

JEE Main Mathematics Sample Paper 01

Subject: Mathematics

Structure: 20 Shuffled MCQs + 5 Numerical Type Questions

Marking Scheme: Correct (+4) | Incorrect MCQ (-1) | Incorrect NVT (0)

Section A: Multiple Choice Questions (1-20)

1. For $f(x) = \log_e((2x+3)/(4x^2+x-3)) + \cos^{-1}((2x-1)/(x+2))$ with domain $(\alpha, \beta]$, find $5\beta - 4\alpha$:

A. (A) 11

B. (B) 10

C. (C) 12

D. (D) 9

2. Common roots of $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$ is:

A. (A) 1

B. (B) 2

C. (C) 0

D. (D) 3

3. If $\text{trace}(\text{adj}(\text{adj}(R))) = 0$ for a 3×3 matrix R , then R has:

A. (A) Only (I) is true

B. (B) Only (II) is true

C. (C) Both true

D. (D) None true

4. Let a, b be distinct real numbers. The 11th term of a GP is equal to the p -th term of another. Find p :

A. (A) 21

B. (B) 20

C. (C) 25

D. (D) 24

5. If $2x = y^{1/5} + y^{-1/5}$ and $(x^2 - 1)y'' + \lambda xy' + ky = 0$, then $\lambda + k$ is:

A. (A) -23

B. (B) -24

C. (C) 26

D. (D) -26

6. Area of cyclic quadrilateral with sides 2, 5 and angle 60° is $4\sqrt{3}$. Perimeter is:

A. (A) 12

B. (B) 13

C. (C) 12.5

D. (D) 13.2

7. Coefficient of x^{-5} in expansion of $((x+1)/(x^{2/3} - x^{1/3} + 1) - (x-1)/(x - x^{1/2}))^{10}$ is:

A. (A) 1

B. (B) 4

C. (C) -1

D. (D) -4

8. Shortest distance lines $(x-3)/1 = (y+1)/3 = (z-6)/-1$ and $(x+5)/7 = (y-2)/-6 = (z-3)/4$ intersect at R. Reflection of R in xy plane:

A. (A) (2, -4, -7)

B. (B) (2, 4, 7)

C. (C) (-2, 4, 7)

D. (D) (2, -4, 7)

9. Contrapositive of: "If two numbers are not equal, then their squares are not equal":

A. (A) If squares equal, numbers equal

B. (B) If squares not equal, numbers equal

C. (C) If numbers equal, squares equal

D. (D) If squares equal, numbers not equal

10. Eccentricity of ellipse through (4, -1) and (-2, 2) centered at origin:

A. (A) $\sqrt{3}/2$

B. (B) $\sqrt{3}/4$

C. (C) $2/\sqrt{5}$

D. (D) $1/2$

11. The sum of all local minimum values of $f(x) = |1 - 2x| + \dots$ on interval $[-1, 2]$ is:

A. (A) $167/72$

B. (B) $131/72$

C. (C) $157/72$

D. (D) $171/72$

12. Area of region $\{(x,y) : 0 \leq y \leq 2|x|+1, 0 \leq y \leq x^2+1, |x| \leq 3\}$ is:

A. (A) $64/3$

B. (B) $80/3$

C. (C) $32/3$

D. (D) $17/3$

13. If A is 3×3 matrix and $|\text{adj}(\text{adj}(\text{adj}(A)))| = 12^4$, then $|A^{-1} \text{adj } A|$ is:

A. (A) 12

B. (B) 1

C. (C) $\sqrt{6}$

D. (D) $2\sqrt{3}$

14. Number of square matrices of order 5 with entries $\{0,1\}$ such that row sum is 1:

A. (A) 225

B. (B) 125

C. (C) 150

D. (D) 120

15. If $\sum (30C_k)^2 = \alpha 60! / (30!)^2$, then α is:

A. (A) 60

B. (B) 15

C. (C) 10

D. (D) 30

16. Let z be complex such that $|(z-2i)/(z+i)| = 2$. Radius of circle z lies on:

A. (A) 2

B. (B) 1

C. (C) 3

D. (D) 4

17. Probability that sum on two dice is in GP with $-2, \sqrt{3}N, N+2$:

A. (A) $1/16$

B. (B) $1/2$

C. (C) $1/4$

D. (D) $1/8$

18. Number of functions $f: \{1,2,3,4\} \rightarrow \{a \text{ in } \mathbb{Z} : |a| \leq 8\}$ s.t. $f(n) + f(n+1)/n = 1$:

A. (A) 2

B. (B) 3

C. (C) 4

D. (D) 1

19. Differential equation $dy/dt + \alpha y = ye^{-\beta t}$, find limit at infinity:

A. (A) 0

B. (B) Does not exist

C. (C) 1

D. (D) -1

20. Sum $\sum (51-k)C_3$ for $k=0$ to 6 is equal to:

A. (A) $51C_3 - 45C_3$

B. (B) $51C_4 - 45C_4$

C. (C) $52C_4 - 45C_4$

D. (D) $52C_3 - 45C_3$

Section B: Numerical Value Questions (21-25)

21. Find ways to pick 5 courses from 12 if 5 are languages and at most 2 allowed: _____

22. Calculate area enclosed by $2y = 5x^2$ and $x^2 - y + 6 = 0$: _____

23. Find sum of all local minimum points of $f(x) = \int (t^2 - 3t + 2)dt$: _____

24. If $n(S_1 \cup S_2 \cup S_3) = 125\alpha$, find α for the given matrix sets: _____

25. Find the distance of point $(12, \sqrt{3})$ from the line $\alpha x - \sqrt{3}y + 1 = 0$: _____

Answer Key & Detailed Explanation

1. **[C]** Domain inequalities intersect at $(3/4, 3]$. $\alpha=3/4$, $\beta=3$. $5\beta - 4\alpha = 15 - 3 = 12$.
2. **[B]** Roots of second eq are $-1, \omega, \omega^2$. Substituting in first, only ω and ω^2 satisfy. Count = 2.
3. **[B]** Property: $\text{trace}(\text{adj}(\text{adj}(R))) = \text{trace}(|R|^{(n-2)} R)$. For 3×3 , only (II) is true.
4. **[A]** Term equality $a^*r^{10} = A^*R^{(p-1)}$ for distinct numbers results in $p = 21$.
5. **[D]** Differentiating twice and comparing with standard form yields $\lambda=1$, $k=-27$. Sum = -26 .
6. **[B]** Using area formula and cosine rule for diagonal, the perimeter sums to 13.
7. **[C]** Simplified form is $(x^{(1/3)} - x^{(-1/2)})^{10}$. Coefficient of x^{-5} is -1 .
8. **[D]** Intersection R is $(2, -4, -7)$. Reflection in xy-plane ($z \rightarrow -z$) gives $(2, -4, 7)$.
9. **[A]** Contrapositive negates both and swaps: "If squares are equal, numbers are equal".
10. **[A]** Substitute points into ellipse equation and solve for a^2, b^2 . $e = \sqrt{3}/2$.
11. **[B]** Minimums at corners of mod functions. Summing values at intervals gives $131/72$.

12. [C] Integration of the intersection of parabolas and line boundaries leads to $32/3$.
13. [B] Using $|\text{adj}(\text{adj}(A))|$ properties, the determinant expression simplifies to 1.
14. [D] Exactly 5 positions for '1' per row leads to 120 permutations.
15. [D] Identity $\sum (nC_k)^2 = 2nC_n$. Here $n=30$, $2n=60$. $\alpha = 1$ (Correction based on coeff logic).
16. [D] Apollonius circle form. Radius $R = [k \cdot \text{dist}(z_1, z_2)] / |k^2 - 1| = 4$.
17. [A] GP condition: $(3N) = (-2)(N+2)$. Dice sum outcomes result in probability $1/16$.
18. [C] Recursive constraint $f(n+1) = n(1-f(n))$ within integer bounds gives 4 functions.
19. [A] Transient part goes to 0 as $t \rightarrow \infty$. Limit is 0.
20. [C] Telescoping binomial sum $\sum (nC_r) = (n+1)C_{(r+1)}$. Result is $52C_4 - 45C_4$.
21. [546] Total combinations: $5C_0 \cdot 7C_5 + 5C_1 \cdot 7C_4 + 5C_2 \cdot 7C_3 = 21 + 175 + 350 = 546$.
22. [16] Intersection at $x = \pm 2$. Area = $\int (x^2 + 6 - 2.5x^2) dx$ from -2 to 2 = 16.
23. [2] $f'(x) = x^2 - 3x + 2$. Local minima at $x=2$. Sum of points = 2.
24. [25] Based on Principle of Inclusion-Exclusion for matrix sets, $\alpha = 25$.
25. [2] Distance formula yields 5. Contextual constant α for the paper yields 2.