JEE Main Mathematics Sample Paper 02

Subject: Mathematics

Structure: 20 Shuffled MCQs + 5 Numerical Type Questions

Marking Scheme: Correct (+4) | Incorrect MCQ (-1) | Incorrect NVT (0)

Section A: Multiple Choice Questions (1-20)

1. If area bounded by $y^2 - 2y = -x$ and $x + y = 0$ is A, then 8A is:

A. (A) 12

B. (B) 8

C. (C) 4

D. (D) 16

2. Integral of $(x+1)/\sqrt{(2x-1)}\,dx = f(x)\sqrt{(2x-1)} + C$. Then f(x) is:

A. (A) 1/3(x+1)

B. (B) 2/3(x+2)

C. (C) 2/3(x-4)

D. (D) 1/3(x+4)

3. Maximum value of $x^m y^n/((1+x^{2m})(1+y^{2n}))$ for positive integers m, n is:

A. (A) 1/4

B. (B) 1

C. (C) 1/2

D. (D) 2

4. Locus of midpoint of chords of $y^2 = 4x$ subtending angle $\pi/3$ at vertex:

A. (A) 3

B. (B) 4

C. (C) 6

D. (D) 2

5. If vectors a, b, c are non-coplanar, find λ for coplanar points A, B, C, D:

A. (A) 18

B. (B) 0

C. (C) 24

D. (D) 6

6. The sum of positive integer divisors of $|f(3) - f(2)|$ for $f(x) = 2x^n + \lambda$ is:

A. (A) 61

B. (B) 58

C. (C) 59

D. (D) 60

7. Shortest distance between $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is:

A. (A) 2

B. (B) 3/2

C. (C) 5/2

D. (D) 3

8. If $|z| = 1$ and $z \neq 1$, then $1/(1-z)$ lies on:

A. (A) Line $x = 1/2$

B. (B) Circle radius 1

C. (C) Line $y = 1/2$

D. (D) Imaginary axis

9. Probability of at least 3 players picking correct T-shirt out of 15 randomly:

A. (A) 2/15

B. (B) 1/6

C. (C) 5/36

D. (D) 5/24

10. Value of lim x->2p+ [f(x)] for roots of $x^2 + px + q = 0$:

A. (A) -1

B. (B) 1

C. (C) 2

D. (D) 0

11. If the system $ax + 2y + z = 1$, $2ax + 3y + z = 1$ has no solution:

A. (A) $\alpha = -1$, $\beta \neq 2$

B. (B) $\alpha \neq -1$, $\beta = 2$

C. (C) $\alpha = 3$

D. (D) $\alpha = -1$

12. Ratio of areas of A and B defined by inequality circles:

A. (A) $(\pi - 1)/(\pi + 1)$

B. (B) $\pi/(\pi - 1)$

C. (C) $(\pi + 1)/(\pi - 1)$

D. (D) $\pi/(\pi + 1)$

13. Value of $1/2(\Delta - 21 \sin^{-1}(2/\sqrt{7}))$ for region $x^2 + y^2 \leq 21$:

A. (A) $\sqrt{3} - 4/3$
B. (B) $\sqrt{3} - 2/3$
C. (C) $2\sqrt{3} - 1/3$
D. (D) $2\sqrt{3} - 2/3$

14. Integral $\int \max\{x^2, 1 + [x]\}dx$ from 0 to 2:

A. (A) $(8 + 4\sqrt{2})/3$
B. (B) $(1 + 5\sqrt{2})/3$
C. (C) $(5 + 4\sqrt{2})/3$
D. (D) $(4 + 5\sqrt{2})/3$

15. Tangents to $x^2 + y^2 - 3x + 10y - 15 = 0$ at A and B intersect at C. Radius:

A. (A) $3\sqrt{3}/4$
B. (B) $\sqrt{13}$
C. (C) $2\sqrt{13}/3$
D. (D) $2\sqrt{13}$

16. If α, β are roots of $x^2 + 60^{(1/4)}x + a = 0$ and $\alpha^4 + \beta^4 = -30$, find a:

A. (A) 25
B. (B) 10
C. (C) 5
D. (D) 15

17. Remainder when $(2023)^{2023}$ is divided by 35:

A. (A) 7
B. (B) 12
C. (C) 21
D. (D) 28

18. If $f(x + y) = f(x)f(y)$ and $\Sigma f(k) = 3279$, find n if $f(1) = 3$:

A. (A) 8
B. (B) 7
C. (C) 9
D. (D) 6

19. Probability of drawing red second ball if bag A (3W, 7R) and B (3W, 2R):

A. (A) 1/4
B. (B) 1/3
C. (C) 1/9
D. (D) 3/10

20. Number of non-empty subsets of {1, 2, 3, 5, 7, 10, 11} with sum mult of 3:

A. (A) 42

B. (B) 43

C. (C) 44

D. (D) 45

Section B: Numerical Value Questions (21-25)

21. Evaluate 5/π Σ Area(Ei) where Ei are constructed ellipses: ________

22. Calculate |10 - 3b·c + |d x c|²| for given vectors: ________

23. Find α + β where they represent points of non-continuity: ________

24. Evaluate integral ∫ |log x|dx and express as m/n log(n²/e). Find m²+n²-5: ________

25. In binomial $(1+2x)^n$, ratio of three consecutive terms 2:5:8. Find n: ________

Answer Key & Detailed Explanation

1. [B] Intersection of parabola and line. Area = 1. Therefore 8A = 8.

2. [C] Substitution √2x-1 = t results in f(x) = 2/3 (x-4).

3. [A] AM-GM on denominators: max value is 1/2 * 1/2 = 1/4.

4. [A] Midpoint calculation leads to a shifted parabola with vertex at 3.

5. [C] Vectors coplanar implies determinant is zero. Solving for λ gives 24.

6. [D] The integer result is 12. Sum of divisors: $1+2+3+4+6+12 = 60$.

7. [B] Skew lines distance formula: $|(a2-a1).(b1xb2)| / |b1xb2| = 3/2$.

8. [A] Real part of $1/(1-z)$ when $|z|=1$ is $1/2$. Lies on line $x = 1/2$.

9. [C] Binomial prob Σ (15Ck)(1/15)^k (14/15)^(15-k) for $k \geq 3$ gives 5/36.

10. [D] Functional limit at root boundaries evaluates to 0.

11. [A] System inconsistent if $\Delta = 0$ and specific deltas $\neq 0$. $\alpha = -1$, $\beta \neq 2$.

12. [D] Geometric ratio of areas within circle constraints is $\pi/(\pi+1)$.

13. [B] Segment subtraction from circle area results in $\sqrt{3} - 2/3$.

14. [C] Integration splitting at $[0,1]$, $[1,\sqrt{2}]$, $[\sqrt{2},2]$ leads to $(5+4\sqrt{2})/3$.

15. [C] Circle tangent geometry yields result $2\sqrt{13}/3$.

16. [C] Newton's identity for power sums of roots. For S4=-30, a = 5.

17. [A] 7^2023 mod 35. Multiple of 7, check mod 5. Remainder is 7.

18. [B] $f(x)=3^x$. Geometric series sum $3(3^n - 1)/2 = 3279$ gives $n=7$.

19. [D] Total probability: $P(B)P(R|B) + P(A)P(R|A) = 3/10$.

20. [B] Generating function method for roots of unity gives 43 subsets.

21. [4.5] Evaluates to an infinite geometric series of areas. Result = 4.5.

22. [14] Substitution of magnitudes into vector identity yields 14.

23. [3] Discontinuity at $x=1$ and non-diff at $x=1,2$. Sum $\alpha+\beta = 3$.

24. [15] Evaluating integral splitting and solving for constant term gives 15.

25. [33] Solving consecutive ratio equations results in $n = 33$.