

Test Date: 31st January 2023 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours

Maximum Marks: 300

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts**. **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B** : Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** contains 20 multiple choice questions which have **only one correct answer**. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.
7. **Section-B (1 – 10)** contains 10 Numerical based questions. The answer to each question is rounded off to the nearest integer value. Each question carries **+4 marks** for correct answer and **–1 mark** for wrong answer.

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Given below are two statements :

Statement I : For transmitting a signal, size of antenna (ℓ) should be comparable to wavelength of signal (at least $\ell = \frac{\lambda}{4}$ in dimension)

Statement II : In amplitude modulation, amplitude of carrier wave remains constant (unchanged).

In the light of the above statements, choose the most appropriate answer from the options given below.

- (A) **Statement I** is incorrect but **Statement II** is correct
(B) Both **Statement I** and **Statement II** are incorrect
(C) **Statement I** is correct but **Statement II** is incorrect
(D) Both **Statement I** and **Statement II** are correct

Q2. Heat energy of 735 J is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but do not oscillate. The increase in the internal energy of the gas will be :

- (A) 525 J (B) 735 J
(C) 572 J (D) 441 J

Q3. The number of turns of the a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be :

- (A) 100% (B) 50%
(C) 75% (D) 0%

Q4. A body weight W , is projected vertically upwards from earth's surface to each a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be :

- (A) $\frac{W}{91}$ (B) $\frac{W}{100}$
(C) $\frac{W}{9}$ (D) $\frac{W}{3}$

Q5. An alternative voltage $V = 260 \sin (628t)$ is connected across a pure inductor of 5 mH. Inductive reactance in the circuit is :

- (A) 0.5Ω (B) 6.28Ω
(C) 0.318Ω (D) 3.14Ω

Q6. Match List I with List II

List I

- A. Angular momentum
B. Torque
C. Stress
D. Pressure gradient

List II

- I. $[ML^2T^{-2}]$
II. $[ML^{-2}T^{-2}]$
III. $[ML^2T^{-1}]$
IV. $[ML^{-1}T^{-2}]$

Choose the correct answer from the options given below :

- (A) A – IV, B – II, C – I, D – III
(B) A – I, B – IV, C – III, D – II
(C) A – II, B – III, C – IV, D – I
(D) A – III, B – I, C – IV, D – II

Q7. Given below are statements :

Statement I : In a typical transistor, all three regions emitter, base collector have same doping level.

Statement II : In a transistor, collectors is the thickest and base is the thinnest segment.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (A) Both **Statement I** and **Statement II** are incorrect
(B) **Statement I** is incorrect but **Statement II** is correct
(C) Both **Statement I** and **Statement II** are correct
(D) **Statement I** is correct but **Statement II** is incorrect

Q8. A body is moving with constant speed, in a circle of radius 10m. The body completes one revolution in 4 s. At the end of 3rd second, the displacement of body (in m) from its starting point is :

- (A) $10\sqrt{2}$
(B) 5π
(C) 15π
(D) 30

Q9. Under the same load, wire A having length 5.0m and cross section $2.5 \times 10^{-5} \text{m}^2$ stretches uniformly by the same amount as another wire B of length 6.0m and a cross section of $3.0 \times 10^{-5} \text{m}^2$ stretches. The ratio of the Young's modulus of wire A to that of wire B will be :

- (A) 1 : 1
(B) 1 : 4
(C) 1 : 10
(D) 1 : 2

Q10. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\frac{5}{3}$ is poured inside the bucket, then microscope have to be raised by 30cm to focus the object again. The height of the liquid in the bucket is :

- (A) 75 cm
(B) 12 cm
(C) 18 cm
(D) 50 cm

Q11. If the metals A and B are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8eV and 2.2 eV. Then choose the correct option.

- (A) Both metals A and B will emit photo-electrons
(B) Metal A will not emit photo-electrons
(C) Both metals A and B will not emit photo-electrons
(D) Metal B will not emit photo-electrons

Q12. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5s due to friction between body and the floor. The value of the coefficient of friction is : (Take acceleration due to gravity $g = 10 \text{ ms}^{-2}$)

- (A) 0.2
(B) 0.5
(C) 0.4
(D) 0.3

Q13. Match List I with List II

List I

- A. Microwaves
B. UV rays
C. Infra-red light
D. X-ray

List II

- I. Physiotherapy
II. Treatment of cancer
III. Lasik eye surgery
IV. Aircraft navigation

Choose the correct answer from the options given below :

- (A) A – II, B – IV, C – III, D – I
(B) A – IV, B – III, C – I, D – II
(C) A – III, B – II, C – I, D – IV
(D) A – IV, B – I, C – II, D – III

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- Q14.** A long conducting wire having a current I following through it is, bent into a circular of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils on both the case. The ratio of the magnetic field in first case to that of second case is :
(A) $n^2 : N^2$ (B) $N^2 : n^2$
(C) $n : N$ (D) $N : n$
- Q15.** For a solid rod, the Young's modulus of elasticity is $3.2 \times 10^{11} \text{ Nm}^{-2}$ and density is $8 \times 10^3 \text{ kg m}^{-3}$. The velocity of longitudinal wave in the rod will be.
(A) $3.65 \times 10^3 \text{ ms}^{-1}$ (B) $6.32 \times 10^3 \text{ ms}^{-1}$
(C) $145.75 \times 10^3 \text{ ms}^{-1}$ (D) $18.96 \times 10^3 \text{ ms}^{-1}$
- Q16.** The H amount of thermal energy is developed by a resistor in 10s when a current of 4A is passed through it. If the current is increased to 16A. the thermal energy developed by the resistor in 10s will be:
(A) $\frac{H}{4}$ (B) $4H$
(C) H (D) $16H$
- Q17.** A stone of mass 1kg is tied to end of a massless string of length 1m. If the breaking tension of the string is 400N, then maximum linear velocity, the stone can have without breaking the string, while rotating in horizontal plane, is :
(A) 20 ms^{-1} (B) 400 ms^{-1}
(C) 40 ms^{-1} (D) 10 ms^{-1}
- Q18.** The radius of electron's second stationary orbit in Bohr's atom is R . The radius of 3rd orbit will be
(A) $\frac{R}{3}$ (B) $9R$
(C) $3R$ (D) $2.25R$
- Q19.** Considering a group of positive charges, which of the following statements is correct?
(A) Net potential of the system at a point can be zero but net electric field can't be zero at the point.
(B) Both the net potential and the net electric field cannot be zero at a point.
(C) Net potential of the system cannot be zero at a point but net electric field can be zero at the point.
(D) Both the net potential and the net field can be zero at a point.
- Q20.** A hypothetical gas expands adiabatically such that its volume changes from 08 liters to 27 liters. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$. Then ratio of $\frac{C_p}{C_v}$ will be.
(A) $\frac{3}{1}$ (B) $\frac{4}{3}$
(C) $\frac{1}{2}$ (D) $\frac{3}{2}$

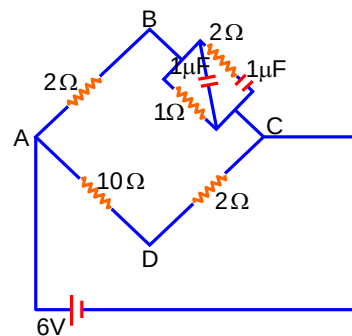
SECTION - B**(Numerical Answer Type)**

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

- Q1.** Two bodies are projected from ground with same speeds 40ms^{-1} at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body projected at an angle of 60° , with horizontal then sum of the maximum heights, attained by the two projectiles, is _____m. (Given $g = 10\text{ms}^{-2}$)
- Q2.** A water heater of power 2000W is used to heat water. The specific heat capacity of water is $4200\text{J kg}^{-1}\text{K}^{-1}$. The efficiency of heater is 70% . Time required to heat 2kg of water from 10°C to 60°C is _____.s.
(Assume that the specific heat capacity of water remains constant over the temperature range of the water).
- Q3.** A ball is dropped from a height of 20m . If the coefficient of restitution for the collision between ball and floor is 0.5 , after hitting the floor, the ball rebounds to a height of _____m.
- Q4.** Two discs of same and different radii are made of different materials such that their thicknesses are 1cm and 0.5cm respectively. The densities of materials are in the ratio $3:5$. The moment of inertia of these disc respectively about their diameters will be in the ratio of $\frac{x}{6}$. The value of x is _____.
- Q5.** If the binding energy of ground state electron in a hydrogen atom is 13.6eV , then, the energy required to remove the electron from the second excited state of Li^{2+} will be : $x \times 10^{-1}\text{eV}$. The value of x is _____.
- Q6.** The displacement equations of two interfering waves are given by $y_1 = 10 \sin\left(\omega t + \frac{\pi}{3}\right)\text{cm}$,
 $y_2 = 5\left[\sin \omega t + \sqrt{3} \cos \omega t\right]\text{cm}$ respectively.
The amplitude of the resultant wave is _____cm.
- Q7.** A series LCR circuit consists of $R = 80\Omega$, $X_L = 100\Omega$ and $X_C = 40\Omega$. The input voltage is $2500\cos(100\pi t)\text{V}$. The amplitude of current, in the circuit, is _____A.
- Q8.** Two parallel plate capacitors C_1 and C_2 each having capacitance of $10\mu\text{F}$ are individually charged by a 100V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between its plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be _____V.
(Assuming Dielectric constant = 10)

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Q9. For the given circuit, in the steady state, $|V_B - V_D| = \underline{\hspace{1cm}} \text{ V}$.



Q10. Two light waves wavelengths 800 and 600nm are used in Young's double slit experiment to obtain interference fringes on a screen placed 7m away from plane of slits. If the two slits are separated by 0.35mm, then shortest distance from the central bright maximum to the where the bright fringes of the two wavelengths coincide will be mm.

PART – B (CHEMISTRY)

SECTION - A

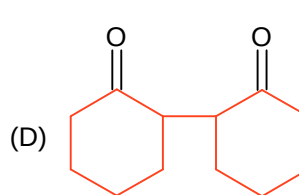
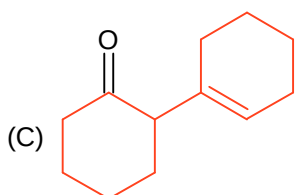
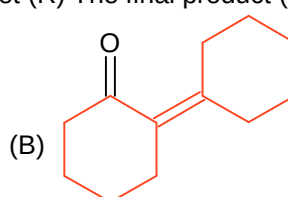
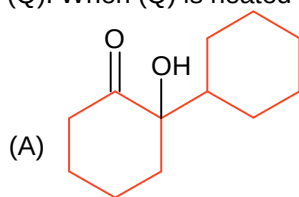
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

- Q1.** In the following halogenated organic compounds the one with maximum number of chlorine atoms in its structure is:
(A) Chloral (B) Gammaxene
(C) Chloropicrin (D) Freon-12
- Q2.** Which one of the following statements is incorrect?
(A) Boron and Indium can be purified by zone refining method.
(B) van Arkel method is used to purify tungsten.
(C) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast.
(D) The malleable iron is prepared from cast iron by oxidising impurities in a reverberatory furnace.
- Q3.** In Dumas method for the estimation of N_2 , the sample is heated with copper oxide and the gas evolved is passed over:
(A) Ni (B) Pd
(C) Copper gauze (D) Copper oxide
- Q4.** Given below are two statements, one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.
Assertion (A): The first ionization enthalpy of 3d series elements is more than that of group 2 metals
Reason (R): In 3d series of elements successive filling of d- orbitals takes place. In the light of the above statements, choose the correct answer from the option given below:
(A) (A) is true but (R) is false
(B) (A) is false but (R) is true
(C) Both (A) and (R) are true but (R) is **not** the correct explanation of (A)
(D) Both (A) and (R) are true and (R) is the correct explanation of (A)
- Q5.** Which of the following compounds are not used as disinfectants?
A. Chloroxylenol
B. Bithional
C. Veronal
D. Prontosil
E. Terpeneol
Choose the **correct** answer from the options given below :
(A) A, B (B) A, B, E
(C) C, D (D) B, D, E
- Q6.** A hydrocarbon 'X' with formula C_6H_8 uses two moles of H_2 on catalytic hydrogenation of its one mole. on ozonolysis. 'X' yield two moles of methane dicarbaldehyde. The hydrocarbon 'X' is :
(A) hexa-1, 3, 5-triene (B) cyclohexa-1, 3- diene
(C) 1- methylcyclopenta-1, 4-diene (D) cyclohexa-1, 4-diene

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- Q7. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dil. NaOH we get (R). The final product (R) is:



- Q8. The Lewis acid character of boron tri halides follows the order:

- (A) $\text{BF}_3 > \text{BCl}_3 > \text{BBR}_3 > \text{BI}_3$ (B) $\text{BI}_3 > \text{BBR}_3 > \text{BCl}_3 > \text{BF}_3$
 (C) $\text{BBR}_3 > \text{BI}_3 > \text{BCl}_3 > \text{BF}_3$ (D) $\text{BCl}_3 > \text{BF}_3 > \text{BBR}_3 > \text{BI}_3$

- Q9. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following?

- (A) $2\text{SO}_2 + \text{O}_2 + 2\text{H}_2\text{O} \rightarrow 2\text{H}_2\text{SO}_4$ (B) $\text{CO}_2 + \text{H}_2\text{O} \rightarrow \text{H}_2\text{CO}_3$
 (C) $4\text{NO}_2 + \text{O}_2 + 2\text{H}_2\text{O} \rightarrow 4\text{HNO}_3$ (D) $\text{N}_2\text{O}_5 + \text{H}_2\text{O} \rightarrow 2\text{HNO}_3$

- Q10. Given below are two statements:

Statement I: Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green

Statement II: The green colour observed is due to the formation of copper (I) metaborate

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (A) **Statement I** is false but **Statement II** is true
 (B) Both **Statement I** and **Statement II** are false
 (C) Both **Statement I** and **Statement II** are true
 (D) **Statement I** is true but **Statement II** is false

- Q11. Given below are two statements:

Statement I: H_2O_2 is used in the synthesis of Cephalosporin.

Statement II: H_2O_2 is used for the restoration of aerobic conditions to sewage wastes.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (A) Both **Statement I** and **Statement II** are correct
 (B) **Statement I** is incorrect but **Statement II** is correct
 (C) **Statement I** is correct but **Statement II** is incorrect
 (D) Both **Statement I** and **Statement II** are incorrect.

- Q12. When a hydrocarbon A undergoes complete combustion it requires 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A?

- (A) C_5H_8 (B) C_{11}H_4
 (C) C_{11}H_8 (D) C_9H_8

- Q13. Evaluate the following statements for their correctness.

- A. The elevation in boiling point temperature of water will be same for 0.1 M NaCl and 0.1 M urea.
 B. Azeotropic mixture boil without change in their composition.
 C. Osmosis always takes place from hypertonic to hypotonic solution.
 D. The density of 32% H_2SO_4 solution having molarity 4.09 M is approximately 1.26 g mL^{-1} .

E. A negatively charged sol is obtained when KI solution is added to silver nitrate solution
Choose the correct answer from the options given below:

- (A) B and D only (B) A, B and D only
(C) A and C only (D) B, D and E only

Q14. Arrange the following orbitals in decreasing order of energy

- A. $n = 3, l = 0, m = 0$
B. $n = 4, l = 0, m = 0$
C. $n = 3, l = 1, m = 0$
D. $n = 3, l = 2, m = 1$

The correct option for the order is :

- (A) $D > B > A > C$ (B) $B > D > C > A$
(C) $A > C > B > D$ (D) $D > B > C > A$

Q15. Incorrect statement for the use of indicators in acid-base titration is:

- (A) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.
(B) Phenolphthalein may be used for a strong acid vs strong base titration.
(C) Methyl orange may be used for a weak acid vs weak base titration.
(D) Methyl orange is a suitable indicator for a strong acid vs weak base titration.

Q16. Which of the following elements have half-filled f-orbitals in their ground state?

(Given: atomic number Sm = 62; Eu = 63; Tb = 65; Gd = 64, Pm = 61)

- A. Sm
B. Eu
C. Tb
D. Gd
E. Pm

Choose the **correct** answer from the options given below :

- (A) C and D only (B) A and B only
(C) B and D only (D) A and E only

Q17. The element playing significant role in neuromuscular function and inter neuronal transmission is:

- (A) Mg (B) Li
(C) Ca (D) Be

Q18. Match List I with List II

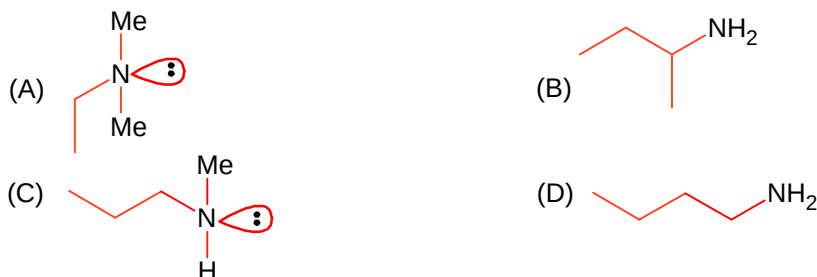
List-I		List-II	
A.	Physisorption	I.	Single Layer Adsorption
B.	Chemisorption	II.	$20-40\text{kJ mol}^{-1}$
C.	$\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \xrightarrow{\text{Fe(s)}} 2\text{NH}_3(\text{g})$	III.	Chromatography
D.	Analytical Application or Adsorption	IV.	Heterogeneous catalysis

Choose the correct answer from the options given below:

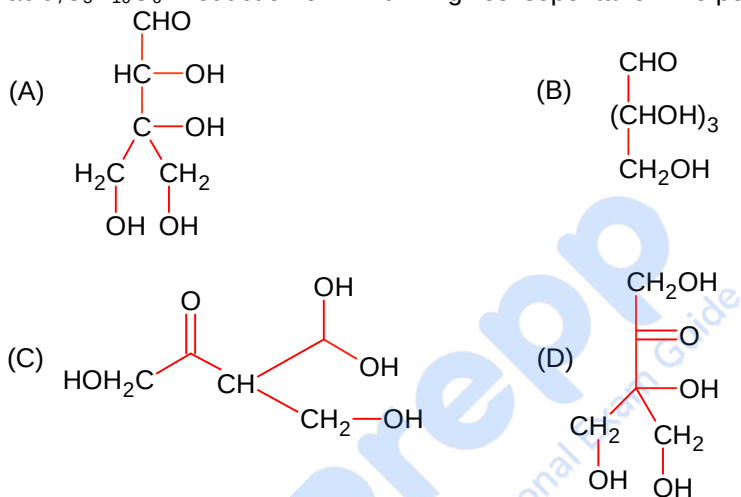
- (A) A- IV, B-II, C-III, D-I (B) A-II, B-I, C-IV, D-III
(C) A-III, B-IV, C-I, D-II (D) A-II, B-III, C-I, D-IV

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- Q19.** An organic compound [A] ($C_4H_{11}N$), shows optical activity and gives N_2 gas on treatment with HNO_2 . The compound [A] reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH . The structure of A is:



- Q20.** Compound A, $C_5H_{10}O_5$, given a tetraacetate with Ac_2O and oxidation of A with Br_2-H_2O gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is:



SECTION - B

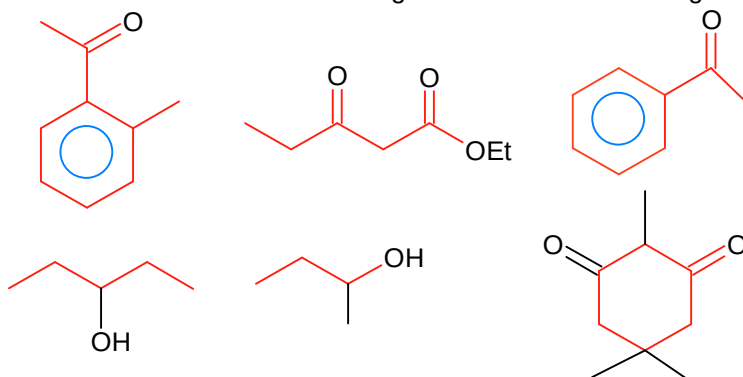
(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

- Q1.** Assume carbon burns according to following equation:
 $2\text{C}_{(s)} + \text{O}_{2(g)} \rightarrow 2\text{CO}_{(g)}$
 When 12 g carbon is burnt in 48 g of oxygen, the volume of carbon monoxide produced is _____ $\times 10^{-1}$ L at STP [Nearest integer]
 [Given: Assume CO as ideal gas, Mass of C is 12 g mol^{-1} , Mass of O is 16 g mol^{-1} and molar volume of an ideal gas at STP is 22.7 L mol^{-1}]
- Q2.** Amongst the following, the number of species having the linear shape is _____.
 $\text{XeF}_2, \text{I}_3^+, \text{C}_3\text{O}_2, \text{I}_3^-, \text{CO}_2, \text{SO}_2, \text{BeCl}_2$ and $\text{BCl}_2^{\ominus}$
- Q3.** The rate constant for a first order reaction is 20 min^{-1} . The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is _____ $\times 10^{-2}$ min.
 (Nearest integer)
 (Given: $\ln 10 = 2.303$
 $\log 2 = 0.3010$)
- Q4.** Enthalpies of formation of $\text{CCl}_4(\text{g})$, $\text{H}_2\text{O}(\text{g})$, $\text{CO}_2(\text{g})$ and $\text{HCl}(\text{g})$ are -105 , -242 , -394 and -92 kJ mol^{-1} respectively. The magnitude of enthalpy of the reaction given below is _____ kJ mol^{-1} (nearest integer)
 $\text{CCl}_4(\text{g}) + 2\text{H}_2\text{O}(\text{g}) \rightarrow \text{CO}_2(\text{g}) + 4\text{HCl}(\text{g})$
- Q5.** The resistivity of a 0.8 M solution of an electrolyte is $5 \times 10^{-3} \Omega \text{ cm}$. Its molar conductivity is _____ $\times 10^4 \Omega^{-1} \text{ cm}^2 \text{ mol}^{-1}$. (Nearest integer)
- Q6.** At 298 K, the solubility of silver chloride in water is $1.434 \times 10^{-3} \text{ g L}^{-1}$. The value of $-\log K_{\text{sp}}$ for silver chloride is _____.
 (Given mass of Ag is 107.9 g mol^{-1} and mass of Cl is 35.5 g mol^{-1})
- Q7.** The number of alkali metal(s), from Li, K, Cs, Rb having ionization enthalpy greater than 400 kJ mol^{-1} and forming stable super oxide is _____.
- Q8.** A sample of a metal oxide has formula $\text{M}_{0.83}\text{O}_{1.00}$. The metal M can exist in two oxidation states +2 and +3. In the sample of $\text{M}_{0.83}\text{O}_{1.00}$, the percentage of metal ions existing in +2 oxidation state is _____ % (nearest integer)
- Q9.** If the CFSE of $[\text{TiH}_2\text{O}_6]^{3+}$ is -96.0 kJ / mol, this complex will absorb maximum at wavelength _____ nm. (nearest integer)
 Assume Plank's constant (h) = 6.4×10^{-34} Js, speed of light (c) = 3.0×10^8 m/s and Avogadro's constant (N_A) = 6×10^{23} / mol.

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Q10. The number of molecules which gives haloform test among the following molecules is _____.



PART - C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. If $\phi(x) = \frac{1}{\sqrt{x}} \int_{\frac{\pi}{4}}^x (4\sqrt{2} \sin t - 3\phi'(t)) dt$, $x > 0$, then $\phi'\left(\frac{\pi}{4}\right)$ is equal to :

(A) $\frac{8}{\sqrt{\pi}}$

(B) $\frac{4}{6 + \sqrt{\pi}}$

(C) $\frac{4}{6 - \sqrt{\pi}}$

(D) $\frac{8}{6 + \sqrt{\pi}}$

Q2. Let the plane $P: 8x + a_1y + a_2z + 12 = 0$ be parallel to the line $L: \frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is :

(A) $\sqrt{\frac{7}{2}}$

(B) $\sqrt{\frac{2}{7}}$

(C) $\frac{6}{\sqrt{14}}$

(D) $\sqrt{14}$

Q3.
$$\lim_{x \rightarrow \infty} \frac{(\sqrt{3x+1} + \sqrt{3x-1})^6 + (\sqrt{3x+1} - \sqrt{3x-1})^6}{(x + \sqrt{x^2-1})^6 + (x - \sqrt{x^2-1})^6}$$

(A) is equal to 27

(B) is equal to $\frac{27}{2}$

(C) does not exist

(D) is equal to 9

Q4. Let $f: \mathbb{R} - \{2, 6\} \rightarrow \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$. Then range of f is

(A) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

(B) $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

(C) $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

(D) $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

Q5. Let $\alpha > 0$. If $\int_0^\alpha \frac{x}{\sqrt{x+\alpha} - \sqrt{x}} dx = \frac{16 + 20\sqrt{2}}{15}$, then α is equal to :

(A) 2

(B) 4

(C) $\sqrt{2}$

(D) $2\sqrt{2}$

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- Q6.** Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is.....
- (A) 3 (B) $\frac{5}{2}$
(C) 2 (D) $\frac{3}{2}$
- Q7.** The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is $(2, a, 4)$, $a \in \mathbb{N}$. If the volume of the tetrahedron OABC is 144 unit^3 , then Which of the following points is NOT on P?
- (A) $(3, 0, 4)$ (B) $(0, 4, 4)$
(C) $(0, 6, 3)$ (D) $(2, 2, 4)$
- Q8.** The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology is :
- (A) 2 (B) 1
(C) 3 (D) 4
- Q9.** Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha (> 0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30 - \alpha$. If the mean and variance of the marks of the combined class of $100 + n$ students are respectively 50 and 350, then the sum of variances of classes A and B is :
- (A) 500 (B) 650
(C) 900 (D) 450
- Q10.** Let $a_1, a_2, a_3, \dots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :
- (A) $\frac{381}{4}$ (B) 9
(C) $\frac{33}{4}$ (D) 24
- Q11.** Among the relations
- $S = \left\{ (a, b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0 \right\}$ and $T = \left\{ (a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z} \right\}$,
- (A) both S and T are symmetric (B) neither S nor T is transitive
(C) S is transitive but T is not (D) T is symmetric but S is not
- Q12.** The complex number $z = \frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ is equal to :
- (A) $\sqrt{2}i \left(\cos \frac{5\pi}{12} - i \sin \frac{5\pi}{12} \right)$ (B) $\cos \frac{\pi}{12} - i \sin \frac{\pi}{12}$
(C) $\sqrt{2} \left(\cos \frac{5\pi}{12} + i \sin \frac{5\pi}{12} \right)$ (D) $\sqrt{2} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$

Q13. If a point $P(\alpha, \beta, \gamma)$ satisfying

$$(\alpha \ \beta \ \gamma) \begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0 \ 0 \ 0) \text{ lies on the plane } 2x + 4y + 3z = 5, \text{ then } 6\alpha + 9\beta + 7\gamma \text{ is equal to :}$$

- (A) -1 (B) 11
(C) $\frac{11}{5}$ (D) $\frac{5}{4}$

Q14. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a

point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$, is equal to :

- (A) $(8, \infty)$ (B) $(0, 4]$
(C) $(4, \infty)$ (D) $(2, 12]$

Q15. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is

- (A) 7 (B) 4
(C) 6 (D) 5

Q16. Let $y = y(x)$ be the solution of the differential equation $(3y^2 - 5x^2)y \, dx + 2x(x^2 - y^2) \, dy = 0$ such that $y(1) = 1$. Then $\left| (y(2))^3 - 12y(2) \right|$ is equal to :

- (A) $32\sqrt{2}$ (B) 32
(C) 64 (D) $16\sqrt{2}$

Q17. Let : $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $25|\vec{r}|^2$ is equal to

- (A) 339 (B) 560
(C) 449 (D) 336

Q18. Let $(a, b) \subset (0, 2\pi)$ be the largest interval for which $\sin^{-1}(\sin \theta) - \cos^{-1}(\sin \theta) > 0$, $\theta \in (0, 2\pi)$, holds.

If $\alpha x^2 + \beta x + \sin^{-1}(x^2 - 6x + 10) + \cos^{-1}(x^2 - 6x + 10) = 0$ and $\alpha - \beta = b - a$, then α is equal to :

- (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{16}$
(C) $\frac{\pi}{48}$ (D) $\frac{\pi}{12}$

Q19. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1, 2]$ is :

- (A) $\frac{3}{2}$ (B) $\frac{3}{4}$
(C) $\frac{5}{4}$ (D) $\frac{1}{4}$

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- Q20.** The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^x + 1 = 0$, $x \in \mathbb{R}$ has :
- (A) no solution
 - (B) four solutions two of which are negative
 - (C) two solutions and both are negative
 - (D) two solutions and only one of them is negative

SECTION - B

(Numerical Answer Type)

This section contains **10** Numerical based questions. The answer to each question is rounded off to the nearest integer value.

- Q1.** Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ is equal to.....
- Q2.** Let A be the event that the absolute difference between two randomly chosen real numbers in the sample space $[0, 60]$ is less than or equal to a. If $P(A) = \frac{11}{36}$, then a is equal to.....
- Q3.** If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$, then $n^2 + n + 15$ is equal to :
- Q4.** If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^\ell}\right)^9$ is -84 and the coefficient of $x^{-3\ell}$ is $2^\alpha \beta$, where $\beta < 0$ is an odd number, then $|\alpha\ell - \beta|$ is equal to.....
- Q5.** Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix $\text{Adj}(2 \cdot \text{Adj}(2A^{-1}))$ is 2^{84} , then n is equal to.....
- Q6.** Let S be the set of all $a \in \mathbb{N}$ such that the area of the triangle formed by the tangent at the point $P(b, c)$, $b, c \in \mathbb{N}$, on the parabola $y^2 = 2ax$ and the lines $x = b, y = 0$ is 16 unit^2 , then $\sum_{a \in S} a$ is equal to.....
- Q7.** The sum $1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + 5 \cdot 9^2 - \dots + 15 \cdot 29^2$ is.....
- Q8.** The coefficient of x^{-6} , in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is.....
- Q9.** Let the area of the region $\{(x, y) : |2x - 1| \leq y \leq |x^2 - x|, 0 \leq x \leq 1\}$ be A. Then $(6A + 11)^2$ is equal to.....
- Q10.** Let $A = [a_{ij}]$, $a_{ij} \in \mathbb{Z} \cap [0, 4]$, $1 \leq i, j \leq 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is.....