Prepp.in

Prepp

Your Personal Exams Guide

NDA	CDS	SSC CGL	CBSE UGC NET
IAS	SSC CHSL	CTET	MPSC
AFCAT	CSIR UDC NET	IBPS PO	UP POLICE
SSC MTS	SBI PO	BPSC	UPTET
IBPS RRB	IBPS CLERK	IES	UPSC CAPF
SSC Stenogr..	RRB NTPC	SSC GD	RBI GRADE B
RBI Assistant	DSSSB		

GATE ME 2011 Question Paper (13-Feb-2011) (Shift 1)

Total Time: 3 Hour **Total Marks:** 100

Instructions

Sl No.	Section Name	No. of Question	Maximum Marks
1	General Aptitude	10	15
2	Mechanical Engineering	55	85

1.) A total of 180 minutes is allotted for the examination.

2.) The server will set your clock for you. In the top right corner of your screen, a countdown timer will display the remaining time for you to complete the exam. Once the timer reaches zero, the examination will end automatically. The paper need not be submitted when your timer reaches zero.

3.) There will, however, be sectional timing for this exam. You will have to complete each section within the specified time limit. Before moving on to the next section, you must complete the current one within the time limits.

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Answers

1. **Answer: b**

Explanation:

The question asks us to identify the word that is most nearly opposite in meaning to the word "Amalgamate". To answer this, we need to understand the meaning of "Amalgamate" and then evaluate each given option.

Amalgamate Definition

The word **Amalgamate** primarily means to combine or unite to form one organization or structure. It implies bringing different things together to create a single, unified whole. Think of it as mixing different elements to become one new substance or entity.

- For example, if two companies **amalgamate**, they become one larger company.
- If different ideas **amalgamate**, they blend into a single, cohesive concept.

Analyzing Options for the Opposite of Amalgamate

Let's look at each option provided and determine its relationship to **Amalgamate**:

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Option	Meaning	Relationship to Amalgamate
Merge	To combine or cause to combine to form a single entity.	This word is very similar in meaning to **Amalgamate**; it is a synonym.
Split	To divide or cause to divide into two or more parts.	This word means the exact opposite of combining or uniting. It implies breaking something that was whole into separate pieces.
Collect	To gather together; accumulate.	While "collect" involves bringing things together, it doesn't necessarily imply they unite to form a single entity. You can collect items without them becoming one. This is not the direct opposite of **Amalgamate**, nor is it a synonym.
Separate	To cause to move or be apart.	This word is also an opposite of **Amalgamate**, as it means to move things apart that might have been together or to keep them from coming together.

Identifying the Best Opposite for Amalgamate

Your Personal Exams Guide

Both "Split" and "Separate" are antonyms of **Amalgamate**. However, we need to find the word that is "most nearly opposite".

- **Amalgamate** suggests a process of forming a unified whole from distinct parts.
- **Split** specifically refers to the action of breaking apart something that was previously unified or whole, or dividing it into distinct components. It directly reverses the action of amalgamating.
- **Separate** is a broader term. While it means to move apart, it doesn't always imply breaking a unified whole. For instance, you can separate items that were merely next to each other, not necessarily combined.

Given that **Amalgamate** implies a strong sense of uniting into a single entity, **Split** provides the most direct and forceful opposite by breaking that single entity back into

Prepp Download Prepp APP GET IT ON Google Play

its constituent parts or dividing it. Therefore, "Split" is the most appropriate antonym.

---

2. **Answer: a**

Explanation:

The question asks us to find the word that is closest in meaning to "**Inexplicable**" from the given options. To do this, we need to understand the precise meaning of "Inexplicable" and then compare it with the definitions of each option.

Inexplicable Definition

The word "**Inexplicable**" is derived from 'in-' (meaning 'not') and 'explicable' (meaning 'able to be explained'). Therefore, "Inexplicable" refers to something that **cannot be explained or accounted for**. It implies a mystery or a situation where the reasons or causes are unknown and cannot be logically presented or understood.

Option Meanings

Let's analyze each of the provided options:

- **Incomprehensible**: This word combines the prefix 'in-' (not) with 'comprehensible' (able to be understood). Thus, "Incomprehensible" means something that is **not able to be understood or grasped**. If something cannot be explained, it often means it cannot be fully understood.
- **Indelible**: This term refers to something that **cannot be removed or erased**. For example, an indelible mark or an indelible memory. It relates to permanence, not to the ability to be explained or understood.
- **Inextricable**: This word describes something from which it is **impossible to escape or disentangle**. For instance, an inextricable problem or an inextricable link between two concepts. It implies a complex, often unavoidable connection, rather than a lack of explanation.
- **Infallible**: This means **incapable of making mistakes or being wrong**. It is commonly used to describe a person or a system that is always correct and

Your Personal Exams Guide

Download Prepp APP

GET IT ON Google Play

never fails. This term relates to accuracy and perfection, not directly to the concept of being explainable or understandable.

Meaning Comparison

When something is **Inexplicable**, it means that its cause, reason, or nature cannot be clarified or put into understandable terms. This inherent inability to explain often directly leads to the inability to **understand** it. If you cannot explain why an event occurred, you are likely unable to fully comprehend it.

Consider the relationship:

- An **Inexplicable** phenomenon (something that cannot be explained) often becomes **Incomprehensible** (something that cannot be understood). The lack of explanation directly impedes understanding.
- While an indelible mark is permanent, its permanence can often be explained (e.g., "The indelible ink is permanent due to its chemical bonding properties.").
- An inextricable problem might be complex, but its intricate nature can still be explained (e.g., "The inextricable link between economic policy and social welfare can be explained through their interdependent effects.").
- An infallible decision can be explained by the process or wisdom that led to it (e.g., "Her infallible judgment is explained by years of experience and meticulous research.").

Prepp Your Personal Exams Guide

Therefore, out of all the options, "**Incomprehensible**" is the closest in meaning to "**Inexplicable**" because both terms denote a state where something cannot be grasped or made sense of, with the former often being a consequence of the latter.

---

3. **Answer: b**

Explanation:

Logarithm Relationship Analysis: Finding the True Equation

Prepp Download Prepp APP GET IT ON Google Play

This problem asks us to determine the correct relationship between variables **P**, **Q**, and **R**, given a specific logarithmic equality. We are given the equation:

$\log(P) = \frac{1}{2} \log(Q) = \frac{1}{3} \log(R)$

Our goal is to manipulate this equation using logarithm properties to match one of the provided options.

Step-by-Step Solution Breakdown

Step 1: Introduce a Constant To make the comparison easier, let's set all parts of the given equation equal to a constant, say k.

- $\log(P) = k$
- $\frac{1}{2} \log(Q) = k$
- $\frac{1}{3} \log(R) = k$

Step 2: Express Variables P, Q, and R Exponentially Now, we can rewrite these logarithmic equations in their equivalent exponential forms. Assuming a consistent base (let's denote it as 'b' for generality, though the base itself won't affect the final relationship):

- From $\log(P) = k$, we get $P = b^k$.
- From $\frac{1}{2} \log(Q) = k$, first multiply by 2 to get $\log(Q) = 2k$. This gives us $Q = b^{2k}$.
- From $\frac{1}{3} \log(R) = k$, first multiply by 3 to get $\log(R) = 3k$. This gives us $R = b^{3k}$.

So, we have derived the following exponential forms: $P = b^k$, $Q = b^{2k}$, and $R = b^{3k}$.

Step 3: Evaluate Each Option Using Derived Expressions Let's substitute these expressions into each of the given options to check which one is true.

- **Option 1:** $P^2 = Q^3 R^2$
  - Left Hand Side (LHS): $P^2 = (b^k)^2 = b^{2k}$
  - Right Hand Side (RHS): $Q^3 R^2 = (b^{2k})^3 \times (b^{3k})^2 = b^{6k} \times b^{6k} = b^{12k}$

Download Prepp APP
GET IT ON Google Play

- Comparison: $b^{2k} \neq b^{12k}$. Thus, Option 1 is false.
- **Option 2:** $Q^2 = P R$
  - LHS: $Q^2 = (b^{2k})^2 = b^{4k}$
  - RHS: $P R = b^k \times b^{3k} = b^{k + 3k} = b^{4k}$
  - Comparison: $b^{4k} = b^{4k}$. Thus, Option 2 is true.
- **Option 3:** $Q^2 = R^3 P$
  - LHS: $Q^2 = (b^{2k})^2 = b^{4k}$
  - RHS: $R^3 P = (b^{3k})^3 \times b^k = b^{9k} \times b^k = b^{10k}$
  - Comparison: $b^{4k} \neq b^{10k}$. Thus, Option 3 is false.
- **Option 4:** $R = P^2 Q^2$
  - LHS: $R = b^{3k}$
  - RHS: $P^2 Q^2 = (b^k)^2 \times (b^{2k})^2 = b^{2k} \times b^{4k} = b^{6k}$
  - Comparison: $b^{3k} \neq b^{6k}$. Thus, Option 4 is false.

Conclusion on Logarithm Equality

After checking all the options, we found that only the relationship $Q^2 = P R$ satisfies the initial logarithmic condition $\log(P) = \frac{1}{2} \log(Q) = \frac{1}{3} \log(R)$. This demonstrates how understanding exponential and logarithmic properties allows us to simplify and verify relationships between variables.

---

4. **Answer: c**

Explanation:

Contemplated: The Right Verb Form for Your Plans

The sentence "I contemplated________Singapore for my vacation but decided against it" requires the most appropriate word to complete it, focusing on correct English grammar, specifically verb forms.

The key to solving this question lies in understanding which grammatical form appropriately follows the verb "contemplate".

Understanding Verb Patterns with 'Contemplate'

Prepp Download Prepp APP GET IT ON Google Play

The verb "contemplate" means to think deeply about something or to consider a possible future action. When "contemplate" is followed by another verb indicating an action, that verb should generally be in the gerund form (the -ing form of the verb).

- **Gerunds:** A gerund is a verb form ending in -ing that functions as a noun. For example, "running," "reading," "visiting."
- **Infinitive:** An infinitive is the base form of a verb, usually preceded by "to" (e.g., "to run," "to read," "to visit").

Many verbs in English are followed by either a gerund or an infinitive, but "contemplate" is one of those verbs that typically takes a gerund.

Evaluating Options: Why 'Visiting' is Correct

Let's examine each option provided to determine its grammatical correctness and suitability in the given sentence:

- **Option 1: to visit** This makes the sentence "I contemplated to visit Singapore." This construction is grammatically incorrect. The verb "contemplate" is not typically followed by an infinitive.
- **Option 2: having to visit** Using "having to visit" results in "I contemplated having to visit Singapore." While "having" is a gerund, the phrase "having to visit" implies an obligation or necessity. This changes the meaning of the original sentence, which implies a consideration or thought, not a necessary action.
- **Option 3: visiting** Inserting "visiting" makes the sentence "I contemplated visiting Singapore." This is grammatically correct and perfectly conveys the intended meaning – that the person considered the action of visiting Singapore. "Visiting" is the gerund form of the verb "visit," and "contemplate" is indeed followed by a gerund.
- **Option 4: for a visit** If we use "for a visit," the sentence reads "I contemplated for a visit Singapore." This phrasing is awkward and ungrammatical in this context. While one might say "I contemplated a visit to Singapore," the phrase "for a visit" does not fit directly after "contemplated" in this sentence structure.

Grammar Rule for 'Contemplate' Explained

Download Prepp APP
GET IT ON Google Play

Based on the analysis, the most appropriate and grammatically correct word to complete the sentence is "visiting." The general rule is that verbs like "contemplate," "consider," "suggest," "recommend," "enjoy," and "finish," are often followed by a gerund (verb-ing) when referring to an action or activity.

Therefore, the complete sentence reads: "I contemplated **visiting** Singapore for my vacation but decided against it."

---

5. **Answer: b**

Explanation:

Understanding How to Make a Strong Impression

The question asks us to find the most suitable word to complete a sentence about making a **strong impression** on an audience. The sentence structure suggests we need a word that, like "understated" and "tentative", is counterproductive to creating such an impression.

Sentence Analysis: Avoiding Weakening Qualities

The sentence states: "If you are trying to make a **strong impression** on your audience, you cannot do so by being understated, tentative or_____________."

- **Goal:** Make a **strong impression**.
- **Ineffective qualities listed:** "Understated" (subtle, not drawing attention) and "tentative" (lacking confidence, uncertain).
- **Requirement:** The missing word should represent another quality that prevents a strong impression, fitting the theme set by "understated" and "tentative".

Evaluating the Options

Let's look at the meaning of each option:

- **1. Hyperbolic:** This means exaggerated or extravagant. Being hyperbolic might actually help create a strong impression, perhaps even an overly dramatic one.

Prepp Download Prepp APP GET IT ON Google Play

It doesn't fit the pattern of qualities that weaken an impression.

- **2. Restrained:** This means being reserved, controlled, or not showing emotion or opinion openly. Being **restrained** is very similar in effect to being "understated" or "tentative"; it implies holding back, which typically prevents a **strong impression**.
- **3. Argumentative:** This means prone to arguing or disagreeing. While being argumentative can make an impression, it's not necessarily the type of **strong impression** desired, and it doesn't align with the "holding back" theme of "understated" and "tentative".
- **4. Indifferent:** This means showing no interest or concern. Being indifferent would certainly prevent a **strong impression**, but it relates more to a lack of care than a lack of boldness or confidence, which is the nuance suggested by "understated" and "tentative".

Conclusion: Selecting the Best Fit

The words "understated" and "tentative" both describe ways of behaving that lack confidence or assertiveness. "**Restrained**" fits this pattern perfectly, describing a similar lack of outward expression or boldness. Therefore, being understated, tentative, or **restrained** would all hinder the goal of making a **strong impression** on an audience.

---

6. **Answer: d**

Explanation:

Spirit Calculation After Repeated Replacements

This problem involves calculating the amount of pure spirit remaining in a container after a series of replacement operations. It's a classic example of mixture problems often encountered in quantitative aptitude.

Understanding the Replacement Process

Initially, the container holds 10 litres of pure spirit. In each step, a certain amount of the mixture is removed and replaced with an equal amount of water. This process

Prepp Download Prepp APP GET IT ON Google Play

dilutes the spirit content with each subsequent replacement.

- **Initial State:** The container has 10 litres of pure spirit.
- **First Replacement:** 1 litre of spirit is removed and 1 litre of water is added.
- **Subsequent Replacements:** From the mixture, 1 litre is removed, and 1 litre of water is added. This is repeated two more times, making a total of three replacement operations.

Formula for Remaining Spirit in Mixture Problems

When a certain quantity of a pure liquid is repeatedly replaced by water, the quantity of the original liquid remaining after 'n' operations can be calculated using the following formula:

$$\text{Quantity of Spirit Left} = \text{Initial Quantity} \times \left(1 - \frac{\text{Quantity Replaced}}{\text{Total Volume}}\right)^n$$

Where:

- **Initial Quantity:** The initial volume of the pure spirit.
- **Quantity Replaced:** The volume of mixture removed and replaced with water in each operation.
- **Total Volume:** The total volume of the container (which remains constant).
- **n:** The number of times the replacement process is repeated.

Prepp Your Personal Exams Guide

Step-by-Step Spirit Calculation

Let's apply the formula to the given problem:

- **Initial Quantity of Spirit (V):** 10 litres
- **Quantity Replaced (x):** 1 litre (1 litre is replaced with 1 litre of water in each step)
- **Total Volume of Container (C):** 10 litres (The volume remains constant at 10 litres)
- **Number of Replacements (n):** 3 times (The process is repeated one more time after the first and subsequent, totaling 3 operations).

Using the formula:

Prepp Download Prepp APP GET IT ON Google Play

$$\text{Spirit Left} = V \times \left(1 - \frac{x}{C}\right)^n$$

Substitute the values:

$$\text{Spirit Left} = 10 \times \left(1 - \frac{1}{10}\right)^3$$

First, calculate the term inside the parenthesis:

$$1 - \frac{1}{10} = \frac{10}{10} - \frac{1}{10} = \frac{9}{10}$$

Now, raise this fraction to the power of 3:

$$\left(\frac{9}{10}\right)^3 = \frac{9^3}{10^3} = \frac{9 \times 9 \times 9}{10 \times 10 \times 10} = \frac{729}{1000}$$

Finally, multiply by the initial quantity of spirit:

$$\text{Spirit Left} = 10 \times \frac{729}{1000}$$

$$\text{Spirit Left} = \frac{7290}{1000}$$

$$\text{Spirit Left} = 7.29 \text{ litres}$$

Conclusion on Spirit Remaining

Prepp Your Personal Exams Guide

After three successive replacements, 7.29 litres of spirit are left in the container. This calculation demonstrates how the concentration of the original liquid diminishes with each dilution process.

---

7. **Answer: c**

Explanation:

Bereavement Unit Introduction

Prepp Download Prepp APP GET IT ON Google Play

The passage highlights that school curricula often lack a unit on **bereavement** and **grief**, despite all students eventually experiencing **losses through death and parting**. This suggests that a unit on **bereavement** would focus on helping students understand, cope with, and manage their feelings and reactions to such **losses**, as well as how to support others who are grieving.

Bereavement Unit Focus

A unit dedicated to **bereavement** and **grief** aims to equip students with practical skills and emotional understanding to navigate the complex process of loss. This typically involves:

- Understanding the emotional and psychological aspects of **grief**.
- Developing coping mechanisms for personal **loss**.
- Learning how to offer appropriate support to others experiencing **bereavement**.
- Recognizing the stages of the healing process.

Topic Analysis for Bereavement Unit

Let's evaluate each option to determine which topic would **not** be included in a unit on **bereavement**, based on its focus on dealing with **grief** and **loss**.

Condolence Letter Writing

Your Personal Exams Guide

How to write a letter of condolence is a topic that fits well within a **bereavement** unit. Writing a condolence letter is a way to express sympathy and support to someone who is grieving. It helps students understand the appropriate etiquette and empathy required when interacting with bereaved individuals. This directly contributes to the unit's goal of teaching students how to deal with and respond to **loss**.

Emotional Stages of Grief

Understanding **what emotional stages are passed through in the healing process** is a core component of a **bereavement** unit. This topic helps students recognize and normalize their own reactions to **grief**, as well as understand what friends and family members might be experiencing. Learning about these stages, such as denial, anger,

Download Prepp APP
GET IT ON Google Play

bargaining, depression, and acceptance, is crucial for effectively dealing with **bereavement** and moving towards healing.

Causes of Death Irrelevance

The topic of **what the leading causes of death are** would **not** typically be included in a unit on **bereavement**. While **death** is the event that triggers **bereavement**, a unit on **bereavement** focuses on the emotional and psychological aftermath of **loss**—the process of grieving and coping—rather than the statistical or medical reasons why people die. Information about causes of death falls more under health education, biology, or public health, not directly under the scope of dealing with **grief** or supporting the bereaved.

Supporting a Grieving Friend

How to give support to a grieving friend is highly relevant and would be a key topic in a **bereavement** unit. This teaches students practical ways to show empathy, offer comfort, and provide meaningful assistance to peers or others experiencing **loss**. Learning how to support someone through **grief** is an essential life skill directly related to helping students deal with the impact of **bereavement** in their social circles.

Conclusion on Bereavement Topics

Prepp Your Personal Exams Guide

Based on the scope of a unit on **bereavement** and **grief**, which focuses on coping with **loss** and supporting others, the topic of **what the leading causes of death are** is an outlier. It pertains to the circumstances of **death** itself, rather than the process of grieving or healing after **loss**. Therefore, this topic would **not** be included in a unit on **bereavement**.

---

8. **Answer: d**

Explanation:

<u>Concept:</u>

Prepp Download Prepp APP GET IT ON Google Play

According to the given information:

Most dangerous microbe ∝ probability that microbe will overcome human immune system.

Most dangerous microbe ∝ area (growth of microbe)

Most dangerous microbe ∝ (quantity required)$^{-1}$

$$So, the\ most\ dangerous\ microbe\ \propto \frac{Probability \times Area}{Quantity\ required}$$

$$So, the\ most\ dangerous\ microbe\ = \frac{K \times Probability \times Area}{Quantity\ required}$$

$$So, the\ most\ dangerous\ microbe\ = \frac{K \times Probability \times \pi d^2}{4 \times Quantity\ required}$$

Calculation:

Given:

For microbe P:

Probability (P) = 0.4, Diameter = 50 mm and Quantity required = 800 milligram of microbe/mass of body in kg.

$$The\ most\ dangerous\ microbe\ = \frac{\pi K}{4} \times \frac{0.4 \times 50^2}{800} \Rightarrow \frac{\pi K}{4} \times \frac{5}{4} = 1.25\frac{\pi K}{4}$$

For microbe Q:

Probability (Q) = 0.5, Diameter = 40 mm and Quantity required = 600 milligram of microbe/mass of body in kg.

$$The\ most\ dangerous\ microbe\ = \frac{\pi K}{4} \times \frac{0.5 \times 40^2}{600} \Rightarrow \frac{\pi K}{4} \times \frac{4}{3} = 1.33\frac{\pi K}{4}$$

For microbe R:

Probability (R) = 0.4, Diameter = 30 mm and Quantity required = 300 milligram of microbe/mass of body in kg.

$$The\ most\ dangerous\ microbe\ = \frac{\pi K}{4} \times \frac{0.4 \times 30^2}{300} \Rightarrow \frac{\pi K}{4} \times \frac{6}{5} = 1.2\frac{\pi K}{4}$$

For microbe S :

Prepp Download Prepp APP GET IT ON Google Play

Probability (S) = 0.8, Diameter = 20 mm and Quantity required = 200 milligram of microbe/mass of body in kg.

$$\text{The most dangerous microbe} = \frac{\pi K}{4} \times \frac{0.8 \times 20^2}{200} \Rightarrow \frac{\pi K}{4} \times \frac{8}{5} = 1.6\frac{\pi K}{4}$$

Microbe S > Microbe Q > Microbe P > Microbe R.

∴ toxicity is more for microbe S and the company should target it in its first attempt.

---

9. **Answer: a**

Explanation:

To determine the number of units that should be produced to minimize the total cost, we need to first formulate the total cost equation based on the given variable cost and fixed cost equations. Then, we will use calculus to find the quantity that yields the minimum total cost.

Cost Equations Breakdown

The problem provides us with two key cost components for manufacturing a product:

- **Variable cost (V)**: This cost changes directly with the quantity produced. The equation given is $V = 4q$, where q represents the quantity of units produced.
- **Fixed cost (F)**: This cost reduces as the quantity produced increases. The equation provided is $F = \frac{100}{q}$.

Total Cost Formulation

The **total cost (TC)** of production is the sum of the variable cost and the fixed cost. So, we can write the total cost equation as:

$$TC = V + F$$

Substituting the given equations for V and F into the total cost formula, we get the total cost function in terms of q:

Prepp Download Prepp APP GET IT ON Google Play

$$TC = 4q + \frac{100}{q}$$

Minimizing Total Cost

To find the quantity q that minimizes the total cost, we use the principles of differential calculus. The general approach involves finding the first derivative of the total cost function with respect to q, setting it to zero, and solving for q. This will give us the quantity that corresponds to a minimum (or maximum) cost.

1. First Derivative of Total Cost

Let's find the first derivative of $TC = 4q + \frac{100}{q}$ with respect to q. For easier differentiation, we can rewrite $\frac{100}{q}$ as $100q^{-1}$.

The derivative of a sum is the sum of the derivatives:

$$\frac{d(TC)}{dq} = \frac{d}{dq}(4q + 100q^{-1})$$

Differentiating each term:

- The derivative of $4q$ with respect to q is 4.
- The derivative of $100q^{-1}$ with respect to q is $100 \cdot (-1)q^{-1-1} = -100q^{-2}$.

Combining these, the first derivative is:

$$\frac{d(TC)}{dq} = 4 - 100q^{-2}$$

Or, expressed with a positive exponent:

$$\frac{d(TC)}{dq} = 4 - \frac{100}{q^2}$$

2. Setting First Derivative to Zero

To find the value of q that minimizes the total cost, we set the first derivative equal to zero:

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

$$4 - \frac{100}{q^2} = 0$$

Now, we solve this equation for q:

$$4 = \frac{100}{q^2}$$

Multiply both sides by q^2:

$$4q^2 = 100$$

Divide by 4:

$$q^2 = \frac{100}{4}$$

$$q^2 = 25$$

Take the square root of both sides:

$$q = \sqrt{25}$$

Since the quantity produced q must be a positive value, we take the positive root:

$$q = 5$$

Prepp Your Personal Exams Guide

3. Second Derivative Check (Optional)

To confirm that $q = 5$ indeed corresponds to a minimum total cost, we can compute the second derivative of the total cost function. If the second derivative is positive at $q = 5$, then it's a minimum.

Recall the first derivative: $\frac{d(TC)}{dq} = 4 - 100q^{-2}$

Now, differentiate this expression with respect to q again:

$$\frac{d^2(TC)}{dq^2} = \frac{d}{dq}(4 - 100q^{-2})$$

$$\frac{d^2(TC)}{dq^2} = 0 - 100(-2q^{-3})$$

Prepp Download Prepp APP GET IT ON Google Play

$$\frac{d^2(TC)}{dq^2} = 200q^{-3}$$

Or, expressed with a positive exponent:

$$\frac{d^2(TC)}{dq^2} = \frac{200}{q^3}$$

Now, substitute $q = 5$ into the second derivative:

$$\frac{d^2(TC)}{dq^2}\Big|_{q=5} = \frac{200}{5^3} = \frac{200}{125} = 1.6$$

Since $1.6 > 0$, the total cost is indeed minimized when $q = 5$ units.

Conclusion: Units for Minimum Cost

Based on our calculations, producing **5 units** will minimize the total cost of manufacturing the product. This quantity balances the increasing variable cost and the decreasing fixed cost to achieve the lowest possible total expenditure.

---

10. **Answer: c**

Explanation:

Prepp Your Personal Exams Guide

This problem involves determining the optimal number of trucks a transporter needs to clear a certain volume of orders within a specific timeframe. We are given two scenarios that help us understand the relationship between the initial pending orders, new orders received daily, and the capacity of each truck. Our goal is to find the minimum number of trucks required for a new scenario.

Orders System Analysis

To begin, let's define the key variables involved in the transporter's order system:

- Let O represent the initial number of pending orders (also known as backlog) that the transporter has at the start.

Prepp Download Prepp APP GET IT ON Google Play

- Let N represent the number of new orders the transporter receives consistently each day.
- Let C represent the processing capacity of one truck per day. This means one truck can clear C orders in a single day.

The fundamental principle we will use is that the total orders to be cleared must equal the total orders shipped by the trucks. This can be expressed as:

Total Orders to Clear = Initial Backlog + (New Orders per Day $\times$ Number of Days)

And this total must be fulfilled by the trucks:

Total Orders Cleared by Trucks = (Number of Trucks $\times$ Truck Capacity per Day $\times$ Number of Days)

Truck Usage Scenario 1: 7 Trucks for 4 Days

According to the first piece of information, if the transporter uses 7 trucks, all orders are cleared at the end of the 4th day.

- During these 4 days, the total new orders received will be $4 \times N$.
- The total capacity of 7 trucks operating for 4 days is $7 \times C \times 4 = 28C$.

Therefore, the total orders that needed to be cleared (initial backlog plus new orders) must equal the total orders shipped by the trucks:

$$O + 4N = 28C \quad \dots (1)$$

Truck Usage Scenario 2: 3 Trucks for 10 Days

The alternative scenario states that if the transporter uses only 3 trucks, all orders are cleared at the end of the 10th day.

- Over these 10 days, the total new orders received will be $10 \times N$.
- The total capacity of 3 trucks operating for 10 days is $3 \times C \times 10 = 30C$.

Similarly, setting up the equation for this scenario:

$$O + 10N = 30C \quad \dots (2)$$

Prepp Download Prepp APP GET IT ON Google Play

Solving for Orders in Terms of Capacity

We now have a system of two linear equations. Our goal is to express O (initial backlog) and N (daily new orders) in terms of C (truck capacity).

Let's subtract Equation (1) from Equation (2) to eliminate O:

$(O + 10N) - (O + 4N) = 30C - 28C$

$6N = 2C$

Now, we can solve for N:

$N = \frac{2C}{6}$

$N = \frac{1}{3}C$

This tells us that the number of new orders received each day is one-third of the orders a single truck can clear in a day.

Next, substitute the value of $N = \frac{1}{3}C$ back into Equation (1) to solve for O:

$O + 4\left(\frac{1}{3}C\right) = 28C$

$O + \frac{4}{3}C = 28C$

Subtract $\frac{4}{3}C$ from both sides of the equation:

$O = 28C - \frac{4}{3}C$

To combine these terms, find a common denominator:

$O = \frac{28 \times 3}{3}C - \frac{4}{3}C$

$O = \frac{84C - 4C}{3}$

$O = \frac{80}{3}C$

This means the initial backlog of orders is $\frac{80}{3}$ times the capacity of one truck.

Minimum Trucks for 5 Days Requirement

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

We need to find the minimum number of trucks, let's call this T, required to clear all orders by the end of the 5th day.

- Over these 5 days, the total new orders received will be $5 \times N$.
- The total capacity of T trucks operating for 5 days is $T \times C \times 5 = 5TC$.

Setting up the equation for this desired scenario:

$$O + 5N = 5TC$$

Now, substitute the expressions for O and N that we found in terms of C:

$$\frac{80}{3}C + 5\left(\frac{1}{3}C\right) = 5TC$$

$$\frac{80}{3}C + \frac{5}{3}C = 5TC$$

Combine the terms on the left side:

$$\frac{80C+5C}{3} = 5TC$$

$$\frac{85C}{3} = 5TC$$

Since C represents the capacity and must be a positive value, we can divide both sides of the equation by C:

$$\frac{85}{3} = 5T$$

Finally, solve for T:

$$T = \frac{85}{3\times5}$$

$$T = \frac{85}{15}$$

Simplify the fraction by dividing both numerator and denominator by 5:

$$T = \frac{17}{3}$$

Converting this fraction to a decimal gives:

$$T \approx 5.66$$

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Conclusion on Trucks Needed

The calculation shows that approximately 5.66 trucks are needed to clear all orders by the end of the 5th day. Since it is impossible to use a fraction of a truck, and to ensure that all orders are indeed cleared (meaning we need at least 5.66 trucks), we must round up to the next whole number.

Therefore, the minimum number of trucks required is 6.

---

11. **Answer: b**

Explanation:

Streamline and Equipotential Line Relationship in Flow Fields

In the study of fluid dynamics, particularly in the context of potential flow, two important concepts help visualize and understand fluid motion: streamlines and equipotential lines. The relationship between a **streamline** and an **equipotential line** in a **flow field** is fundamental to analyzing various fluid flow phenomena.

Prepp Your Personal Exams Guide

Understanding Streamlines in Fluid Flow

A **streamline** is an imaginary line in a flow field that is drawn such that the velocity vector of the fluid at every point on the line is tangent to the line at that instant. This means that if you were to follow a tiny fluid particle, its path would trace out a streamline.

- **Definition:** A line that is everywhere tangent to the instantaneous velocity vector of the fluid.
- **Key Property:** No fluid crosses a streamline. This is because the velocity component normal to the streamline is zero.
- **Purpose:** Streamlines provide a visual representation of the direction of fluid flow at different points in space.

Prepp Download Prepp APP GET IT ON Google Play

Understanding Equipotential Lines in Fluid Flow

An **equipotential line** (or equipotential surface in three dimensions) is a line connecting points in a **flow field** that have the same value of velocity potential. The velocity potential, denoted by ϕ, is a scalar function whose negative gradient gives the velocity vector $\vec{V}$ for irrotational flow (a type of potential flow).

Mathematically, the velocity vector $\vec{V}$ is related to the velocity potential ϕ by:

$$\vec{V} = -\nabla\phi = -\left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)$$

where $\nabla\phi$ is the gradient of ϕ.

- **Definition:** A line joining points of equal velocity potential ($\phi = \text{constant}$).
- **Key Property:** The gradient of a scalar function (like ϕ) is always normal (perpendicular) to the surfaces (or lines) of constant value. Therefore, the velocity vector $\vec{V}$ is always perpendicular to the equipotential lines.
- **Purpose:** Equipotential lines help characterize the scalar potential field, which in turn defines the velocity field.

Perpendicular Relationship Between Streamlines and Equipotential Lines

Now, let's bring the two concepts together to understand their relationship. We know two crucial facts:

Prepp Your Personal Exams Guide

- A **streamline** is, by definition, tangential to the velocity vector $\vec{V}$ at every point.
- An **equipotential line** is, by property, perpendicular (normal) to the velocity vector $\vec{V}$ at every point.

Given these two facts, it logically follows that wherever a **streamline** intersects an **equipotential line** in a **flow field**, they must do so at a right angle, meaning they are **perpendicular to each other**. This perpendicularity is a fundamental characteristic of potential flow and is crucial for constructing flow nets, which are graphical representations of flow fields using intersecting streamlines and equipotential lines.

Think of it like this: if the velocity vector points in a certain direction, the streamline follows that direction, while the equipotential line cuts across it at 90 degrees.

Prepp Download Prepp APP GET IT ON Google Play

Conclusion on Flow Field Elements

In summary, for a valid **flow field**, a **streamline** and an **equipotential line** will always intersect at 90 degrees. This specific relationship is a hallmark of irrotational, incompressible flow, often referred to as potential flow, where the velocity can be derived from a scalar potential function.

---

12. **Answer: d**

Explanation:

Analyzing Moist Air in an Airtight Vessel

Let's consider a mass of moist air contained within an airtight vessel. When this air is heated to a higher temperature, we need to understand how its properties change, specifically focusing on specific humidity and relative humidity.

Understanding Specific Humidity

Specific humidity (ω) is defined as the ratio of the mass of water vapor (m_v) to the mass of dry air (m_a) in a given sample of moist air. Mathematically, it is expressed as:

$\omega = \frac{m_v}{m_a}$

Since the vessel is airtight, no mass can enter or leave the vessel. This means the total mass of dry air inside the vessel remains constant $(m_a$ is constant), and the total mass of water vapor inside the vessel also remains constant $(m_v$ is constant). Therefore, the ratio of these two constant masses, which is the specific humidity, must remain constant.

Heating the air does not change the amounts of water vapor or dry air present in the airtight vessel. Thus, specific humidity does not change.

Understanding Relative Humidity

Download Prepp APP
GET IT ON Google Play

Relative humidity (RH) is a measure of the amount of water vapor actually present in the air compared to the maximum amount of water vapor the air can hold at the same temperature and pressure. It is usually expressed as a percentage. One common definition is:

$$RH = \frac{\text{Actual partial pressure of water vapor}(p_v)}{\text{Saturation partial pressure of water vapor at the same temperature}(p_s)} \times 100\%$$

Alternatively, and perhaps more intuitively for this case, it can be understood in terms of mass:

$$RH \propto \frac{\text{Actual mass of water vapor}}{\text{Maximum mass of water vapor air can hold at that temperature}}$$

In our scenario:

- The vessel is airtight, so the actual mass of water vapor in the air remains constant as the air is heated.
- However, the capacity of air to hold water vapor (the saturation point or p_s) increases significantly as the temperature increases. Warmer air can hold much more water vapor than colder air.

Since the actual amount of water vapor is constant, but the maximum amount the air can hold increases with temperature, the ratio (Relative Humidity) must decrease.

As the moist air in the airtight vessel is heated to a higher temperature, the denominator in the relative humidity calculation (the capacity to hold water vapor) increases while the numerator (actual water vapor) stays the same. This leads to a decrease in relative humidity.

Conclusion

Based on the analysis:

- Specific humidity remains constant because the masses of water vapor and dry air are fixed in the airtight vessel.
- Relative humidity decreases because the air's capacity to hold water vapor increases with temperature while the actual amount of water vapor stays constant.

Prepp Download Prepp APP GET IT ON Google Play

Therefore, when a mass of moist air in an airtight vessel is heated to a higher temperature, the relative humidity of the air decreases.

---

13. **Answer: b**

Explanation:

This question asks to calculate the Logarithmic Mean Temperature Difference (LMTD) for a condenser in a power plant, given the steam condensation temperature and the inlet and outlet temperatures of the cooling water.

Understanding Condenser Temperature Differences

In a power plant condenser, heat is transferred from the steam to the cooling water. The steam condenses at a constant temperature, while the cooling water temperature increases as it absorbs heat. The LMTD is a measure used to determine the driving force for heat transfer in heat exchangers like condensers. It accounts for the changing temperature difference along the exchanger.

LMTD Formula and Calculation

The formula for the Logarithmic Mean Temperature Difference (LMTD) is:

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

Where:

- ΔT_1 is the temperature difference at one end of the heat exchanger.
- ΔT_2 is the temperature difference at the other end of the heat exchanger.
- $\ln$ denotes the natural logarithm.

Calculating Temperature Differences

We are given:

Prepp Download Prepp APP GET IT ON Google Play

- Steam condensation temperature (T_{steam}): $60^{\circ}\mathrm{C}$
- Cooling water inlet temperature ($T_{cw,in}$): $30^{\circ}\mathrm{C}$
- Cooling water outlet temperature ($T_{cw,out}$): $45^{\circ}\mathrm{C}$

Assuming a counter-flow arrangement for the condenser (which is common), we calculate the temperature differences at the two ends:

1. **Temperature difference at the end where cooling water exits (hotter water) and steam is condensing:**
   $\Delta T_1 = T_{steam} - T_{cw,out}$
   $\Delta T_1 = 60^{\circ}\mathrm{C} - 45^{\circ}\mathrm{C}$
   $\Delta T_1 = 15^{\circ}\mathrm{C}$
2. **Temperature difference at the end where cooling water enters (cooler water) and steam is condensing:**
   $\Delta T_2 = T_{steam} - T_{cw,in}$
   $\Delta T_2 = 60^{\circ}\mathrm{C} - 30^{\circ}\mathrm{C}$
   $\Delta T_2 = 30^{\circ}\mathrm{C}$

Applying the LMTD Formula

Now, substitute these values into the LMTD formula:

$$\mathrm{LMTD} = \frac{15^{\circ}\mathrm{C} - 30^{\circ}\mathrm{C}}{\ln\left(\frac{15^{\circ}\mathrm{C}}{30^{\circ}\mathrm{C}}\right)}$$

Your Personal Exams Guide

$$\mathrm{LMTD} = \frac{-15^{\circ}\mathrm{C}}{\ln(0.5)}$$

Using a calculator, $\ln(0.5) \approx -0.6931$.

$$\mathrm{LMTD} = \frac{-15^{\circ}\mathrm{C}}{-0.6931}$$

$$\mathrm{LMTD} \approx 21.64^{\circ}\mathrm{C}$$

Final LMTD Value

The calculated Logarithmic Mean Temperature Difference (LMTD) for the condenser is approximately $21.6^{\circ}\mathrm{C}$. This value represents the average temperature difference

Prepp Download Prepp APP GET IT ON Google Play

driving heat transfer from the steam to the cooling water throughout the condenser.

---

14. **Answer: a**

Explanation:

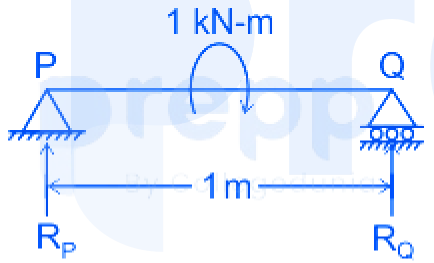
Concept:

Equilibrium Conditions:

$\sum F_v = 0$ and $\sum M_P = 0$

Calculation:

Given:



Now, we know that

Balancing vertical forces by

$\sum F_v = 0$

$R_P + R_Q = 0$ .... (1)

Now,

Taking moment about point 'P'

$\sum M_P = 0$

$1 = R_Q \times 1 \Rightarrow$ **R_Q = 1 kN (upward)**

Then, from equation (1), we get

Prepp Download Prepp APP GET IT ON Google Play

$R_P = 0 - 1$

∴ R_p = −1 kN = 1 kN (downward)

---

15. **Answer: c**

Explanation:

Concept:

Mobility of the mechanism = Degree of Freedom = [3(l - 1) - 2j - h] - F_r

where, l = no. of links, j = no. of the binary joint, h = no. of higher pairs, F_r = redundant link

Here the middle link is acting as a dummy link.

Calculation:

Given:

l = 4, j = 4, h = 0, F_r = 0

DOF = [3(4 - 1) - 2 × 4 - 0] - 0 = 9 - 8 = 1

So degree of freedom of double – parallelogram mechanism is one.

Prepp Your Personal Exams Guide

---

16. **Answer: a**

Explanation:

Cold Rolling: Maximum Possible Draft Explained

Cold rolling is a metal forming process where a metal sheet or strip is passed through a pair of rolls at a temperature below its recrystallization temperature. This

Prepp Download Prepp APP GET IT ON Google Play

process reduces the thickness of the material and improves its surface finish, strength, and dimensional accuracy. A key parameter in rolling is the "draft," which is the amount of thickness reduction achieved in a single pass.

Draft in Rolling Process

The draft (Δh) is defined as the difference between the initial thickness (h_i) and the final thickness (h_f) of the sheet after a rolling pass. Mathematically, it is expressed as:

$$\Delta h = h_i - h_f$$

However, there is a maximum possible draft that can be achieved in a single pass, which is limited by the friction between the rolls and the workpiece, as well as the geometry of the rolls.

Maximum Draft and Biting Condition

For the rolling process to occur, the rolls must be able to "bite" or pull the sheet into the roll gap. This condition, known as the biting condition, dictates the maximum possible draft. The maximum possible draft (Δh_{max}) that can be achieved without the sheet slipping is given by the formula:

$$\Delta h_{max} = \mu^2 R$$

Prepp Your Personal Exams Guide

Where:

- Δh_{max} is the maximum possible draft (the largest reduction in thickness possible in one pass).
- μ (mu) is the coefficient of friction between the rolls and the workpiece material.
- R is the radius of the rolls.

Friction: Impact on Maximum Possible Draft

From the formula $\Delta h_{max} = \mu^2 R$, it is clear that the maximum possible draft is directly proportional to the square of the coefficient of friction. This means:

Prepp Download Prepp APP GET IT ON Google Play

- **Increase in coefficient of friction (μ):** If the coefficient of friction between the rolls and the sheet increases, the rolls can exert a greater tangential force on the sheet. This increased gripping force allows the rolls to pull in a thicker sheet, overcoming the horizontal component of the normal force that tries to push the sheet out. Consequently, a larger reduction in thickness (greater draft) can be achieved without the sheet slipping.
- **Decrease in coefficient of friction (μ):** Conversely, if the coefficient of friction decreases, the rolls will have less grip on the sheet, making it harder to pull the material into the gap. This would lead to a decrease in the maximum possible draft.

Roll Radius: Influence on Maximum Possible Draft

The maximum possible draft is also directly proportional to the roll radius (R).

- **Increase in roll radius (R):** A larger roll radius means a larger contact area between the rolls and the workpiece, which can also help in pulling the material into the roll gap more effectively. This allows for a greater maximum possible draft.
- **Decrease in roll radius (R):** A smaller roll radius would reduce the maximum possible draft.

Analyzing the Options for Maximum Draft

Let's evaluate the given options based on the principles discussed:

Prepp Download Prepp APP GET IT ON Google Play

Option	Analysis	Impact on Maximum Possible Draft (Δh_{max})
Increase in coefficient of friction	As per the formula $\Delta h_{max} = \mu^2 R$, an increase in μ directly increases Δh_{max}. Higher friction enhances the gripping ability of the rolls.	**Increases**
Decrease in coefficient of friction	A decrease in μ would reduce the gripping force, leading to a smaller Δh_{max}.	Decreases
Decrease in roll radius	A decrease in R would directly reduce Δh_{max} as they are directly proportional.	Decreases
Increase in roll velocity	Roll velocity primarily affects the rolling force, power consumption, and production rate. It does not directly determine the maximum possible draft, which is limited by friction and roll geometry (biting condition). Excessive velocity might lead to issues like skidding, but it's not a primary factor for maximum draft.	No direct impact on maximum possible draft limit

Prepp
Your Personal Exams Guide

Based on the analysis, an increase in the coefficient of friction directly leads to an increase in the maximum possible draft in cold rolling of a sheet.

---

17. **Answer: c**

Explanation:

In the field of material science, specifically within powder metallurgy, several distinct operations are used to create and enhance components. The question focuses on a specific operation where oil is introduced into the pores of a product made through powder metallurgy. Let's break down the options and identify the correct process.

Prepp Download Prepp APP GET IT ON Google Play

Impregnation: Oil Permeation in Powder Metallurgy

The operation in which oil is permeated into the pores of a powder metallurgy product is known as **impregnation**.

- **Definition:** Impregnation is a secondary operation performed on porous powder metallurgy (PM) parts. It involves filling the interconnected pores of the PM component with a fluid, most commonly oil or a resin.
- **Process:** The process typically involves placing the porous component in a vacuum chamber, evacuating the air from its pores, and then introducing the impregnating fluid (like oil) which is then drawn into the pores by atmospheric pressure or slight positive pressure.
- **Purpose:**
    - When oil is permeated, the primary purpose is to provide self-lubrication for components such as bearings. The oil stored within the pores can slowly release during operation, providing continuous lubrication.
    - When resins are permeated, it is often to seal the pores, preventing leakage of fluids or to improve machining characteristics.

Powder Metallurgy: Distinguishing Operations

It's important to understand why the other options are not correct for the described operation:

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Operation	Description	Relevance to Question
Mixing	This is the initial step in powder metallurgy where various metal powders (and sometimes lubricants or binders) are thoroughly blended to achieve a uniform composition.	It prepares the raw materials but does not involve permeating oil into the pores of a finished product.
Sintering	Sintering is a core process in powder metallurgy where compacted powder parts (green compacts) are heated to a high temperature (below the melting point of the main constituent) in a controlled atmosphere. This causes the individual powder particles to bond together, increasing strength and reducing porosity, but it doesn't involve adding oil.	It is a thermal treatment to consolidate the powder particles, not an oil permeation process. While it influences porosity, it doesn't involve filling pores with oil.
Infiltration	Infiltration is a process where the pores of a pre-sintered or fully sintered porous compact are filled with a molten metal or alloy that has a lower melting point than the base material. The molten infiltrant flows into the interconnected pores due to capillary action.	This operation involves filling pores with a *molten metal*, not oil. The purpose is typically to enhance density, strength, and toughness, not lubrication. It is often confused with impregnation due to the "filling pores" aspect, but the material being permeated is different.

Based on the definitions, **Impregnation** is the precise term for the process of permeating oil into the pores of a powder metallurgy product to achieve lubrication or other desired properties.

---

18. Answer: c

Prepp Download Prepp APP GET IT ON Google Play

Explanation:

Determining the Fit Type for Hole and Shaft Assembly

This solution explains how to determine the type of fit between a hole and a shaft based on their given dimensions. We will analyze the tolerances provided for both components to calculate the range of possible clearances or interferences.

Analyzing Hole Dimensions

The given dimension for the hole is $\phi 9^{+0.015}_{+0}\ mm$. This means:

- The **basic size** of the hole is 9 mm.
- The **upper limit** of the hole size is $9 + 0.015 = 9.015\ mm$.
- The **lower limit** of the hole size is $9 + 0 = 9.000\ mm$.
- The **tolerance** for the hole (the difference between the upper and lower limits) is $9.015\ mm - 9.000\ mm = 0.015\ mm$.

Analyzing Shaft Dimensions

The given dimension for the shaft is $\phi 9^{+0.010}_{+0.001}\ mm$. This means:

- The **basic size** of the shaft is 9 mm.
- The **upper limit** of the shaft size is $9 + 0.010 = 9.010\ mm$.
- The **lower limit** of the shaft size is $9 + 0.001 = 9.001\ mm$.
- The **tolerance** for the shaft (the difference between the upper and lower limits) is $9.010\ mm - 9.001\ mm = 0.009\ mm$.

Calculating Clearance and Interference

The fit between the hole and the shaft depends on the difference between their sizes. We need to calculate the maximum possible clearance and the minimum possible clearance (which could be an interference if negative).

Maximum Clearance Calculation

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Maximum clearance occurs when the hole is at its largest size and the shaft is at its smallest size.

Maximum Clearance = (Upper Limit of Hole) - (Lower Limit of Shaft)

Maximum Clearance = $9.015\ mm - 9.001\ mm = 0.014\ mm$

Minimum Clearance Calculation

Minimum clearance occurs when the hole is at its smallest size and the shaft is at its largest size.

Minimum Clearance = (Lower Limit of Hole) - (Upper Limit of Shaft)

Minimum Clearance = $9.000\ mm - 9.010\ mm = -0.010\ mm$

A negative value for minimum clearance indicates interference.

Classifying the Resulting Fit

Now, let's classify the fit based on the calculated values:

- **Clearance Fit:** Occurs when there is always clearance (positive difference) between the hole and the shaft for all combinations of limits.
- **Interference Fit:** Occurs when there is always interference (negative difference) between the hole and the shaft for all combinations of limits.
- **Transition Fit:** Occurs when the fit can result in either clearance or interference, depending on the actual sizes of the mating parts within their tolerances. This happens when the maximum clearance is positive, and the minimum clearance is negative.
- **Running Fit / Sliding Fit:** These are types of clearance fits, further categorized by the amount of clearance (e.g., loose running, close running).

In this specific case:

- Maximum Clearance = 0.014 mm (positive)
- Minimum Clearance = -0.010 mm (negative, indicating interference)

Prepp Download Prepp APP GET IT ON Google Play

Since the range of possible fits includes both positive clearance (0.014 mm) and negative clearance (-0.010 mm), the resulting assembly has a **Transition Fit**.

---

19. **Answer: d**

Explanation:

Understanding Heat and Work as Path Functions in Thermodynamics

In thermodynamics, processes involve energy transfer, most commonly as heat and work. Understanding the nature of these quantities is crucial.

What are Thermodynamic Properties?

Thermodynamic properties describe the state of a system. They can be classified as:

- **Intensive Properties:** Properties that do not depend on the amount of substance in the system, such as temperature, pressure, and density.
- **Extensive Properties:** Properties that depend on the amount of substance in the system, such as mass, volume, and internal energy.

Prepp Your Personal Exams Guide

Point Functions vs. Path Functions

Thermodynamic quantities can also be classified based on whether they depend on the state of the system or the process (path) taken between states.

- **Point Functions (State Functions):** These quantities depend only on the initial and final states of the system. The change in a point function is determined solely by the difference between its value at the final state and its value at the initial state, regardless of the path taken. Examples include internal energy (U), enthalpy (H), entropy (S), temperature (T), and pressure (P). For a state function ϕ, the change $\Delta\phi = \phi_{\text{final}} - \phi_{\text{initial}}$. The differential of a point function is an exact differential.

Prepp Download Prepp APP GET IT ON Google Play

- **Path Functions:** These quantities depend on the specific process or path taken to go from one state to another. The amount of a path function transferred during a process depends on how the process is carried out. Heat (Q) and Work (W) are the prime examples of path functions. Their differentials are inexact differentials, often denoted as δQ and δW. The total heat transfer or work done during a process is the integral of the differential along the specific path.

Why are Heat and Work Path Functions?

Consider a system changing from state 1 to state 2. There can be many different ways (paths) to achieve this change of state. The amount of heat added to the system and the amount of work done by or on the system during this process can be different for each path, even if the initial and final states are the same.

For example, consider compressing a gas from a certain volume to a smaller volume:

- You could do it rapidly (adiabatically), where work is done on the gas, increasing its internal energy and temperature, with little or no heat transfer.
- You could do it slowly (isothermally), where work is done, but heat is simultaneously removed from the gas to keep the temperature constant. The work done in this case will be different from the adiabatic case.

Prepp Your Personal Exams Guide

Since the amounts of heat and work depend on the specific manner in which the process is executed (the path), they are classified as path functions.

Analyzing the Options

Based on the definitions:

- Option 1: Intensive properties - Incorrect. Heat and work are not properties like temperature or pressure.
- Option 2: Extensive properties - Incorrect. While the amount of heat or work might scale with the system size in some contexts, their fundamental nature is defined by the process path, not just the system's size or amount of substance.

Prepp Download Prepp APP GET IT ON Google Play

- Option 3: Point functions - Incorrect. Heat and work depend on the path, not just the initial and final points.
- Option 4: Path functions - Correct. Heat and work are quintessential examples of quantities whose values depend entirely on the process path taken between thermodynamic states.

Therefore, heat and work are path functions.

---

20. **Answer: b**

Explanation:

Determining the Column Slenderness Ratio

This explanation details the calculation for the **slenderness ratio** of a given column, focusing on its dimensions and length.

Understanding the Slenderness Ratio Formula

The **slenderness ratio** (λ) is a measure used in structural engineering to predict buckling of columns. It is defined as the ratio of the effective length of the column to its least radius of gyration.

The formula is:

$$\lambda = \frac{L_e}{r}$$

Where:

- L_e = Effective length of the column (depends on end support conditions).
- r = Least radius of gyration of the column's cross-section.

Calculating the Effective Length (L_e)

The problem states the column length (L) is 1 m. The end support conditions are not explicitly mentioned. A common assumption in such cases is that the column is

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

pinned at both ends. For a column pinned at both ends, the effective length (L_e) is equal to its actual length (L).

Convert the length to millimeters:

$L = 1 \text{ m} = 1000 \text{ mm}$

Assuming pinned-pinned ends:

$L_e = L = 1000 \text{ mm}$

Calculating the Least Radius of Gyration (r)

The radius of gyration (r) is calculated using the formula $r = \sqrt{I/A}$, where I is the relevant moment of inertia and A is the cross-sectional area. We need the *least* radius of gyration, which corresponds to the *least* moment of inertia.

1. Area Calculation (A)

The column has a **rectangular cross-section** of 10 mm x 20 mm.

Let width $b = 10$ mm and depth $d = 20$ mm.

The cross-sectional area A is:

$$A = b \times d = 10 \text{ mm} \times 20 \text{ mm} = 200 \text{ mm}^2$$

2. Moment of Inertia Calculation (I)

For a rectangular section, the moments of inertia (I) about the centroidal axes are:

- Moment of inertia about the axis parallel to the 20 mm side (I_x):

$$I_x = \frac{bd^3}{12} = \frac{10 \times (20)^3}{12} = \frac{10 \times 8000}{12} = \frac{80000}{12} \text{ mm}^4$$

- Moment of inertia about the axis parallel to the 10 mm side (I_y):

$$I_y = \frac{db^3}{12} = \frac{20 \times (10)^3}{12} = \frac{20 \times 1000}{12} = \frac{20000}{12} \text{ mm}^4$$

Prepp Download Prepp APP GET IT ON Google Play

The **least moment of inertia** (I_{min}) is the smaller of I_x and I_y. Since $b < d$, I_y will be smaller.

$$I_{min} = I_y = \frac{20000}{12}\text{ mm}^4$$

3. Least Radius of Gyration Calculation (r)

Now, calculate the least radius of gyration (r):

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{20000/12}{200}}$$

$$r = \sqrt{\frac{20000}{12 \times 200}} = \sqrt{\frac{20000}{2400}} = \sqrt{\frac{200}{24}}$$

Simplify the fraction:

$$r = \sqrt{\frac{25}{3}}\text{ mm} = \frac{5}{\sqrt{3}}\text{ mm}$$

Final Slenderness Ratio Calculation

Now, substitute the values of L_e and r into the slenderness ratio formula:

$$\lambda = \frac{L_e}{r} = \frac{1000\text{ mm}}{5/\sqrt{3}\text{ mm}}$$

Prepp Your Personal Exams Guide

$$\lambda = \frac{1000 \times \sqrt{3}}{5} = 200 \times \sqrt{3}$$

Using the approximate value of $\sqrt{3} \approx 1.732$:

$$\lambda \approx 200 \times 1.732 \approx 346.4$$

Conclusion

The calculated **slenderness ratio** for the column is approximately 346.4. Comparing this result to the options provided, the closest value is 346.

---

21. **Answer: b**

Prepp Download Prepp APP GET IT ON Google Play

Explanation:

Understanding Series Expansions for Functions

A series expansion, like the Taylor or Maclaurin series, represents a function as an infinite sum of terms calculated from the values of its derivatives at a single point. The Maclaurin series is a special case of the Taylor series centered at zero $(\theta = 0)$. For a function $f(\theta)$ that has derivatives of all orders at $\theta = 0$, the Maclaurin series is given by:

$$f(\theta) = f(0) + f'(0)\theta + \frac{f''(0)}{2!}\theta^2 + \frac{f'''(0)}{3!}\theta^3 + \ldots + \frac{f^{(n)}(0)}{n!}\theta^n + \ldots$$

Finding the Series Expansion for sin θ

To find the Maclaurin series for the function $f(\theta) = \sin\theta$, we need to evaluate the function and its derivatives at $\theta = 0$. Let's calculate the first few terms:

- $f(\theta) = \sin\theta \Rightarrow f(0) = \sin(0) = 0$
- $f'(\theta) = \cos\theta \Rightarrow f'(0) = \cos(0) = 1$
- $f''(\theta) = -\sin\theta \Rightarrow f''(0) = -\sin(0) = 0$
- $f'''(\theta) = -\cos\theta \Rightarrow f'''(0) = -\cos(0) = -1$
- $f^{(4)}(\theta) = \sin\theta \Rightarrow f^{(4)}(0) = \sin(0) = 0$
- $f^{(5)}(\theta) = \cos\theta \Rightarrow f^{(5)}(0) = \cos(0) = 1$

We can see a pattern emerging for the values of the derivatives at $\theta = 0$: 0, 1, 0, -1, 0, 1, 0, -1, ... This pattern repeats every four terms.

Constructing the Maclaurin Series for sin θ

Now, substitute these values into the Maclaurin series formula:

$$\sin\theta = f(0) + f'(0)\theta + \frac{f''(0)}{2!}\theta^2 + \frac{f'''(0)}{3!}\theta^3 + \frac{f^{(4)}(0)}{4!}\theta^4 + \frac{f^{(5)}(0)}{5!}\theta^5 + \ldots$$

Substitute the calculated values:

$$\sin\theta = 0 + (1)\theta + \frac{0}{2!}\theta^2 + \frac{-1}{3!}\theta^3 + \frac{0}{4!}\theta^4 + \frac{1}{5!}\theta^5 + \ldots$$

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Simplifying the terms, we get:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

This is the series expansion for sin θ.

Comparing with Options

Let's compare the derived series expansion with the given options:

- Option 1: $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$ (This is the series expansion for $\cos\theta$)
- Option 2: $\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$ (This matches our derived series)
- Option 3: $1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \dots$ (This is the series expansion for e^θ)
- Option 4: $\theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \dots$ (This is the series expansion for $\sinh\theta$)

Therefore, the series expansion for sin θ is given by option 2.

Function	Maclaurin Series Expansion
e^θ	$1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \dots$
$\sin\theta$	$\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$
$\cos\theta$	$1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$
$\sinh\theta$	$\theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \frac{\theta^7}{7!} + \dots$

Revision Table: Trigonometric Series Expansions

Here's a quick summary of common series expansions to remember:

Function	Series Expansion
$\sin\theta$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\theta^{2n+1} = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$
$\cos\theta$	$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\theta^{2n} = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots$
e^θ	$\sum_{n=0}^{\infty} \frac{1}{n!}\theta^n = 1 + \theta + \frac{\theta^2}{2!} + \dots$

Prepp Download Prepp APP GET IT ON Google Play

Additional Information: Properties of sin θ Series

The Maclaurin series for sin θ has several notable properties:

- It includes only odd powers of θ. This is because sin θ is an odd function.
- The signs of the terms alternate (+, −, +, −, ...).
- The denominators are the factorials of the corresponding odd powers (3!, 5!, 7!, ...).
- This series converges for all real values of θ.

Understanding the properties of the function (like being odd or even) can help recall or verify its series expansion.

---

22. **Answer: d**

Explanation:

Green Sand Mould Explained

A **green sand mould** is a type of casting mould that is widely used in the metal casting industry. The term "green" in this context does not refer to the colour of the mould, but rather to its condition. It indicates that the mould is in its natural, undried state and contains moisture. This moisture is a crucial component that contributes to the mould's properties.

Key Characteristics of Green Sand Mould

The composition of a typical green sand mixture includes:

- **Silica sand:** This forms the bulk of the mould material.
- **Clay:** Bentonite clay is commonly used as a binder to hold the sand grains together.
- **Water (moisture):** This activates the clay, giving the sand mixture its plasticity and strength, allowing it to be compacted and retain its shape after the

Prepp Download Prepp APP GET IT ON Google Play

pattern is removed.
- **Additives:** Small amounts of other materials like cereals or coal dust might be added to improve mould properties.

The presence of moisture is what makes the mould "green." Without sufficient moisture, the clay would not bind the sand particles effectively, and the mould would lack the necessary strength and integrity for casting.

Analyzing the Options for Green Sand Mould

Let's evaluate each given option to understand why mould contains moisture is the correct interpretation for a green sand mould:

- **Option 1: Polymeric mould has been cured**

  This option is incorrect. Green sand moulds do not use polymeric binders. They rely on natural binders like clay activated by water. Polymeric moulds are a different category of moulding processes, often involving chemical reactions for curing (hardening).

- **Option 2: Mould has been totally dried**

  This option is incorrect. If a mould were totally dried, it would be referred to as a "dry sand mould" or "baked mould." Drying removes the moisture, which is the defining characteristic of a green sand mould. Drying is sometimes done to improve surface hardness or eliminate moisture-related defects, but then it's no longer a "green" mould.

- **Option 3: Mould is green in colour**

  This option is incorrect. As explained earlier, the term "green" refers to the moisture content and the undried state of the mould, not its actual colour. The colour of a sand mould typically depends on the type of sand and binders used, often appearing brownish or grayish.

- **Option 4: Mould contains moisture**

  This option is correct. The defining characteristic of a green sand mould is that it contains moisture. This moisture is essential for the clay binder to work,

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

providing the mould with sufficient strength and plasticity to hold the cavity shape created by the pattern during the casting process.

Conclusion on Green Sand Mould Meaning

In summary, when we talk about a **green sand mould**, we are specifically referring to a mould that is prepared using a mixture of sand, clay, and most importantly, water (moisture). This moisture gives the mould its characteristic workability and strength in its undried state, making it ready for immediate pouring of molten metal.

---

23. **Answer: d**

Explanation:

Limit Introduction

The question asks us to find the value of the limit $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$. This is a very important and fundamental limit in calculus, especially when dealing with derivatives of trigonometric functions.

Understanding the Limit of $\frac{\sin \theta}{\theta}$

When we evaluate limits, we are looking at what value a function approaches as its input approaches a certain point. In this case, we are interested in what value the expression $\frac{\sin \theta}{\theta}$ approaches as θ gets very, very close to zero.

- If we try to substitute $\theta = 0$ directly into the expression, we get $\frac{\sin 0}{0} = \frac{0}{0}$, which is an indeterminate form. This means we cannot find the limit by simple substitution and need to use other methods.
- This specific limit, $\lim_{\theta \to 0} \frac{\sin \theta}{\theta}$, is a standard result in calculus. It forms the basis for proving the derivatives of trigonometric functions like $\sin x$ and $\cos x$.
- There are a few ways to prove this limit, including using a geometric argument (the Squeeze Theorem), or by using L'Hôpital's Rule.

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Evaluating the Limit $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$

Geometric Proof (Squeeze Theorem):

One common way to understand this limit is through a geometric argument involving a unit circle. For small angles θ (in radians):

- The area of a triangle formed by the origin, a point on the unit circle, and the x-axis is $\frac{1}{2}\cos\theta\sin\theta$.
- The area of the sector of the unit circle is $\frac{1}{2}\theta$.
- The area of a larger triangle formed by the origin, a point on the unit circle, and the tangent at that point is $\frac{1}{2}\tan\theta$.

By comparing these areas, we can establish the inequality:

$$\cos\theta \leq \frac{\sin\theta}{\theta} \leq \frac{1}{\cos\theta}$$

As $\theta \to 0$, we know that $\cos\theta \to \cos 0 = 1$. According to the Squeeze Theorem, since $\frac{\sin\theta}{\theta}$ is "squeezed" between $\cos\theta$ and $\frac{1}{\cos\theta}$, and both the lower and upper bounds approach 1, the limit of $\frac{\sin\theta}{\theta}$ must also be 1.

L'Hôpital's Rule:

Since substituting $\theta = 0$ gives the indeterminate form $\frac{0}{0}$, we can apply L'Hôpital's Rule. This rule states that if $\lim_{x \to c} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$, provided the latter limit exists.

Here, $f(\theta) = \sin\theta$ and $g(\theta) = \theta$.

- The derivative of $f(\theta) = \sin\theta$ is $f'(\theta) = \cos\theta$.
- The derivative of $g(\theta) = \theta$ is $g'(\theta) = 1$.

Applying L'Hôpital's Rule:

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \lim_{\theta \to 0} \frac{\frac{d}{d\theta}(\sin\theta)}{\frac{d}{d\theta}(\theta)}$$

Prepp Download Prepp APP GET IT ON Google Play

$$= \lim_{\theta \to 0} \frac{\cos\theta}{1}$$

Now, substitute $\theta = 0$ into the expression:

$$= \frac{\cos 0}{1}$$

$$= \frac{1}{1}$$

$$= 1$$

Both methods confirm that the limit $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$ is equal to 1.

Conclusion

The fundamental limit $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$ is a crucial concept in calculus and its value is always 1, assuming θ is in radians. This result is widely used in many mathematical derivations.

---

24. Answer: c

Explanation:

Prepp Your Personal Exams Guide

Eigenvalues of Real Symmetric Matrices

In linear algebra, understanding the properties of different types of matrices is crucial. One significant type is the **real symmetric matrix**, and its **eigenvalues** exhibit a very specific and important characteristic.

What is a Real Symmetric Matrix?

A matrix is considered **symmetric** if it is equal to its own transpose. That is, if A is a matrix, then A is symmetric if $A = A^T$. A matrix is a **real matrix** if all its entries are real numbers. Therefore, a **real symmetric matrix** is a square matrix whose entries are all real numbers, and it remains unchanged when its rows and columns are swapped (i.e., it equals its transpose).

Prepp Download Prepp APP GET IT ON Google Play

For example, the following is a real symmetric matrix:

2	1	3
1	0	4
3	4	-5

Eigenvalues of Real Symmetric Matrices are Always Real

A fundamental property in linear algebra states that the **eigenvalues of a real symmetric matrix are always real numbers**. This is a very important result with wide applications in various fields, including physics, engineering, and statistics.

Mathematical Justification for Real Eigenvalues

Let's consider a **real symmetric matrix** A. Let λ be an **eigenvalue** of A and V be its corresponding eigenvector. By definition, we have:

$$AV = \lambda V$$

Now, let's take the complex conjugate transpose (also known as the Hermitian conjugate) of both sides. The conjugate transpose of a vector V is denoted as V^H.

Prepp Your Personal Exams Guide

$$(AV)^H = (\lambda V)^H$$

Using the property $(AB)^H = B^H A^H$ and $(cV)^H = \bar{c}V^H$ where c is a scalar:

$$V^H A^H = \bar{\lambda} V^H$$

Since A is a **real symmetric matrix**, we know that $A = A^T$. For a real matrix, the complex conjugate of each element is the element itself, so $A^H = (\bar{A})^T = A^T$. Therefore, $A^H = A$.

Substituting $A^H = A$ into the equation:

$$V^H A = \bar{\lambda} V^H$$

Prepp Download Prepp APP GET IT ON Google Play

Now, multiply this equation by V from the right:

$$V^H AV = \bar{\lambda} V^H V \quad (\text{Equation } 1)$$

Recall our original **eigenvalue** equation: $AV = \lambda V$. Multiply this equation by V^H from the left:

$$V^H AV = \lambda V^H V \quad (\text{Equation } 2)$$

By comparing Equation 1 and Equation 2, we can see that their left-hand sides are identical. Therefore, their right-hand sides must also be equal:

$$\lambda V^H V = \bar{\lambda} V^H V$$

Rearranging the terms:

$$(\lambda - \bar{\lambda}) V^H V = 0$$

Since V is an eigenvector, it must be a non-zero vector, which means $V^H V$ (which is the squared norm of V) is always a positive real number ($V^H V = ||V||^2 > 0$).

For the product $(\lambda - \bar{\lambda}) V^H V$ to be zero, it must be that $(\lambda - \bar{\lambda}) = 0$.

$$\lambda = \bar{\lambda}$$

This condition, $\lambda = \bar{\lambda}$, implies that λ must be a **real number**. If λ were complex (e.g., $a + bi$ where $b \neq 0$), then $\bar{\lambda}$ would be $a - bi$, and $\lambda \neq \bar{\lambda}$. Hence, for $\lambda = \bar{\lambda}$, the imaginary part must be zero, meaning λ is real.

Why Other Options Are Incorrect

- **Positive or Negative:** While **real symmetric matrices** can have all positive **eigenvalues** (positive definite matrices) or all negative **eigenvalues** (negative definite matrices), their **eigenvalues** are not *always* positive or *always* negative. For example, a real symmetric matrix can have both positive and negative **eigenvalues**, or even zero **eigenvalues**. For instance, the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is real symmetric and has eigenvalues 1 and -1. The matrix $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is real symmetric and has eigenvalues 0 and 0.

Prepp Download Prepp APP GET IT ON Google Play

- **Complex:** As demonstrated by the mathematical proof above, the **eigenvalues** of a **real symmetric matrix** cannot be complex numbers with a non-zero imaginary part. They are strictly **real numbers**. Complex **eigenvalues** (that are not real) typically arise from non-symmetric matrices.

Therefore, the only always true statement about the **eigenvalues of a real symmetric matrix** is that they are **real**.

---

25. **Answer: c**

Explanation:

Heat Transfer and Insulation for Pipes

Understanding heat transfer and the role of insulation is crucial for efficient energy management, especially in systems like steam pipes. Insulation is primarily added to reduce heat loss from a hot surface or heat gain by a cold surface, thereby improving thermal efficiency and ensuring safety.

Critical Radius of Insulation Explained

For a cylindrical pipe, there's a specific concept called the **critical radius of insulation**. This is the radius at which, for a given insulation material and external convection coefficient, adding insulation actually maximizes heat transfer rather than reducing it. Beyond this critical radius, adding more insulation will then start to reduce the heat transfer.

The formula for the critical radius (r_{crit}) for a cylinder is given by:

$$r_{crit} = \frac{k}{h}$$

Where:

- k is the thermal conductivity of the insulation material (in W/mK).
- h is the convective heat transfer coefficient between the outer surface of the insulation and the surroundings (in W/m^{2}K).

Prepp Download Prepp APP GET IT ON Google Play

The effect of adding insulation depends on the initial outer radius of the pipe (r_{pipe}) compared to the critical radius (r_{crit}):

- If $r_{pipe} < r_{crit}$: Adding insulation initially **increases** the heat loss until the total radius reaches r_{crit}. Beyond r_{crit}, further insulation will reduce heat loss.
- If $r_{pipe} > r_{crit}$: Adding any amount of insulation will **always reduce** the heat loss.

Pipe Parameters and Calculation

Let's consider the given parameters for the steam pipe:

- Outer diameter of the pipe = 25 mm
- Outer radius of the pipe (r_{pipe}) = 25 mm / 2 = 12.5 mm = 0.0125 m
- Heat transfer coefficient between the cylinder and surroundings (h) = 5 W/m^{2}K
- Thermal conductivity of the insulation (k) = 0.05 W/mK

Now, let's calculate the critical radius of insulation:

$$ r_{crit} = \frac{k}{h} = \frac{0.05 \text{ W/mK}}{5 \text{ W/m}^2\text{K}} = 0.01 \text{ m} $$

Comparing the pipe's outer radius with the calculated critical radius:

- Pipe outer radius (r_{pipe}) = 0.0125 m
- Critical radius (r_{crit}) = 0.01 m

Since $0.0125 \text{ m} > 0.01 \text{ m}$, it means that the pipe's outer radius is greater than the critical radius of insulation ($r_{pipe} > r_{crit}$).

Analyzing Insulation Statements

Based on our calculation and understanding of the critical radius concept, we can evaluate each statement concerning the insulation for the pipe:

- **Statement 1: The outer radius of the pipe is equal to the critical radius**
  This statement is false because $0.0125 \text{ m} \neq 0.01 \text{ m}$.
- **Statement 2: The outer radius of the pipe is less than the critical radius**
  This statement is also false because $0.0125 \text{ m} \not< 0.01 \text{ m}$. In fact, it is greater.

Prepp Your Personal Exams Guide

Download Prepp APP
GET IT ON Google Play

- **Statement 3: Adding the insulation will reduce the heat loss**
  This statement is true. As established, when the pipe's outer radius (r_{pipe}) is greater than the critical radius (r_{crit}), adding any amount of insulation will always lead to a reduction in heat loss from the pipe.
- **Statement 4: Adding the insulation will increase the heat loss**
  This statement is false. An increase in heat loss would occur only if the pipe's outer radius was less than the critical radius ($r_{pipe} < r_{crit}$) and insulation was added such that the total outer radius did not exceed r_{crit}. Since $r_{pipe} > r_{crit}$, this is not the case here.

Conclusion on Heat Loss Reduction

Given that the outer radius of the pipe (0.0125 m) is greater than the critical radius of insulation (0.01 m), adding the specified insulation will effectively reduce the heat loss from the steam pipe. This aligns with the principles of thermal insulation designed for energy efficiency.

---

26. **Answer: a**

Explanation:

This problem involves applying the First Law of Thermodynamics to a system consisting of a well-insulated tank and its contents. We need to determine the rates of heat transfer (Q), work done (W), and change in internal energy (ΔU) during the process.

Let's break down the problem step-by-step, considering the given information and thermodynamic conventions:

Thermodynamic System Analysis

The system under consideration is the "tank along with its contents". This is a closed system as no mass is entering or leaving it. The process involves heating by an electrical resistor.

Prepp
Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

- **Well-insulated tank:** This is a crucial piece of information. A well-insulated tank implies that there is no heat exchange between the system and its surroundings. Therefore, the heat transfer rate (Q) for this process is zero.

Therefore, we can state:

Heat Transfer (Q)	0 kW (since the tank is well-insulated)

Work Done Calculation

The system is heated by a resistor with a resistance of 23 ohm, through which a current of 10 A is flowing. The electrical power dissipated by the resistor is a form of work done on the system.

- **Electrical Power (P):** The rate at which electrical energy is converted into heat within the resistor is given by the formula $P = I^2R$, where I is the current and R is the resistance.

Let's calculate the electrical power:

$$P = I^2R$$

Given:

- Current (I) = 10 A
- Resistance (R) = 23 ohm

Substituting the values:

$$P = (10 \text{ A})^2 \times (23 \text{ ohm})$$

$$P = 100 \times 23 \text{ W}$$

$$P = 2300 \text{ W}$$

To convert watts (W) to kilowatts (kW), divide by 1000:

$$P = \frac{2300}{1000} \text{ kW}$$

Prepp Download Prepp APP GET IT ON Google Play

$$P = 2.3\,\mathrm{kW}$$

- **Work Done (W) Sign Convention:** The problem states, "The work done by the system is positive." In this scenario, electrical energy is being supplied to the resistor, which is *inside* the system. This means work is being done *on* the system, not *by* the system. According to the given sign convention, work done *on* the system is negative.

Therefore, we can state:

Work Done (W)	-2.3 kW (work done on the system)

Internal Energy Change

Now, we apply the First Law of Thermodynamics for a closed system, which relates the change in internal energy (ΔU) to the heat transfer (Q) and work done (W):

$$\Delta U = Q - W$$

We have calculated Q and W:

- $Q = 0\,\mathrm{kW}$
- $W = -2.3\,\mathrm{kW}$

Prepp Your Personal Exams Guide

Substitute these values into the First Law equation:

$$\Delta U = 0\,\mathrm{kW} - (-2.3\,\mathrm{kW})$$

$$\Delta U = 0\,\mathrm{kW} + 2.3\,\mathrm{kW}$$

$$\Delta U = +2.3\,\mathrm{kW}$$

Therefore, the change in internal energy is positive, indicating that the internal energy of the system is increasing due to the electrical work done on it.

Summary of Rates

Prepp Download Prepp APP GET IT ON Google Play

Based on our calculations, the rates of heat, work, and change in internal energy during the process are:

Parameter	Rate (kW)
Heat (Q)	0
Work (W)	-2.3
Change in Internal Energy (ΔU)	+2.3

These values match the first option provided.

---

27. **Answer: b**

Explanation:

Explanation:

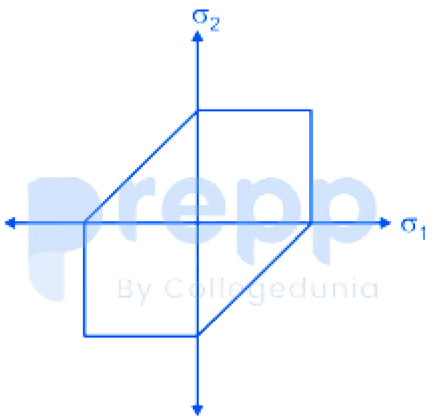
Maximum shear stress theory

(Guest & Tresca's Theory)

Your Personal Exams Guide

According to this theory, failure of the specimen subjected to any combination of a load when the maximum shearing stress at any point reaches the failure value equal to that developed at the yielding in an axial tensile or compressive test of the same material.

Prepp Download Prepp APP GET IT ON Google Play



Graphical Representation

$\tau_{\max} \leq \frac{\sigma_y}{2}$ For no failure

$\sigma_1 - \sigma_2 \leq \left(\frac{\sigma_y}{FOS}\right)$ For design

σ1 and σ2 are maximum and minimum principal stress respectively.

Here, τmax = Maximum shear stress

σy = permissible stress

This theory is well justified for ductile materials.

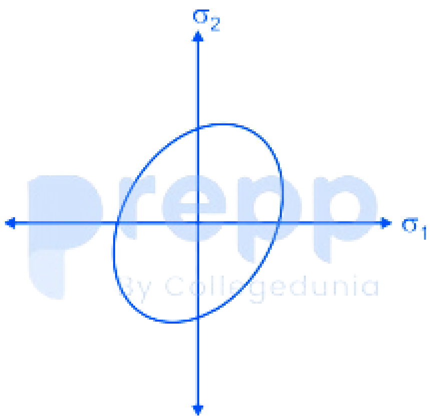
Maximum shear strain energy / Distortion energy theory / Mises – Henky theory.

It states that inelastic action at any point in body, under any combination of stress begging, when the strain energy of distortion per unit volume absorbed at the point is equal to the strain energy of distortion absorbed per unit volume at any point in a bar stressed to the elastic limit under the state of uniaxial stress as occurs in a simple tension/compression test.

for no failure

For design

prepp Download Prepp APP GET IT ON Google Play



(Ellipse)

It cannot be applied for material under hydrostatic pressure.

All theories will give the same results if loading is uniaxial.

Maximum principal stress theory (Rankine's theory)

According to this theory, the permanent set takes place under a state of complex stress, when the value of maximum principal stress is equal to that of yield point stress as found in a simple tensile test.

For the design criterion, the maximum principal stress (σ_1) must not exceed the working stress 'σ_y' for the material.

$\sigma_{1,2} \leq \sigma_{\mathrm{y}}$ for no failure

$\sigma_{1,2} \leq \frac{\sigma}{\mathrm{FOS}}$ for design

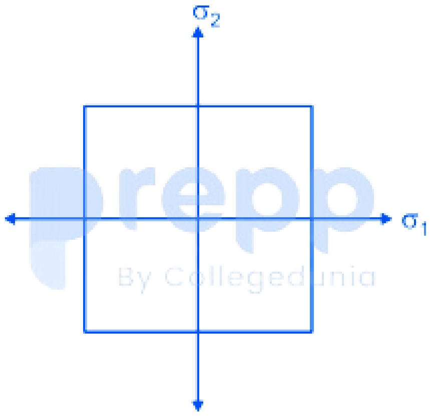
Note: For no shear failure $\tau \leq 0.57\ \sigma_y$

Graphical representation

For brittle material, which does not fail by yielding but fail by brittle fracture, this theory gives a satisfactory result.

The graph is always square even for different values of σ_1 and σ_2.

Prepp Download Prepp APP GET IT ON Google Play



(Rectangular)

⭐ Important Points

Maximum principal strain theory (ST. Venant's theory)

According to this theory, a ductile material begins to yield when the maximum principal strain reaches the strain at which yielding occurs in simple tension.

$\epsilon_{1,2} \leq \frac{\sigma_y}{E_1}$ For no failure in uniaxial loading.

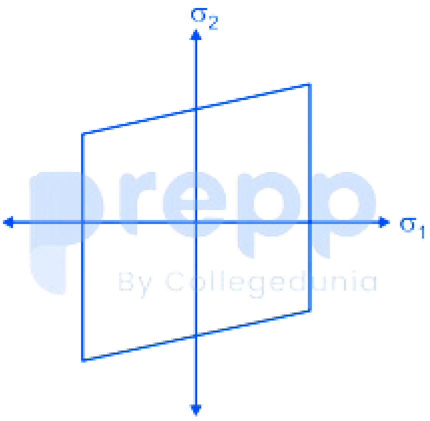
$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \leq \frac{\sigma_y}{E}$ For no failure in triaxial loading.

$\sigma_1 - \mu\sigma_2 - \mu\sigma_3 \leq \left(\frac{\sigma_y}{FOS}\right)$ For design, Here, ϵ = Principal strain

σ1, σ2, and σ3 = Principal stresses

Graphical Representation

This theory overestimates the elastic strength of ductile material.



(Rhombus)

Prepp Download Prepp APP GET IT ON Google Play

Maximum strain energy theory (Haigh's theory)

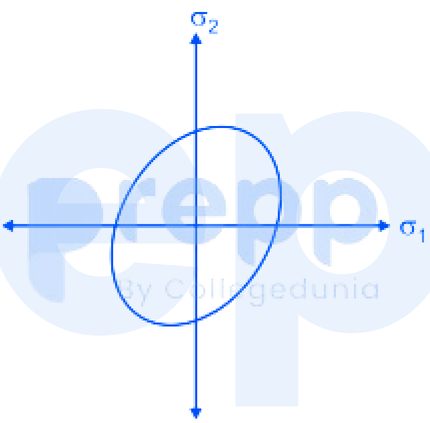
According to this theory, a body complex stress fails when the total strain energy at the elastic limit in simple tension.

Graphical Representation.

for no failure

for design

This theory does not apply to brittle material for which elastic limit stress in tension and in compression are quite different.



(Ellipse)

Your Personal Exams Guide

---

28. **Answer: a**

Explanation:

Complex Numbers Product Calculation

Understanding how to find the product of two **complex numbers** is a fundamental concept in mathematics. This problem asks us to multiply two specific **complex numbers**: $1 + i$ and $2 - 5i$. Let's break down the process step by step to find their **product**.

Complex Number Definition and Structure

prepp Download Prepp APP GET IT ON Google Play

A **complex number** is a number that can be expressed in the form $a + bi$, where a and b are real numbers, and i is the imaginary unit, defined as $i = \sqrt{-1}$. The term a is called the **real part**, and b is called the **imaginary part**.

- The **real part** corresponds to the numbers on the standard number line.
- The **imaginary part** involves the imaginary unit i, where $i^2 = -1$. This property is crucial for simplifying expressions during **complex number multiplication**.

Product of Complex Numbers: Method Explained

To find the **product** of two **complex numbers**, say $(a + bi)$ and $(c + di)$, we use a method similar to multiplying two binomials in algebra, often referred to as the FOIL method (First, Outer, Inner, Last).

The steps involved are:

- Multiply the **First** terms of each binomial.
- Multiply the **Outer** terms.
- Multiply the **Inner** terms.
- Multiply the **Last** terms.
- Substitute $i^2 = -1$ wherever it appears.
- Combine the **real parts** and the **imaginary parts** separately to express the final **product** in the standard $a + bi$ form.

Product Calculation for (1 + i) and (2 - 5i)

Let's apply the method to find the **product** of the given **complex numbers**, $(1 + i)$ and $(2 - 5i)$.

Let $z_1 = 1 + i$ and $z_2 = 2 - 5i$.

We want to calculate $z_1 \times z_2$:

$(1 + i)(2 - 5i)$

Now, let's perform the multiplication using the FOIL method:

1. **First** terms: $1 \times 2 = 2$
2. **Outer** terms: $1 \times (-5i) = -5i$

Prepp Download Prepp APP GET IT ON Google Play

3. **Inner** terms: $i \times 2 = 2i$
4. **Last** terms: $i \times (-5i) = -5i^2$

Combining these results, we get:

$2 - 5i + 2i - 5i^2$

Next, we substitute the value of i^2, which is -1:

$2 - 5i + 2i - 5(-1)$

$2 - 5i + 2i + 5$

Finally, we group and combine the **real parts** and the **imaginary parts**:

- **Real parts**: $2 + 5 = 7$
- **Imaginary parts**: $-5i + 2i = -3i$

So, the **product** of the two **complex numbers** is:

$7 - 3i$

This is the final **complex number product** in the standard $a + bi$ form.

Summary of Complex Number Multiplication

Term 1	Term 2	Product	Simplified Product
1	2	2	2
1	$-5i$	$-5i$	$-5i$
i	2	$2i$	$2i$
i	$-5i$	$-5i^2$	$-5(-1) = 5$
Total Product (Real + Imaginary)			$(2+5) + (-5i+2i) = 7 - 3i$

---

29. Answer: d

Prepp Download Prepp APP GET IT ON Google Play

Explanation:

This problem involves the application of queuing theory, specifically an M/M/1 queuing model. In this model, arrivals follow a Poisson distribution, service times follow an exponential distribution, and there is a single server. Our goal is to determine the average waiting time a car spends in the queue at steady state.

Queuing Parameters Identification

Let's first list the given information and convert units to ensure consistency for calculations. It's often easiest to work with a common time unit, such as minutes, since the final answer options are in minutes.

- **Car Arrival Rate:** The mean arrival rate of cars is given as 5 per hour. This is our λ (lambda).
- **Service Time Per Car:** The mean service time for each car is 10 minutes. This helps us find μ (mu), the service rate.

Arrival Rate (λ) Calculation

The arrival rate (λ) needs to be converted from cars per hour to cars per minute:

$$\lambda = \frac{5 \text{ cars}}{1 \text{ hour}} = \frac{5 \text{ cars}}{60 \text{ minutes}} = \frac{1}{12} \text{ cars/minute}$$

Your Personal Exams Guide

Service Rate (μ) Calculation

The mean service time is 10 minutes per car. The service rate (μ) is the number of cars that can be served per unit of time, which is the reciprocal of the mean service time:

$$\mu = \frac{1}{\text{Mean Service Time}} = \frac{1}{10} \text{ cars/minute}$$

Traffic Intensity (ρ) Determination

Traffic intensity, denoted by ρ (rho), is a crucial parameter in queuing theory. It represents the proportion of time the server is busy. For a stable system (i.e., for a

Prepp Download Prepp APP GET IT ON Google Play

steady state to exist), the traffic intensity must be less than 1 ($\rho < 1$).

The formula for traffic intensity in an M/M/1 model is:

$$\rho = \frac{\lambda}{\mu}$$

Substituting the calculated values of λ and μ:

$$\rho = \frac{1/12 \text{ cars/minute}}{1/10 \text{ cars/minute}} = \frac{1}{12} \times \frac{10}{1} = \frac{10}{12} = \frac{5}{6}$$

Since $\rho = 5/6$, which is indeed less than 1, the system is stable, and we can proceed to calculate the steady-state average waiting time.

Average Waiting Time in the Queue (W_q)

For an M/M/1 queuing system, the average waiting time in the queue (W_q) is given by the formula:

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

Now, let's plug in the values for λ and μ:

$$W_q = \frac{1/12}{(1/10)(1/10 - 1/12)}$$

First, calculate the difference inside the parenthesis:

$$1/10 - 1/12 = \frac{6}{60} - \frac{5}{60} = \frac{6-5}{60} = \frac{1}{60}$$

Substitute this back into the W_q formula:

$$W_q = \frac{1/12}{(1/10)(1/60)}$$

$$W_q = \frac{1/12}{1/600}$$

To simplify, multiply the numerator by the reciprocal of the denominator:

Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

$$W_q = \frac{1}{12} \times 600$$

$$W_q = \frac{600}{12}$$

$$W_q = 50 \text{ minutes}$$

Summary of Queuing System Metrics

Here is a concise summary of the parameters and the calculated average waiting time:

Parameter	Symbol	Value	Units
Arrival Rate	λ	$1/12$	cars/minute
Service Rate	μ	$1/10$	cars/minute
Traffic Intensity	ρ	$5/6$	(dimensionless)
Average Waiting Time in Queue	W_q	50	minutes

Therefore, at steady state, the average waiting time in the queue for a car is 50 minutes.

Your Personal Exams Guide

---

30. **Answer: b**

Explanation:

Kanban System: Understanding its Core Association

The term **kanban** is a key concept in lean manufacturing and production management, deeply rooted in the Toyota Production System. To understand its most appropriate association, let's explore what **kanban** signifies and how it functions.

Prepp Download Prepp APP GET IT ON Google Play

What is Kanban?

Kanban is a scheduling system for lean manufacturing and just-in-time (JIT) production. It is a visual system used to manage work as it moves through a process. The word "**kanban**" itself is Japanese for "visual signal" or "card." It acts as a signaling system to trigger action, typically the production or movement of items.

- The core idea of **kanban** is to pull production based on actual demand, rather than pushing it based on forecasts.
- A **kanban** card or signal indicates that a certain material or product has been consumed and needs to be replenished.
- This system helps to minimize overproduction and reduce inventory levels.

Kanban's Link to Just-in-Time (JIT) Production

The most appropriate association for **kanban** is undoubtedly with **Just-in-Time (JIT) production**. **JIT production** is a manufacturing philosophy that aims to reduce waste and improve efficiency by producing goods only when they are needed, in the exact quantities required, and delivering them just in time for the next process step or for customer demand. **Kanban** is a fundamental tool and a cornerstone of achieving **JIT production** goals.

- **Kanban** cards serve as the "pull" signals in a **JIT production** system. When a component is used in a later stage of production, a **kanban** card associated with that component is sent back to the preceding stage, authorizing the production or delivery of another component.
- This ensures that production is directly linked to consumption, preventing the build-up of excess inventory, which is a major waste in **JIT production**.
- By using **kanban**, companies can achieve significant reductions in lead times, work-in-process inventory, and overall production costs, all of which are primary objectives of **JIT production**.

Why Other Options are Less Appropriate

While the other options are related to operations management, they do not directly represent the core association of **kanban**:

Prepp Download Prepp APP GET IT ON Google Play

Option	Explanation
Economic Order Quantity (EOQ)	**EOQ** is an inventory management formula used to determine the optimal order quantity that minimizes total inventory costs (holding costs and ordering costs). While both deal with inventory, **kanban** focuses on demand-pull production and waste reduction, whereas **EOQ** is a specific formula for traditional inventory optimization.
Capacity Planning	**Capacity planning** involves determining the production capacity an organization needs to meet changing demands. While **kanban** influences the flow of work, it is not primarily a tool for long-term capacity planning; rather, it manages the immediate flow within existing capacity.
Product Design	**Product design** is the process of creating new products to be sold by a business to its customers. This field is distinct from process management tools like **kanban**, which focus on how products are made, not what products are made.

Prepp
Your Personal Exams Guide

In summary, **kanban** is a visual control system that enables the smooth and efficient flow of materials and information in a pull-based system, making it an indispensable tool for implementing and sustaining **Just-in-Time production**.

---

31. **Answer: d**

Explanation:

Even Function Definition and Properties

An **even function** is a type of function $f(x)$ that satisfies the condition $f(-x) = f(x)$ for all values of x in its domain. This property means that the function's output remains the same whether the input is x or $-x$. Geometrically, the graph of an **even function** is perfectly symmetric about the y-axis.

Prepp Download Prepp APP GET IT ON Google Play

For example, functions like $f(x) = x^2$, $f(x) = \cos(x)$, or $f(x) = |x|$ are all **even functions** because substituting $-x$ for x does not change their value.

A **definite integral**, denoted as \(\mathop \smallint \limits_a^b f\left(x \right)dx\), represents the signed area under the curve of the function $f(x)$ from $x = a$ to $x = b$.

Integral Property for Even Functions

When dealing with **definite integrals** of functions over a **symmetric interval**, such as from $-a$ to a, specific properties arise depending on whether the function is even or odd. For an **even function** $f(x)$, integrating from $-a$ to a simplifies significantly.

The general property of definite integrals allows us to split the integral over an interval into a sum of integrals over sub-intervals. For the integral \(\mathop \smallint \limits_{ - a}^a f\left(x \right)dx\), we can write it as:

\[\mathop \smallint \limits_{ - a}^a f\left(x \right)dx = \mathop \smallint \limits_{ - a}^0 f\left(x \right)dx + \mathop \smallint \limits_0^a f\left(x \right)dx \]

Deriving the Definite Integral Result

To evaluate the integral \(\mathop \smallint \limits_{ - a}^a f\left(x \right)dx\) for an **even function** $f(x)$, we need to analyze the first term, \(\mathop \smallint \limits_{ - a}^0 f\left(x \right)dx\).

Let's use a substitution method for the integral \(\mathop \smallint \limits_{ - a}^0 f\left(x \right)dx\):

- Let $x = -t$.
- Then, the differential $dx = -dt$.
- We also need to change the limits of integration:
    - When $x = -a$, then $-a = -t \implies t = a$.
    - When $x = 0$, then $0 = -t \implies t = 0$.

Now, substitute these into the integral:

Download Prepp APP
GET IT ON Google Play

\[\mathop \smallint \limits_{ - a}^0 f\left(x \right)dx = \mathop \smallint \limits_a^0 f\left(-t \right)\left({ - dt} \right) \]

We can pull the negative sign outside the integral and reverse the limits of integration (which also changes the sign of the integral):

\[\mathop \smallint \limits_a^0 f\left(-t \right)\left({ - dt} \right) = - \mathop \smallint \limits_a^0 f\left(-t \right)dt = \mathop \smallint \limits_0^a f\left(-t \right)dt \]

Since $f(x)$ is an **even function**, we know that $f(-t) = f(t)$. Substituting this property into our transformed integral:

\[\mathop \smallint \limits_0^a f\left(-t \right)dt = \mathop \smallint \limits_0^a f\left(t \right)dt \]

The definite integral's value does not depend on the variable of integration, so \(\mathop \smallint \limits_0^a f\left(t \right)dt\) is the same as \(\mathop \smallint \limits_0^a f\left(x \right)dx\). Therefore, we have:

\[\mathop \smallint \limits_{ - a}^0 f\left(x \right)dx = \mathop \smallint \limits_0^a f\left(x \right)dx \]

Now, substitute this result back into our original split integral:

\[\mathop \smallint \limits_{ - a}^a f\left(x \right)dx = \mathop \smallint \limits_{ - a}^0 f\left(x \right)dx + \mathop \smallint \limits_0^a f\left(x \right)dx \]

\[\mathop \smallint \limits_{ - a}^a f\left(x \right)dx = \mathop \smallint \limits_0^a f\left(x \right)dx + \mathop \smallint \limits_0^a f\left(x \right)dx \]

\[\mathop \smallint \limits_{ - a}^a f\left(x \right)dx = 2\mathop \smallint \limits_0^a f\left(x \right)dx \]

This powerful property shows that for an **even function**, the integral over a **symmetric interval** $[-a, a]$ is simply twice the integral from 0 to a.

Final Answer for Even Function Integral

Prepp Your Personal Exams Guide

prepp Download Prepp APP GET IT ON Google Play

Based on the derivation, if $f(x)$ is an **even function** and a is a **positive real number**, then the definite integral \(\mathop \smallint \limits_{ - a}^a f\left(x \right)dx\) is equal to \(2\mathop \smallint \limits_0^a f\left(x \right)dx\).

---

32. **Answer: a**

Explanation:

Coefficient of Restitution Explained: Perfectly Plastic Impact

The concept of a **coefficient of restitution** is crucial in understanding how objects behave during a collision. It is a dimensionless quantity that measures the "bounciness" or elasticity of an impact. This **coefficient of restitution** provides a ratio of the relative speeds of the objects after and before a collision.

Mathematically, the **coefficient of restitution** (denoted as e) is defined as:

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

Or, if we consider two objects with initial velocities u_1 and u_2 before collision, and final velocities v_1 and v_2 after collision, the formula becomes:

$$e = \frac{(v_2 - v_1)}{(u_1 - u_2)}$$

Perfectly Plastic Impact Defined

A **perfectly plastic impact** (also known as a perfectly inelastic impact) is a specific type of collision where the colliding objects stick together after the impact. In such an impact, there is the maximum possible loss of kinetic energy, although momentum is still conserved.

Let's analyze what happens in a **perfectly plastic impact** in terms of velocities:

- Before the collision, the objects approach each other with velocities u_1 and u_2. The relative velocity of approach is $(u_1 - u_2)$.

Prepp Download Prepp APP GET IT ON Google Play

- After the collision, since the objects stick together, they move as a single unit. This means their final velocities are identical. Let's say their common final velocity is V. So, $v_1 = V$ and $v_2 = V$.
- Therefore, the relative velocity of separation after a **perfectly plastic impact** is $(v_2 - v_1) = (V - V) = 0$.

Calculating the Coefficient of Restitution for Perfectly Plastic Impact

Now, let's substitute these values into the formula for the **coefficient of restitution**:

$e = \frac{(v_2 - v_1)}{(u_1 - u_2)}$

For a **perfectly plastic impact**, we established that $(v_2 - v_1) = 0$.

$e = \frac{0}{(u_1 - u_2)}$

$e = 0$

This result confirms that for a **perfectly plastic impact**, where objects merge and move together, the **coefficient of restitution** is 0. This signifies that there is no "bounce" or relative separation velocity between the objects after they collide.

Summary of Coefficient of Restitution Values

Your Personal Exams Guide

To put it into perspective, here's a quick summary of the **coefficient of restitution** for different types of impacts:

- **Perfectly Elastic Impact:** $e = 1$ (Kinetic energy is conserved, objects separate cleanly).
- **Inelastic Impact:** $0 < e < 1$ (Kinetic energy is not conserved, objects separate but with reduced relative speed).
- **Perfectly Plastic Impact:** $e = 0$ (Objects stick together after collision, maximum kinetic energy loss).

Thus, for a **perfectly plastic impact**, the **coefficient of restitution** is indeed 0.

---

33. **Answer: b**

prepp Download Prepp APP GET IT ON Google Play

Explanation:

Thin Cylinder Hoop Stress Calculation

Understanding the stress distribution in thin-walled cylinders, like pressure vessels or pipes, is a fundamental concept in mechanics of materials. When a thin cylinder is subjected to internal pressure, two primary types of stresses are developed: circumferential (or hoop) stress and longitudinal (or axial) stress. This problem focuses on calculating the average circumferential (hoop) stress.

Given Parameters for Thin Cylinder Stress

Let's identify the given values for the thin cylinder from the problem statement:

- Inner radius of the cylinder, $r = 500 \text{ mm}$
- Thickness of the cylinder wall, $t = 10 \text{ mm}$
- Internal pressure applied to the cylinder, $p = 5 \text{ MPa}$

Formula for Circumferential (Hoop) Stress

For a thin-walled cylinder, where the ratio of the inner radius to the wall thickness (r/t) is typically greater than 10, the average circumferential or hoop stress (σ_h) can be calculated using a simplified formula. This formula assumes uniform stress distribution across the wall thickness, which is a reasonable approximation for thin cylinders.

The formula for the average circumferential (hoop) stress in a thin cylinder is:

$$\sigma_h = \frac{pr}{t}$$

Where:

- σ_h is the circumferential (hoop) stress
- p is the internal pressure
- r is the inner radius
- t is the wall thickness

Prepp Download Prepp APP GET IT ON Google Play

Step-by-Step Calculation of Hoop Stress

Now, let's substitute the given values into the formula to find the average circumferential (hoop) stress. It's important to ensure that all units are consistent. In this case, pressure is in MPa, and dimensions are in mm, which will yield stress in MPa.

Parameter	Value
Internal Pressure (p)	5 MPa
Inner Radius (r)	500 mm
Thickness (t)	10 mm

Prepp
Your Personal Exams Guide

Substitute these values into the formula:

$$\sigma_h = \frac{(5 \text{ MPa}) \times (500 \text{ mm})}{10 \text{ mm}}$$

First, multiply the pressure by the inner radius:

$$\sigma_h = \frac{2500 \text{ MPa} \cdot \text{mm}}{10 \text{ mm}}$$

Now, divide by the thickness:

$$\sigma_h = 250 \text{ MPa}$$

Final Result for Circumferential Stress

The calculated average circumferential (hoop) stress in the thin cylinder subjected to an internal pressure of 5 MPa is **250 MPa**. This value represents the tensile stress acting along the circumference of the cylinder due to the internal pressure.

---

34. Answer: c

Prepp Download Prepp APP GET IT ON Google Play

Explanation:

In the field of metal joining, understanding different **welding processes** and their components is crucial. One key component in many arc **welding processes** is the electrode, which can be either consumable or **non-consumable**. The question asks to identify the **welding process** that uses a **non-consumable electrode**.

Welding Electrodes Explained

A **welding electrode** is a conductor that carries current to the **welding** arc. Depending on the **welding process**, electrodes are categorized into two main types:

- **Consumable Electrodes:** These electrodes melt during the **welding process** and become part of the weld metal. They are consumed as the weld progresses, requiring continuous feeding.
- **Non-Consumable Electrodes:** These electrodes resist melting and maintain their shape during the **welding process**. They primarily serve to establish and maintain the arc, with filler metal (if needed) added separately.

Gas Tungsten Arc Welding (GTAW) Explanation

Gas Tungsten Arc Welding (GTAW), often known by its older name Tungsten Inert Gas (TIG) **welding**, is the **welding process** that uses a **non-consumable electrode**. Here's why:

- **Electrode Type:** GTAW employs a tungsten electrode, which has a very high melting point (approx. $3422^\circ\mathrm{C}$). This high melting point allows the tungsten electrode to withstand the intense heat of the arc without melting, thus making it **non-consumable**.
- **Filler Material:** In most GTAW applications, a separate filler rod is manually or automatically fed into the weld puddle. This means the electrode's primary function is to create the arc, not to provide filler material.
- **Shielding Gas:** An inert shielding gas (like Argon or Helium) protects the weld pool, electrode, and arc from atmospheric contamination, ensuring a high-quality weld.

Prepp Download Prepp APP GET IT ON Google Play

GTAW is known for producing very clean, precise welds with excellent control, making it suitable for thin materials and critical applications.

Gas Metal Arc Welding (GMAW) Overview

Gas Metal Arc Welding (GMAW), commonly known as MIG (Metal Inert Gas) **welding**, utilizes a **consumable electrode**. In GMAW:

- **Electrode Type:** A continuously fed wire electrode, which is made of a material similar to the base metal, melts into the weld puddle. This wire serves as both the electrode and the filler material.
- **Shielding Gas:** An external shielding gas (usually a mixture of Argon and CO_2) protects the arc and weld puddle.

GMAW is widely used for its speed and efficiency in joining various metals.

Submerged Arc Welding (SAW) Details

Submerged Arc Welding (SAW) is another **welding process** that uses a **consumable electrode**. Key features include:

- **Electrode Type:** A continuously fed bare wire electrode is used. This electrode melts and becomes part of the weld.
- **Flux Shielding:** The arc and weld pool are 'submerged' under a blanket of granular flux. This flux melts to form a protective slag and adds alloying elements to the weld.

SAW is known for its high deposition rates and deep penetration, often used in heavy fabrication.

Flux Coated Arc Welding (FCAW) Basics

Flux Coated Arc Welding typically refers to Shielded Metal Arc Welding (SMAW), also known as "stick **welding**," where the electrode is coated with flux. Alternatively, it can refer to Flux-Cored Arc Welding (FCAW), which uses a tubular wire filled with flux. Both are **consumable electrode** processes:

Download Prepp APP
GET IT ON Google Play

- **SMAW (Stick Welding):** Uses a flux-coated stick electrode that is consumed during **welding**. The flux coating creates a shielding gas and slag.
- **FCAW (Flux-Cored Arc Welding):** Uses a tubular wire electrode filled with flux. This wire is continuously fed and consumed, with the flux generating shielding gas and slag.

These processes are versatile and widely used in construction and fabrication.

Welding Process Electrode Comparison

To summarize the types of electrodes used in these **welding processes**:

Welding Process	Electrode Type	Description
Gas Tungsten Arc Welding (GTAW)	**Non-consumable**	Uses a tungsten electrode that does not melt; filler metal is added separately.
Gas Metal Arc Welding (GMAW)	Consumable	Uses a continuously fed wire electrode that melts and becomes filler.
Submerged Arc Welding (SAW)	Consumable	Uses a bare wire electrode submerged under granular flux, which melts.
Flux Coated Arc Welding (FCAW/SMAW)	Consumable	Uses a flux-coated stick or flux-cored wire electrode that melts.

Prepp Your Personal Exams Guide

Based on the analysis, **Gas Tungsten Arc Welding** is the distinct **welding process** among the options that employs a **non-consumable electrode**.

---

35. **Answer: b**

Explanation:

Austenite Crystal Structure Explained

Prepp Download Prepp APP GET IT ON Google Play

Austenite, also known as gamma-iron (γ-Fe), is a metallic, non-magnetic allotrope of iron or a solid solution of carbon in iron with a face-centered cubic (FCC) crystal structure. This specific arrangement of atoms is crucial to its properties and behavior in various alloys, especially steel.

Face Centered Cubic (FCC) Structure of Austenite

The **crystal structure** of **austenite** is characterized by a **Face Centered Cubic** (FCC) lattice. In an FCC unit cell:

- There are atoms located at each of the eight corners of the cube.
- There is an atom located at the center of each of the six faces of the cube.

This arrangement results in a higher packing density compared to other common crystal structures like Body Centered Cubic (BCC). The coordination number for an atom in an FCC structure is 12, meaning each atom is surrounded by 12 nearest neighbors. The atomic packing factor (APF) for FCC is approximately 0.74, which is the maximum possible for spheres of uniform size.

Austenite's Role in Materials Science

Austenite is a very important phase in metallurgy, particularly in the study and heat treatment of steel. It is typically stable at high temperatures. For plain carbon steels, austenite forms when iron is heated above its critical temperature (typically above 727 °C, but it varies with carbon content). Its FCC structure allows for greater solubility of carbon compared to the Body Centered Cubic (BCC) structure of ferrite, another phase of iron. This higher carbon solubility is vital for various heat treatment processes, such as hardening and annealing, which alter the mechanical properties of steel.

Comparing Austenite's FCC with Other Crystal Structures

Understanding the **Face Centered Cubic** (FCC) structure of **austenite** is clearer when compared to other common crystal structures:

prepp Download Prepp APP GET IT ON Google Play

- **Body Centered Cubic (BCC):** This structure is found in alpha-iron (ferrite) at lower temperatures. In BCC, there are atoms at the corners and one atom at the very center of the cube. It has a lower packing density and coordination number (8) than FCC.
- **Hexagonal Close-Packed (HCP):** This structure is found in metals like zinc and titanium but is not characteristic of common iron phases. It also has a high packing density (0.74), similar to FCC, but with a different geometry.
- **Body Centered Tetragonal (BCT):** This structure is found in martensite, which is a very hard, brittle phase formed when austenite is rapidly cooled (quenched). BCT is a distorted version of BCC, where one axis is longer or shorter than the other two, often due to interstitial atoms like carbon.

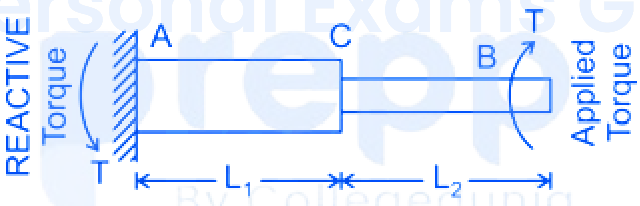
The unique **Face Centered Cubic** arrangement of **austenite** provides it with distinct properties, including increased ductility and a non-magnetic nature, which differentiate it from other iron phases.

---

36. Answer: b

Explanation:

Concept:



Shafts in series

For shafts in series, both the shafts carry the same torque T and the total angle of twist at the resisting end is the sum of separate angles of twist of two shafts

- $T_1 = T_2$
- $\theta = \theta_1 + \theta_2$

The angle of twist θ is given by,

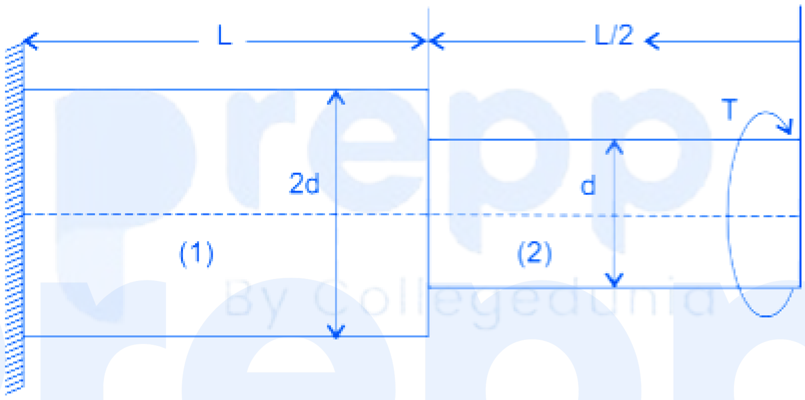
prepp Download Prepp APP GET IT ON Google Play

$\theta = \frac{TL}{GJ}$

where, T = Torque applied, L = length of the shaft, G = Modulus of rigidity J = Polar section modulus

For the solid circular shaft, $J = \frac{\pi}{32} d^4$

Calculation:
Given:



$T_1 = T_2 = T$, $L_1 = L$, $L_2 = L/2$, $d_1 = 2d$, $d_2 = d$

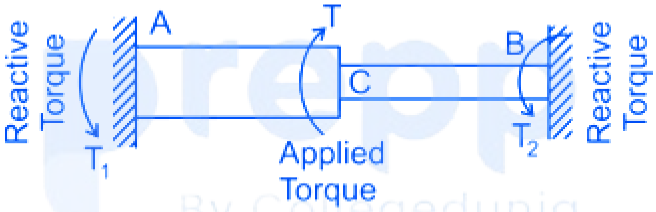
$\theta = \theta_1 + \theta_2$

$$\theta = \frac{T_1 L_1}{G J_1} + \frac{T_2 L_2}{G J_2} = \frac{TL}{G \times \frac{\pi}{32} \times (2d)^4} + \frac{T \times \frac{L}{2}}{G \times \frac{\pi}{32} \times d^4} = \frac{18TL}{G\pi d^4}$$

$$d = \left(\frac{18TL}{G\pi\theta}\right)^{\frac{1}{4}}$$

Shafts in parallel:

prepp Download Prepp APP GET IT ON Google Play



Shafts in parallel

- If two or more shafts are rigidly fixed together such that the applied torque is shared between them then the composite shaft so formed is said to be connected in parallel
- The angle of twist for both the shafts is the same
- $\theta_1 = \theta_2$ and $T = T_1 + T_2$

---

37. Answer: c

Explanation:

Concept:

The velocity of flow is given by:

$V = \sqrt{2gh}$ and $h = x(\frac{\rho_m}{\rho} - 1)$

where, h = Difference of pressure head, x = Difference of the manometric fluid level in differential U tube manometer, ρ_m = density of the manometric fluid, ρ = density of fluid flow through the pipe.

Calculation:

Given:

$\rho_m = 1000\ kg/m^3$, $\rho = 1.2\ kg/m^3$, x = 10 mm = 0.01 m

we know that,

$h = x(\frac{\rho_m}{\rho} - 1)$

$h = 0.01(\frac{1000}{1.2} - 1) = 8.323\ m$

prepp Download Prepp APP GET IT ON Google Play

Velocity of flow is:

$V = \sqrt{2gh}$

$V = \sqrt{2 \times 9.8 \ \times 8.323} = 12.77 \ m/s$

---

38. **Answer: a**

Explanation:

In a Rankine cycle, the steam turbine is a crucial component where high-pressure, high-temperature steam expands to produce work. The efficiency of a steam turbine, and consequently the overall Rankine cycle, is often assessed by parameters like specific steam consumption. This question asks us to determine the specific steam consumption in kg/kW-hour, given the enthalpy values at the inlet and outlet of the steam turbine and neglecting pump work.

To calculate the specific steam consumption, we first need to determine the work output produced by the steam turbine. The work output of the steam turbine is the difference in enthalpy between the steam entering and leaving the turbine.

Steam Turbine Work Output Calculation

Your Personal Exams Guide

The enthalpy of the steam at the inlet of the turbine (h_{inlet}) is given as 2800 kJ/kg.

The enthalpy of the steam at the outlet of the turbine (h_{outlet}) is given as 1800 kJ/kg.

The work done by the steam turbine per unit mass of steam (W_t) can be calculated using the following formula:

$$W_t = h_{inlet} - h_{outlet}$$

Substituting the given values:

$$W_t = 2800 \text{ kJ/kg} - 1800 \text{ kJ/kg}$$

$$W_t = 1000 \text{ kJ/kg}$$

Prepp Download Prepp APP GET IT ON Google Play

Since pump work is neglected, the net work output of the Rankine cycle per unit mass of steam is equal to the turbine work output.

$$\text{Net Work Output} = 1000 \text{ kJ/kg}$$

Specific Steam Consumption Determination

Specific steam consumption (SSC) is defined as the mass flow rate of steam required to produce a unit of power output. It is typically expressed in kg/kW-hour. The formula to calculate specific steam consumption is given by:

$$\text{SSC} = \frac{\text{Number of kJ/kW-hour}}{\text{Net Work Output in kJ/kg}}$$

We know that 1 kW-hour is equivalent to 3600 kJ (since 1 kW = 1 kJ/s and 1 hour = 3600 seconds, so 1 kW-hour = 1 kJ/s $\times$ 3600 s = 3600 kJ).

Using this conversion factor:

$$\text{SSC} = \frac{3600 \text{ kJ/kW-hour}}{1000 \text{ kJ/kg}}$$

$$\text{SSC} = 3.60 \text{ kg/kW-hour}$$

Specific Steam Consumption Summary

Your Personal Exams Guide

The calculation steps can be summarized as follows:

Parameter	Value	Unit
Enthalpy at turbine inlet (h_{inlet})	2800	kJ/kg
Enthalpy at turbine outlet (h_{outlet})	1800	kJ/kg
Turbine work output ($W_t = h_{inlet} - h_{outlet}$)	1000	kJ/kg
Conversion factor (1 kW-hour)	3600	kJ
Specific Steam Consumption (SSC = 3600 / W_t)	3.60	kg/kW-hour

Prepp Download Prepp APP GET IT ON Google Play

Therefore, the specific steam consumption in kg/kW-hour is 3.60.

---

39. **Answer: c**

Explanation:

To evaluate the integral \(\mathop \smallint \limits_1^3 \frac{1}{x}dx\) using Simpson's 1/3 rule with two equal subintervals, we need to follow a series of steps. Simpson's 1/3 rule is a numerical method for approximating definite integrals. It is particularly effective for functions where the data points are at equally spaced intervals.

Simpson's 1/3 Rule Formula Explained

Simpson's 1/3 rule is given by the formula:

\[\mathop \smallint \limits_a^b f(x)dx \approx \frac{h}{3} [y_0 + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) + y_n] \]

For the case of two subintervals (n=2), which is required by the problem statement, the formula simplifies significantly to:

\[\mathop \smallint \limits_a^b f(x)dx \approx \frac{h}{3} [y_0 + 4y_1 + y_2] \]

Here, h is the width of each subinterval, and y_0, y_1, y_2 are the values of the function $f(x)$ at the points x_0, x_1, x_2, respectively. Note that Simpson's 1/3 rule requires an even number of subintervals.

Integral Parameters Determination

From the given integral \(\mathop \smallint \limits_1^3 \frac{1}{x}dx\), we can identify the following parameters:

- The function is $f(x) = \frac{1}{x}$.
- The lower limit of integration is $a = 1$.
- The upper limit of integration is $b = 3$.

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

- The number of equal subintervals is $n = 2$.
- The length of each subinterval is given as $h = 1$. We can verify this using the formula $h = \frac{b-a}{n}$:

$$h = \frac{3-1}{2} = \frac{2}{2} = 1$$

This confirms the given length of each subinterval.

Function Values Calculation

Next, we need to determine the x-values for each subinterval and calculate the corresponding function values, y_i.

- For the first point, $x_0 = a = 1$.
- For the second point, $x_1 = x_0 + h = 1 + 1 = 2$.
- For the third point, $x_2 = x_1 + h = 2 + 1 = 3$. This should be equal to the upper limit b.

Now, we calculate the y values by evaluating $f(x) = \frac{1}{x}$ at these x points:

- $y_0 = f(x_0) = f(1) = \frac{1}{1} = 1$
- $y_1 = f(x_1) = f(2) = \frac{1}{2} = 0.5$
- $y_2 = f(x_2) = f(3) = \frac{1}{3} \approx 0.3333$

We can summarize these values in a table:

x_i	$y_i = f(x_i) = \frac{1}{x_i}$
1	1
2	0.5
3	0.3333...

Applying Simpson's Rule for Evaluation

Prepp Download Prepp APP GET IT ON Google Play

Now, we substitute these values into the simplified Simpson's 1/3 rule formula for $n = 2$:

\[\mathop \smallint \limits_1^3 \frac{1}{x}dx \approx \frac{h}{3} [y_0 + 4y_1 + y_2] \]

Substitute $h = 1$, $y_0 = 1$, $y_1 = 0.5$, and $y_2 = \frac{1}{3}$:

\[\mathop \smallint \limits_1^3 \frac{1}{x}dx \approx \frac{1}{3} \left[1 + 4(0.5) + \frac{1}{3} \right] \]

Perform the multiplication:

$$\approx \frac{1}{3}\left[1 + 2 + \frac{1}{3}\right]$$

Add the terms inside the brackets:

$$\approx \frac{1}{3}\left[3 + \frac{1}{3}\right]$$

Combine the terms within the bracket into a single fraction:

$$\approx \frac{1}{3}\left[\frac{9}{3} + \frac{1}{3}\right]$$

$$\approx \frac{1}{3}\left[\frac{10}{3}\right]$$

Your Personal Exams Guide

Finally, multiply the fractions:

$$\approx \frac{10}{9}$$

Converting this fraction to a decimal gives:

$$\frac{10}{9} \approx 1.1111...$$

Therefore, the integral \(\mathop \smallint \limits_1^3 \frac{1}{x}dx\), when evaluated using Simpson's 1/3 rule on two equal subintervals, equals approximately 1.111.

The final answer is 1.111.

prepp Download Prepp APP GET IT ON Google Play

40. **Answer: b**

Explanation:

When two identical ball bearings, P and Q, operate under different loads, their operational life is inversely related to the applied load. This relationship is crucial in mechanical design and bearing selection.

Bearing Life Fundamentals

The life of a ball bearing, often referred to as fatigue life or L10 life, is the number of revolutions or hours that 90% of a group of identical bearings will complete or exceed before the first evidence of fatigue develops. For ball bearings, the relationship between life (L), dynamic load capacity (C), and equivalent dynamic load (P) is given by a specific formula:

$$L = \left(\frac{C}{P}\right)^k$$

Where:

- L is the bearing life (e.g., in million revolutions or hours).
- C is the basic dynamic load rating (dynamic load capacity) of the bearing, which is constant for identical bearings.
- P is the equivalent dynamic load acting on the bearing.
- k is the life exponent. For ball bearings, $k = 3$. For roller bearings, $k = 10/3$.

Analyzing Identical Ball Bearings

Since ball bearings P and Q are identical, their basic dynamic load capacity (C) is the same. Therefore, we can write the life formula for both bearings as:

- For bearing P:

$$L_P = \left(\frac{C}{P_P}\right)^3$$

Prepp Download Prepp APP GET IT ON Google Play

- For bearing Q:

$$L_Q = \left(\frac{C}{P_Q}\right)^3$$

Applying Given Load Values

We are given the loads for bearing P and bearing Q:

Bearing	Load (P)
P	30 kN
Q	45 kN

Calculating Bearing Life Ratio

To find the ratio of the life of bearing P to the life of bearing Q (L_P/L_Q), we divide the life equation for P by the life equation for Q:

$$\frac{L_P}{L_Q} = \frac{\left(\frac{C}{P_P}\right)^3}{\left(\frac{C}{P_Q}\right)^3}$$

This simplifies to:

$$\frac{L_P}{L_Q} = \left(\frac{C}{P_P} \cdot \frac{P_Q}{C}\right)^3$$

$$\frac{L_P}{L_Q} = \left(\frac{P_Q}{P_P}\right)^3$$

Now, substitute the given load values:

$$P_P = 30 \text{ kN}$$

$$P_Q = 45 \text{ kN}$$

$$\frac{L_P}{L_Q} = \left(\frac{45 \text{ kN}}{30 \text{ kN}}\right)^3$$

Prepp Download Prepp APP GET IT ON Google Play

Simplify the fraction inside the parenthesis:

$$\frac{45}{30} = \frac{3 \times 15}{2 \times 15} = \frac{3}{2}$$

Substitute this simplified fraction back into the ratio equation:

$$\frac{L_P}{L_Q} = \left(\frac{3}{2}\right)^3$$

Calculate the cube:

$$\left(\frac{3}{2}\right)^3 = \frac{3^3}{2^3} = \frac{27}{8}$$

Therefore, the ratio of the life of bearing P to the life of bearing Q is $27/8$.

---

41. Answer: d

Explanation:

Concept:

The given linkage is a 4-bar mechanism with 4-revolute joints.

Your Personal Exams Guide

When the input and output link is parallel to each other and perpendicular to AD.

$V_{AB} = V_{CD}$

$\therefore \omega_{AB} \times AB = \omega_{CD} \times CD.$

Calculation:

Given:

ω_{AB} = 1 rad/s, CD = 1.5 × AB

$V_{AB} = V_{CD}$

Prepp Download Prepp APP GET IT ON Google Play

$\omega_{AB} \times AB = \omega_{CD} \times CD$

$\therefore 1 \times AB = \omega_{CD} \times 1.5 \times AB$

$\therefore \omega_{CD} = \frac{2}{3}$ rad/s.

---

42. Answer: b

Explanation:

Concept:

When the force applied is continuous in nature, the work done by the force for small displacement "dx" is given by:

$$W =\mathop \smallint \limits_{x_1}^{x_2} {F_x}dx$$

The energy stored in the rubber band is equal to the work done by the stone.

Calculation:

Given:

$F_x = 300x^2$, $x_1 = 0$ m, $x_2 = 0.1$ m.

Prepp Your Personal Exams Guide

Energy stored in the bar = Work done by the stone

$$W=\mathop \smallint \limits_0^{0.1} {F_x}dx = \mathop \smallint \limits_0^{0.1} 300{x^2}dx = \left[{300 \times \frac{{{x^3}}}{3}} \right]_0^{0.1} = 100 \times {0.1^3} = 0.1\ J$$

---

43. Answer: d

Explanation:

Prepp Download Prepp APP GET IT ON Google Play

Understanding the Differential Equation Problem

The problem asks us to find the **general solution** of a given **differential equation**, which involves finding the function y in terms of x and an arbitrary constant C. The specific differential equation we need to solve is:

$$\frac{dy}{dx} = (1 + y^2)\, x$$

We will use the method of separation of variables to find the solution.

Step-by-Step Solution for the Differential Equation

We follow these steps to solve the differential equation:

1. **Write the differential equation:**

$$\frac{dy}{dx} = (1 + y^2)\, x$$

2. **Separate the variables:** To solve this equation, we rearrange it so that all terms involving y are on one side with dy, and all terms involving x are on the other side with dx.

   Divide both sides by $(1 + y^2)$ and multiply by dx:

$$\frac{1}{1 + y^2}\, dy = x\, dx$$

3. **Integrate both sides:** Now, we integrate both sides of the equation.

$$\int \frac{1}{1 + y^2}\, dy = \int x\, dx$$

4. **Evaluate the integrals:**
    - The integral of $\frac{1}{1+y^2}$ with respect to y is $\tan^{-1}(y)$ (or $\arctan(y)$).
    - The integral of x with respect to x is $\frac{x^2}{2}$.

   So, the equation becomes:

$$\tan^{-1}(y) = \frac{x^2}{2} + c$$

Download Prepp APP

GET IT ON Google Play

Here, c represents the constant of integration, which is the arbitrary constant mentioned in the problem.

5. **Solve for y:** To get the final form of the general solution, we need to isolate y. We do this by applying the tangent function to both sides of the equation:

$$y = \tan\left(\frac{x^2}{2} + c\right)$$

Comparing the Solution with the Options

Now we compare our derived general solution, $y = \tan\left(\frac{x^2}{2} + c\right)$, with the given options:

- Option 1: $y = \tan\frac{x^2}{2} + \tan c$ - This is incorrect as the constant c is added inside the tangent function, not added after the tangent of $x^2/2$.
- Option 2: $y = \tan^2\left(\frac{x}{2} + c\right)$ - This is incorrect. The argument of the tangent function is $x/2$, not $x^2/2$, and the tangent is squared.
- Option 3: $y = \tan^2\left(\frac{x}{2}\right) + c$ - This is incorrect for similar reasons as option 2.
- Option 4: $y = \tan\left(\frac{x^2}{2} + c\right)$ - This exactly matches our derived general solution.

Therefore, the correct general solution for the given differential equation is represented by Option 4.

Your Personal Exams Guide

---

44. Answer: d

Explanation:

Probability Basics for Coin Tosses

When an **unbiased coin** is tossed, there are two equally likely outcomes: a **head** (H) or a **tail** (T). The probability of getting a head is $P(H) = \frac{1}{2}$, and the probability of getting a tail is $P(T) = \frac{1}{2}$.

The question involves an **unbiased coin tossed five times**. Each toss is an independent event. To find the total number of possible outcomes when tossing a

Prepp Download Prepp APP GET IT ON Google Play

coin five times, we multiply the number of outcomes for each toss:

- Number of outcomes for 1 toss = 2 (Head or Tail)
- Number of outcomes for 5 tosses = $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$

So, there are 32 possible distinct sequences of heads and tails when an **unbiased coin** is tossed five times.

At Least One Head Explained

We are asked to find the **probability of getting at least one head**. The phrase "at least one head" means that the outcome includes one head, two heads, three heads, four heads, or five heads. Calculating the probabilities for all these individual cases and then adding them can be quite lengthy.

A more efficient approach is to use the concept of **complementary probability**. The complementary event to "getting at least one head" is "getting no heads at all." If there are no heads, it means all the tosses must be tails.

The formula for complementary probability is:

$$P(\text{event}) = 1 - P(\text{complement of event})$$

In this case:

$$P(\text{at least one head}) = 1 - P(\text{no heads})$$

Calculating Probability of No Heads

The event "no heads" means all five tosses result in a **tail** (TTTTT).

Since each toss is independent, the probability of getting a tail on any single toss is $P(T) = \frac{1}{2}$. To find the probability of getting five consecutive tails, we multiply the probabilities of each individual tail:

$$P(\text{no heads}) = P(\text{TTTTT}) = P(T) \times P(T) \times P(T) \times P(T) \times P(T)$$

$$P(\text{no heads}) = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$$

Download Prepp APP
GET IT ON Google Play

$P(\text{no heads}) = \left(\frac{1}{2}\right)^5$

$P(\text{no heads}) = \frac{1}{32}$

This means there is only 1 outcome (TTTTT) out of the 32 total possible outcomes where you get no heads.

Final Probability Calculation

Now, we can use the complementary probability formula to find the **probability of getting at least one head**:

$P(\text{at least one head}) = 1 - P(\text{no heads})$

$P(\text{at least one head}) = 1 - \frac{1}{32}$

To subtract, we find a common denominator:

$P(\text{at least one head}) = \frac{32}{32} - \frac{1}{32}$

$P(\text{at least one head}) = \frac{32-1}{32}$

$P(\text{at least one head}) = \frac{31}{32}$

Therefore, the probability of getting at least one head when an **unbiased coin** is **tossed five times** is $\frac{31}{32}$.

Your Personal Exams Guide

---

45. **Answer: a**

Explanation:

<u>Concept:</u>

Natural frequency of the system

$\omega_n = \sqrt{\frac{k_e}{m}}\, rad/s$ and $f_n = \frac{\omega_n}{2\pi}\, Hz$

where k_e = equivalent stiffness, m = mass attach to the system

Prepp Download Prepp APP GET IT ON Google Play

Calculation:

Given:

k = 20 kN/m = 20000 N/m, m = 1 kg

As the arrangement of springs are in parallel,

therefore, k_e = k + k = 2 k = 40000 N/m

$\omega_n = \sqrt{\frac{40 \times 1000}{1}} = 200$ rad/s

And

$f_n = \frac{200}{2\pi} = 31.83\ Hz \approx 32\ Hz$

---

46. **Answer: c**

Explanation:

Blanking Force Calculation for Sheet Metal

Understanding the forces involved in sheet metal operations like blanking is crucial in manufacturing engineering. Blanking is a cutting process where a punch cuts a desired shape (the blank) from a larger piece of sheet metal. The force required for this operation primarily depends on the material's shear strength, the perimeter of the cut, and the thickness of the sheet.

Given Data for Blanking Operation

To calculate the blanking force, we are provided with the following parameters:

- **Shear strength** of the sheet metal (τ_s) = 300 MPa
- **Diameter** of the blank (D) = 100 mm
- **Thickness** of the sheet (t) = 1.5 mm

Formula for Blanking Force

Prepp Download Prepp APP GET IT ON Google Play

The total blanking force (F) required for a blanking operation can be calculated using the following formula:

$$F = \tau_s \times P \times t$$

Where:

- F is the blanking force in Newtons (N).
- τ_s is the shear strength of the material in Pascals (Pa) or N/mm^2.
- P is the perimeter of the blank in millimeters (mm).
- t is the thickness of the sheet in millimeters (mm).

For a circular blank, the perimeter (P) is given by the formula:

$$P = \pi \times D$$

Where D is the diameter of the blank.

Step-by-Step Blanking Force Calculation

Let's calculate the blanking force using the given values.

1. Calculate the Perimeter of the Blank

Given the diameter (D) = 100 mm, the perimeter (P) will be:

$$P = \pi \times D$$

$$P = 3.14159 \times 100 \text{ mm}$$

$$P \approx 314.159 \text{ mm}$$

2. Convert Shear Strength Units (if necessary)

The shear strength is given as 300 MPa. Since 1 MPa = 1 N/mm^2, we can use it directly in N/mm^2:

$$\tau_s = 300 \text{ MPa} = 300 \text{ N/mm}^2$$

3. Calculate the Blanking Force

Prepp Download Prepp APP GET IT ON Google Play

Now, we substitute the values into the blanking force formula:

$$F = \tau_s \times P \times t$$

$$F = 300 \text{ N/mm}^2 \times 314.159 \text{ mm} \times 1.5 \text{ mm}$$

$$F = 141371.55 \text{ N}$$

4. Convert the Force to KiloNewtons (kN)

Since the options are in kN, we convert the calculated force from Newtons to KiloNewtons (1 kN = 1000 N):

$$F = \frac{141371.55}{1000} \text{ kN}$$

$$F \approx 141.37 \text{ kN}$$

Conclusion on Blanking Force

The calculated blanking force is approximately 141.37 kN. Comparing this value with the provided options:

- 45 kN
- 70 kN
- 141 kN
- 3500 kN

The value 141.37 kN is very close to 141 kN. Therefore, the blanking force required is close to **141 kN**.

---

47. **Answer: a**

Explanation:

Understanding Laminar Flow Characteristics and Dimensionless Numbers

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

This question focuses on the relationship between hydrodynamic and thermal boundary layers in laminar flow over a flat plate. We need to determine the Prandtl number (Pr) and Nusselt number (Nu) for fluid Q, given information about fluid P and the relevant physical principles.

Key Concepts Explained

- **Hydrodynamic Boundary Layer Thickness (δ_{hydro}):** This is the distance from the surface where the fluid velocity reaches approximately 99% of the free stream velocity. It's influenced by viscosity.
- **Thermal Boundary Layer Thickness (δ_{th}):** This is the distance from the surface where the fluid temperature difference reaches approximately 99% of the difference between the surface temperature and the free stream temperature. It's influenced by thermal diffusivity.
- **Reynolds Number (Re):** Defined as $Re = \frac{\rho UL}{\mu}$, it represents the ratio of inertial forces to viscous forces, indicating the flow regime. Here, $Re = 10^4$ signifies laminar flow.
- **Prandtl Number (Pr):** Defined as $Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$, it's the ratio of momentum diffusivity to thermal diffusivity. It links the hydrodynamic and thermal boundary layer behavior.
- **Nusselt Number (Nu):** Defined as $Nu = \frac{hL}{k}$, it represents the ratio of convective heat transfer to conductive heat transfer across the boundary layer.

Prepp Your Personal Exams Guide

Governing Relationships

For laminar flow over a flat plate, two key relationships are generally used:

1. **Thickness Ratio and Prandtl Number:** The ratio of the hydrodynamic boundary layer thickness to the thermal boundary layer thickness is related to the Prandtl number:

$$\frac{\delta_{hydro}}{\delta_{th}} \approx Pr^{\frac{1}{3}}$$

2. **Nusselt Number Correlation:** The average Nusselt number is typically correlated with the Reynolds and Prandtl numbers as:

Prepp Download Prepp APP GET IT ON Google Play

$$Nu_L = K \cdot Re_L^{\frac{1}{2}} \cdot Pr^{\frac{1}{3}}$$

where K is a constant.

Analyzing Fluid P Data

We are given the following data for fluid P:

- Ratio: $\frac{\delta_{hydro,P}}{\delta_{th,P}} = \frac{1}{2}$
- Reynolds Number: $Re_P = 10^4$
- Prandtl Number: $Pr_P = \frac{1}{8}$
- Nusselt Number: $Nu_P = 35$

First, let's verify the thickness ratio using the Prandtl number:

$$Pr_P^{\frac{1}{3}} = \left(\frac{1}{8}\right)^{\frac{1}{3}} = \frac{1}{2}$$

This confirms that the relationship $\frac{\delta_{hydro}}{\delta_{th}} \approx Pr^{\frac{1}{3}}$ holds for fluid P.

Next, we use the Nusselt number correlation to find the constant K:

$$Nu_P = K \cdot Re_P^{\frac{1}{2}} \cdot Pr_P^{\frac{1}{3}}$$

Substituting the values for fluid P:

Your Personal Exams Guide

$$35 = K \cdot (10^4)^{\frac{1}{2}} \cdot \left(\frac{1}{8}\right)^{\frac{1}{3}}$$

$$35 = K \cdot 100 \cdot \frac{1}{2}$$

$$35 = K \cdot 50$$

Solving for K:

$$K = \frac{35}{50} = 0.7$$

Determining Properties for Fluid Q

We are given the following data for fluid Q:

Prepp Download Prepp APP GET IT ON Google Play

- Ratio: $\frac{\delta_{hydro,Q}}{\delta_{th,Q}} = 2$
- Reynolds Number: $Re_Q = 10^4$

1. Calculate Prandtl Number (Pr_Q)

Using the thickness ratio relationship:

$$\frac{\delta_{hydro,Q}}{\delta_{th,Q}} \approx Pr_Q^{\frac{1}{3}}$$

Substituting the given ratio:

$$2 = Pr_Q^{\frac{1}{3}}$$

To find Pr_Q, we cube both sides:

$$Pr_Q = 2^3 = 8$$

2. Calculate Nusselt Number (Nu_Q)

Now, we use the Nusselt number correlation with the determined constant $K = 0.7$, $Re_Q = 10^4$, and the calculated $Pr_Q = 8$:

$$Nu_Q = K \cdot Re_Q^{\frac{1}{2}} \cdot Pr_Q^{\frac{1}{3}}$$

$$Nu_Q = 0.7 \cdot (10^4)^{\frac{1}{2}} \cdot (8)^{\frac{1}{3}}$$

$$Nu_Q = 0.7 \cdot 100 \cdot 2$$

$$Nu_Q = 0.7 \cdot 200$$

$$Nu_Q = 140$$

Final Results and Option Matching

For fluid Q, the calculated Prandtl number is $Pr_Q = 8$ and the Nusselt number is $Nu_Q = 140$.

Comparing these results with the provided options:

- Option 1: 8 and 140
- Option 2: 8 and 70

Prepp Download Prepp APP GET IT ON Google Play

- Option 3: 4 and 70
- Option 4: 4 and 35

Option 1 provides the correct values for the Prandtl and Nusselt numbers for fluid Q.

---

48. **Answer: d**

Explanation:

Swept Volume Calculation for an IC Engine

The swept volume of an internal combustion (IC) engine cylinder is a crucial parameter that indicates the volume displaced by the piston as it moves from its bottom-most position (Bottom Dead Center - BDC) to its top-most position (Top Dead Center - TDC). This volume is directly related to the engine's capacity or displacement.

Engine Parameters Provided

We are given the following dimensions for the single-cylinder IC engine:

- **Crank Radius (r):** 60 mm
- **Cylinder Diameter (D):** 80 mm

We need to calculate the swept volume of the cylinder in cubic centimeters (cm^3).

Converting Units to Centimeters

Since the final answer is required in cm^3, it is necessary to convert the given dimensions from millimeters (mm) to centimeters (cm) before performing calculations. We know that 1 cm = 10 mm.

- **Crank Radius (r):** $60 \text{ mm} = \frac{60}{10} \text{ cm} = 6 \text{ cm}$
- **Cylinder Diameter (D):** $80 \text{ mm} = \frac{80}{10} \text{ cm} = 8 \text{ cm}$

Determining Piston Stroke

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

For a single-cylinder internal combustion engine, the piston stroke (L) is the total distance the piston travels from BDC to TDC. This distance is precisely twice the crank radius (r).

Piston Stroke (L): $L = 2 \times r$

Substituting the converted crank radius value:

$L = 2 \times 6 \text{ cm}$

$L = 12 \text{ cm}$

Calculating Cylinder Bore Area

The cross-sectional area of the cylinder bore (A) is the area of the circle formed by the cylinder's inner wall. It can be calculated using the formula for the area of a circle, which uses the cylinder diameter (D).

Cylinder Bore Area (A): $A = \frac{\pi D^2}{4}$

Substituting the converted cylinder diameter value:

$A = \frac{\pi \times (8 \text{ cm})^2}{4}$

$A = \frac{\pi \times 64 \text{ cm}^2}{4}$

$A = 16\pi \text{ cm}^2$

Final Swept Volume Calculation

The swept volume (V_s) is the product of the cylinder bore area (A) and the piston stroke (L). This volume represents the displacement of the piston during one complete stroke within the cylinder.

Swept Volume (V_s): $V_s = A \times L$

Substituting the calculated values for A and L:

$V_s = (16\pi \text{ cm}^2) \times (12 \text{ cm})$

Prepp Download Prepp APP GET IT ON Google Play

$V_s = 192\pi \text{ cm}^3$

Using the approximate value of $\pi \approx 3.14159$:

$V_s = 192 \times 3.14159 \text{ cm}^3$

$V_s \approx 603.185 \text{ cm}^3$

Rounding the result to the nearest whole number, the swept volume of the cylinder is approximately 603 cm^3.

Summary of Key Engine Dimensions and Calculations

Parameter	Given Value (mm)	Converted Value (cm)
Crank Radius (r)	60	6
Cylinder Diameter (D)	80	8
Piston Stroke $(L = 2r)$	120	12

Calculation Step	Formula Used	Result
Cylinder Bore Area (A)	$A = \frac{\pi D^2}{4}$	$16\pi \text{ cm}^2$
Swept Volume (V_s)	$V_s = A \times L$	$192\pi \text{ cm}^3 \approx 603 \text{ cm}^3$

Thus, the swept volume of the cylinder for the given IC engine is 603 cm^3.

---

49. **Answer: d**

Explanation:

To determine the isentropic specific work done by a pump that handles a liquid, we utilize a fundamental principle of fluid mechanics and thermodynamics. Since the liquid is considered incompressible, its density remains constant throughout the

Prepp Download Prepp APP GET IT ON Google Play

pumping process. The specific work done by the pump in an isentropic (reversible and adiabatic) process can be calculated based on the change in pressure and the liquid's specific volume.

Pump Work Formula for Incompressible Fluids

For an incompressible fluid undergoing an isentropic process, the specific work done (work per unit mass) is given by the product of the specific volume and the change in pressure. The formula is:

$$W = \nu \Delta P = \frac{1}{\rho}(P_2 - P_1)$$

Where:

- W represents the isentropic specific work done by the pump, typically expressed in Joules per kilogram (J/kg) or kiloJoules per kilogram (kJ/kg).
- ν is the specific volume of the liquid, which is the reciprocal of its density $(1/\rho)$ and is measured in cubic meters per kilogram (m^3/kg).
- ρ is the density of the liquid, given in kilograms per cubic meter (kg/m^3).
- P_1 is the initial pressure of the liquid before entering the pump, measured in Pascals (Pa).
- P_2 is the final pressure of the liquid after leaving the pump, also measured in Pascals (Pa).
- ΔP is the pressure difference, calculated as $P_2 - P_1$.

Liquid Pump Given Parameters

Based on the question, we are provided with the following parameters:

- Initial pressure of the liquid, $P_1 = 1 \text{ bar}$
- Final pressure of the liquid, $P_2 = 30 \text{ bar}$
- Density of the liquid, $\rho = 990 \text{ kg/m}^3$

Pressure Conversion Steps

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Before applying the formula, it is essential to ensure all units are consistent. Pressure is given in bar, so we must convert it to Pascals (Pa), which is the standard unit in SI for pressure ($1 \text{ bar} = 10^5 \text{ Pa}$).

- Initial pressure: $P_1 = 1 \text{ bar} = 1 \times 10^5 \text{ Pa}$
- Final pressure: $P_2 = 30 \text{ bar} = 30 \times 10^5 \text{ Pa}$

Next, we calculate the pressure difference (ΔP):

$$\Delta P = P_2 - P_1 = (30 \times 10^5 \text{ Pa}) - (1 \times 10^5 \text{ Pa})$$

$$\Delta P = (30 - 1) \times 10^5 \text{ Pa} = 29 \times 10^5 \text{ Pa}$$

Isentropic Work Calculation

Now we can substitute the density and the calculated pressure difference into the specific work formula:

$$W = \frac{1}{\rho} \Delta P$$

$$W = \frac{1}{990 \text{ kg/m}^3} \times (29 \times 10^5 \text{ Pa})$$

$$W = \frac{29 \times 10^5}{990} \text{ J/kg}$$

Your Personal Exams Guide

Performing the division:

$$W \approx 2929.2929 \text{ J/kg}$$

Work Unit Conversion to kJ/kg

The question specifically asks for the work done in kiloJoules per kilogram (kJ/kg). To convert from Joules per kilogram (J/kg) to kiloJoules per kilogram (kJ/kg), we divide by 1000 (since 1 kJ = 1000 J):

$$W_{\text{kJ/kg}} = \frac{2929.2929 \text{ J/kg}}{1000}$$

$$W_{\text{kJ/kg}} \approx 2.9292929 \text{ kJ/kg}$$

Prepp Download Prepp APP GET IT ON Google Play

Rounding the result to two decimal places, the isentropic specific work done by the pump is 2.93 kJ/kg.

Summary of Isentropic Specific Work Calculation

Parameter	Value	Unit
Initial Pressure (P_1)	1	bar
Final Pressure (P_2)	30	bar
Density (ρ)	990	kg/m^3
Pressure Difference (ΔP)	29×10^5	Pa
Isentropic Specific Work (W)	2.93	kJ/kg

Prepp

Thus, the isentropic specific work done by the pump is 2.93 kJ/kg.

---

50. **Answer: d**

Explanation:

Your Personal Exams Guide

Steel Ball Temperature Difference Analysis

This problem involves understanding heat transfer mechanisms, specifically conduction within a spherical steel ball and convection from its surface to the surrounding environment. The key to determining the temperature difference between the center and the surface of the steel ball lies in evaluating the Biot number. The Biot number helps us compare how easily heat moves inside the object versus how easily it moves from the object's surface to the surroundings.

Heat Transfer Key Concepts

Prepp Download Prepp APP GET IT ON Google Play

- **Conduction Resistance:** This represents the opposition to heat flow through a material by conduction. If a material has high thermal conductivity (like steel), heat can conduct very easily through it, meaning its internal conduction resistance is low. This leads to a small internal temperature difference within the material.
- **Convection Resistance:** This represents the opposition to heat flow from a solid surface to a surrounding fluid (liquid or gas). If the heat transfer coefficient is high, the convection resistance is low, meaning heat can easily transfer away from the surface.
- **Biot Number (Bi):** The Biot number is a dimensionless quantity that compares the internal thermal resistance of a body to the external thermal resistance at its surface. It helps in deciding whether the lumped capacitance analysis can be applied, which assumes uniform temperature throughout the body. Mathematically, the Biot number is given by:

$$Bi = \frac{\text{Internal Conduction Resistance}}{\text{External Convection Resistance}} = \frac{hL_c}{k}$$

Here, h is the convective heat transfer coefficient, L_c is the characteristic length of the body, and k is the thermal conductivity of the body material.

For a sphere, the characteristic length L_c is defined as the ratio of its volume (V) to its surface area (A_s):

$$L_c = \frac{V}{A_s} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{D}{6}$$

Where r is the radius and D is the diameter of the sphere.

Significance of Biot Number:

If $Bi \ll 0.1$, the internal conduction resistance is much smaller than the external convection resistance. This implies that temperature gradients within the body are negligible, and the lumped capacitance method can be used, meaning the temperature throughout the body is nearly uniform.

If $Bi > 0.1$, the internal conduction resistance is significant, and there will be a noticeable temperature gradient within the body.

Prepp Download Prepp APP GET IT ON Google Play

Steel Ball Problem Parameters

Parameter	Value
Diameter of steel ball (D)	12 mm = 0.012 m
Convective heat transfer coefficient (h)	5 W/m²K
Thermal conductivity of steel (k)	20 W/mK
Initial temperature	1000 K
Surrounding temperature	300 K

Biot Number Calculation for the Steel Ball

First, let's calculate the characteristic length (L_c) for the spherical steel ball:

$$L_c = \frac{D}{6} = \frac{0.012 \text{ m}}{6} = 0.002 \text{ m}$$

Now, we can calculate the Biot number (Bi) using the given values:

$$Bi = \frac{hL_c}{k} = \frac{(5 \text{ W/m}^2\text{K}) \times (0.002 \text{ m})}{20 \text{ W/mK}}$$

$$Bi = \frac{0.01}{20} = 0.0005$$

Biot Number Interpretation

The calculated Biot number is $Bi = 0.0005$.

- Since $0.0005 \ll 0.1$, this indicates that the internal conduction resistance of the steel ball is significantly smaller than the external convective resistance at its surface.
- This means that heat can conduct very quickly through the steel material, ensuring that the temperature throughout the ball remains nearly uniform at

Prepp Download Prepp APP GET IT ON Google Play

any given instant.
- The major temperature drop will occur at the interface between the steel ball's surface and the surrounding fluid due to the higher convective resistance.
- Consequently, the temperature difference between the center and the surface of the steel ball will be very small. The steel ball essentially acts as if its temperature is uniform throughout its volume as it cools. This condition allows for the use of the lumped capacitance method in transient heat transfer analysis.

Temperature Difference Conclusion

Based on the calculated Biot number of 0.0005, which is much less than 0.1, it is evident that the internal conduction resistance of the spherical steel ball is far less than the external convective resistance. This means heat transfers very efficiently within the ball compared to how it transfers from the ball's surface to the surrounding. Therefore, the temperature difference between the center and the surface of the steel ball will be **small**.

---

51. **Answer: a**

Explanation:

Brayton Cycle Analysis: Determining Compression and Expansion Temperatures

An ideal Brayton cycle is a thermodynamic cycle that describes the operation of constant-pressure heat engines. It is often used to model gas turbines. The cycle consists of four key processes: isentropic compression, constant pressure heat addition, isentropic expansion, and constant pressure heat rejection.

Given Parameters for the Brayton Cycle

We are provided with the following parameters for the ideal Brayton cycle:

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Parameter	Symbol	Value
Minimum Pressure (Inlet to Compressor)	P_1	1 bar
Maximum Pressure (Outlet from Compressor / Inlet to Turbine)	$P_2 = P_3$	6 bar
Minimum Temperature (Inlet to Compressor)	T_1	300 K
Maximum Temperature (Inlet to Turbine)	T_3	1500 K
Ratio of Specific Heats of Working Fluid	γ	1.4

Our goal is to find the temperature at the end of the compression process (T_2) and the temperature at the end of the expansion process (T_4).

Isentropic Processes in Brayton Cycle

For an ideal Brayton cycle, both the compression (process 1-2) and expansion (process 3-4) are considered isentropic processes. For an isentropic process, the relationship between temperature and pressure is given by:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

And for the expansion process:

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$$

In an ideal Brayton cycle, the pressure ratio for compression (P_2/P_1) is equal to the pressure ratio for expansion (P_3/P_4), because the heat addition and rejection occur at constant pressure. Let's denote this pressure ratio as r_p.

$$r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

Calculating Temperatures

Pressure Ratio Calculation

Prepp Download Prepp APP GET IT ON Google Play

First, let's calculate the pressure ratio (r_p):

- $P_1 = 1 \text{ bar}$
- $P_2 = 6 \text{ bar}$
- $$r_p = \frac{P_2}{P_1} = \frac{6 \text{ bar}}{1 \text{ bar}} = 6$$

Exponent Calculation

Next, we calculate the exponent $\frac{\gamma-1}{\gamma}$:

- $\gamma = 1.4$
- $$\frac{\gamma - 1}{\gamma} = \frac{1.4 - 1}{1.4} = \frac{0.4}{1.4} = \frac{4}{14} = \frac{2}{7} \approx 0.2857$$

Compression Temperature (T_2) Calculation

Now, we can find the temperature at the end of the compression process, T_2, using the isentropic relation:

- $T_1 = 300 \text{ K}$
- $$T_2 = T_1 \times (r_p)^{\frac{\gamma-1}{\gamma}}$$
- $$T_2 = 300 \text{ K} \times (6)^{\frac{2}{7}}$$
- $$T_2 \approx 300 \text{ K} \times 1.66816$$
- $$T_2 \approx 500.448 \text{ K}$$
- Rounding to the nearest whole number, the approximate final temperature at the end of the compression process is **500 K**.

Expansion Temperature (T_4) Calculation

Finally, we find the temperature at the end of the expansion process, T_4, using the isentropic relation. Since $\frac{T_3}{T_4} = r_p^{\frac{\gamma-1}{\gamma}}$, we can write $T_4 = \frac{T_3}{r_p^{\frac{\gamma-1}{\gamma}}}$.

Prepp Download Prepp APP GET IT ON Google Play

- $T_3 = 1500\ \text{K}$
- $$T_4 = \frac{T_3}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$
- $$T_4 = \frac{1500\ \text{K}}{(6)^{\frac{2}{7}}}$$
- $$T_4 \approx \frac{1500\ \text{K}}{1.66816}$$
- $$T_4 \approx 899.19\ \text{K}$$
- Rounding to the nearest whole number, the approximate final temperature at the end of the expansion process is **900 K**.

Summary of Final Temperatures

Based on our calculations, the approximate final temperatures are:

- Temperature at the end of compression (T_2): 500 K
- Temperature at the end of expansion (T_4): 900 K

Therefore, the final temperatures at the end of the compression and expansion processes are respectively 500 K and 900 K.

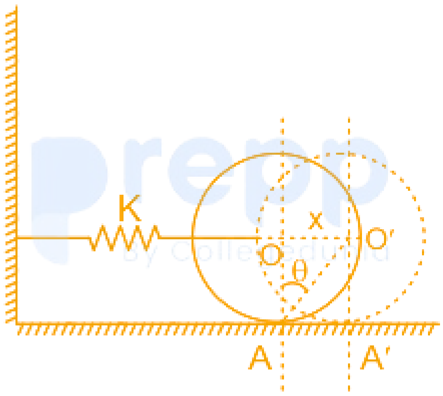
Prepp Your Personal Exams Guide

52. Answer: c

Explanation:

Calculation:

Prepp Download Prepp APP GET IT ON Google Play



Taking moments about instantaneous centre 'A'

I A$\ddot{\theta}$ + (kx) r = 0

⇒ (I$_O$+ mr^2) $\ddot{\theta}$ + k (θr)r = 0

$$\Rightarrow \left(\frac{1}{2}mr^2 + mr^2\right)\ddot{\theta} + k\left(\theta r^2\right) = 0$$
$$\Rightarrow \ddot{\theta} + \frac{kr^2}{\frac{3}{2}mr^2}\theta = 0 \Rightarrow \ddot{\theta} + \frac{2k}{3m}\theta = 0; \ \therefore f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$$

Alternate Method

Total energy of the system is:

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$$
$$E = \frac{1}{2}kx^2 + \frac{1}{2}mV^2 + \frac{1}{2}\frac{Mr^2}{2} \times \frac{V^2}{r^2}$$
$$E = \frac{1}{2}kx^2 + \frac{\frac{3}{2}}{2}mV^2$$
$$\frac{dE}{dt} = 0 \Rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{k}{\frac{3}{2}m}}$$
$$\Rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{2k}{3m}}$$

★ **Shortcut Trick**

For a cylinder of radius 'r' with surface contact at point 'p',

natural frequency is given by

$$\mathbf{f_n} = \frac{1}{2\pi}\sqrt{\frac{\mathbf{k}\cdot(\mathbf{r})^2}{\frac{3}{2}\mathbf{mr}^2}} = \frac{1}{2\pi}\sqrt{\frac{2\mathrm{k}}{3\mathrm{m}}}$$

Download Prepp APP
GET IT ON Google Play

53. **Answer: b**

Explanation:

Concept:

The friction force is given by:

f = μN

where **μ** is the coefficient of friction between the surfaces in contact, **N** is the normal force perpendicular to friction force.

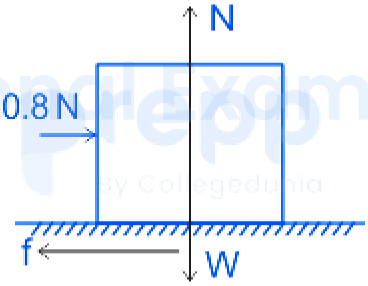
Calculation:

Given:

μ = 0.1, m = 1 kg, F = 0.8 N

Now, we know that

From the FBD as shown below



Normal reaction, N = mg = 1 × 9.81 = 9.81 N

Limiting friction force between the block and the surface, f = μN = 0.1 × 9.81 = 0.98 N

But the applied force is 0.8 N which is less than the limiting friction force.

∴ The friction force for the given case is 0.8 N.

Prepp Download Prepp APP GET IT ON Google Play

54. **Answer: c**

Explanation:

System of Equations Analysis

To determine the nature of solutions for a given system of linear equations, we first need to represent it in matrix form. The given system is a homogeneous system because all the constant terms on the right-hand side are zero.

The system of equations is:

1. $2x_1 + x_2 + x_3 = 0$
2. $x_2 - x_3 = 0$
3. $x_1 + x_2 = 0$

This system can be written in the matrix form $AX = 0$, where A is the coefficient matrix, X is the column vector of variables, and 0 is the zero vector.

Coefficient Matrix Formulation

Let's form the coefficient matrix A from the given system of equations:

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

Determinant Calculation

For a homogeneous system of linear equations $AX = 0$, the nature of the solution depends on the determinant of the coefficient matrix A:

- If $\det(A) \neq 0$, the system has a **unique solution**, which is always the trivial solution $(x_1 = 0, x_2 = 0, x_3 = 0)$.

Prepp Download Prepp APP GET IT ON Google Play

- If $\det(A) = 0$, the system has **infinite number of solutions**, including the trivial solution and non-trivial solutions.

Let's calculate the determinant of matrix A:

$$\det(A) = 2\begin{vmatrix}1 & -1 \\ 1 & 0\end{vmatrix} - 1\begin{vmatrix}0 & -1 \\ 1 & 0\end{vmatrix} + 1\begin{vmatrix}0 & 1 \\ 1 & 1\end{vmatrix}$$

Breaking down the calculation:

- For the first term: $2 \times ((1)(0) - (-1)(1)) = 2 \times (0+1) = 2 \times 1 = 2$
- For the second term: $-1 \times ((0)(0) - (-1)(1)) = -1 \times (0+1) = -1 \times 1 = -1$
- For the third term: $+1 \times ((0)(1) - (1)(1)) = +1 \times (0-1) = 1 \times (-1) = -1$

Adding these values together:

$$\det(A) = 2 + (-1) + (-1) = 2 - 1 - 1 = 0$$

Since the determinant of the coefficient matrix $\det(A) = 0$, the system of equations has an **infinite number of solutions**.

Solving the System (Optional Verification)

We can further verify this by solving the system. From the equations:

- From equation (2): $x_2 - x_3 = 0 \implies x_2 = x_3$
- From equation (3): $x_1 + x_2 = 0 \implies x_1 = -x_2$

Your Personal Exams Guide

Now, substitute $x_2 = x_3$ into $x_1 = -x_2$, which gives $x_1 = -x_3$.

Substitute $x_1 = -x_3$ and $x_2 = x_3$ into equation (1):

$$\begin{gathered} 2x_1 + x_2 + x_3 = 0 \\ 2(-x_3) + (x_3) + x_3 = 0 \\ -2x_3 + 2x_3 = 0 \\ 0 = 0 \end{gathered}$$

Since we arrived at an identity $(0 = 0)$, it indicates that the equations are linearly dependent, and there are free variables. Let's express the solutions in terms of a parameter, say k.

Prepp Download Prepp APP GET IT ON Google Play

Let $x_3 = k$, where k is any real number.

- Then, from $x_2 = x_3$, we get $x_2 = k$.
- And from $x_1 = -x_2$, we get $x_1 = -k$.

Thus, the solutions are of the form $(x_1, x_2, x_3) = (-k, k, k)$ for any real number k. Since k can take infinitely many real values, there are infinitely many solutions to this system of equations.

---

55. **Answer: b**

Explanation:

Shear Angle Calculation in Orthogonal Machining

The question asks us to determine the **shear angle** in an orthogonal machining operation. We are provided with the cutting tool's **rake angle**, the **uncut thickness** (also known as depth of cut), and the resulting **chip thickness**. Understanding these parameters is crucial for analyzing metal cutting processes.

In orthogonal machining, material is removed by a cutting tool that moves perpendicular to the cutting edge. The deformation of the material occurs primarily along a plane called the shear plane, and the angle this plane makes with the cutting direction is the **shear angle** (ϕ).

Given Parameters for Machining

Let's list the known values provided in the problem statement for the machining of the steel workpiece:

- **Rake Angle (α):** This is the angle between the rake face of the cutting tool and a plane perpendicular to the cutting velocity vector. Here, $\alpha = 12^\circ$.
- **Uncut Thickness (t_1):** This is the thickness of the material before it is cut, sometimes referred to as the depth of cut in orthogonal machining. Here, $t_1 = 0.81 \text{ mm}$.

Prepp Download Prepp APP GET IT ON Google Play

- **Chip Thickness (t_2):** This is the thickness of the chip formed after the material has been cut and deformed. Here, $t_2 = 1.8 \text{ mm}$.

Chip Thickness Ratio Determination

Before calculating the **shear angle**, we first need to determine the **chip thickness ratio**, often denoted as r. The chip thickness ratio is a fundamental concept in metal cutting and represents the ratio of the uncut chip thickness to the chip thickness. It helps quantify the amount of deformation that the material undergoes during the cutting process.

The formula for the chip thickness ratio (r) is given by:

$$r = \frac{\text{Uncut Thickness (depth of cut)}}{\text{Chip Thickness}} = \frac{t_1}{t_2}$$

Substituting the given values for t_1 and t_2:

$$r = \frac{0.81 \text{ mm}}{1.8 \text{ mm}}$$

$$r = 0.45$$

So, the chip thickness ratio for this orthogonal machining operation is 0.45.

Shear Angle Calculation Steps

Now that we have the **chip thickness ratio** (r) and the **rake angle** (α), we can use a widely accepted formula to calculate the **shear angle** (ϕ). This formula is derived from geometric considerations of the cutting process and relates these three key parameters:

$$\tan\phi = \frac{r\cos\alpha}{1 - r\sin\alpha}$$

We have $r = 0.45$ and $\alpha = 12^\circ$. Let's find the values of $\cos(12^\circ)$ and $\sin(12^\circ)$:

- $\cos(12^\circ) \approx 0.9781$
- $\sin(12^\circ) \approx 0.2079$

Now, substitute these values into the formula for $\tan\phi$:

Prepp Download Prepp APP GET IT ON Google Play

$$\tan\phi = \frac{0.45 \times 0.9781}{1 - 0.45 \times 0.2079}$$

$$\tan\phi = \frac{0.440145}{1 - 0.093555}$$

$$\tan\phi = \frac{0.440145}{0.906445}$$

$$\tan\phi \approx 0.48557$$

To find the **shear angle** (ϕ), we take the inverse tangent (arctan) of this value:

$$\phi = \arctan(0.48557)$$

$$\phi \approx 25.9^\circ$$

Rounding this value to the nearest whole degree, the approximate **shear angle** is 26°.

Summary of Orthogonal Machining Parameters

Parameter	Value
Rake Angle (α)	12°
Uncut Thickness (t_1)	0.81 mm
Chip Thickness (t_2)	1.8 mm
Calculated Chip Thickness Ratio (r)	0.45
Calculated Shear Angle (ϕ)	26° (approximately)

Based on our calculations, the **shear angle** under the given **orthogonal machining** conditions is approximately 26°. This value is essential for understanding the mechanics of chip formation and optimizing cutting processes for better performance and tool life.

Prepp Download Prepp APP GET IT ON Google Play

56. **Answer: a**

Explanation:

Non-traditional machining processes are advanced manufacturing methods used to machine difficult-to-cut materials or to achieve complex geometries that are challenging with conventional machining. Each process utilizes a unique mechanism for removing material from the workpiece. Understanding these mechanisms is crucial for selecting the appropriate process for a given application.

Let's delve into the specific material removal mechanisms for each non-traditional machining process mentioned:

Chemical Machining Mechanism

Chemical machining (CM) is a material removal process where material is removed by controlled chemical dissolution. The workpiece is exposed to a chemical reagent, known as an etchant, which reacts with the material to form soluble products that are then washed away. This process is fundamentally a **corrosive reaction**, where the material is dissolved at the atomic or molecular level.

- **Process:** Chemical Machining (CM)
- **Mechanism:** Corrosive reaction (material dissolution by chemical etchants).

Prepp
Your Personal Exams Guide

Electrochemical Machining Mechanism

Electrochemical machining (ECM) is based on the principle of reverse electroplating or electrolytic dissolution. The workpiece acts as an anode, and the tool acts as a cathode, separated by a flowing electrolyte. When a DC current is passed through the electrolyte, workpiece material is removed atom by atom through an electrochemical reaction, leading to **ion displacement** from the workpiece surface into the electrolyte.

- **Process:** Electrochemical Machining (ECM)
- **Mechanism:** Ion displacement (electrochemical dissolution).

Prepp Download Prepp APP GET IT ON Google Play

Electro-discharge Machining Mechanism

Electro-discharge machining (EDM), also known as spark machining, removes material through a series of rapid, repetitive electrical discharges between an electrode (tool) and the workpiece. These discharges generate intense localized heat, causing the workpiece material to melt and vaporize. The molten and vaporized material is then flushed away by the dielectric fluid. Therefore, the primary material removal mechanism for EDM is **fusion and vaporization**.

- **Process:** Electro-discharge Machining (EDM)
- **Mechanism:** Fusion and vaporization (thermal energy from sparks).

Ultrasonic Machining Mechanism

Ultrasonic machining (USM) utilizes high-frequency vibrations of a tool to abrade material from the workpiece. Abrasive particles suspended in a liquid slurry are introduced between the vibrating tool and the workpiece. The tool's high-frequency vibrations cause the abrasive particles to impact the workpiece surface, leading to micro-chipping, abrasion, and brittle fracture of the material. This action is best described as **erosion**.

- **Process:** Ultrasonic Machining (USM)
- **Mechanism:** Erosion (abrasive action of suspended particles).

Prepp
Your Personal Exams Guide

Machining Process & Material Removal Summary

To summarize the relationships between the non-traditional machining processes and their material removal mechanisms:

Prepp Download Prepp APP GET IT ON Google Play

Machining Process	Mechanism of Material Removal
P. Chemical Machining	2. Corrosive reaction
Q. Electrochemical Machining	3. Ion displacement
R. Electro–discharge Machining	4. Fusion and vaporization
S. Ultrasonic Machining	1. Erosion

Based on this analysis, the correct matching is P-2, Q-3, R-4, S-1.

---

57. **Answer: a**

Explanation:

Casting Shrinkage and Contraction Explained

This problem involves calculating the final dimensions of a cubic casting after it undergoes two types of volume changes: volumetric solidification shrinkage and volumetric solid contraction. Understanding these concepts is crucial in foundry practices to achieve desired final part dimensions. We will consider the initial volume, then apply the solidification shrinkage, and finally the solid contraction to determine the final volume and subsequently, the final side length of the cube.

Initial Casting Dimensions

First, let's establish the initial state of the cubic casting before any shrinkage or contraction occurs.

- The initial side length of the cubic casting is given as **50 mm**.
- We assume uniform cooling in all directions, which means the cube will remain a cube throughout the process, just with a smaller side length.
- No riser is used, implying that all volumetric losses due to shrinkage and contraction will directly affect the final dimensions of the casting.

Prepp Download Prepp APP GET IT ON Google Play

The initial volume of the cubic casting V_0 can be calculated using the formula for the volume of a cube:

$$V_0 = (\text{side length})^3$$

$$V_0 = (50 \text{ mm})^3$$

$$V_0 = 125000 \text{ mm}^3$$

Volumetric Solidification Shrinkage

Volumetric solidification shrinkage occurs as the molten metal transforms into a solid state. This shrinkage is typically compensated by a riser, but since no riser is used here, the casting itself will experience this volume reduction.

- The given volumetric solidification shrinkage is **4%**. This means the volume reduces by 4% of its initial volume during solidification.

Let V_1 be the volume of the cube after solidification shrinkage.

$$V_1 = V_0 - (V_0 \times \text{solidification shrinkage percentage})$$

$$V_1 = V_0(1 - \text{solidification shrinkage percentage})$$

$$V_1 = 125000 \text{ mm}^3(1 - 0.04)$$

$$V_1 = 125000 \text{ mm}^3(0.96)$$

$$V_1 = 120000 \text{ mm}^3$$

So, after solidification, the volume of the casting is 120000 mm^3.

Volumetric Solid Contraction

After solidification, as the solid casting cools down from its solidification temperature to room temperature, it undergoes further contraction. This is known as solid contraction (or thermal contraction in the solid state).

- The given volumetric solid contraction is **6%**. This percentage applies to the volume of the solidified casting.

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

Let V_f be the final volume of the cube after solid contraction.

$$V_f = V_1 - (V_1 \times \text{solid contraction percentage})$$

$$V_f = V_1(1 - \text{solid contraction percentage})$$

$$V_f = 120000 \text{ mm}^3(1 - 0.06)$$

$$V_f = 120000 \text{ mm}^3(0.94)$$

$$V_f = 112800 \text{ mm}^3$$

The final volume of the cubic casting after both shrinkage and contraction is 112800 mm^3.

Final Casting Side Length

Since the casting is still a cube and undergoes uniform cooling in all directions, its final side length can be found by taking the cube root of the final volume.

Let L_f be the final side length of the cube.

$$L_f = (V_f)^{1/3}$$

$$L_f = (112800 \text{ mm}^3)^{1/3}$$

$$L_f \approx 48.3248 \text{ mm}$$

Prepp Your Personal Exams Guide

Rounding to two decimal places, the side of the cube after solidification and contraction is approximately **48.32 mm**.

Prepp Download Prepp APP GET IT ON Google Play

Parameter	Value
Initial Side Length (L_0)	50 mm
Initial Volume (V_0)	125000 mm^3
Volumetric Solidification Shrinkage	4%
Volume After Solidification (V_1)	120000 mm^3
Volumetric Solid Contraction	6%
Final Volume (V_f)	112800 mm^3
Final Side Length (L_f)	48.32 mm

---

58. **Answer: b**

Explanation:

Adiabatic Flow Analysis Between Stations P and Q

Your Personal Exams Guide

This solution explains the calculation for the maximum possible pressure at station Q, given the initial conditions at station P and the temperature change during an adiabatic flow process.

Understanding Adiabatic Processes

An **adiabatic process** is a thermodynamic process where no heat is transferred into or out of the system. In this scenario, air flows between two stations, P and Q, under adiabatic conditions. For an ideal gas undergoing a reversible adiabatic (isentropic) process, the relationship between pressure (P) and temperature (T) is given by the following equation:

Prepp Download Prepp APP GET IT ON Google Play

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

where:

- P_1, T_1 are the pressure and temperature at the initial state (Station P).
- P_2, T_2 are the pressure and temperature at the final state (Station Q).
- γ is the ratio of the specific heat at constant pressure (C_P) to the specific heat at constant volume (C_V).

The question asks for the maximum possible pressure at station Q. Assuming the process is reversible adiabatic (isentropic), we can use the formula above. The phrase "maximum possible value" typically implies an isentropic process in the context of such problems unless specific irreversibilities are mentioned.

Given Parameters

- Station P Conditions: $P_P = 150$ kPa, $T_P = 350$ K
- Station Q Condition: $T_Q = 300$ K
- Specific heat at constant pressure (C_P): 1.005 kJ/kgK
- Specific heat at constant volume (C_V): 0.718 kJ/kgK
- Characteristic gas constant (R): 0.287 kJ/kgK

Step-by-Step Calculation

Your Personal Exams Guide

Step 1: Calculate the Specific Heat Ratio (γ)

The ratio of specific heats (γ) is calculated as:

$$\gamma = \frac{C_P}{C_V}$$

Substituting the given values:

$$\gamma = \frac{1.005 \text{ kJ/kgK}}{0.718 \text{ kJ/kgK}} \approx 1.3997$$

Step 2: Calculate the Exponent Term $\frac{\gamma}{\gamma-1}$

Prepp Download Prepp APP GET IT ON Google Play

Next, we calculate the exponent $\frac{\gamma}{\gamma-1}$:

$$\gamma - 1 = 1.3997 - 1 = 0.3997$$

$$\frac{\gamma}{\gamma - 1} = \frac{1.3997}{0.3997} \approx 3.502$$

Step 3: Calculate the Pressure at Station Q (P_Q)

Now, we use the adiabatic process formula to find P_Q. We rearrange the formula as:

$$P_Q = P_P \times \left(\frac{T_Q}{T_P}\right)^{\frac{\gamma}{\gamma-1}}$$

Substitute the known values:

$$P_Q = 150 \text{ kPa} \times \left(\frac{300 \text{ K}}{350 \text{ K}}\right)^{3.502}$$

First, calculate the temperature ratio:

$$\frac{T_Q}{T_P} = \frac{300}{350} = \frac{6}{7} \approx 0.85714$$

Now, raise this ratio to the power of the exponent:

$$\left(\frac{6}{7}\right)^{3.502} \approx (0.85714)^{3.502} \approx 0.5827$$

Finally, calculate P_Q:

$$P_Q = 150 \text{ kPa} \times 0.5827 \approx 87.405 \text{ kPa}$$

Conclusion

The calculated pressure at station Q is approximately 87.405 kPa. Comparing this value to the given options:

- Option 1: 50 kPa
- Option 2: 87 kPa
- Option 3: 128 kPa
- Option 4: 150 kPa

Prepp Download Prepp APP GET IT ON Google Play

The calculated value of 87.405 kPa is closest to **87 kPa**.

---

59. **Answer: c**

Explanation:

This problem asks us to calculate the change in entropy for air flowing adiabatically between two distinct stations, P and Q. We are provided with the pressure and temperature conditions at both stations, along with the specific thermodynamic properties of air. The core of this problem lies in applying the appropriate entropy change formula for an ideal gas under the given conditions.

Air Flow Scenario Analysis

We are considering an experimental setup where air moves from an initial state at station P to a final state at station Q. The key condition mentioned is that the flow is adiabatic, which means there is no heat transfer into or out of the system during this process. Understanding these initial and final states, along with the properties of air, is crucial for determining the change in entropy ($S_Q - S_P$).

Given Parameters for Air Flow

Prepp Your Personal Exams Guide

The following detailed parameters and properties are provided in the question:

- **Station P Conditions:**
- Pressure at P (P_P) = 150 kPa
- Temperature at P (T_P) = 350 K
- **Station Q Conditions:**
- Pressure at Q (P_Q) = 50 kPa
- Temperature at Q (T_Q) = 300 K
- **Air Properties:**
- Specific heat at constant pressure (C_P) = 1.005 kJ/kgK
- Specific heat at constant volume (C_V) = 0.718 kJ/kgK
- Characteristic gas constant (R) = 0.287 kJ/kgK

Prepp Download Prepp APP GET IT ON Google Play

Entropy Change Calculation for Adiabatic Air Flow

To calculate the change in entropy (Δs) for an ideal gas, when both temperature and pressure changes are known, we use the following fundamental thermodynamic relation:

$$\Delta s = s_2 - s_1 = C_P \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

In our case, station P represents the initial state (subscript 1) and station Q represents the final state (subscript 2). Therefore, the change in entropy from P to Q (S_Q - S_P) can be expressed as:

$$S_Q - S_P = C_P \ln\left(\frac{T_Q}{T_P}\right) - R\ln\left(\frac{P_Q}{P_P}\right)$$

Now, we substitute the given numerical values into this equation:

- C_P = 1.005 kJ/kgK
- R = 0.287 kJ/kgK
- T_Q = 300 K
- T_P = 350 K
- P_Q = 50 kPa
- P_P = 150 kPa

Prepp
Your Personal Exams Guide

The calculation proceeds as follows:

$$S_Q - S_P = 1.005\ln\left(\frac{300\text{ K}}{350\text{ K}}\right) - 0.287\ln\left(\frac{50\text{ kPa}}{150\text{ kPa}}\right)$$

First, simplify the ratios:

$$\frac{300}{350} = \frac{6}{7} \approx 0.85714286$$

$$\frac{50}{150} = \frac{1}{3} \approx 0.33333333$$

Next, calculate the natural logarithms of these ratios:

- $\ln\left(\frac{6}{7}\right) \approx \ln(0.85714286) \approx -0.154150679$

Prepp Download Prepp APP GET IT ON Google Play

- $\ln\left(\frac{1}{3}\right) \approx \ln(0.33333333) \approx -1.098612289$

Now, substitute these logarithmic values back into the entropy change equation:

$$S_Q - S_P = 1.005 \times (-0.154150679) - 0.287 \times (-1.098612289)$$

Perform the multiplications:

- $1.005 \times (-0.154150679) \approx -0.154923432$
- $0.287 \times (-1.098612289) \approx -0.315206096$

Finally, complete the subtraction:

$$S_Q - S_P = -0.154923432 - (-0.315206096)$$

$$S_Q - S_P = -0.154923432 + 0.315206096$$

$$S_Q - S_P = 0.160282664 \text{ kJ/kgK}$$

Final Entropy Result

Rounding the calculated change in entropy to three decimal places, we get:

$$S_Q - S_P \approx 0.160 \text{ kJ/kgK}$$

---

60. **Answer: a**

Explanation:

Resource Allocation Problem: Understanding Dual Prices

This question asks us to determine the unit worth of resource R2, also known as its dual price or shadow price, within a manufacturing scenario. This is a classic linear programming problem (LPP) where we aim to maximize profit given limited resources. The dual price tells us how much the total profit would increase if we had one more unit of a specific resource.

1. Primal Problem Formulation

Your Personal Exams Guide

First, let's set up the primal linear programming problem. We need to define variables, the objective function (what we want to maximize), and the constraints (resource limitations).

- Let x_1 be the number of units of product P1 to be manufactured.
- Let x_2 be the number of units of product P2 to be manufactured.

Objective Function (Maximize Profit):

The profit per unit for P1 is Rs. 2000, and for P2 is Rs. 3000.

Maximize $Z = 2000x_1 + 3000x_2$

Constraints (Resource Availability):

We have two resources, R1 and R2, with limited quantities.

- **Resource R1 Constraint:** One unit of P1 requires 3 kg of R1. One unit of P2 requires 2 kg of R1. Total R1 available is 90 kg.

  $3x_1 + 2x_2 \leq 90$

- **Resource R2 Constraint:** One unit of P1 requires 1 kg of R2. One unit of P2 requires 2 kg of R2. Total R2 available is 100 kg.

  $x_1 + 2x_2 \leq 100$

- **Non-negativity Constraints:** The number of units produced cannot be negative.

  $x_1 \geq 0, x_2 \geq 0$

2. Primal Problem Solution (Graphical Method)

To find the optimal production mix that maximizes profit, we can use the graphical method. We plot the constraint lines and identify the feasible region, then test the corner points.

Constraint Lines:

- For $3x_1 + 2x_2 = 90$:

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

- If $x_1 = 0$, then $2x_2 = 90 \Rightarrow x_2 = 45$. Point: (0, 45)
- If $x_2 = 0$, then $3x_1 = 90 \Rightarrow x_1 = 30$. Point: (30, 0)

- For $x_1 + 2x_2 = 100$:
  - If $x_1 = 0$, then $2x_2 = 100 \Rightarrow x_2 = 50$. Point: (0, 50)
  - If $x_2 = 0$, then $x_1 = 100$. Point: (100, 0)

Identifying the Feasible Region:

The feasible region is the area that satisfies all constraints, including non-negativity ($x_1 \geq 0, x_2 \geq 0$). We check the corner points of this feasible region.

- **(0,0):** $Z = 2000(0) + 3000(0) = 0$
- **(30,0):** This point satisfies $3x_1 + 2x_2 \leq 90$ ($3(30) + 0 = 90 \leq 90$) and $x_1 + 2x_2 \leq 100$ ($30 + 0 = 30 \leq 100$).

  $Z = 2000(30) + 3000(0) = 60000$

- **(0,45):** This point satisfies $3x_1 + 2x_2 \leq 90$ ($3(0) + 2(45) = 90 \leq 90$) and $x_1 + 2x_2 \leq 100$ ($0 + 2(45) = 90 \leq 100$).

  $Z = 2000(0) + 3000(45) = 135000$

The point (0,50) from the second constraint is not feasible because $3(0) + 2(50) = 100$, which is not ≤ 90. Similarly, (100,0) is not feasible as $3(100) + 2(0) = 300$, which is not ≤ 90.

Your Personal Exams Guide

Comparing the profit values at the feasible corner points, the maximum profit is Rs. 135,000, achieved at $x_1 = 0$ and $x_2 = 45$. This is our optimal primal solution.

3. Binding Constraints Analysis

A constraint is "binding" if the resource is completely used up at the optimal solution (i.e., the inequality becomes an equality). If there's leftover resource, the constraint is "non-binding" and has "slack".

Let's check the resource utilization at the optimal solution $(x_1^*, x_2^*) = (0, 45)$:

- **Resource R1:** Usage = $3x_1 + 2x_2 = 3(0) + 2(45) = 90$ kg. Available = 90 kg. Since Usage = Available, Resource R1 constraint is **binding**.

Prepp Download Prepp APP GET IT ON Google Play

- **Resource R2:** Usage = $x_1 + 2x_2 = 1(0) + 2(45) = 90$ kg. Available = 100 kg. Since Usage < Available $(90 < 100)$, Resource R2 constraint is **non-binding**. There is a slack of $100 - 90 = 10$ kg of R2.

4. Dual Price Concept

The dual price (or shadow price) of a resource represents the marginal increase in the optimal objective function value (profit in this case) if one additional unit of that resource becomes available, assuming all other conditions remain constant. It tells us the "worth" of an additional unit of a resource.

5. Complementary Slackness Theorem Application

A key principle in linear programming, the Complementary Slackness Theorem, directly relates the primal and dual solutions:

- If a primal resource constraint is non-binding (meaning there is slack in the resource at the optimal solution), then its corresponding dual variable (the dual price for that resource) must be **zero**.
- Conversely, if a primal resource constraint is binding (meaning the resource is fully utilized), its corresponding dual variable can be non-zero.

From our analysis in step 3, we found that the Resource R2 constraint ($x_1 + 2x_2 \leq 100$) is **non-binding** at the optimal solution. This means there is an excess of resource R2. If we were to obtain an additional unit of resource R2, it would not help increase our profit because we are not even using all of the currently available R2.

Therefore, according to the Complementary Slackness Theorem, the dual price of resource R2 must be **0**.

6. Conclusion

Based on the analysis of the primal problem and the application of the Complementary Slackness Theorem, the unit worth of resource R2, i.e., the dual price of resource R2, is Rs. 0 per kg.

The final answer is $\boxed{0}$.

Prepp Download Prepp APP GET IT ON Google Play

61. **Answer: b**

Explanation:

Maximizing Profit with Resource Constraints using Linear Programming

This problem involves finding the maximum profit achievable by producing two products, P1 and P2, given limited resources, R1 and R2. We can solve this using the principles of **linear programming**.

1. Defining the Problem Variables and Objective

Let x be the number of units of product P1 produced. Let y be the number of units of product P2 produced.

The goal is to maximize the total profit (Z). The profit function is given by:

$$Z = 2000x + 3000y$$

2. Formulating the Constraints

The production is limited by the available amounts of resource R1 and resource R2.

Resource R1 Constraint: Each unit of P1 requires 3 kg of R1, and each unit of P2 requires 2 kg of R1. The total available R1 is 90 kg.

$$3x + 2y \leq 90$$

Resource R2 Constraint: Each unit of P1 requires 1 kg of R1, and each unit of P2 requires 2 kg of R1. The total available R2 is 100 kg.

$$x + 2y \leq 100$$

Non-negativity Constraints: The number of units produced cannot be negative.

$$x \geq 0$$

Prepp Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play

$$y \geq 0$$

3. Analyzing the Feasible Region and Corner Points

The feasible region is determined by the intersection of the areas defined by the constraints in the first quadrant ($x \geq 0, y \geq 0$). We need to find the corner points of this region.

The constraints are:

- $3x + 2y \leq 90$
- $x + 2y \leq 100$
- $x \geq 0$
- $y \geq 0$

Let's analyze the constraints. Consider the first constraint $3x + 2y \leq 90$. Any point (x, y) satisfying this constraint and $x \geq 0, y \geq 0$ will also satisfy $x + 2y \leq 100$. This is because $x \geq 0$ implies $x \leq 3x$, so $x + 2y \leq 3x + 2y$. Since $3x + 2y \leq 90$, it follows that $x + 2y \leq 90$. As $90 \leq 100$, the condition $x + 2y \leq 100$ is always met if $3x + 2y \leq 90$. Therefore, the second constraint ($x + 2y \leq 100$) is redundant, and the feasible region is solely defined by $3x + 2y \leq 90$, $x \geq 0$, and $y \geq 0$.

The corner points of this feasible region are:

- The origin: (0, 0)
- The x-intercept of $3x + 2y = 90$ (where $y = 0$): $3x = 90 \implies x = 30$. Point: (30, 0)
- The y-intercept of $3x + 2y = 90$ (where $x = 0$): $2y = 90 \implies y = 45$. Point: (0, 45)

So, the corner points are (0, 0), (30, 0), and (0, 45).

4. Calculating Profit at Corner Points

We evaluate the profit function $Z = 2000x + 3000y$ at each corner point:

Prepp Download Prepp APP GET IT ON Google Play

Corner Point (x, y)	Profit Calculation ($Z = 2000x + 3000y$)	Profit (Z)
(0, 0)	$Z = 2000(0) + 3000(0)$	0
(30, 0)	$Z = 2000(30) + 3000(0)$	60000
(0, 45)	$Z = 2000(0) + 3000(45)$	135000

5. Determining Maximum Profit

Comparing the profits calculated at the corner points, the maximum profit occurs at the point (0, 45).

The maximum profit is Rs. 135,000.

---

62. **Answer: b**

Explanation:

<u>Concept:</u>

Width change per unit length is b/l.

Your Personal Exams Guide

width at distance x:

$$\omega_x = \frac{b}{l}x$$
$$\Rightarrow I_x = \frac{\left(\frac{bx}{l}\right)\times t^3}{12}$$

---

63. **Answer: d**

Explanation:

<u>Concept:</u>

The beam is having uniform thickness t but varying width b,

Prepp Download Prepp APP GET IT ON Google Play

Therefore, the width at any point in beam is $b_x = b(\frac{x}{l})$

For deflection Castigliano's theorem is, $\Delta = \frac{\delta U}{\delta P}$

where P = load acting at the end, U = strain energy of the beam,

\(U = \mathop \smallint \limits_0^l \frac{{{{\left({{M_{x - x}}} \right)}^2}dx}}{{2EI}}\), where, M_{x-x} = moment at cross-section, E = Young’s modulus, I = MOI = $\frac{b_x t^3}{12} = \frac{bt^3}{12l}(x)$

Calculation:

Strain energy, \(U = \mathop \smallint \limits_0^l \frac{{{{\left({Px} \right)}^2}dx}}{{2E\left({\frac{{b{t^3}x}}{{12l}}} \right)}} = \frac{{3{P^2}{l^3}}}{{Eb{t^3}}}\)

From Castigliano's theorem,

$$\Delta = \frac{\delta U}{\delta P} = \frac{6Pl^3}{Ebt^3}$$

---

64. Answer: c

Explanation:

Prepp
Your Personal Exams Guide

Concept:

The relation between pressure and temperature for the isentropic process.

$$\frac{T_2}{T_2} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Calculation:

Given:

T_1 = 400 K, P_1 = 300 kPa, P_2 = 50 kPa, R = 0.289 kJ/kgK

γ = 1.4, A_2 = 0.005 m^2

Prepp Download Prepp APP
GET IT ON Google Play

The process followed from entrance to exit is isentropic process, therefore

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 400\left(\frac{50}{300}\right)^{\frac{0.4}{1.4}} = 239.5\ K$$

From the perfect gas equation

$$\rho = \frac{P}{RT}\ or\ \rho_2 = \frac{P_2}{RT_2} = \frac{50}{0.287\times 239.5} \Rightarrow \rho_2 = 0.727\ \frac{kg}{m^3}$$

---

65. **Answer: d**

Explanation:

Concept:

The mass flow rate through the nozzle is, m = $(\rho AV)_i = (\rho AV)_o$

The velocity of flow through nozzle is find from steady flow energy equation,

$$h_i + \frac{V_i^2}{2} + Q = h_o + \frac{V_o^2}{2} + W$$

where, V_i = 0 , W = 0 ,Q = 0 ...adiabatic

Therefore, $h_i - h_o = \frac{V_o^2}{2} \Rightarrow C_p(T_i - T_o) = \frac{V_o^2}{2}$

Calculation:

Given:

T_1 = 400 K, P_1 = 300 kPa, P_2 = 50 kPa, R = 0.289 kJ/kgK, γ = 1.4, A_2 = 0.005 m^2, ρ = 0.727 kg/m^3

The process followed from entrance to exit is isentropic process, therefore

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = 400\left(\frac{50}{300}\right)^{\frac{0.4}{1.4}} = 239.5\ K$$

Therefore, for exit velocity,

1.005 (400 - 239.5) × 2000 = V_o^2

Prepp Download Prepp APP GET IT ON Google Play

V_o = 567.98 m/s

The mass flow rate, m = 0.727 × 0.005 × 567.98 = **2.06 kg/sec**

Prepp
Your Personal Exams Guide

Prepp Download Prepp APP GET IT ON Google Play