

JEE Main Mathematics Sample Paper 03

Section A: Multiple Choice Questions (1-10)

1. Value of $|z|$ if $z + |z| = 3 + i$:

- | | |
|----------------------|----------------------|
| A. (A) $5/3$ | B. (B) $5/4$ |
| C. (C) $\sqrt{34}/3$ | D. (D) $\sqrt{41}/4$ |

2. Let α, β be roots of $x^2 \sin \theta - x(\sin \theta \cos \theta + 1) + \cos \theta = 0$. Sum of GP is:

- | | |
|--|--|
| A. (A) $1/(1+\cos\theta) - 1/(1-\sin\theta)$ | B. (B) $1/(1-\cos\theta) - 1/(1+\sin\theta)$ |
| C. (C) Addition form | D. (D) Multiplication form |

3. If A, B invertible 3×3 and $\det(ABAT)=8$, $\det(AB^{-1})=8$, then $\det(BA^{-1}BT)$ is:

- | | |
|-----------|---------------|
| A. (A) 1 | B. (B) $1/4$ |
| C. (C) 16 | D. (D) $1/16$ |

4. If a_i is in AP and $a_1 + a_3 = 10$. If mean is $19/2$, variance is:

- | | |
|------------|------------|
| A. (A) 200 | B. (B) 210 |
| C. (C) 220 | D. (D) 105 |

5. Convergence of series integral evaluates to:

- | | |
|-------------------------|----------------------------|
| A. (A) $(\sqrt{2}+1)/2$ | B. (B) $3(\sqrt{2}+1)$ |
| C. (C) $2\sqrt{2}$ | D. (D) $3/2(\sqrt{2} + 1)$ |

6. Foot of perpendicular from $(2,0,5)$ on line (α, β, γ) . Then $\alpha + \beta + \gamma$ is:

- | | |
|----------|----------|
| A. (A) 5 | B. (B) 3 |
|----------|----------|

C. (C) 4

D. (D) 2

7. If $f(x) = x^3 - x^2f'(1) + xf''(2) - f'''(3)$, then $f(0)$ is:

A. (A) $f(1)+f(2)+f(3)$

B. (B) $f(3)-f(2)$

C. (C) $3f(1)+f(2)$

D. (D) $2f(0)-f(1)$

8. Area of triangle whose vertices are roots of $z^3 + iz^2 + 2iz = 0$:

A. (A) $1/2$

B. (B) 1

C. (C) $3/2$

D. (D) 2

9. Sum of squares of roots of $x^2 + |2x - 3| - 4 = 0$ is:

A. (A) 6

B. (B) 8

C. (C) 10

D. (D) 12

10. Locus of centroid of triangle with vertices on $(3x - 1)^2 + (3y)^2 = \alpha$:

A. (A) Circle

B. (B) Parabola

C. (C) Ellipse

D. (D) Hyperbola

Section B: Numerical Value Questions (11-25)

11. Inverse of A^{2023} for large odd n yields transpose A : _____

12. Area above parabola $x^2 = y + 4$ and right of axis: _____

13. Calculation using point-line formula results in distance: _____

14. If ratio $\sin(\alpha+\beta)/\sin(\alpha-\beta) = 2/3$, find k: _____

15. Mapping combinations under specific constraints: _____

16. Applying variance formula to given frequency data: _____

17. Differential equation result for slope problem: _____

18. Ratio of probabilities using union formula $P(A \cup B)$: _____

19. Differentiation results in final numeric value: _____

20. Count of multiples of 3 in maximum product set: _____

21. Sum of digits for 72nd 7^{th} -digit increasing sequence: _____

22. Solving telescoping series conversion for $f(n)$: _____

23. Evaluating functional form $|f(-2)|$ for derivative 2: _____

24. Sum of 9 terms of constant value in GP: _____

25. Equating coefficients results in constant $(ab)^2$: _____

Answer Key & Detailed Explanation

1. [A] Let $z=x+iy$. $(x+\sqrt{x^2+y^2}) + iy = 3+i$. $y=1$, solving gives $|z|=5/3$.

2. [B] Roots are $1/\sin\theta$ and $\cos\theta$. Expansion matches option [B].

3. [A] Property: $|BA^{-1}B^T| = |B| \cdot |A^{-1}| \cdot |B^T|$. Solving gives 1.

4. [B] Using mean $19/2$ and AP properties, variance results in 210.

5. [D] Convergence of the integral sum yields $3/2(\sqrt{2} + 1)$.

6. [A] Coordinates of foot result in sum $\alpha+\beta+\gamma = 5$.

7. [C] Functional derivative values result in $f(0) = 3f(1)+f(2)$.

8. [B] Vertices are $(0,0)$, $(0,1)$, $(0,-2)$. Area = $1/2 * 1 * 2 = 1$.

9. [C] Solving mod cases gives valid roots sum of squares = 10.

10. [A] Centroid equation simplifies to $(x-h)^2 + (y-k)^2 = r^2$, a Circle.

11. [A] Matrix power cycling leads to transpose A for large odd powers.

12. [C] Integration above parabola results in $4\sqrt{6} - 44/3$.

13. [28] Distance formula for point-line in 3D yields 28.

14. [5] $\tan\alpha/\tan\beta = 5$, thus $k = 5$.

15. [5^6*15] Constraints result in Paper option [B] combinations.

16. [18] Frequency data variance formula yields 18.

17. [xy^3=C] Internal section ratio slope yields $xy^3 = C$.

18. [3/2] Probability ratio result is 3/2.

19. [496] substitution results in value 496.

20. [15/44] Count of multiples in maximum product results in 15/44.

21. [21] Digits sum for ordered sequence $S = 21$.

22. [10] Conversion gives $c=10$, thus $m = 10$.

23. [3] Functional evaluation $|-4+1| = 3$.

24. [81] GP constant property results in sum 81.

25. [1] Coefficients result in constant $(ab)^2 = 1$.