

MATHEMATICS

1. If $|\vec{a}| = 3$ then value of

$$|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = \underline{\hspace{2cm}}.$$

- (A) 9 (B) 18
(C) 27 (D) 36

Answer (B)

Sol. Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{Then } |\vec{a}|^2 = x^2 + y^2 + z^2$$

$$\text{Now, } |\vec{a} \times \hat{i}|^2 = y^2 + z^2, \quad |\vec{a} \times \hat{j}|^2 = x^2 + z^2$$

$$\text{and } |\vec{a} \times \hat{k}|^2 = x^2 + y^2$$

$$\therefore |\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2(x^2 + y^2 + z^2) \\ = 2|\vec{a}|^2 = 18$$

2. The co-ordinates of the foot of perpendicular drawn from origin to the plane $2x - 3y + 4z - 6 = 0$ is _____

- (A) $\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$ (B) $\left(\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$
(C) $\left(\frac{12}{29}, \frac{18}{29}, \frac{24}{29}\right)$ (D) $\left(-\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$

Answer (A)

Sol. Let (x_1, y_1, z_1) be foot of perpendicular from origin to plane $2x - 3y + 4z - 6 = 0$.

$$\therefore \frac{x_1 - 0}{2} = \frac{y_1 - 0}{-3} = \frac{z_1 - 0}{4} = -\frac{(-6)}{2^2 + 3^2 + 4^2}$$

$$\therefore \frac{x_1}{2} = \frac{y_1}{-3} = \frac{z_1}{4} = \frac{6}{29}$$

$$\therefore (x_1, y_1, z_1) = \left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$$

3. The angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$ is _____.

- (A) $\cos^{-1} \frac{8}{21}$ (B) $\tan^{-1} \frac{8}{\sqrt{377}}$
(C) $\sin^{-1} \frac{8}{\sqrt{377}}$ (D) $\sin^{-1} \left(\frac{21}{8}\right)$

Answer (B)

Sol. Let angle between line and plane be θ .

$$\text{Line is } \frac{x+1}{2} = \frac{y-0}{3} = \frac{z-3}{6}$$

with direction ratios (2, 3, 6)

And plane is $10x + 2y - 11z - 3 = 0$ direction ratios of normal to plane is (10, 2, -11)

$$\therefore \sin \theta = \left| \frac{20 + 6 - 66}{\sqrt{4 + 9 + 36} \sqrt{100 + 4 + 121}} \right|$$

$$\sin \theta = \left| \frac{-40}{7 \times 15} \right|$$

$$\sin \theta = \frac{8}{21} \text{ or } \tan \theta = \frac{8}{\sqrt{377}}$$

$$\therefore \theta = \tan^{-1} \left(\frac{8}{\sqrt{377}} \right)$$

4. If the points $(1, 1, p)$ and $(-3, 0, 1)$ be equidistant from the plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$ then the values of p are _____.

- (A) $1, \frac{7}{3}$ (B) $1, \frac{4}{3}$
(C) $2, \frac{4}{3}$ (D) $\frac{7}{3}, 2$

Answer (A)

Sol. Equation of plane is $3x + 4y - 12z + 13 = 0$

Now as $(1, 1, p)$ and $(-3, 0, 1)$ are equidistant from the plane

$$\left| \frac{3 + 4 - 12p + 13}{\sqrt{9 + 16 + 144}} \right| = \left| \frac{-9 + 0 - 12 + 13}{\sqrt{9 + 16 + 144}} \right|$$

$$|20 - 12p| = -8 \Rightarrow 20 - 12p = 8 \Rightarrow p = 1$$

$$\text{and } 20 - 12p = -8 \Rightarrow 12p = 28 \Rightarrow p = \frac{7}{3}$$

5. The maximum value of $Z = 3x + 4y$ subject to constraints $x + y \leq 4, x \geq 0, y \geq 0$ is _____.

- (A) 16 (B) 12
(C) 0 (D) not possible

Answer (A)

Sol. $z = 3x + 4y$

At $(0, 0)$; $z = 0$

At $(4, 0)$; $z = 12$

At $(0, 4)$; $z = 16$

$\therefore$ Maximum value = 16

6. If A and B are independent events such that $P(A) = p$, $P(B) = 2p$ and $P(\text{Exactly one of } A \text{ and } B) = \frac{5}{9}$, then $p =$ _____.

- (A) $\frac{1}{3}, \frac{5}{12}$ (B) $\frac{1}{2}, \frac{3}{4}$
(C) $\frac{1}{12}, \frac{5}{3}$ (D) $\frac{2}{15}, \frac{5}{12}$

Answer (A)

Sol. As A and B are independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = 2p^2$$

and $P(\text{exactly one of } A \text{ and } B)$

$$= P(A \cup B) - P(A \cap B)$$

$$= P(A) + P(B) - 2P(A \cap B) = \frac{5}{9}$$

$$\Rightarrow p + 2p - 2 \cdot 2p^2 = \frac{5}{9}$$

$$\Rightarrow 36p^2 - 27p + 5 = 0$$

$$\Rightarrow (12p - 5)(3p - 1) = 0$$

$$\therefore p = \frac{5}{12}, \frac{1}{3}$$

7. For the probability distribution

X	1	2	3	4
$P(X)$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$$E(X^2) = \text{_____}$$

- (A) 7 (B) 5
(C) 3 (D) 10

Answer (D)

Sol. $E(X^2) = 1 \times \frac{1}{10} + 4 \times \frac{1}{5} + 9 \times \frac{3}{10} + 16 \times \frac{2}{5}$

$$= \frac{1}{10} + \frac{4}{5} + \frac{27}{10} + \frac{32}{5}$$

$$= \frac{1 + 8 + 27 + 64}{10}$$

$$= \frac{100}{10} = 10$$

8. If A and B are any two events such that $P(A) + P(B) - P(A \cap B) = P(A)$ then _____.

- (A) $P\left(\frac{B}{A}\right) = 1$ (B) $P\left(\frac{A}{B}\right) = 0$
(C) $P\left(\frac{B}{A}\right) = 0$ (D) $P\left(\frac{A}{B}\right) = 1$

Answer (D)

Sol. $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

9. Let $f: R \rightarrow R$ be defined by $f(x) = 2x^2 - 5$ and $g: R \rightarrow R$ by $g(x) = \frac{x}{x^2 + 1}$, then $g \circ f$ is _____.

- (A) $\frac{2x^2 - 5}{4x^4 + 20x^2 + 26}$ (B) $\frac{2x^2 - 5}{4x^4 - 20x^2 + 26}$
(C) $\frac{2x^2}{x^4 + 2x^2 - 4}$ (D) $\frac{2x^2}{4x^4 - 20x^2 + 26}$

Answer (B)

Sol. $g(f(x)) = \frac{f(x)}{(f(x))^2 + 1}$

$$= \frac{2x^2 - 5}{(2x^2 - 5)^2 + 1}$$

$$= \frac{2x^2 - 5}{4x^4 - 20x^2 + 26}$$

10. Let $f: [2, \infty) \rightarrow R$ be the function defined by $f(x) = x^2 - 4x + 5$. Then the range of f is _____.

- (A) $[1, \infty)$ (B) $[4, \infty)$
(C) R (D) $[5, \infty)$

Answer (A)

Sol. $f(x) = (x-2)^2 + 1$
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 As $x \in [2, \infty)$

$$0 \leq x-2 < \infty$$

$$1 \leq (x-2)^2 + 1 < \infty$$

$\therefore$ Range is $[1, \infty)$

11. On R , binary operation $*$ is defined by $a * b = a + b + ab$ then identity and inverse of $*$ are respectively.

(A) $0, \frac{a}{1-a}$ (B) $1, \frac{a}{1+a}$

(C) $0, -\frac{a}{1+a}$ (D) $1, \frac{a}{1-a}$

Answer (C)

Sol. If $a * e = e * a = a$ then e is identity

$$a + e + ae = a$$

$$e(1+a) = 0$$

$$e = 0 \quad (\text{Identity})$$

Now if $a * b = e$ then b is inverse of a

$$a + b + ab = 0$$

$$a = -b(1+a)$$

$$\text{inverse } b = \frac{-a}{1+a}$$

12. $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) = \underline{\hspace{2cm}}$

(A) $\cos^{-1}\left(\frac{84}{85}\right)$ (B) $\cos^{-1}\left(\frac{24}{85}\right)$

(C) $\sin^{-1}\left(\frac{24}{85}\right)$ (D) $\sin^{-1}\left(\frac{84}{85}\right)$

Answer (A)

Sol. $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right)$

$$= \sin^{-1}\left(\frac{3}{5} \times \frac{15}{17} - \frac{4}{5} \times \frac{8}{17}\right)$$

$$= \sin^{-1}\left(\frac{13}{85}\right)$$

$$= \cos^{-1}\left(\frac{84}{85}\right)$$

13. $\tan^2(\sec^{-1} 3) + \operatorname{cosec}^2(\cot^{-1} 2) +$

$$\cos^2\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{2}{3}\right) = \underline{\hspace{2cm}}$$

(A) 15 (B) 16

(C) 14 (D) 13

Answer (D)

Sol. $\tan^2(\sec^{-1} 3) + \operatorname{cosec}^2(\cot^{-1} 2) +$

$$\cos^2\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{2}{3}\right)$$

$$= \tan^2(\tan^{-1}(2\sqrt{2})) +$$

$$\operatorname{cosec}^2(\operatorname{cosec}^{-1}\sqrt{5}) + \cos^2\left(\frac{\pi}{2}\right)$$

$$= 8 + 5 = 13$$

14. If $A = \begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$ then

$$\underline{\hspace{2cm}}$$

(A) $1 - a^2 + bc = 0$ (B) $1 + a^2 + bc = 0$

(C) $1 + a^2 - bc = 0$ (D) $1 - a^2 - bc = 0$

Answer (D)

Sol. $A^2 = I$

$$\begin{bmatrix} a & b \\ c & -a \end{bmatrix} \begin{bmatrix} a & b \\ c & -a \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2 + bc & ab - ab \\ ac - ac & bc + a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow a^2 + bc = 1$$

$$\Rightarrow 1 - a^2 - bc = 0$$

15. If A is a square matrix such that $A^2 = I$ then $(A-I)^3 + (A+I)^3 - 7A$ is equal to _____.

(A) $I + A$

(B) $I - A$

(C) A

(D) $3A$

Answer (C)

Sol. $(A-I)^3 + (A+I)^3 - 7A$

$$= A^3 - I - 3A^2 + 3A + A^3 + I + 3A^2 + 3A - 7A$$

$$= 2A^3 - A$$

$$= A(2A^2 - I)$$

$$= A(2I - I) = A$$

16. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and inverse of A is

$$\frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ x & -3 & 1 \end{bmatrix} \text{ then } x = \underline{\hspace{2cm}}.$$

- (A) 5 (B) 3
(C) 2 (D) 4

Answer (A)

Sol. We Know that $AA^{-1} = I$

Solving, we get $x = 5$

17. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2 \tan t & t & 2t \\ \tan t & t & t \end{vmatrix}$. Then $\lim_{t \rightarrow 0} \frac{f(t)}{t^2}$ is equal

to $\underline{\hspace{2cm}}$.

- (A) 3 (B) 1
(C) -1 (D) 0

Answer (D)

Sol. $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2 \tan t & t & 2t \\ \tan t & t & t \end{vmatrix}$

$$\Rightarrow f(t) = t[-t \cos t - (2t \tan t - 2t \tan t) + 1(\tan t)]$$

$$\Rightarrow f(t) = -t^2 \cos t + t \tan t$$

$$\therefore \lim_{t \rightarrow 0} \frac{f(t)}{t^2} = \lim_{t \rightarrow 0} \left(-\cos t + \frac{\tan t}{t} \right) = 0$$

18. If $x, y \in \mathbb{R}$ and $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix}$

$= 2y + 6$, then $y = \underline{\hspace{2cm}}$.

- (A) 0 (B) 3
(C) -3 (D) 6

Answer (C)

Sol. Applying $C_1 \rightarrow C_1 - C_2$, we get

$$\begin{vmatrix} 4 & (a^x - a^{-x})^2 & 1 \\ 4 & (b^x - b^{-x})^2 & 1 \\ 4 & (c^x - c^{-x})^2 & 1 \end{vmatrix} = 2y + 6$$

$$\Rightarrow 2y + 6 = 0$$

$$\Rightarrow y = -3$$

19. For $\triangle ABC$, the value of

$$\begin{vmatrix} 0 & \sin A & \tan B \\ -\sin(B+C) & 0 & \cos C \\ \tan(A+C) & -\cos C & 0 \end{vmatrix} = \underline{\hspace{2cm}}.$$

- (A) -1 (B) 0
(C) 1 (D) $\sin A \cos C$

Answer (B)

Sol. Given determinant is of skew symmetric matrix of odd order so value = 0

20. If function $f(\alpha) = \begin{cases} \frac{1 - \cos 6\alpha}{36\alpha^2} & \text{if } \alpha \neq 0 \\ k & \text{if } \alpha = 0 \end{cases}$ is continuous at $\alpha = 0$ then $k = \underline{\hspace{2cm}}$.

- (A) $-\frac{1}{2}$ (B) 1
(C) $\frac{1}{2}$ (D) 0

Answer (C)

Sol. $k = \lim_{\alpha \rightarrow 0} \frac{1 - \cos 6\alpha}{36\alpha^2} = \frac{1}{2}$

21. If $y = \sin^{-1} \left(\frac{2^{x-1}}{1+4^x} \right)$ and $\frac{dy}{dx} = \frac{2^{x-1} \log 2}{f(x)}$ then $f(0) = \underline{\hspace{2cm}}$.

- (A) 0 (B) -2
(C) 2 (D) $2 \log 2$

Answer (C)

Sol. Now $y = \sin^{-1} \left(\frac{2 \cdot 2^x}{(2^x)^2 + 1} \right)$

$$\text{Let } 2^x = \tan \theta$$

$$= \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$\Rightarrow \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow y = 2 \tan^{-1}(2^x)$$

$$\therefore \frac{dy}{dx} = \frac{2 \cdot 2^x \log 2}{1 + 4^x} = \frac{2^{x+1} \log 2}{4^x + 1}$$

$$\therefore f(x) = 4^x + 1$$

$$\Rightarrow f(0) = 2$$

22. For function $f(x) = x + \frac{1}{x}$, $x \in [1, 2]$, the value of C for mean value theorem is _____.

(A) 2 (B) $\sqrt{2}$
(C) 1 (D) $\sqrt{3}$

Answer (B)

Sol. We know

$$f'(C) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow 1 - \frac{1}{C^2} = \frac{f(2) - f(1)}{2 - 1}$$

$$\Rightarrow 1 - \frac{1}{C^2} = \frac{1}{2}$$

$$\Rightarrow \frac{1}{C^2} = \frac{1}{2} = C = \pm\sqrt{2}$$

Now as $C \in (1, 2)$

$$\Rightarrow C = \sqrt{2}$$

23. The interval in which $y = x^2 e^{-x}$ is increasing is _____.

(A) $(0, 2)$
(B) $(2, \infty)$
(C) $(-\infty, \infty)$
(D) $(-2, 0)$

Answer (A)

Sol. The interval in which $y = x^2 \cdot e^{-x}$ is increasing

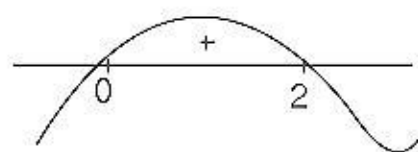
$$y = x^2 \cdot e^{-x}$$

$$\Rightarrow \frac{dy}{dx} = 2xe^{-x} + x^2(-e^{-x})$$

$$= e^{-x} [2x - x^2]$$

$$= -xe^{-x} [x - 2]$$

For increasing function $\frac{dy}{dx} > 0$



$\therefore$ Increasing in interval $(0, 2)$.

24. The rate of change of volume of sphere with respect to its radius r at $r = 2$ is _____.

(A) 24π
(B) 32π
(C) 16π
(D) 8π

Answer (C)

Sol. $V = \frac{4}{3}\pi r^3$

$$\Rightarrow \frac{dV}{dr} = \frac{4}{3} \times 3\pi r^2$$

$$= 4\pi r^2$$

Now at $(r = 2) = 16\pi$

25. The tangent to the curve given by $x = e^\theta \cdot \cos \theta$, $y = e^\theta \cdot \sin \theta$ at $\theta = \frac{\pi}{4}$ makes an angle with X-axis is _____.

(A) $\frac{\pi}{2}$ (B) 0
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

Answer (A)

Sol. $x = e^\theta \cos \theta$

$$\Rightarrow \frac{dx}{d\theta} = e^\theta \cos \theta - e^\theta \sin \theta$$

$$= e^\theta (\cos \theta - \sin \theta)$$

$$y = e^\theta \sin \theta$$

$$\Rightarrow \frac{dy}{d\theta} = e^\theta \sin \theta + e^\theta \cos \theta$$

$$= e^\theta (\sin \theta + \cos \theta)$$

$$= \frac{dy}{dx} = \frac{\sin \theta + \cos \theta}{\cos \theta - \sin \theta}$$

Not defined at $\theta = \frac{\pi}{4}$

$\therefore \theta = \frac{\pi}{2}$ (Angle formed by tangent with X-axis)

26. The minimum value of $f(x) = x \log x$ is _____.

- (A) 0 (B) $-\frac{1}{e}$
(C) $\frac{1}{e}$ (D) e

Answer (B)

Sol. $y = x \log x$

$$\Rightarrow \frac{dy}{dx} = \log x + \left(x \times \frac{1}{x} \right)$$

$$\Rightarrow \frac{dy}{dx} = 1 + \log x$$

$$\therefore \text{Minima at } x = \frac{1}{e}$$

$$\therefore f\left(\frac{1}{e}\right) = -\frac{1}{e}$$

27. If $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx = \frac{x^3}{3} + f(x) + C$, then $f(1) =$ _____.

- (A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$
(D) $\frac{1}{2}$

Answer (B)

Sol. $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx$

$$\int \left(x^2 + \frac{1}{x^2 + 1} \right) dx$$

$$\frac{x^3}{3} + \tan^{-1}(x) + c$$

$$\therefore f(x) = \tan^{-1}(x)$$

$$f(1) = \tan^{-1}(1)$$

$$\frac{\pi}{4}$$

28. $\int \frac{x+100}{(x+101)^2} e^x dx = \text{_____} + C$.

- (A) $\frac{1}{x+101} e^x$ (B) $\frac{x}{x+101} e^x$
(C) $\frac{1}{x+100} e^x$ (D) $(x+101) e^x$

Answer (A)

Sol. $\int \frac{(x+101)-1}{(x+101)^2} e^x dx$

Now we know $\int e^x (f(x) + f'(x)) dx$

$$= e^x f(x) + c$$

$$= \frac{e^x}{x+101} + c$$

29. $\int \frac{\sqrt{\cot x}}{\cos x \sin x} dx = \text{_____} + C$.

- (A) $-2\sqrt{\cot x}$ (B) $-2\sqrt{\tan x}$
(C) $2\sqrt{\cot x}$ (D) $\frac{1}{\sqrt{\cot x}}$

Answer (A)

Sol. $\int \frac{\sqrt{\cot x}}{\cos x \sin x} dx$

$$= \int \frac{\sqrt{\cos x}}{\sqrt{\sin x} \sin x \cos x} dx$$

$$= \int (\cos x)^{-\frac{1}{2}} (\sin x)^{-\frac{3}{2}} dx$$

As $m + n$ negative even integer put

$$\therefore \tan x = t$$

$$= \int (\tan x)^{-\frac{3}{2}} (\cos x)^{-2} dx$$

$$= \int (t)^{-\frac{3}{2}} dt$$

$$= \frac{t^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + c$$

$$= \frac{-2}{\sqrt{\tan x}} + c$$

$$= -2\sqrt{\cot x} + c$$

30. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2019-x}{2019+x}\right) dx = \underline{\hspace{2cm}}$.

(A) π (B) 0
(C) $\frac{\pi}{2}$ (D) 1

Answer (B)

Sol. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2019-x}{2019+x}\right) dx$

as we know $\int_{-a}^a f(x) = 0$

if $f(x) + f(-x) = 0$ (odd function)

$\therefore 0$

31. $\int_4^9 \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx = \underline{\hspace{2cm}}$.

(A) $\frac{19}{66}$ (B) $\frac{19}{33}$
(C) $\frac{38}{99}$ (D) $\frac{19}{99}$

Answer (D)

Sol. $\int_4^9 \frac{\sqrt{x}}{\left(30 - x^{\frac{3}{2}}\right)^2} dx \dots (1)$

Let $30 - x^{\frac{3}{2}} = t$

$\Rightarrow \frac{-3}{2} (x)^{\frac{1}{2}} dx = dt$

$\Rightarrow \sqrt{x} dx = \frac{-2}{3} dt$

Now using (1),

$= \frac{-2}{3} \int_{22}^3 \frac{1}{t^2} dt$

$= \frac{2}{3} \left[\frac{1}{t} \right]_{22}^3$

$= \frac{2}{3} \left[\frac{1}{3} - \frac{1}{22} \right]$

$= \frac{19}{99}$

32. If $f(a+b-x) = f(x)$ then $\int_a^b x \cdot f(x) dx$ is equal to $\underline{\hspace{2cm}}$.

(A) $\frac{a+b}{2} \int_a^b f(x) dx$ (B) $\frac{a+b}{2} \int_a^b f(b+x) dx$

(C) $\frac{a+b}{2} \int_a^b f(b-x) dx$ (D) $\frac{b-a}{2} \int_a^b f(x) dx$

Answer (A)

Sol. $I = \int_a^b x \cdot f(x) dx$

$= \int (a+b-x) f(a+b-x) = I$

add both

$\Rightarrow 2I = \int_a^b (a+b) f(x) dx$

$= \left(\frac{a+b}{2} \right) \int_a^b f(x) dx$

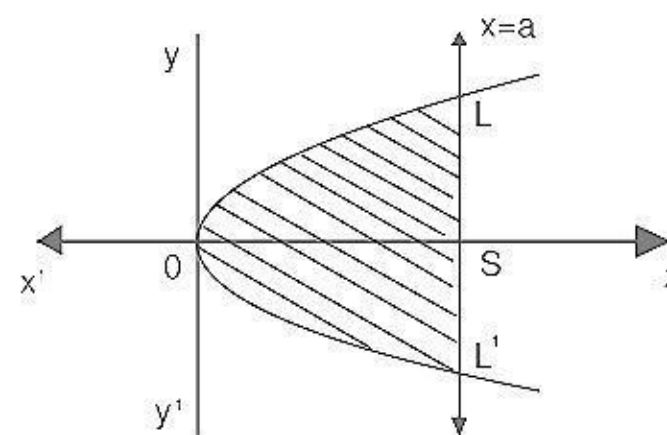
33. The area of the parabola $y^2 = 4ax$ bounded by its latus rectum is $\underline{\hspace{2cm}}$.

(A) $\frac{16}{3} a^2$ (B) $\frac{4}{3} a^2$

(C) $\frac{8}{3} a^2$ (D) $4a^2$

Answer (C)

Sol. For parabola $y^2 = 4ax$



Area required = area OLSL'

$= 2 \times \text{Area OSL}$

$= 2 \times \int_0^a y dx$

Now parabola equation is

$y^2 = 4ax$

$\Rightarrow y = \pm \sqrt{4ax}$

Since OSL is in 1st quadrant

$$y = \sqrt{4ax}$$

$$\begin{aligned} \text{Area required} &= 2 \times \int_0^a \sqrt{4ax} dx \\ &= 2\sqrt{4a} \int_0^a \sqrt{x} dx \\ &= 4\sqrt{a} \int_0^a \sqrt{x} dx \\ &= \frac{8}{3} a^2 \end{aligned}$$

34. The area enclosed by the curve $x = 4\cos\theta$, $y = 3\sin\theta$ is _____.

- (A) 4π (B) 6π
(C) 8π (D) 12π

Answer (D)

$$\text{Sol. } \left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1$$

And area of ellipse = πab

$$= \pi \times 4 \times 3$$

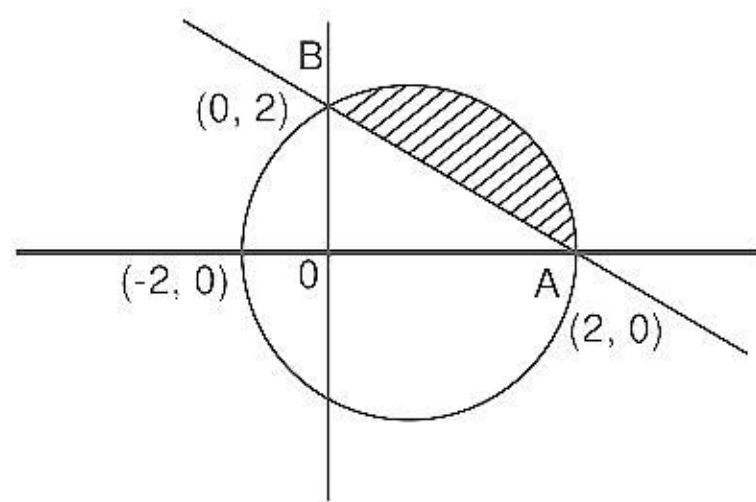
$$= 12\pi$$

35. The smallest area enclosed by circle $x^2 + y^2 = 4$ and line $x + y = 2$ is _____.

- (A) $\pi + 2$ (B) $\pi - 2$
(C) π (D) 2π

Answer (B)

Sol. The Smallest area enclosed by circle $x^2 + y^2 = 4$ and line $x + y = 2$



Required area

$$= \frac{1}{4} (\text{Area of circle}) - \text{area of triangle } \triangle OAB$$

$$= \text{Area} = \frac{\pi}{4} \times (2)^2 - \frac{1}{2} \times 2 \times 2 = \pi - 2$$

36. The order and degree of differential equation

$$\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}} = \frac{d^2y}{dx^2} \text{ are } p \text{ and } q \text{ respectively}$$

then $p + q =$ _____.

- (A) 6 (B) 4
(C) 2 (D) 5

Answer (B)

$$\text{Sol. } \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \left(\frac{d^2y}{dx^2}\right)^2$$

$$\therefore \text{Order} = 2 = p$$

$$\text{Degree} = 2 = q$$

$$\therefore p + q = 4$$

37. Integrating factor of differential equation $(\tan^{-1} y - x) dy = (1 + y^2) dx$ is _____.

- (A) e^{1+y^2} (B) e^y
(C) $e^{\tan^{-1} x}$ (D) $e^{\tan^{-1} y}$

Answer (D)

$$\text{Sol. } \frac{dy}{dx} = \frac{(1+y^2)}{(\tan^{-1} y - x)}$$

$$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1} y}{1+y^2} - \frac{x}{1+y^2}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$$

$$\text{Now integrating factor} = e^{\int \frac{dy}{1+y^2}}$$

$$e^{\tan^{-1} y}$$

38. The differential equation $y \frac{dy}{dx} + x = k$ represents

- (A) circles (B) hyperbolas
(C) parabolas (D) ellipses

Answer (A)

Sol. $\frac{dy}{dx} = \frac{k-x}{y}$

$$y \, dy = dx(k-x) \quad \frac{y^2}{2} = kx - \frac{x^2}{2} + c$$

$$x^2 + y^2 = 2kx + 2c$$

∴ Circle

39. If $\vec{a} = 2\hat{i} - \hat{j} + k$, $\vec{b} = \hat{i} + \hat{j} - 2k$, $\vec{c} = \hat{i} + 3\hat{j} - k$, if $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$, then the value of λ is

- (A) 0 (B) 2
(C) -2 (D) 3

Answer (C)

Sol. As $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$

$$\Rightarrow \vec{a} \cdot (\lambda\vec{b} + \vec{c}) = 0$$

$$\Rightarrow (2\hat{i} - \hat{j} + k) \cdot (\lambda[\hat{i} + \hat{j} - 2k] + [\hat{i} + 3\hat{j} - k]) = 0$$

$$\Rightarrow 2(\lambda + 1) - (\lambda + 3) + 1(-2\lambda - 1) = 0$$

$$\Rightarrow 2\lambda + 2 - \lambda - 3 - 2\lambda - 1 = 0$$

$$\Rightarrow \lambda = -2$$

40. For three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfies $\vec{a} + \vec{b} + \vec{c} = 0$

and $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 2$ then

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \underline{\hspace{2cm}}$$

- (A) 29
(B) $\frac{29}{2}$
(C) $-\frac{9}{2}$
(D) $-\frac{29}{2}$

Answer (D)

Sol. $(\vec{a} + \vec{b} + \vec{c})^2 =$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$\Rightarrow (9 + 16 + 4) + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = \frac{-29}{2}$$

