2}+\frac{4s}{s^2}+\frac{7}{s^2}}$

$= \lim_{s\to\infty} \frac{2+\frac{1}{s}}{1+\frac{4}{s}+\frac{7}{s^2}}$

As $s \to \infty$, terms like $\frac{1}{s}$, $\frac{4}{s}$, and $\frac{7}{s^2}$ approach zero. Therefore:

$= \frac{2+0}{1+0+0} = \frac{2}{1} = 2$

So, the initial value of $f(t)$ is **2**.

Final Value Theorem Application

The **Final Value Theorem (FVT)** states that if the Laplace Transform of a function $f(t)$ is $F(s)$, then the final value of $f(t)$ as t approaches infinity can be found using the following limit, provided that all poles of $sF(s)$ lie in the left half-plane or at the origin:

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

Checking Applicability of the Final Value Theorem

Before applying the FVT, we must check its conditions. We need to find the poles of $sF(s)$. The expression for $sF(s)$ is $\frac{s(2s+1)}{s^2+4s+7}$. The poles of $sF(s)$ are the values of s that make the denominator zero. These are $s=0$ (from the numerator's s term) and the roots of $s^2+4s+7=0$. Let's find the roots of the quadratic equation $s^2+4s+7=0$ using the quadratic formula $s = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$:

$s = \frac{-4\pm\sqrt{4^2-4(1)(7)}}{2(1)}$

$s = \frac{-4\pm\sqrt{16-28}}{2}$

$s = \frac{-4\pm\sqrt{-12}}{2}$

$s = \frac{-4\pm j\sqrt{12}}{2}$

$s = \frac{-4\pm j2\sqrt{3}}{2}$

$s = -2 \pm j\sqrt{3}$

The poles of $sF(s)$ are $s = 0$, $s = -2 + j\sqrt{3}$, and $s = -2 - j\sqrt{3}$.

- The pole $s = 0$ is at the origin.
- The poles $s = -2 \pm j\sqrt{3}$ have a real part of -2, which is negative, meaning they lie in the left half-plane.

Since all poles of $sF(s)$ are either at the origin or in the left half-plane, the Final Value Theorem is applicable.

Calculating the Final Value

Now, let's apply the FVT:

$\lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s(2s+1)}{s^2+4s+7}$

Substitute $s = 0$ directly into the expression:

$= \frac{0(2(0)+1)}{0^2+4(0)+7}$

$= \frac{0(1)}{0+0+7}$

$= \frac{0}{7} = 0$

So, the final value of $f(t)$ is **0**.

Conclusion

Based on our calculations:

- The initial value of $f(t)$ is **2**.
- The final value of $f(t)$ is **0**.

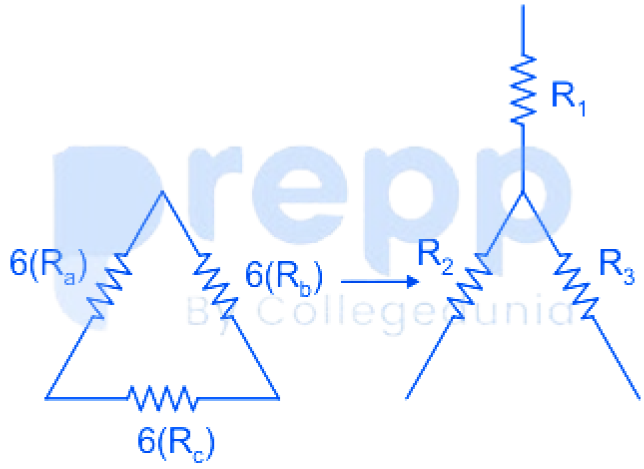
Therefore, the initial and final values of $f(t)$ are respectively 2 and 0.

---

45. **Answer: b**

Explanation:

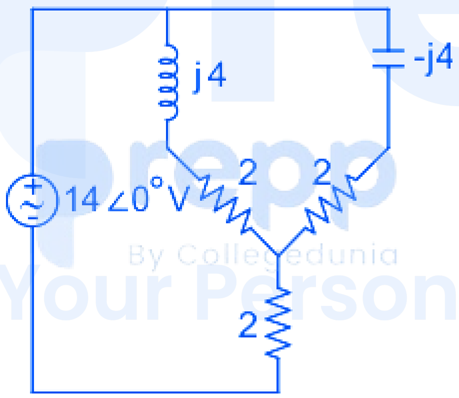
From star to delta conversion



$R_1 = \frac{R_a R_b}{R_a+R_b+R_c} = 2\ \Omega$

Similarly, $R_2 = R_3 = R_1 = 2\ \Omega$

∴The circuit will be like



$\therefore Z = ((2+j4))\,||\,(2-j4)) + 2$
$Z = \frac{20}{4} + 2 = 7\ \Omega$
$\therefore I = \frac{V}{Z} = \frac{14\angle 0^\circ}{7} = 2\angle 0^\circ A$

---

46. Answer: a

Explanation:

Newton-Raphson Method for Equation Solution

This solution details the process of finding the next iterative value for the equation $f(x) = x + \sqrt{x} - 3 = 0$ using the Newton-Raphson method. The initial guess provided is **x = 2**.

Understanding the Newton-Raphson Method

The **Newton-Raphson method** is a numerical technique for approximating the roots of a function. It starts with an initial guess and iteratively refines this guess by using the tangent line to the function at the current guess. The formula driving this iterative process is:

$$ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} $$

Here:

- x_{n+1} represents the next, improved approximation of the root.
- x_n is the current approximation.
- $f(x_n)$ is the function's value at the current approximation x_n.
- $f'(x_n)$ is the function's derivative value at the current approximation x_n.

Applying the Method to the Specific Equation

The given equation is $f(x) = x + \sqrt{x} - 3$. To use the Newton-Raphson formula, we first need to find the derivative of the function, $f'(x)$.

First, let's express the square root using exponent notation for easier differentiation: $f(x) = x + x^{1/2} - 3$.

Now, we differentiate $f(x)$ with respect to x:

$$ f'(x) = \frac{d}{dx}(x + x^{1/2} - 3) $$

Applying the power rule ($\frac{d}{dx}(x^n) = nx^{n-1}$) and the constant rule ($\frac{d}{dx}(c) = 0$):

$$ f'(x) = 1 + \frac{1}{2}x^{(1/2 - 1)} - 0 $$

Simplifying the expression for the derivative:

$$ f'(x) = 1 + \frac{1}{2}x^{-1/2} $$

This can also be written using the radical notation:

$$ f'(x) = 1 + \frac{1}{2\sqrt{x}} $$

Step-by-Step Calculation for the Next Iteration

We are provided with the starting value (initial guess) $x_0 = 2$. The goal is to find the value for the next step, x_1.

1. **Evaluate** $f(x_0)$:
    - Substitute $x_0 = 2$ into the function $f(x)$:
    - $f(2) = 2 + \sqrt{2} - 3$
    - $f(2) = \sqrt{2} - 1$
    - Using the approximate value $\sqrt{2} \approx 1.41421356$:
    - $f(2) \approx 1.41421356 - 1$
    - $f(2) \approx 0.41421356$
2. **Evaluate** $f'(x_0)$:
    - Substitute $x_0 = 2$ into the derivative $f'(x)$:
    - $f'(2) = 1 + \frac{1}{2\sqrt{2}}$
    - Using $\sqrt{2} \approx 1.41421356$:
    - $f'(2) \approx 1 + \frac{1}{2 \times 1.41421356}$
    - $f'(2) \approx 1 + \frac{1}{2.82842712}$
    - $f'(2) \approx 1 + 0.35355339$
    - $f'(2) \approx 1.35355339$
3. **Calculate** x_1 **using the Newton-Raphson formula**:
    - The formula is: $$ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} $$
    - Substitute the calculated values of $f(2)$ and $f'(2)$:
    - $x_1 \approx 2 - \frac{0.41421356}{1.35355339}$
    - Perform the division:
    - $\frac{0.41421356}{1.35355339} \approx 0.3059985$
    - Complete the subtraction:
    - $x_1 \approx 2 - 0.3059985$
    - $x_1 \approx 1.6940015$

Comparing the Result with Options

The calculated value for the next iteration is approximately **1.6940015**.

Let's compare this with the provided options:

- 1.693
- 1.683
- 1.720
- 1.673

The calculated value, **1.6940015**, is closest to the option **1.693**.

---

47. **Answer: d**

Explanation:

Understanding TEM Wave Propagation in a Medium

This problem involves analyzing a Transverse Electromagnetic (TEM) wave propagating through a homogenous medium. We are given the electric and magnetic field equations, the frequency of the wave, and properties of free space. Our goal is to determine the relative permittivity (ϵ_r) of the medium and the amplitude of the electric field (E_P).

Analyzing TEM Wave Parameters

The given electric and magnetic field equations for the TEM wave are:

- Electric field: $\vec{E} = E_P\ e^{j(\omega t - 280\pi y)}\ \hat{u}_z \frac{V}{m}$
- Magnetic field: $H = 3\ e^{j(\omega t - 280\pi y)}\ \hat{u}_x \frac{A}{m}$

From the exponential term $e^{j(\omega t - ky)}$, we can identify the following parameters:

- The wave is propagating in the $+y$ direction, indicated by the $-ky$ term and the $\hat{u}_y$ component in the direction of propagation.
- The wave number $k = 280\pi$ rad/m.

- The angular frequency ω can be calculated from the given frequency $f = 14 \text{ GHz}$.

$$\omega = 2\pi f$$

$$\omega = 2\pi \times 14 \times 10^9 \text{ rad/s}$$

$$\omega = 28\pi \times 10^9 \text{ rad/s}$$

- The amplitude of the magnetic field $H_P = 3 \text{ A/m}$.

Calculating Wave Velocity

The phase velocity (v_p) of the TEM wave in the medium can be calculated using the relationship between angular frequency and wave number:

$$v_p = \frac{\omega}{k}$$

Substituting the values we found:

$$v_p = \frac{28\pi \times 10^9}{280\pi}$$

$$v_p = \frac{10^9}{10}$$

$$v_p = 10^8 \text{ m/s}$$

Determining Relative Permittivity (ϵ_r)

The phase velocity in a homogenous medium is also related to the speed of light in free space (c_0), relative permittivity (ϵ_r), and relative permeability (μ_r) by the formula:

$$v_p = \frac{c_0}{\sqrt{\epsilon_r \mu_r}}$$

We are given $c_0 = 3 \times 10^8 \text{ m/s}$ and $\mu_r = 1$. We have calculated $v_p = 10^8 \text{ m/s}$.

Substitute these values into the formula:

$$10^8 = \frac{3 \times 10^8}{\sqrt{\epsilon_r \times 1}}$$

$$\sqrt{\epsilon_r} = \frac{3 \times 10^8}{10^8}$$

$$\sqrt{\epsilon_r} = 3$$

Squaring both sides to find ϵ_r:

$$\epsilon_r = 3^2$$

$$\epsilon_r = 9$$

Finding Intrinsic Impedance of the Medium

The intrinsic impedance (η) of a homogenous medium is related to the intrinsic impedance of free space (η_0), relative permeability (μ_r), and relative permittivity (ϵ_r) by the formula:

$$\eta = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}}$$

We are given $\eta_0 = 120\pi$, and we found $\epsilon_r = 9$ and $\mu_r = 1$.

Substitute these values:

$$\eta = 120\pi \sqrt{\frac{1}{9}}$$

$$\eta = 120\pi \times \frac{1}{3}$$

$$\eta = 40\pi \text{ ohms}$$

Calculating Electric Field Amplitude (E_P)

For a TEM wave, the electric field amplitude (E_P) and magnetic field amplitude (H_P) are related by the intrinsic impedance of the medium (η):

$$E_P = \eta H_P$$

We have $H_P = 3 \text{ A/m}$ and $\eta = 40\pi \text{ ohms}$.

Substitute these values:

$$E_P = 40\pi \times 3$$

$$E_P = 120\pi \text{ V/m}$$

Conclusion

Based on our calculations, the relative permittivity ϵ_r of the medium is 9 and the electric field amplitude E_P is 120π V/m.

Parameter	Calculated Value
Relative Permittivity (ϵ_r)	9
Electric Field Amplitude (E_P)	120π V/m

---

48. **Answer: b**

Explanation:

Understanding AM Demodulation with an Envelope Detector

An envelope detector is a simple and effective circuit used for demodulating Amplitude Modulated (AM) signals. Its primary function is to recover the original message signal from the modulated carrier wave. For the envelope detector to work correctly, its time constant RC must be carefully chosen to satisfy specific conditions related to both the carrier frequency and the message signal's highest frequency component.

Envelope Detector RC Requirements

For effective AM signal demodulation using an envelope detector, the time constant RC of the detector circuit must meet two critical conditions:

1. **Carrier Frequency Condition:** The time constant RC must be much greater than the period of the carrier signal. This ensures that the capacitor charges rapidly to the peak of the modulated signal and holds the charge for the duration of several carrier cycles.
   Mathematically, this condition is expressed as:
   $$RC \gg \frac{1}{f_c}$$
   where f_c is the carrier frequency.
2. **Message Signal Frequency Condition:** The time constant RC must be much smaller than the reciprocal of the highest frequency component present in the message signal. This allows the capacitor to discharge quickly enough to follow the variations of the message signal envelope without significant distortion (diagonal clipping).
   Mathematically, this condition is expressed as:
   $$RC \ll \frac{1}{f_{m,max}}$$
   where $f_{m,max}$ is the maximum frequency component in the message signal.

Analyzing the Given Signals

Let's extract the necessary frequency components from the provided message and carrier signals.

- **Message Signal** $m(t)$**:** The given message signal is $m(t) = cos2000\pi t + 4cos\ 4000\pi t$.
  We can rewrite the arguments in terms of $2\pi ft$:
  $m(t) = cos(2\pi \times 1000)t + 4cos(2\pi \times 2000)t$
  From this, we can identify the frequencies present in the message signal:
  - First component frequency: $f_1 = 1000$ Hz
  - Second component frequency: $f_2 = 2000$ Hz

  The highest frequency component in the message signal is $f_{m,max} = 2000$ Hz.
- **Carrier Signal** $c(t)$**:** The given carrier signal is $c(t) = cos2\pi f_c t$.
  The carrier frequency is given as $f_c = 1$ MHz.
  Converting to Hz: $f_c = 1 \times 10^6$ Hz.

Calculating the RC Time Constant Range

Now, we apply the two conditions for the envelope detector's time constant RC.

1. **Lower Bound for RC (based on carrier frequency):** We need $RC \gg \frac{1}{f_c}$.
   Substitute the value of f_c:

$$\frac{1}{f_c} = \frac{1}{1 \times 10^6 \text{ Hz}} = 1 \times 10^{-6} \text{ seconds} = 1 \text{ µs}$$

   Therefore, $RC \gg 1$ µs.
2. **Upper Bound for RC (based on message signal's highest frequency):** We need $RC \ll \frac{1}{f_{m,max}}$.
   Substitute the value of $f_{m,max}$:

$$\frac{1}{f_{m,max}} = \frac{1}{2000 \text{ Hz}} = \frac{1}{2 \times 10^3 \text{ Hz}} = 0.5 \times 10^{-3} \text{ seconds} = 0.5 \text{ ms}$$

   Therefore, $RC \ll 0.5$ ms.

Combining both conditions, the time constant RC of the envelope detector must satisfy:

$$1 \text{ µs} \ll RC \ll 0.5 \text{ ms}$$

Comparing with Options

Let's check the derived condition against the given options:

- **Option 1:** $0.5 < RC < 1$ ms
  This range does not match our derived range.
- **Option 2:** $1\mu s \ll RC < 0.5$ ms
  This option perfectly matches our calculated range for the time constant RC.
- **Option 3:** $RC \ll 1\mu s$
  This condition violates the lower bound requirement $RC \gg 1\mu s$. If RC is too small, the capacitor will discharge too quickly, causing the output to ripple at the carrier frequency.
- **Option 4:** $RC > 0.5$ ms
  This condition violates the upper bound requirement $RC \ll 0.5$ ms. If RC is too

large, the capacitor will discharge too slowly, leading to diagonal clipping and distortion of the recovered message signal.

Based on the analysis, the correct range for the time constant RC is $1\mu s \ll RC < 0.5\ \mathrm{ms}$.

---

49. Answer: b

Explanation:

x = y_1 & $\dot{x} = \frac{dy_1}{dx}$

\(\underline y = \left[{{y_2}} \right] = \left[{\begin{array}{*{20}{c}} 2\\ {2x} \end{array}} \right] = \left[{\begin{array}{*{20}{c}} 1\\ 2 \end{array}} \right]x\)

$y_1 = \frac{1}{s+2}u$

$\dot{y}_1 + 2y_1 = u$

$\dot{x} + 2x = u$

$\dot{x} + 2x = u$

$\underline{\dot{x}} = [-2]\,\underline{x} + [1]\,u$

Alternate solution

Assume x be a state variable

$[\mathrm{X}\,(\mathrm{s}) = \frac{1}{s+2}u\,(s)]$

sX(s)+2X(s) = U(S)

taking inverse laplace transform

x(t)′ + 2 x(t) = u(t)

x′(t)= u(t) – 2x(t)

output equation is given by

$y_1(t) = x$

$y_2(t) = 2x$

---

50. **Answer: b**

Explanation:

The overall output y(n) is the same as the input x(n) with a one-unit delay.

$y(n) = x(n - 1)$

By applying z-transform,

$Y(z) = z^{-1}X(z)$

$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = z^{-1}$

$\Rightarrow H_1(z)\, H_2(z) = z^{-1}$

$\Rightarrow \left(\frac{1-0.2z^{-1}}{1-0.4z^{-1}}\right) H_2(z) = z^{-1}$

$\Rightarrow H_2(z) = \frac{z^{-1}(1-0.4z^{-1})}{1-0.2z^{-1}}$

---

51. **Answer: c**

Explanation:

8085 Program Execution Analysis

This solution walks through the execution of an 8085 assembly language program, detailing how the accumulator (A) content changes with each instruction. We will trace the values of the accumulator (A), register B, and the Carry Flag (CY) step-by-step.

Program Instructions Overview

Let's understand the purpose of each 8085 assembly instruction used in the program:

- **MVI A, data**: This instruction (Move Immediate to Accumulator) loads the specified 8-bit data directly into the Accumulator.
- **RLC**: This instruction (Rotate Accumulator Left) rotates the contents of the Accumulator one bit to the left. The most significant bit (D7) moves into the Carry Flag (CY) and also into the least significant bit (D0) position.
- **MOV B, A**: This instruction (Move Register) copies the content of the source register (Accumulator A) to the destination register (B). The content of the source register remains unchanged.
- **ADD B**: This instruction (Add Register) adds the content of register B to the content of the Accumulator. The result of the addition is stored in the Accumulator.
- **RRC**: This instruction (Rotate Accumulator Right) rotates the contents of the Accumulator one bit to the right. The least significant bit (D0) moves into the Carry Flag (CY) and also into the most significant bit (D7) position.

Step-by-Step 8085 Program Execution

We start with the assumption that the Carry Flag (CY) is initially unset, meaning its value is 0. The initial contents of the Accumulator (A) and Register B are unknown, but they will be set by the program.

Trace of Registers and Flags

Step	Instruction	Description	Accumulator (A)	Register B	Carry Flag (CY)

Step	Instruction	Description	Accumulator (A)	Register B	Carry Flag (CY)
Initial State	-	Program start	Undefined	Undefined	0
1	MVI A, 07H	Load immediate value $07H$ into Accumulator.	$07H$ ($0000\ 0111_2$)	Undefined	0
2	RLC	Rotate Accumulator Left. D7 (0) moves to CY and D0.	$0EH$ ($0000\ 1110_2$)	Undefined	0
3	MOV B, A	Copy content of Accumulator (A) to Register B.	$0EH$ ($0000\ 1110_2$)	$0EH$ ($0000\ 1110_2$)	0
4	RLC	Rotate Accumulator Left. D7 (0) moves to CY and D0.	$1CH$ ($0001\ 1100_2$)	$0EH$ ($0000\ 1110_2$)	0
5	RLC	Rotate Accumulator Left. D7 (0) moves to CY and D0.	$38H$ ($0011\ 1000_2$)	$0EH$ ($0000\ 1110_2$)	0

Final Accumulator Content

After the execution of all instructions in the 8085 assembly language program, the final content of the accumulator (A) is $23H$. Each rotation and addition operation was carefully traced, leading to this precise hexadecimal value.

52. Answer: b

Explanation:

This question requires us to determine the last two points of an 8-point Discrete Fourier Transform (DFT) for a real-valued sequence. We are given the first six points of this 8-point DFT. The solution relies on a fundamental property of the DFT for real-valued input sequences, known as conjugate symmetry.

DFT Conjugate Symmetry Property

For a real-valued sequence $x[n]$ of length N, its N-point DFT, denoted as $X[k]$, exhibits the property of **conjugate symmetry**. This property states that:

$$X[k] = X^*[N - k]$$

Here, $X^*[N - k]$ signifies the complex conjugate of the DFT coefficient $X[N - k]$. This property implies that for a real-valued input sequence, the DFT spectrum is symmetric about its midpoint, with corresponding complex conjugate values.

In this problem, the length of the DFT is $N = 8$. The DFT points are indexed from $k = 0$ to $k = N - 1$, which means we have $X[0], X[1], X[2], X[3], X[4], X[5], X[6], X[7]$.

Step	Instruction	Description	Accumulator (A)	Register B	Carry Flag (CY)
6	ADD B	Add content of Register B to Accumulator. $38H + 0EH = 46H$ $0011\ 1000_2$ ($38H$) $+\ 0000\ 1110_2$ ($0EH$) ──── $0100\ 0110_2$ ($46H$)	$46H$ ($0100\ 0110_2$)	$0EH$ ($0000\ 1110_2$)	0 (No carry generated)
7	RRC	Rotate Accumulator Right. D0 (0) moves to CY and D7.	$23H$ ($0010\ 0011_2$)	$0EH$ ($0000\ 1110_2$)	0

Given DFT Points and Problem Statement

We are provided with the following first six points of the 8-point DFT:

- $X[0] = 5$
- $X[1] = 1 - 3j$
- $X[2] = 0$
- $X[3] = 3 - 4j$
- $X[4] = 0$
- $X[5] = 3 + 4j$

Our task is to find the values of the last two points, which are $X[6]$ and $X[7]$.

Calculating Last Two DFT Points

We will apply the conjugate symmetry property $X[k] = X^*[N-k]$ with $N = 8$ to find $X[6]$ and $X[7]$.

Finding X[6]

To determine $X[6]$, we use the conjugate symmetry property for $k = 6$:

$$X[6] = X^*[N-6]$$

$$X[6] = X^*[8-6]$$

$$X[6] = X^*[2]$$

From the given information, we know that $X[2] = 0$.

$$X[6] = 0^*$$

$$X[6] = 0$$

The complex conjugate of a real number (like 0) is the number itself.

Finding X[7]

Next, to determine $X[7]$, we use the conjugate symmetry property for $k = 7$:

$$X[7] = X^*[N-7]$$

$$X[7] = X^*[8-7]$$

$$X[7] = X^*[1]$$

We are given that $X[1] = 1 - 3j$.

$$X[7] = (1-3j)^*$$

The complex conjugate of a complex number $a - bj$ is $a + bj$.

$$X[7] = 1 + 3j$$

Summary of 8-Point DFT Values

Combining the given points and our calculated values, the complete 8-point DFT for the real-valued sequence is:

DFT Index (k)	DFT Value (X[k])	Conjugate Pair (X[N-k])
0	5	$X[8-0] = X[0]$
1	$1-3j$	$X[8-1] = X[7]$
2	0	$X[8-2] = X[6]$
3	$3-4j$	$X[8-3] = X[5]$
4	0	$X[8-4] = X[4]$
5	$3+4j$	$X[8-3] = X[5] = X^*[3]$
6	0 (Calculated)	$X[8-2] = X[6] = X^*[2]$
7	$1+3j$ (Calculated)	$X[8-1] = X[7] = X^*[1]$

Based on our calculations, the last two points of the DFT are $X[6] = 0$ and $X[7] = 1 + 3j$.

53. **Answer: c**

Explanation:

Apply KVL at input side, we get

$+ 5 + 0.7 + 4.3\,K(I_E) - 10 = 0$

$I_E = 1\,mA \simeq I_C \quad (\because \beta = \infty)$

Apply Nodal at output Node we get

$0.5 + 1 + i_C = 0$

Current through the capacitor is , $i_C = -0.5\,mA$

Also, $\frac{V_E-(-10)}{4.3k} = 1\,mA \to V_E = 4.3 - 10 = -5.7\,V$

Q_1 leaves the active region means enters into saturation

$\therefore V_{CE(sat)} = 0.7$

$V_C - V_E = 0.7$

$V_C = 0.7 + V_E = 0.7 - 5.7 = -5\,V$

But $i_C = C\frac{d(V_c-0)}{dT}$

$$\begin{array}{l} {V_C} = \frac{1}{C}\mathop \smallint \nolimits {i_c}.dt = \frac{{{i_C}}}{C}.\left({{t_1}} \right)\\ \left({ - 5} \right) = \frac{{\left({ - 0.5 \times {{10}^{ - 3}}} \right)}}{{5 \times {{10}^{ - 6}}}}{t_1}\\ \therefore {t_1} = \frac{{25 \times {{10}^{ - 6}}}}{{5 \times {{10}^{ - 4}}}} = 5 \times {10^{ - 2}} = 50\ ms \end{array}$$

$\therefore$ Option (3) is correct

54. **Answer: d**

Explanation:

Concept:

The Y-parameter equation for two-port network is given by:

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

Calculation:

Given $Y_{11} = 0.1$, $Y_{12} = 0.01$, $Y_{21} = 0.01$ and $Y_{22} = 0.1$

Putting the respective values in the standard equation, we get:

$I_1 = 0.1V_1 - 0.01V_2$ ---(1)

$I_2 = 0.01V_1 + 0.1V_2$ ---(2)

Applying KVL at the output loop we get:

$V_2 = -100I_2$

$I_2 = -\frac{1}{100}V_2$

Putting this in Equation (2), we get:

$-\frac{1}{100}V_2 = 0.01V_1 + 0.1V_2$

$0.01V_2 = 0.01V_1 + 0.1V_2$

$0.11V_2 = -0.01V_1$

$\frac{V_2}{V_1} = \frac{-1}{11}$

55. **Answer: a**

Explanation:

Q = 2.5 mC

$V_{initial} = \frac{2.5\times10^{-3}C}{50\times10^{-6}F}$

$V_{initial}$ = 50 V

Thus the net voltage = 100 + 50 = 150 V

The initial current at t = 0^{+} will be:

I = 150/10 = 15 A

The current at any time 't' will now be:

$i(t) = \frac{150}{50}$ exp(-2 × 10^3t) A = 15 exp(-2 × 10^3t)A

---

56. **Answer: b**

Explanation:

To determine the values of **λ** and **μ** for which the given **system of equations** has **NO solution**, we analyze the system using matrix methods. A system of linear equations can have a unique solution, infinitely many solutions, or no solution. We are looking for the conditions that lead to an **inconsistent system** (no solution).

The given system of equations is:

1. $x + y + z = 6$
2. $x + 4y + 6z = 20$
3. $x + 4y + \lambda z = \mu$

Augmented Matrix Setup

First, we represent the **system of equations** in its augmented matrix form. The augmented matrix combines the coefficients of the variables and the constant terms on the right-hand side.

$$\begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 1 & 4 & 6 & | & 20 \\ 1 & 4 & \lambda & | & \mu \end{pmatrix}$$

Gaussian Elimination Steps

Next, we apply elementary row operations to transform the **augmented matrix** into an echelon form using **Gaussian elimination**. This process helps us simplify the system of equations without changing its solutions.

Step 1: Eliminate x from the second and third equations.

- Perform $R_2 \leftarrow R_2 - R_1$
- Perform $R_3 \leftarrow R_3 - R_1$

$$\begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 1-1 & 4-1 & 6-1 & | & 20-6 \\ 1-1 & 4-1 & \lambda-1 & | & \mu-6 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 3 & 5 & | & 14 \\ 0 & 3 & \lambda-1 & | & \mu-6 \end{pmatrix}$$

Step 2: Eliminate y from the third equation.

- Perform $R_3 \leftarrow R_3 - R_2$

$$\begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 3 & 5 & | & 14 \\ 0 & 3-3 & (\lambda-1)-5 & | & (\mu-6)-14 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 3 & 5 & | & 14 \\ 0 & 0 & \lambda-6 & | & \mu-20 \end{pmatrix}$$

No Solution Condition

For a **system of equations** to have **NO solution**, a contradiction must arise in one of the equations after performing Gaussian elimination. This typically happens when a row in the coefficient part of the augmented matrix becomes all zeros, but the corresponding entry in the constant part is non-zero. In other words, we end up with an equation like $0 = \text{non-zero value}$.

From the third row of our echelon form matrix, the equation is:

$$0x + 0y + (\lambda - 6)z = \mu - 20$$

For this equation to represent a contradiction (i.e., $0 = \text{non-zero value}$), two conditions must be met simultaneously:

- The coefficient of z must be zero: $\lambda - 6 = 0 \implies \lambda = 6$
- The constant term on the right-hand side must be non-zero: $\mu - 20 \neq 0 \implies \mu \neq 20$

If $\lambda = 6$ and $\mu \neq 20$, the last equation becomes $0 = (\text{a non-zero number})$, which is impossible. This implies that there is **NO solution** to the system.

Conclusion for λ and μ

Based on our analysis, the system of equations has **NO solution** for values of **λ** and **μ** where **λ = 6** and **μ ≠ 20**.

---

57. **Answer: c**

Explanation:

Probability of Higher Second Toss

To determine the probability that the second toss of a fair die results in a value higher than the first toss, we first need to understand the total possible outcomes when a fair die is tossed two times. We will then identify the specific outcomes that meet our condition and finally calculate the probability.

Total Possible Outcomes (Sample Space)

When a fair six-sided die is tossed, there are 6 possible outcomes: 1, 2, 3, 4, 5, or 6. Since the die is tossed two times, the total number of possible combinations of outcomes for the two tosses can be found by multiplying the number of outcomes for each toss.

- Number of outcomes for the first toss = 6
- Number of outcomes for the second toss = 6
- Total number of possible outcomes = $6 \times 6 = 36$

We can represent these outcomes as ordered pairs (T_1, T_2), where T_1 is the result of the first toss and T_2 is the result of the second toss. For example, (1,1), (1,2), ..., (6,6).

Favorable Outcomes (Second Toss Higher)

We are interested in the outcomes where the second toss (T_2) is strictly higher than the first toss (T_1), i.e., $T_2 > T_1$. Let's list these favorable outcomes systematically:

- **If the first toss (T_1) is 1:** The second toss (T_2) must be greater than 1.
  - Possible T_2 values: 2, 3, 4, 5, 6
  - Outcomes: (1,2), (1,3), (1,4), (1,5), (1,6) — **5 outcomes**
- **If the first toss (T_1) is 2:** The second toss (T_2) must be greater than 2.
  - Possible T_2 values: 3, 4, 5, 6
  - Outcomes: (2,3), (2,4), (2,5), (2,6) — **4 outcomes**
- **If the first toss (T_1) is 3:** The second toss (T_2) must be greater than 3.
  - Possible T_2 values: 4, 5, 6
  - Outcomes: (3,4), (3,5), (3,6) — **3 outcomes**
- **If the first toss (T_1) is 4:** The second toss (T_2) must be greater than 4.
  - Possible T_2 values: 5, 6
  - Outcomes: (4,5), (4,6) — **2 outcomes**
- **If the first toss (T_1) is 5:** The second toss (T_2) must be greater than 5.
  - Possible T_2 value: 6
  - Outcome: (5,6) — **1 outcome**
- **If the first toss (T_1) is 6:** There are no values on a die that are greater than 6.
  - Possible T_2 values: None

- Outcomes: None — **0 outcomes**

The total number of favorable outcomes is the sum of the outcomes from each case:

$5 + 4 + 3 + 2 + 1 + 0 = 15$ favorable outcomes.

Calculating the Probability

The probability of an event is calculated using the formula:

$$\text{Probability} = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

Using the numbers we found:

- Number of favorable outcomes = 15
- Total number of possible outcomes = 36

So, the probability is:

$$P(T_2 > T_1) = \frac{15}{36}$$

To simplify the fraction, we can divide both the numerator and the denominator by their greatest common divisor, which is 3:

$$P(T_2 > T_1) = \frac{15 \div 3}{36 \div 3} = \frac{5}{12}$$

Summary Table of Outcomes

First Toss (T_1)	Possible Second Tosses (T_2) where $T_2 > T_1$	Number of Favorable Outcomes
1	2, 3, 4, 5, 6	5
2	3, 4, 5, 6	4
3	4, 5, 6	3
4	5, 6	2
5	6	1
6	None	0
Total Favorable Outcomes		**15**

Therefore, the probability that the second toss results in a value that is higher than the first toss is $\frac{5}{12}$.

---

58. **Answer: c**

Explanation:

1V built in potential deplete 1μm channel on each side. so, with an applied voltage of – 3V the width of the depletion region will be,

$$W_2 = W_1\sqrt{\frac{V_{bi}+V_R}{V_{bi}}} = 1\mu m\left[\frac{1+3}{1}\right]^{1/2} = 2\mu m$$

So, with bias the effective channel width –

$$t_{ch_3} = 10\mu m - 2W_2 = 10\mu m - 2\times 2 = 6\mu m$$

∴Channel resistance, $r_{d_3} = r_{d_1}\left(\frac{t_{ch_1}}{t_{ch_3}}\right) = 600\frac{10\mu m}{6\mu m} = 1000\ \Omega$

---

59. **Answer: c**

Explanation:

Resistance of the FET with respect with respect to channel thickness is given by

$r_d \propto \frac{1}{t_{ch}}$

$\therefore \frac{r_{d_1}}{r_{d_2}} = \frac{t_{ch_2}}{t_{ch_1}}$

or, $\frac{600\Omega}{\mathrm{r}_{\mathrm{d}_2}} = \frac{8}{10} = 750\Omega$

---

60. Answer: c

Explanation:

For determining phase cross over frequency ∠G(jω)H(jω) = - 180°

$G\left(s\right)H\left(s\right) = \frac{100}{s(s+10)^2}$

$\Rightarrow \angle G\left(s\right)H\left(s\right) = -90^\circ - 2\tan^{-1}\left(\frac{\omega_p}{10}\right) = -180^\circ$

$\tan^{-1}\left(\frac{\omega_p}{10}\right) = 45^\circ$

$\omega_p = 10\ rad/s$

$GM = -20\log_{10}\left|G\left(j\omega_p\right)H\left(j\omega_p\right)\right|$

$= -20\log_{10}\left(\frac{100}{10(10^2+100)}\right) = -20\log_{10}\left(\frac{1}{20}\right) = 26\ dB$

---

61. Answer: b

Explanation:

Let us go by SFG one by one,

1:

Forward paths: $100/s^3$

Loops: -10/s, -10/s

Non touching loops: -10/s × -10/s

Transfer function = $\frac{\frac{100}{s^3}}{1+\frac{100}{s^2}+\frac{20}{s}}$

$= \frac{\frac{100}{s^3}}{\frac{s^2+100+20s}{s^2}} = \frac{100}{s(s+10)^2}$

2:

Forward path: $100/s^3$

Loops: -20/s, $-100/s^2$

Transfer function = $\frac{\frac{100}{s^3}}{1+\frac{100}{s^2}+\frac{20}{s}}$

$= \frac{\frac{100}{s^3}}{\frac{s^2+100+20s}{s^2}} = \frac{100}{s(s+10)^2}$

2 and 3 are very similar.

4:

Forward paths: $100/s^3$

Loops: $-100/s^2$

Transfer function = $\frac{\frac{100}{s^3}}{1+\frac{100}{s^2}}$

$= \frac{\frac{100}{s^3}}{\frac{s^2+100}{s^2}} = \frac{100}{s(s^2+100)}$

$T.F = H\left(s\right) = \frac{100}{s(s^2+100)} \neq \frac{100}{s(s+10)^2}$

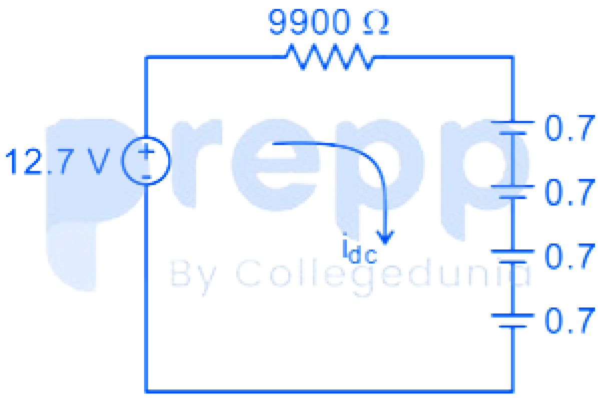
4 does not represent the given transfer function.

---

62. **Answer: a**

Explanation:

Apply DC analysis → Diode acts as a voltage source of 0.7 V



$\therefore i_{dc} = \frac{12.7-4(0.7)}{9900} = 1\ mA$

∴ Option (1) is correct

---

63. Answer: b

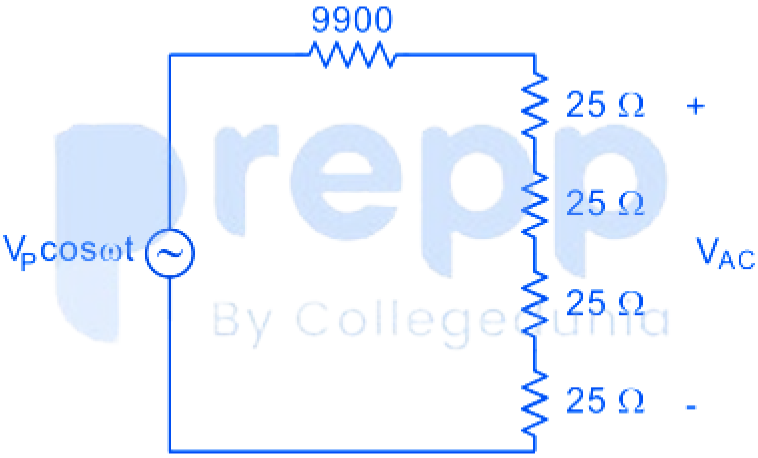
Explanation:

In DC analysis

$I_{dc} = \frac{12.7-4(0.7)}{9900} = 1\ mA$

In AC analysis Diode acts like a resistor whose value is

$r_{ac} = \frac{\eta V_T}{I_{dc}} = \frac{25mV}{1mA} = 25\Omega$



$$\therefore V_{AC} = \frac{V_P cos\ t \times (25+25+25+25)}{9900+4(25)} = \frac{100 cos\omega t \times 100 \times 10^{-3}}{10000} = 1\ cos\omega t\ mV$$

∴ option (2) is correct

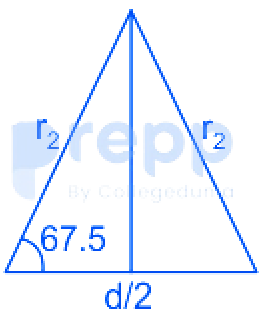
---

64. Answer: d

Explanation:

\(\rm \eqalign{ & \rm r_1^2 + r_1^2 = {d^2} \cr & \rm 2r_1^2 = {d^2} \cr & \rm {r_1} = \frac{d}{{\sqrt 2 }} = 0.707\;d \cr} \)

For constellation 2, two radii lines from the center to two consecutive points and the line between them form an isosceles triangle as shown below



\(\eqalign{ & \rm \cos 67.5 = \frac{{d/2}}{{{r_2}}} \cr & \rm {r_2} = 1.307d \cr} \)

---

65. **Answer: d**

Explanation:

PSK Modulation Fundamentals

Phase Shift Keying (PSK) is a digital modulation technique that conveys data by changing (modulating) the phase of a reference signal (the carrier wave). In M-ary PSK, M different phase shifts are used to represent $k = \log_2 M$ bits per symbol. Higher values of M allow more bits to be transmitted per symbol, increasing spectral efficiency. However, increasing M also makes the phases closer to each other, making the system more susceptible to noise and requiring higher signal energy to achieve the same error performance.

The question specifically asks about the additional average transmitted **signal energy** required by 8-ary PSK compared to 4-ary PSK to achieve the same **error probability** under high Signal-to-Noise Ratio (SNR) conditions.

Error Probability in M-ary PSK (High SNR)

For M-ary PSK modulation over an Additive White Gaussian Noise (AWGN) channel, the symbol error probability (P_s) under high SNR conditions is approximated by:

$$P_s \approx 2 \cdot Q\left(\sqrt{2 \cdot \frac{E_s}{N_0}} \cdot \sin\left(\frac{\pi}{M}\right)\right)$$

Where:

- $Q(x)$ is the Q-function, which represents the tail probability of the standard normal distribution.
- E_s is the average **signal energy** per symbol.
- N_0 is the single-sided power spectral density of the noise.
- M is the number of phases (e.g., 4 for QPSK, 8 for 8-PSK).

For the **same error probability** (P_s), the argument of the Q-function must be constant. Let this constant argument be K.

$$K = \sqrt{2 \cdot \frac{E_s}{N_0}} \cdot \sin\left(\frac{\pi}{M}\right)$$

Squaring both sides, we get:

$$K^2 = 2 \cdot \frac{E_s}{N_0} \cdot \sin^2\left(\frac{\pi}{M}\right)$$

This implies that for a given error probability, the product $E_s \cdot \sin^2\left(\frac{\pi}{M}\right)$ must be constant (assuming N_0 is constant).

Energy Comparison: 4-ary PSK vs. 8-ary PSK

4-ary PSK (QPSK) Analysis

For 4-ary PSK (QPSK), $M_1 = 4$. Let the required average **signal energy** per symbol be $E_{s,4}$.

The argument of the Q-function for 4-ary PSK is:

$$K_1 = \sqrt{2 \cdot \frac{E_{s,4}}{N_0}} \cdot \sin\left(\frac{\pi}{4}\right)$$

We know that $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

$$K_1 = \sqrt{2 \cdot \frac{E_{s,4}}{N_0}} \cdot \frac{1}{\sqrt{2}} = \sqrt{\frac{E_{s,4}}{N_0}}$$

Therefore, $K_1^2 = \frac{E_{s,4}}{N_0}$.

8-ary PSK Analysis

For 8-ary PSK, $M_2 = 8$. Let the required average **signal energy** per symbol be $E_{s,8}$.

The argument of the Q-function for 8-ary PSK is:

$$K_2 = \sqrt{2 \cdot \frac{E_{s,8}}{N_0}} \cdot \sin\left(\frac{\pi}{8}\right)$$

Therefore, $K_2^2 = 2 \cdot \frac{E_{s,8}}{N_0} \cdot \sin^2\left(\frac{\pi}{8}\right)$.

Energy Calculation for Same Error Probability

To achieve the **same error probability**, we must have $K_1 = K_2$, which implies $K_1^2 = K_2^2$.

$$\frac{E_{s,4}}{N_0} = 2 \cdot \frac{E_{s,8}}{N_0} \cdot \sin^2\left(\frac{\pi}{8}\right)$$

We can cancel N_0 from both sides:

$$E_{s,4} = 2 \cdot E_{s,8} \cdot \sin^2\left(\frac{\pi}{8}\right)$$

We need to find the ratio $\frac{E_{s,8}}{E_{s,4}}$.

$$\frac{E_{s,8}}{E_{s,4}} = \frac{1}{2 \cdot \sin^2\left(\frac{\pi}{8}\right)}$$

Calculating $\sin^2\left(\frac{\pi}{8}\right)$

We use the trigonometric identity $\cos(2\theta) = 1 - 2\sin^2(\theta)$. Let $\theta = \frac{\pi}{8}$, so $2\theta = \frac{\pi}{4}$.

$$\cos\left(\frac{\pi}{4}\right) = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$$

We know that $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

$$\frac{1}{\sqrt{2}} = 1 - 2\sin^2\left(\frac{\pi}{8}\right)$$

$$2\sin^2\left(\frac{\pi}{8}\right) = 1 - \frac{1}{\sqrt{2}} = 1 - \frac{\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$$

$$\sin^2\left(\frac{\pi}{8}\right) = \frac{2-\sqrt{2}}{4}$$

Calculating the Energy Ratio

Substitute the value of $\sin^2\left(\frac{\pi}{8}\right)$ back into the energy ratio equation:

$$\frac{E_{s,8}}{E_{s,4}} = \frac{1}{2 \cdot \left(\frac{2-\sqrt{2}}{4}\right)} = \frac{1}{\frac{2-\sqrt{2}}{2}} = \frac{2}{2-\sqrt{2}}$$

To rationalize the denominator, multiply the numerator and denominator by the conjugate $2+\sqrt{2}$:

$$\frac{E_{s,8}}{E_{s,4}} = \frac{2}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{2(2+\sqrt{2})}{2^2-(\sqrt{2})^2} = \frac{2(2+\sqrt{2})}{4-2} = \frac{2(2+\sqrt{2})}{2} = 2+\sqrt{2}$$

Using the approximate value $\sqrt{2} \approx 1.414$:

$$\frac{E_{s,8}}{E_{s,4}} \approx 2 + 1.414 = 3.414$$

Converting Energy Ratio to Decibels (dB)

The additional average transmitted **signal energy** required by 8-ary PSK, expressed in **dB**, is:

$$\text{Energy in dB} = 10\log_{10}\left(\frac{E_{s,8}}{E_{s,4}}\right)$$

$$\text{Energy in dB} = 10\log_{10}(2+\sqrt{2})$$

$$\text{Energy in dB} \approx 10\log_{10}(3.414)$$

Calculating the logarithm:

$$\log_{10}(3.414) \approx 0.5332$$

Therefore:

$$\text{Energy in dB} \approx 10 \times 0.5332 \approx 5.332 \text{ dB}$$

Conclusion

To achieve the same **error probability** under high SNR conditions, 8-ary PSK requires approximately **5.33 dB** more average transmitted **signal energy** per symbol compared to 4-ary PSK. This increase in energy is necessary to compensate for the reduced distance between constellation points in 8-ary PSK, which makes it more susceptible to noise for a given energy level.