

GUJCET-ME-2023

Test Booklet No.

0900437

Test Booklet Set No.

09**This booklet contains 16 pages.****DO NOT open this Test Booklet until you are asked to do so.****Important Instructions :**

- 1) The Mathematics test consists of 40 questions. Each question carries 1 mark. For each correct response, the candidate will get 1 mark. For each incorrect response, $\frac{1}{4}$ mark will be deducted. The maximum marks are 40.
- 2) This Test is of 1 hour duration.
- 3) Use **Black Ball Point Pen** only for writing particulars on OMR Answer Sheet and marking answers by darkening the circle '●'.
- 4) Rough work is to be done on the space provided for this purpose in the Test Booklet only.
- 5) **On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room / Hall. The candidates are allowed to take away this Test Booklet with them.**
- 6) The Set No. for this Booklet is **09**. Make sure that the Set No. printed on the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
- 7) The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
- 8) Do not write your Seat No. anywhere else, except in the specified space in the Test Booklet / Answer Sheet.
- 9) Use of White fluid for correction is not permissible on the Answer Sheet.
- 10) Each candidate must show on demand his / her Admission Card to the Invigilator.
- 11) No candidate, without special permission of the Superintendent or Invigilator, should leave his / her seat.
- 12) Use of Simple (Manual) Calculator is permissible.
- 13) The candidate should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak - 01). Cases where a candidate has not signed the Attendance Sheet (Patrak - 01) will be deemed not to have handed over the Answer Sheet and will be dealt with as an unfair means case.
- 14) The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
- 15) No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
- 16) The candidates will write the Correct Test Booklet Set No. as given in the Test Booklet / Answer Sheet in the Attendance Sheet. (Patrak - 01)

MATHEMATICS

1) $\begin{vmatrix} \sin \frac{11\pi}{36} & \cos \frac{11\pi}{36} \\ \sin \frac{2\pi}{9} & \cos \frac{2\pi}{9} \end{vmatrix} = \underline{\hspace{2cm}}.$

(A) $\cos \frac{\pi}{12}$

(B) $\sin \frac{2\pi}{9}$

(C) $\cos \frac{5\pi}{12}$

(D) $\sin \frac{7\pi}{12}$

- 2) If A(K, 1), B(2, 4) and C(1, 1) are the vertices of the ΔABC , such that area of the ΔABC is 6 unit, then K = .

(A) -5 and 3

(B) 5 and -3

(C) 3 and -1

(D) 5 and 3

3) $\left\{ \frac{d}{dx} (x^x + x^{x+1} + x^{x+2}) \right\}_{x=e} = \underline{\hspace{2cm}}.$

(A) $e^e (1 + e^2 + 2e)$

(B) $e^e (3e^2 + 2e + 2)$

(C) $e^e (2e^2 + 4e + 3)$

(D) $e^e (1 + 4e + 2e^2)$

(Space for Rough Work)

4) If $x = a \cos \theta$, $y = a \sin \theta$, then $\frac{d^2 y}{dx^2} = \underline{\hspace{2cm}}$. (where $a \neq 0$, $\theta \neq k\pi$, $k \in \mathbb{Z}$)

(A) $-\frac{1}{a} \operatorname{cosec}^2 \theta \sec \theta$

(B) $\frac{1}{a} \cot^3 \theta$

(C) $\operatorname{cosec}^2 \theta$

(D) $-\frac{1}{a} \operatorname{cosec}^3 \theta$

5) If $y = \sqrt{\sin^{-1} x + y}$, then $\frac{dy}{dx} = \underline{\hspace{2cm}}$ (where $x \in (0, 1)$)

(A) $\frac{1}{(2y-1)\sqrt{1-x^2}}$

(B) $\frac{1}{(1-2y)\sqrt{1-x^2}}$

(C) $\frac{1}{(2y-1)\sqrt{x^2-1}}$

(D) $\frac{1}{(2y+1)\sqrt{1-x^2}}$

6) For the function $f(x) = x + x^{-1}$, $x \in [1, 3]$, the value of C for mean value theorem is :

(A) $\sqrt{3}$

(B) 1

(C) $\sqrt{2}$

(D) $\sqrt{5}$

(Space for Rough Work)

7) Rate of change in the volume of a sphere of a radius r w.r.t. its diameter = _____

(A) $4\pi r^2$

(B) $2\pi r^2$

(C) $\frac{2}{3}\pi r^2$

(D) $8\pi r^2$

8) Which of the following function is decreasing on $\left(0, \frac{\pi}{8}\right)$?

(A) $\sin x$

(B) $-\cos x$

(C) $\cos 4x$

(D) $\tan 4x$

9) An approximate value of $(81.5)^{1/4}$ is :

(A) 3.0436

(B) 3.0033

(C) 3.0046

(D) 3.0465

10) Equation of the normal to the curve $x^{2/3} + y^{2/3} = 2$ at $(1, 1)$ is :

(A) $2x - y - 1 = 0$

(B) $x + y - 2 = 0$

(C) $x + y = 0$

(D) $x - y = 0$

11) If $\int \left\{ \cos^{-1} x - (1-x^2)^{-1/2} \right\} K dx = K \cdot \cos^{-1} x + C$, then $K =$ _____

(A) e^x

(B) $-e^x$

(C) e^{-x}

(D) $e^{\cos^{-1} x}$

(Space for Rough Work)

12) $\int \frac{\tan x}{\cos x(\sec x - 1)(\sec x - 2)} dx = \underline{\hspace{2cm}} + C.$

(A) $\log \left| \frac{\sec x - 2}{\sec x - 1} \right|$

(B) $\log \left| \frac{\sec x + 2}{\sec x - 1} \right|$

(C) $\log \left| \frac{\operatorname{cosec} x + 2}{\operatorname{cosec} x - 1} \right|$

(D) $\log \left| \frac{\cos x + 1}{\cos x - 2} \right|$

13) $\int x^{2019} \cdot e^{x^{2020}} dx = \underline{\hspace{2cm}} + C.$

(A) $\frac{1}{2019} e^{x^{2019}}$

(B) $\frac{1}{2020} e^{x^{2019}}$

(C) $e^{x^{2020}}$

(D) $\frac{1}{2020} e^{x^{2020}}$

14) $\int_{\pi/6}^{\pi/3} \frac{1}{1 + \tan^4 x} dx = \underline{\hspace{2cm}}.$

(A) $\pi/12$

(B) $\pi/4$

(C) $\pi/2$

(D) $\pi/6$

(Space for Rough Work)

15) $\int_0^1 (0.001)^{x/3} \cdot e^x dx = \underline{\hspace{2cm}}.$

(A) $\frac{10-10e}{(1+\log_e 10)}$

(B) $\frac{10-e}{e(1-\log_e 10)}$

(C) $\frac{e-10}{10(1-\log_e 10)}$

(D) $\frac{e-10}{10(1+\log_{10} e)}$

16) Area of the region bounded by the ellipse $9x^2 + 4y^2 = 1$ in the first quadrant is :

(A) $\pi/6$

(B) $\pi/24$

(C) $3\pi/2$

(D) 6π

17) Find the area of the region bounded by the line $y = 3 - x$, the X-axis and the ordinates $x = 2$ and $x = 5$.

(A) 3

(B) $1/2$

(C) $5/2$

(D) $3/2$

18) Area of the region enclosed by the parabola $y = x^2$ and the line $y = x + 2$ is :

(A) $9/2$

(B) $11/2$

(C) $5/2$

(D) $7/2$

(Space for Rough Work)

19) The order and degree of the differential equation $\sqrt[4]{\left(\frac{d^3y}{dx^3}\right)^5} = \sqrt[3]{\left(\frac{d^2y}{dx^2}\right)^4}$ is :

(A) 2 and 16

(B) 3 and 15

(C) 3 and 16

(D) 2 and 12

20) The integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = \sec x$ is :

(A) $\tan x$

(B) $e^{\sec x}$

(C) $\cos x$

(D) $\sec x$

21) Particular solution of the differential equation $\frac{dy}{dx} = -4xy^2$, Given that $y = 1$, where $x = 0$ is :

(A) $y = \frac{1}{2x^2 + 1}$

(B) $x = \frac{1}{2y^2 + 1}$

(C) $y = 2x^2 + 1$

(D) $y = \frac{x}{2x^2 + 1}$

(Space for Rough Work)

22) For any vector $\vec{a} \in R^3$, $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = \underline{\hspace{2cm}}$.

(A) $2|\vec{a}|^2$

(B) $|\vec{a}|$

(C) $|\vec{a}|^2$

(D) $3|\vec{a}|^2$

23) For the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$, $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = \underline{\hspace{2cm}}$.

(A) 11

(B) -11

(C) 5

(D) -5

24) Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$ and $\vec{a} \times \vec{b}$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

25) Cartesian equation of the line through (5, -2, 4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$ is :

(A) $\frac{x-5}{3} = \frac{y+2}{2} = \frac{z-4}{-8}$

(B) $\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$

(C) $\frac{x-3}{5} = \frac{y-2}{2} = \frac{z+8}{-4}$

(D) $\frac{x-3}{5} = \frac{y+2}{-2} = \frac{z+8}{4}$

(Space for Rough Work)

- 26) If a , b and c are intercepts on X-axis, Y-axis and Z-axis respectively of the plane $x + 2y + 3z = 1$, then $2a + 4b + 3c =$ _____.

(A) 19

(B) 5

(C) 6

(D) 17

$$2 \times \frac{1}{1} + 4 \times \frac{1}{2} + 3 \times \frac{1}{3}$$

$$2 + 2 + 1 = 5$$

$$\frac{2}{6} + \frac{4 \times 2}{3 \times 2} + \frac{3 \times 1}{2 \times 3}$$

$$\frac{2}{6} + \frac{8}{6} + \frac{3}{6} = \frac{13}{6}$$

- 27) Measure of the angle between the line $\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) (\lambda \in R)$ and the plane $10x + 2y - 11z = 3$ is :

(A) $\pi/2$ (B) $\cos^{-1}(8/21)$ (C) $\sin^{-1}(8/21)$ (D) $\sin^{-1}(1/21)$

- 28) The coordinates of the corner points of the bounded feasible region are $(0, 10)$, $(5, 5)$, $(15, 15)$ and $(0, 20)$. The maximum of the objective function $Z = 10x + 20y$ is :

(A) 600

(B) 550

(C) 400

(D) 450

[Space for Rough Work]

29) Minimise objective function $z = 3x + 2y$ subject to the constraints :

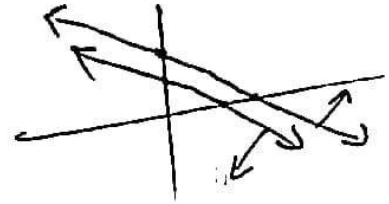
$$x + y \geq 8, x + y \leq 5, x \geq 0, y \geq 0 \text{ is :}$$

(A) 15

(B) 6

(C) 24

(D) No feasible region and hence no feasible solution



30) For independent events A and B $P(A) = P$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$, then $P =$ _____.

(A) $\frac{1}{10}$

(B) $\frac{1}{5}$

(C) $\frac{1}{2}$

(D) $\frac{2}{5}$

31) If a fair coin is tossed 5 times, then the probability of getting exactly 3 heads is :

(A) $\frac{5}{16}$

(B) $\frac{3}{32}$

(C) $\frac{1}{32}$

(D) $\frac{5}{32}$

(Space for Rough Work)

32) If the probability distribution of X is :

X	1	2	3	4	5	6
$P(X)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

then $\text{Var.}(X) = \underline{\hspace{2cm}}$.

(A) $\frac{91}{6}$

(B) $\frac{21}{6}$

(C) $\frac{35}{12}$

(D) $\frac{35}{3}$

33) Relation R in the set $\{\pi, \pi^2, \pi^3\}$ defined by

$R = \{(\pi, \pi), (\pi^2, \pi^2), (\pi^3, \pi^3), (\pi, \pi^2), (\pi^2, \pi^3)\}$ is :

(A) Reflexive but neither symmetric nor transitive

(B) Symmetric but neither reflexive nor transitive

(C) Transitive but neither reflexive nor symmetric

(D) Only symmetric and transitive

34) If $m * n = \frac{mn}{2}$, $\forall m, n \in \mathbb{Q}^+$, then $(4 * 3)^{-1} = \underline{\hspace{2cm}}$.

(A) $\frac{1}{6}$

(B) $\frac{3}{2}$

(C) 2

(D) $\frac{2}{3}$

(Space for Rough Work)

35) $\cos^{-1} \left\{ \cot \left(\sum_{i=1}^3 \cot^{-1} i \right) \right\} = \underline{\hspace{2cm}}$

(A) 0

(B) $\pi/2$

(C) π

(D) $-\pi/2$

36) $\cos(\sec^{-1} 2) + \tan(\cot^{-1} \sqrt{3}) + \sin(\operatorname{cosec}^{-1} \frac{2}{\sqrt{3}}) = \underline{\hspace{2cm}}$

(A) $\frac{3+\sqrt{3}}{5\sqrt{3}}$

(B) $\frac{7+\sqrt{3}}{5\sqrt{3}}$

(C) $\frac{5+\sqrt{3}}{2\sqrt{3}}$

(D) $\frac{7-\sqrt{3}}{2\sqrt{3}}$

37) If $\cos \left(\cos^{-1} \frac{\sqrt{3}}{2} + \sin^{-1} x \right) = 1$, then value of x is :

(A) $-1/2$

(B) $\sqrt{3}/2$

(C) $-\sqrt{3}/2$

(D) 0

(Space for Rough Work)

38) If $\begin{bmatrix} x+y & -2 \\ 7+z & x-y \end{bmatrix} = \begin{bmatrix} -7 & -2 \\ 5 & 0 \end{bmatrix}$, then $2x + 4y + 2z =$ _____

(A) -9

(B) 17

(C) -25

(D) -14

39) If $A = \begin{bmatrix} 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, then $(BA)' =$ _____

(A) $\begin{bmatrix} 3 & 6 \\ 4 & 8 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 8 \\ 6 & 3 \end{bmatrix}$

(D) [11]

40) If $A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & -5 & 0 \\ -5 & 0 & 0 \end{bmatrix}$, then $A^2 =$ _____

(A) -5 I

(B) 5 A

(C) 25 A

(D) 25 I

(Space for Rough Work)