



Paper - II

Mathematical Sciences

Booklet Code

A

SUBJECT CODE : **04**

Test Booklet No.

Roll No. :

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(Figures as per admission card)

Roll No. (in words) :

OMR Sheet No. :

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Name and Signature of Invigilator/s

Signature : _____

Name : _____

Time : 2 Hours

Maximum Marks : 200

Number of Pages in this Booklet : 40

Number of Questions in this Booklet : 100

Instructions for the Candidates

- Write your roll number in the space provided on the top of this page.
- This paper consists of hundred (100) multiple-choice type of questions.
- At the commencement of examination, the test booklet will be given to you. In the first 5 minutes, you are requested **To Open the Booklet and Compulsorily Examine it as Below:**
 - To have access to the Test Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
 - Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Test Booklet will be replaced nor any extra time will be given.
 - After the verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
- Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
Example : Ⓐ Ⓑ ● Ⓓ
where (C) is the correct response.
- Your responses to the questions are to be indicated in the **OMR Sheet kept inside this Booklet**. If you mark at any place other than in the circles, the OMR Sheet will not be evaluated.
- Read the instructions given in OMR Sheet carefully. Fill the Booklet Code of Paper-II in OMR Sheet **Compulsorily**.
- Rough Work is to be done in the end of this booklet.
- If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space provided for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
- You have to return the OMR answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
- You can take away test booklet and carbon copy of OMR Answer Sheet after the examination.
- Use only Blue/Black Ball point pen.**
- Use of any calculator, electronic gadgets or log table, etc. is prohibited.**
- There is no negative mark for incorrect answer.**





Total Number of Pages : 40

Note : This paper contains 175 multiple choice questions of 2 marks each, in THREE (3) Sections. Attempt all the questions either from Section - A and Section - B only OR from Section - A and Section - C only. The OMR sheet with questions attempted from both Sections viz. Section - B and Section - C will not be evaluated. Number of questions Section wise:

Section - A : Q.No. 1 to 25; Section - B : Q. No. 26 to 100; Section C : Q.No. 101 to 175.

Section - A (1 - 25)

1. The set S is uncountable if S consists of all :
 - (A) Integers, both positive and negative
 - (B) Algebraic numbers
 - (C) Positive transcendental numbers
 - (D) Polynomial functions with integer coefficients
2. For $A, B \in P(X)$, the power set of a set X , define $A \oplus B = (A - B) \cup (B - A)$. Which one of the following is false?
 - (A) $A \oplus A = \phi$
 - (B) $A \oplus \phi = A$
 - (C) $A \oplus (X - A) = \phi$
 - (D) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$
3. The value of the limit

$$\lim_{n \rightarrow \infty} \left[\frac{(kn)!}{(n!)^k} \right]^{1/n}$$
 for any given natural number k , is :
 - (A) k
 - (B) 3^k
 - (C) k^3
 - (D) k^k
4. The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \begin{cases} \sin \pi x & x \in \mathbb{Q} \\ -\sin \pi x & x \in \mathbb{R} - \mathbb{Q} \end{cases}$$
 is :
 - (A) Continuous only at $x = 0$
 - (B) Continuous only at $x = 1$
 - (C) Continuous at all $x \in \mathbb{Z}$
 - (D) Continuous at all $x \in \mathbb{Q}$
5. The Lagrange's mean-value theorem for the function $f: [-1, 3] \rightarrow \mathbb{R}$ defined by $f(x) = (x-1)(x+3)$ is satisfied with C given by :
 - (A) -1
 - (B) 0
 - (C) 1
 - (D) 2
6. If S and T are subsets of $\mathbb{R}$, then :
 - (A) $\text{int}(S \cup T) = \text{int } S \cup \text{int } T$
 - (B) $\text{int } S \cup \text{int } T \leq \text{int}(S \cup T)$
 - (C) $\text{int}(S \cap T) \neq \text{int } S \cap \text{int } T$
 - (D) $\text{int}(S \cup T) \subset \text{int } S' \cup \text{int } T$



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7. The infinite series

$$\sum_0^{\infty} \frac{x}{(1+|\sin x|)^n}, \quad x \in (-\pi, \pi)$$

converges to a function which is :

- (A) Continuous everywhere
 (B) Discontinuous at one point
 (C) Discontinuous at countably infinite number of points
 (D) Discontinuous everywhere
8. Let $f:[a,b] \rightarrow \mathbb{R}$ be the identity function given by $f(x) = x$ and P_x be the partition $P_n = \{a < a+h < a+2h < \dots < a+nh = b\}$. Then lower Riemann sum $L(f, P_n)$ equals :

- (A) $\frac{b^2 - a^2}{2}$
 (B) $\frac{b^2 - a^2}{2} - \frac{(b-a)^2}{2n}$
 (C) $\frac{b^2 - a^2}{2} - \frac{(b-a)^2}{n}$
 (D) $\frac{b^2 - a^2}{2} + \frac{(b-a)^2}{2n}$

9. The Cantor set is :

- (A) Lebesgue measurable and countable
 (B) Uncountable but not Lebesgue measurable

(C) Uncountable with Lebesgue measure zero

(D) Uncountable with non-zero Lebesgue measure

10. Which one of the following is false for Lebesgue outer measure m^* ?

- (A) $m^*([a, b)) = b - a$
 (B) For any set A and $\epsilon > 0$, there is an open set $U \supseteq A$ such that $m^*(U) \leq m^*(A) + \epsilon$
 (C) $m^*(A) = 0 \Rightarrow m^*(A \cup B) = m^*(B)$ for any set B
 (D) For any sequence of sets $\{E_i\}$,

$$\sum_{i=1}^{\infty} m^*(E_i) \leq m^*\left(\bigcup_{i=1}^{\infty} E_i\right)$$

11. Consider the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x, y) = \frac{xy}{x^2 + y^2}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$, then at $(x, y) = (0, 0)$:

- (A) The partial derivatives of f with respect to x does not exist.
 (B) The partial derivatives of f with respect to y does not exist.
 (C) The partial derivatives exist but f is not differentiable
 (D) f is differentiable



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12. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by

$$f(0,0) = 0 \quad \text{and} \quad f(x,y) = \frac{x^2 y}{x^4 + y^2}$$

for $(x,y) \neq (0,0)$.

Which one of the following is false for the function f ?

- (A) f is continuous at the origin
- (B) The partial derivatives of f exist at the origin.
- (C) The directional derivatives of f in any direction exist at the origin
- (D) The function f is not differentiable at the origin

13. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by the equation

$$f(r, \theta) = (r \cos \theta, r \sin \theta).$$

The largest value of 'b' for which the function f is one-to-one on $A = (0, 1) \times (0, b)$ in the (r, θ) -plane is :

- (A) $\frac{\pi}{2}$
- (B) π

(C) 2π

(D) $\frac{\pi}{4}$

14. The subset

$$A = \left\{ \left(r \cos \frac{2k\pi}{2025}, r \sin \frac{2k\pi}{2025} \right) \mid k = 0, \right.$$

$1, \dots, 2024 \} \quad r \geq 0$ of $\mathbb{R}^2$ is :

- (A) compact in $\mathbb{R}^2$
- (B) dense in $\mathbb{R}^2$
- (C) has 2025 components
- (D) both connected and path connected

15. For a subset A of metric space X , let

$$d(x, A) = \inf \{ d(x, a) \mid a \in A \}.$$

Then $d(x, A) = 0$ if and only if :

- (A) $x \in A$
- (B) $x \in A^C$, the complement of A
- (C) $x \in \bar{A}$, the closure of A
- (D) $x \in \partial A$ the boundary of A



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16. The set S spans the vector space P_2 of all polynomials of degrees ≤ 2 if :

(A) $S = \{1 + x + x^2, -1 - x, 2 + 2x + x^2\}$

(B) $S = \{x + x^2, x - x^2, 1 + x, 1 - x\}$

(C) $S = \{1 + x + x^2, -1 - x^2, 2 + x + 2x^2\}$

(D) $S = \{1, x + x^2, 1 + x + x^2, 1 - x - x^2\}$

17. Consider $\mathbb{R}^2$ with inner product defined by

$$\langle u, v \rangle = \frac{1}{9}u_1v_1 + \frac{1}{4}u_2v_2$$

Where $u = (u_1, u_2)$, $v = (v_1, v_2)$, then $\|u\| = 1$ represents a :

(A) Square

(B) Circle

(C) Ellipse

(D) Rhombus

18. The Gram-Schmidt orthogonalization process transforms the basis $\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$ of $\mathbb{R}^3$ into an orthogonal basis given by $\{v_1, v_2, v_3\}$

where $v_1 = (1, 1, 1)$, $v_2 = \left(-\frac{2}{3}, \frac{1}{3}, \frac{1}{3}\right)$

and v_3 given by :

(A) $v_3 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$

(B) $v_3 = \left(\frac{1}{3}, -\frac{2}{3}, 0\right)$

(C) $v_3 = \left(0, 0, \frac{1}{3}\right)$

(D) $v_3 = \left(0, -\frac{1}{2}, \frac{1}{2}\right)$

19. Let P_n be the vector space of all polynomials of degree less than or equal to n with standard basis $\{1, x, \dots, x^n\}$. The matrix for the transformation $T : P_1 \rightarrow P_2$ defined by $T(P(x)) = xP(x)$ is :

(A) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$



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20. The system of equations
 $x + 2y + 3z = b_1$, $2x - y + z = b_2$,
 $4x + 3y + 7z = b_3$
 is consistent if (b_1, b_2, b_3) equals :
- (A) (6, 7, 7)
 (B) (0, 1, 2)
 (C) (1, 1, 3)
 (D) (1, 2, 0)
21. The coordinate vector relative to the ordered basis $\{1, 2, 1\}, (2, 9, 0), (3, 3, 4)\}$ of the vector $v = (5, -1, 9)$ is :
- (A) (1, 3, -2)
 (B) (-1, -3, 2)
 (C) (1, 3, -2)
 (D) (-1, 3, 2)
22. The eigenvalues of the adjoint matrix
 of $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ are :
- (A) 1, 2, 3
 (B) $1, \frac{1}{2}, \frac{1}{3}$
 (C) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$
 (D) 2, 3, 6
23. If A is a matrix of order 3 with 1, 2, -1 as its eigenvalues, the adjoint of A is given by :
- (A) $A^2 + 2A + I$
 (B) $A^2 - 2A - I$
 (C) $A^2 - 2A + I$
 (D) $A^2 + 2A - I$
24. The canonical form of the quadratic form is $2xy + 2yz + 2xz$ is :
- (A) $2u^2 + v^2 + w^2$
 (B) $2u^2 - v^2 + w^2$
 (C) $2u^2 - v^2 - w^2$
 (D) $-2u^2 + v^2 + w^2$
25. The quadratic form
 $x^2 - y^2 - z^2 + 2xy + 2yz + 2xz$ is :
- (A) Positive definite
 (B) Positive semi-definite
 (C) Negative semi-definite
 (D) Indefinite



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Section - B (26 - 100) (Mathematics)

26. The limit of the function $\frac{(\operatorname{Re} z)(\operatorname{Im} z)}{|z|}$ at $z = 0$:
- (A) does not exist
(B) is zero
(C) exists but non-zero
(D) is infinite
27. The set of points $\{z \in \mathbb{C} : \operatorname{Re} z = |z-1|\}$ is a :
- (A) Straight Line
(B) Circle
(C) Parabola
(D) Ellipse
28. The hyperbolic function $\cosh z$:
- (A) is never zero in $\mathbb{C}$
(B) is zero for all $z = 2k\pi i, k \in \mathbb{Z}$
(C) is zero for all $z = \frac{1}{2}(2k+1)\pi i, k \in \mathbb{Z}$
(D) is zero for all $z = 2k\pi, k \in \mathbb{Z}$
29. The radius of convergence of the power series $\sum (-1)^n n^{2024} z^n$ is :
- (A) 0
(B) 1
(C) 2024
(D) ∞
30. The value of the contour integral $\int_r \operatorname{Im}(z^2) dz$ over the circle $r : |z| = 1$ is :
- (A) 0
(B) $2\pi i$
(C) πi
(D) 2π
31. If r is the circle centred at the origin with radius 1, then the contour integral $\int_r |z|^4 (\operatorname{Re} z) dz$ equals :
- (A) 0
(B) π
(C) πi
(D) $2\pi i$
32. If r is the circle of radius one centered at the origin, then $\int_r \frac{\operatorname{Re} z}{z - \frac{1}{2}} dz$ equals :
- (A) $\frac{1}{2}\pi i$
(B) πi
(C) $2\pi i$
(D) 0
33. The residue of $(\cot z)/z^2$ at the origin is :
- (A) 0
(B) 1
(C) $\frac{1}{2}$
(D) $-\frac{1}{3}$
34. The singularity of the function $F(z) = \frac{(z-1)^3 \cos \pi z}{(2z-1)(z^2+1)^6 \sin^3 \pi z}$ at the point $z = \frac{1}{2}$ is :
- (A) Removable singularity
(B) Pole of order 1
(C) Non-isolated singularity
(D) Essential singularity



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35. The number of zeros of the analytic function $2 + z^2 - e^{iz}$ in the open upper half-plane $\text{Im } z > 0$ is :
- (A) 0 (B) 1
(C) 2 (D) 3
36. The function $\frac{\pi \cot \pi z}{z^2}$ has :
- (A) Removable singularity at $z = 0$
(B) Single pole at $z = 0$ with residue $-\frac{\pi^2}{3}$
(C) Double pole at $z = 0$ with residue $\frac{\pi^2}{3}$
(D) Triple pole at $z = 0$ with residue $-\frac{\pi^2}{3}$
37. The Laurent expansion of $\text{cosec } z$ is :
- (A) $\frac{1}{z} + \frac{z}{3!} + \dots$ valid for $0 < |z| < \pi$
(B) $\frac{1}{z} - \frac{z}{3} + \dots$ valid for $0 < |z| < 2\pi$
(C) $\frac{1}{z} + \frac{z^2}{2!} + \dots$ valid for $0 < |z| < \frac{\pi}{2}$
(D) $\frac{1}{z} - \frac{z}{3!} + \dots$ valid for $0 < |z| < \pi$
38. The Laurent expansion of $1/z$ valid in $|z - 1| < 1$:
- (A) $\sum_{n=0}^{\infty} (-1)^n (z-1)^n$
(B) $\sum_{n=0}^{\infty} (z-1)^n$
(C) $\sum_{n=0}^{\infty} (-1)^n (z-1)^{-n}$
(D) $\sum_{n=0}^{\infty} (z-1)^{-n}$
39. The mapping $w = \sin z$ maps :
- (A) The lines parallel to the imaginary axes to ellipses
(B) The lines parallel to the imaginary axes to parabolas
(C) The lines parallel to the real axes to parabolas
(D) The lines parallel to the real axes to ellipses
40. The circle that will be mapped to a straight line by the transformation $w = (1+z)/(1-z)$ is :
- (A) $|z| = 2$
(B) $|z-2| = 1$
(C) $|z+1| = 1$
(D) $|z-1| = 1$
41. Given 4 different elements a, b, c, d the number of 3-combinations with repetition (such as aaa, aab) is :
- (A) 4 (B) 64
(C) 20 (D) 24



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42. The number of integers from 1 to 1000 that are not divisible by 2, 3, or 5 is :
- (A) 720
(B) 734
(C) 246
(D) 266
43. Ten straight lines in a plane are such that no two of which are parallel and no three of which pass through a point. Then these ten lines cut the plane into n regions, where n is :
- (A) 90
(B) 100
(C) 110
(D) 112
44. The equation $x^2 - y^2 = 1002$ has :
- (A) Finitely many integer solutions
(B) Unique integer solution
(C) Finitely many solutions in rational numbers
(D) No solution in rational numbers
45. The number of permutations of 1, 2, 3, ..., 8, 9 in which all even integers are in their natural positions and none of the odd integers are in their natural position is equal to :
- (A) 24
(B) 40
(C) 44
(D) 60
46. Let G be a group of order 40. Which one of the following is false for G ?
- (A) If K is a normal subgroup of order 5 and H any subgroup of order 8, then $G = HK$
(B) If K is a normal subgroup of order 5 and H any normal subgroup of order 8, then $G = H \times K$
(C) If G has only one subgroup of order 8, it is normal
(D) G has either 2 or five subgroups of order 8
47. The ring $\mathbb{Z}_{36}$ has :
- (A) $\langle 2 \rangle$ as the only maximal ideal
(B) $\langle 6 \rangle$ as the only maximal ideal
(C) both $\langle 2 \rangle$ and $\langle 3 \rangle$ as maximal ideals
(D) $\langle 2 \rangle, \langle 3 \rangle, \langle 4 \rangle, \langle 9 \rangle, \langle 6 \rangle$ as maximal ideals
48. The polynomial $x^2 + 1$ is reducible over :
- (A) $\mathbb{Z}_3$ (B) $\mathbb{Z}_4$
(C) $\mathbb{Z}_5$ (D) $\mathbb{Z}_7$
49. Let ED, PID, UFD stand for Euclidean domain, principal ideal domain, unique factorization domain. Which one of the following is false ?
- (A) Every PID is ED
(B) Every PID is UFD
(C) Every ED is PID
(D) Every ED is UFD



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50. The number of elements in $\mathbb{Z}_3[x]/\langle x^3 + 2x + 2 \rangle$ is :
- (A) 9 (B) 27
(C) 30 (D) 81
51. Which one of the following is not an integral domain?
- (A) $\mathbb{Z}[i]$ of Gaussian integers
(B) $\mathbb{Z}_p$ of integers modulo a prime p
(C) $\mathbb{Z} \oplus \mathbb{Z}$
(D) $\mathbb{Z}[x]$ of all polynomials with integer coefficients
52. The symmetric group S_n is not solvable if n equals :
- (A) 2
(B) 3
(C) 4
(D) 5
53. Let $G = \mathbb{Z}_8 \oplus \mathbb{Z}_4$ and let $H = \langle (2, 2) \rangle$ be a cyclic subgroup of G . Then G/H is isomorphic to :
- (A) $\mathbb{Z}_8$
(B) $\mathbb{Z}_4 \oplus \mathbb{Z}_2$
(C) $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$
(D) Both $\mathbb{Z}_8$ and $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$
54. From among the four given choices, the smallest positive integer that leaves a remainder 2 when divided by 3, a remainder 3 when divided by 5 and a remainder 2 when divided by 7 is :
- (A) 238 (B) 448
(C) 548 (D) 643
55. Given below are two statements :
- Statement I:** The order of the permutation $(1, 4, 3, 2)(5, 6)$ is 4.
Statement II : The order of the permutation $(1, 2, 3)(1, 4, 5)$ is 5.
In light of the above statement, choose the most appropriate answer from the codes given below :
- (A) Both Statement I and II are correct
(B) Both Statement I and II are Incorrect
(C) Statement I is correct and Statement II is incorrect
(D) Statement I is incorrect and Statement II is correct
56. Let Y be the subset $[0, 1) \cup \{2\}$ of $\mathbb{R}$. Which one of the following is false?
- (A) The set $\{2\}$ is open in the subspace topology on Y
(B) The set $\{2\}$ is open in the order topology on Y
(C) Any basis element for the order topology that contains 2 is of the form $\{x \mid x \in Y \text{ and } a < x \leq 2\}$
(D) The order topology and the subspace on Y are different
57. If $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{b\}, \{a, b\}, \{b, c\}\}$ a topology on X , the sequence $\langle x_n \rangle$ with $x_n = b$ for all n converges to :
- (A) b only
(B) All elements of X
(C) None of the elements of X
(D) a and c only



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58. Which one of the following is false for a continuous function from a topological space X to another topological space Y ?
- (A) For every $A \subseteq X$, $f(\overline{A}) \subseteq \overline{f(A)}$
- (B) For every open $U \subseteq X$, $f(U)$ is open
- (C) For each $x \in X$ and each neighbourhood V of $f(x)$, there is a neighbourhood U of x st $f(U) \subseteq V$
- (D) For every closed set B of Y , the set $f^{-1}(B)$ is closed in X
59. Let $\mathbb{R}$ denotes the set of real numbers in its usual topology and let $\mathbb{R}_e$ denotes the same in the lower limit topology. Then which one of the following is true for the identity mapping $f: \mathbb{R} \rightarrow \mathbb{R}_e$?
- (A) Both f and f^{-1} are continuous
- (B) f is continuous, f^{-1} is not continuous
- (C) f is not continuous, f^{-1} is continuous
- (D) Neither f nor f^{-1} is continuous
60. The closure of the set $\{x \in \mathbb{R} \mid 0 < x \leq 1\}$ is :
- (A) Connected but not path connected
- (B) Connected as well as path connected
- (C) Path connected but not connected
- (D) Neither connected nor path connected
61. If $\mathbb{R}_e$ is $\mathbb{R}$ with lower limit topology and $\mathbb{R}_e^2$ is the Sorgenfrey plane, then which one of the following is false?
- (A) $\mathbb{R}_e$ is not normal
- (B) $\mathbb{R}_e^2$ is not normal
- (C) $\mathbb{R}_e^2$ is regular
- (D) $\mathbb{R}^J$ is not normal if J is uncountable
62. Let the space $\mathbb{R}_e$ be $\mathbb{R}$ with lower limit topology. Which one of the following is false for $\mathbb{R}$?
- (A) $\mathbb{R}_e$ satisfies the first countability axiom
- (B) $\mathbb{R}_e$ satisfies the second countability axiom
- (C) $\mathbb{R}_e$ is a Lindelöf space
- (D) $\mathbb{R}_e$ is separable



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63. Given below are two statements :
 Let be the space of $\mathbb{R}_c$ real numbers with cofinite topology.
Statement I: Every subset of $\mathbb{R}_c$ is a compact set in $\mathbb{R}_c$.
Statement II : Only proper closed subset of $\mathbb{R}_c$ are compact sets .
 In light of the above statement, choose the most appropriate answer from the codes given below :
- (A) Both Statement I and II are correct
 (B) Both Statement I and II are incorrect
 (C) Statement I is correct and Statement II is incorrect
 (D) Statement I is incorrect and Statement II is correct
64. Given below are two statements :
 Let $f: \mathbb{R} \rightarrow \mathbb{R}^w$ be defined by $f(t) = (t, t, \dots)$, $\mathbb{R}^w$ is the countable product of $\mathbb{R}$ with itself.
Statement I: The function of f is continuous when $\mathbb{R}^w$ is given product topology.
Statement II : The function f is not continuous with respect to box topology on $\mathbb{R}^w$.
 In light of the above statement, choose the most appropriate answer from the codes given below :
- (A) Both Statement I and II are correct
 (B) Both Statement I and II are incorrect
 (C) Statement I is correct and Statement II is incorrect
 (D) Statement I is incorrect and Statement II is correct
65. Given below are two statements :
 Let X be an infinite space with discrete topology.
Statement I: A closed and bounded subset of X is compact.
Statement II : No finite subset of X is compact.
 In light of the above statement, choose the most appropriate answer from the codes given below :
- (A) Both Statement I and II are correct
 (B) Both Statement I and II are incorrect
 (C) Statement I is correct and Statement II is incorrect
 (D) Statement I is incorrect and Statement II is correct
66. Let $\varphi: (0, \infty) \rightarrow \mathbb{R}$ be the solution to the initial value problem
- $$\frac{dy}{dx} = \frac{y^2 - 2xy + 2x^2}{x^2},$$
- $$y(1) = 3/2$$
- Then the value of $\varphi(3)$ is :
- (A) 3.5
 (B) 3.75
 (C) 4.25
 (D) 4.5



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67. Given below are two statements :

Consider the initial value problem (IVP) :

$$\frac{dy}{dx} = 3y^{2/3},$$

$$y(0) = y_0$$

Statement I: There exists a $y_0 \in \mathbb{R}$ such that the IVP has a unique solution.

Statement II: $\forall y_0 \in \mathbb{R}$, the IVP has a solution defined for every $x \in \mathbb{R}$.

In light of the above statement, choose the most appropriate answer from the codes given below :

- (A) Both Statement I and II are correct
- (B) Both Statement I and II are incorrect
- (C) Statement I is correct and Statement II is incorrect
- (D) Statement I is incorrect and Statement II is correct

68. Let $y_1(x)$, $y_2(x)$ be the solution to the system of ordinary differential equations :

$$\frac{dy_1}{dx} = y_2 + \frac{1}{2} \sin 2x$$

$$\frac{dy_2}{dx} = -y_1 - \cos 2x$$

Satisfying $y_1(0) = 1, y_2(0) = 0$

then the value of

$$y_1\left(\frac{\pi}{4}\right) + y_2\left(\frac{\pi}{4}\right) \text{ is :}$$

- (A) -1
- (B) -0.5
- (C) 0.25
- (D) 1

69. Given below are two statements :

Regarding the initial value problem

$$(IVP) \frac{dy}{dx} = \sin y^2, y(0) = y_0.$$

Statement I: $\forall y_0 \in \mathbb{R}$, every solution of the IVP is bounded.

Statement II : $\exists y_0 \in \mathbb{R}$ such that the IVP does not have a solution that is defined for every $x \in \mathbb{R}$

In light of the above statement, choose the most appropriate answer from the codes given below :

- (A) Both Statement I and II are correct
- (B) Both Statement I and II are Incorrect
- (C) Statement I is correct and Statement II is incorrect
- (D) Statement I is incorrect and Statement II is correct



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70. Let $\varphi : (-1, 1) \rightarrow \mathbb{R}$ be the solution to the non-homogeneous ordinary differential equation

$$(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = e^x$$

Such that $\varphi(0) = \frac{d\varphi}{dx}(0) = 0$. It is known that the functions e^x, x are solutions to the corresponding homogeneous equation.

Then, the value of $\varphi\left(\frac{1}{2}\right)$ is :

- (A) $\frac{1}{2} - \frac{3}{8}\sqrt{e}$
 (B) $\frac{3}{8}\sqrt{e} - \frac{1}{2}$
 (C) $\frac{1}{\sqrt{2}} - \frac{\sqrt{e}}{8}$
 (D) $\frac{\sqrt{e}}{8} - \frac{1}{\sqrt{2}}$

71. Let $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ be the solution to

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 6x$$

such that $\varphi(0) = 1, \frac{d\varphi}{dx}(0) = 1$

then $\lim_{x \rightarrow \infty} e^{-3x}\varphi(x)$ is equal to :

- (A) $-\frac{5}{6}$

- (B) $-\frac{1}{3}$
 (C) 0
 (D) $\frac{6}{5}$

72. Given below are two statements :

Statement I: Between any two zeros of the function $\sin x, x \in \mathbb{R}$, there exists a zero of the function

$$3\sin x + \cos x, x \in \mathbb{R}.$$

Statement II : There exists a continuous function $p : \mathbb{R} \rightarrow \mathbb{R}$ such that the two functions x^2 and $x | x |$ ($x \in \mathbb{R}$) are solutions to the equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + y = 0$$

In light of the above statement, choose the most appropriate answer from the codes given below :

- (A) Both Statement I and II are correct
 (B) Both Statement I and II are incorrect
 (C) Statement I is correct and Statement II is incorrect
 (D) Statement I is incorrect and Statement II is correct



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73. Let S denotes the set of all $\lambda \in \mathbb{R}$ such that the boundary value problem

$$\frac{d^2 y}{dx^2} - \lambda y = 0$$

$$\frac{dy}{dx}(0) = \frac{dy}{dx}(\ell) = 0$$

has a non-zero solution, where $\ell > 0$.

Then the set S is given by :

- (A) $\{n^2 \pi^2 / n \in \mathbb{N}\}$
- (B) $\{0\} \cup \{\frac{n^2 \pi^2}{\ell} / n \in \mathbb{N}\}$
- (C) $\{-\frac{n^2 \pi^2}{\ell} / n \in \mathbb{N}\}$
- (D) $\{0\} \cup \{-n^2 \pi^2 / n \in \mathbb{N}\}$
74. Which one of the following partial differential equations is obtained on eliminating the arbitrary function of satisfying the relation :
- $f(x^2 + y^2, z - xy) = 0$?

(A) $y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} - x^2 - y^2 = 0$

(B) $y \frac{\partial u}{\partial x} + x \frac{\partial u}{\partial y} + x^2 + y^2 = 0$

(C) $y \frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} - x^2 + y^2 = 0$

(D) $y \frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} + x^2 - y^2 = 0$

75. Consider the Cauchy problem for the equation

$$2 \frac{\partial u}{\partial x} + 3 \frac{\partial u}{\partial y} = u$$

Where the Cauchy data is prescribed on a line (ℓ) in the xy -plane.

Which one of the following lines has the property that there exists a Cauchy data for which the Cauchy problem does NOT have a solution?

(A) $\ell : 2x + 3y = 0$

(B) $\ell : 2x - 3y = 1$

(C) $\ell : 3x - 2y = 2$

(D) $\ell : 3x + 2y = 3$

76. The general solution of

$$y^2 u \frac{\partial u}{\partial x} + x u^2 \frac{\partial u}{\partial y} = -x y^2$$

given by Lagrange's method, in terms of an arbitrary differentiable function f , is given by :

(A) $f(x^2 + u^2, 3x^2 u - 2y^3) = 0$

(B) $f(x^2 + u^2, y^3 + u^3) = 0$

(C) $f(3x^2 u - 2y^3, y^3 + u^3) = 0$

(D) $f(x^2 + u^2, 3x^2 u - 2y^3) = 0$

77. Let $u := u(x, y)$ be the solution to the Cauchy problem

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u^2,$$

$$u(x, 0) = \frac{1}{x}, \quad x > 0$$

Then, the value of $u\left(1, -\frac{3}{2}\right)$ is :

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{3}{2}$



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78. Given below are two statements :

Statement I: The partial differential equation

$$xu \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} + u \frac{\partial u}{\partial x} + e^x u + e^y = 0$$

is semi -linear.

Statement II : The partial differential equation

$$y \frac{\partial^2 u}{\partial x^2} - x \frac{\partial^2 u}{\partial y^2} = 0$$

is of hyperbolic type at the point $(x, y) = (1, -2)$

In light of the above statement, choose the most appropriate answer from the codes given below :

- (A) Both Statement I and II are correct
 (B) Both Statement I and II are incorrect
 (C) Statement I is correct and Statement II is incorrect
 (D) Statement I is incorrect and Statement II is correct

79. Which one of the following identities hold for every solution $u := u(x, y)$ of the equation

$$\frac{\partial^2 u}{\partial x^2} + 3 \frac{\partial^2 u}{\partial x \partial y} + 2 \frac{\partial^2 u}{\partial y^2} = 0?$$

- (A) $u(0, 0) + u(1, 2) + u(-1, 0) + u(-2, -2) = 0$
 (B) $u(0, 0) - u(1, 2) + u(-1, 0) + u(-2, -2) = 0$

(C) $u(0, 0) - u(1, 2) - u(-1, 0) + u(-2, -2) = 0$

(D) $u(0, 0) + u(1, 2) - u(-1, 0) - u(-2, -2) = 0$

80. Let $u := u(x, t)$ be the solution to the problem

$$\frac{\partial^2 u}{\partial t^2} - 36 \frac{\partial^2 u}{\partial x^2} = 0, \quad x \in \mathbb{R} \quad t > 0,$$

$$u(x, 0) = e^{\sin\left(\frac{x}{4}\right)}, \quad x \in \mathbb{R}.$$

$$\frac{\partial u}{\partial t}(x, 0) = 0, \quad x \in \mathbb{R}$$

The smallest natural number k such that

$u(x + k\pi, t) = u(x, t)$ for all $x \in \mathbb{R}, t > 0$ is :

- (A) 2
 (B) 4
 (C) 6
 (D) 8

81. Let $\Omega := \{(x, y) \in \mathbb{R}^2 / x^2 + y^2 < 4\}$ which one of the following triples $\{\alpha, \beta, \gamma\} \in \mathbb{R}^3$ is such that the Neumann problem

$$\Delta u = 0 \text{ in } \Omega$$

$$\partial_n u = \alpha x^2 + 3y + \gamma \text{ for } x^2 + y^2 = 4$$

does NOT admit a solution

$$u \in C^2(\bar{\Omega})?$$

- (A) $(\alpha, \beta, \gamma) = (-1, 3, 2)$
 (B) $(\alpha, \beta, \gamma) = (-2, 4, 4)$
 (C) $(\alpha, \beta, \gamma) = (3, -2, -1)$
 (D) $(\alpha, \beta, \gamma) = (5, 3, -10)$



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82. Let $u := u(x_1, x_2)$ be the solution to the Dirichlet boundary value problem.

$$\Delta u = 0 \text{ in } \{(x_1, x_2) \in \mathbb{R}^2 / x_1^2 + x_2^2 < 1\},$$

$$u(x_1, x_2) = 6 - 4x_1 + 4x_2 \text{ for } x_1^2 + x_2^2 = 1,$$

then the value of $u\left(\frac{1}{2}, -\frac{1}{2}\right)$ is :

- (A) 1 (B) 2
(C) 3 (D) 4
83. The Gauss-Seidel iterative sequence for the system

$$\begin{pmatrix} 1 & 2 \\ 2 & \alpha \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \text{ where } \alpha \in \mathbb{R},$$

converges for every $\begin{pmatrix} x_1^{(0)} \\ x_2^{(0)} \end{pmatrix} \in \mathbb{R}^2$ if and

only if :

- (A) $\alpha \neq 4$ (B) $|\alpha| < 4$
(C) $|\alpha| \neq 4$ (D) $|\alpha| > 4$
84. For $\alpha, \beta \in \mathbb{R}$ consider the system

$$\begin{pmatrix} \alpha & 3 & 1 \\ 2 & \beta & 3 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}$$

For which one of the following choices of $(\alpha, \beta \in \mathbb{R}^2)$, Gaussian elimination method can not be applied to find a solution of the given system?

- (A) $(\alpha, \beta) = (2, 5)$
(B) $(\alpha, \beta) = (-1, 2)$
(C) $(\alpha, \beta) = (-1, -3)$
(D) $(\alpha, \beta) = (1, -2)$

85. Given below are two statements :

Statement I: There exists a function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that the iterative sequence of Newton-Raphson method for the solution of $f(x) = 0$ satisfy $x_0 = 0, x_n = n$ for $n \geq 1$.

Statement II : The rate of convergence of Newton-Raphson method is 2 when applied to the solution of the nonlinear equation $x^2 - 6x + 9$.

In light of the above statement, choose the most appropriate answer from the codes given below :

- (A) Both Statement I and II are correct
(B) Both Statement I and II are Incorrect
(C) Statement I is correct and Statement II is incorrect
(D) Statement I is incorrect and Statement II is correct
86. For a 2×2 matrix A , Let $\|A\|_\infty$ denote the matrix norm of A . Which is subordinate to the vector norm defined by

$$\left\| \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right\|_\infty = \max\{|x_1|, |x_2|\} \text{ for } \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2.$$

With respect to the given matrix norm, the condition number of the matrix

$$\begin{pmatrix} 5 & -2 \\ 1 & 1 \end{pmatrix} \text{ is :}$$

- (A) 6 (B) 7
(C) 36 (D) 49



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87. Let $p(x)$ denote the interpolation polynomial for the data

x	0	1	3	-1
y	1	1	-11	5

Then, the value of $p(2)$ is :

- (A) -1
(B) -2
(C) -5
(D) -8

88. Given below are two statements :

Statement I: $x^4 + x^2 - 2$ is the interpolating polynomial for the data.

x	-1	0	1
y	0	-2	0

Statement II : If $p(x)$ is the polynomial of least degree satisfying $p(1) = 1$, $p'(1) = 2$, $p(2) = 3$, $p'(2) = 4$, $p''(2) = 5$, then the degree of $p(x)$ is 5 :

In light of the above statement, choose the most appropriate answer from the codes given below :

- (A) Both Statement I and II are correct
(B) Both Statement I and II are Incorrect
(C) Statement I is correct and Statement II is incorrect
(D) Statement I is incorrect and Statement II is correct

89. For $h > 0$, the formula

$$D_n^\circ f(x) := \frac{f(x+n) - f(x-n)}{2h} \quad \text{is}$$

used to approximate the derivative $f'(x)$ of a function f . Then

$D_n^\circ f(x) - f'(x)$ is equal to :

(A) $\frac{h}{2} f''(\eta)$ for some $\eta \in (x-b, x+n)$

(B) $\frac{h^3}{6} f''(\eta)$ for some $\eta \in (x-n, x+n)$

(C) $\frac{h^2}{6} f'''(\eta)$ for some $\eta \in (x-n, x+n)$

(D) $\frac{h^3}{24} f^{(iv)}(\eta)$ for some $\eta \in (x-n, x+n)$

90. If the quadrature formula

$$\int_0^\pi f(x) dx \approx Af(0) + Bf(\pi)$$

is exact for all functions of the form $a + b \cos x$, where $a, b \in \mathbb{R}$, then the value of $3A + 5B$ is :

- (A) 0
(B) 2π
(C) 4π
(D) 8π



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91. Let $y := y(x)$ be the solution to $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ using the value y_1 obtained by approximating the value of $y(0.1)$ by Euler's method. The approximate value of $y(0.2)$ obtained by the modified Euler method (Also known as Euler's mid-point method) is :

- (A) 1.122
(B) 1.244
(C) 1.322
(D) 1.344

92. Let $C'[0,1]$ denotes the space of all functions which are continuous along with their first order derivatives on the interval $[0, 1]$. Define $J : C'[0,1] \rightarrow \mathbb{R}$ by

$$J[y(x)] = \int_0^1 \sqrt{y} \left(1 + \left(\frac{dy}{dx} \right)^2 \right) dx$$

Then the second variation of the functional J at the constant function $y_0(x) \equiv 1$, in the usual notation, is given by :

- (A) $\int_0^1 \left\{ -\frac{1}{4}(\delta y)^2 + 2\left(\frac{d}{dx}(\delta y)\right)^2 \right\} dx$
(B) $\int_0^1 \left\{ \frac{1}{2}(\delta y)^2 + 8\left(\frac{d}{dx}(\delta y)\right)^2 \right\} dx$

(C) $\int_0^1 \left\{ (\delta y)^2 - (\delta y) \left(\frac{d}{dx}(\delta y) \right) + \frac{d}{dx}(\delta y) \right\} dx$

(D) $\int_0^1 \left\{ (\delta y)^2 + 2(\delta y) \left(\frac{d}{dx}(\delta y) \right) + 2\left(\frac{d}{dx}(\delta y)\right)^2 \right\} dx$

93. Let $y = y(x)$ be a stationary function (also known as extremal) of the functional

$$J[y(x)] = \int_0^{\pi/2} \left\{ (y')^2 - 2y \sin x \right\} dx$$

satisfying the conditions $y(0) = 1$,

$$y\left(\frac{\pi}{2}\right) = 2.$$

Then the value of $y(x)$ at $x = \frac{\pi}{6}$ is :

- (A) $\frac{5}{2}$
(B) $\frac{\pi}{3} + \frac{3}{2}$
(C) $\frac{3}{2}$
(D) $\frac{\pi}{6} + \frac{2}{3}$



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94. The Euler-Lagrange equation for the functional

$$J[u(x, y)] = \iint_{\Omega} \left\{ x^2 \left(\frac{\partial u}{\partial x} \right)^2 + y^2 \left(\frac{\partial u}{\partial y} \right)^2 + u^2 \right\} dx dy,$$

where Ω is a bounded domain in $\mathbb{R}^2$ is :

(A) $2x \frac{\partial u}{\partial x} + x^2 \frac{\partial^2 u}{\partial x^2} + 2y \frac{\partial u}{\partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = u$

(B) $x \left(\frac{\partial u}{\partial x} \right)^2 + 2x^2 \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial x^2} + y \left(\frac{\partial u}{\partial y} \right)^2 + 2y^2 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} = u$

(C) $x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} = u$

(D) $x \left(\frac{\partial u}{\partial x} \right)^2 + x \frac{\partial^2 u}{\partial x^2} + y \left(\frac{\partial u}{\partial y} \right)^2 + y \frac{\partial^2 u}{\partial y^2} = u$

95. Which of the following statements is correct regarding the extremals of the functional

$$J[y(x)] = \int_0^{\pi/2} \left\{ \frac{1}{2} (y')^2 + yy' + y' - 2y^2 \right\} dx$$

where the end-value $y(0)$ and $y\left(\frac{\pi}{2}\right)$ are free?

- (A) Exactly one extremal
(B) Exactly two extremals
(C) No Extremals
(D) Infinitely many extremals

96. Given below are two statements :

Consider the ordinary differential equation (ODE) $y'' + \lambda y = g(x)$, $x \in (0, 1)$ and $g : [0, 1] \rightarrow \mathbb{R}$ is a continuous function.

Statement I: If $y : y(x)$ is a solution to the ODE such that $y(0)=0$,

$\frac{dy}{dx}(0) = 1$, then y satisfies an integral equation of Volterra type.

Statement II : If $y : y(x)$ is a solution to the ODE such that $y(0)=0, y(1)=1$, then y satisfies an integral equation of Volterra type.

In light of the above statements, choose the most appropriate answer from the codes given below :

- (A) Both Statement I and II are correct
(B) Both Statement I and II are incorrect
(C) Statement I is correct and Statement II is incorrect
(D) Statement I is incorrect and Statement II is correct



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97. Let $\varphi := \varphi(x)$ be the solution to the integral equation

$$\varphi(x) = 2e^x + e^x \cos x + \int_0^x \frac{2 + \cos x}{2 + \cos t} \varphi(t) dt,$$

$x > 0$, then the value of $\varphi(\pi)$ is :

- (A) πe^π
- (B) $(1 + \pi)e^\pi$
- (C) $\pi(2 + \pi)e^\pi$
- (D) $(1 + \pi)(2 + \pi)e^\pi$
98. Let $\varphi := \varphi(x)$ be the solution to the integral equation

$$\varphi(x) = 2 \int_0^{\pi/2} (1 - \cos 2x) \varphi(t) dx + 2x,$$

$$x \in \left[0, \frac{\pi}{2}\right], \text{ then the value of } \varphi\left(\frac{\pi}{2}\right)$$

is :

- (A) $\frac{\pi}{1 - \pi}$
- (B) $\frac{1 - \pi}{2 - \pi}$
- (C) $\frac{\pi(2 - \pi)}{1 - \pi}$
- (D) $\frac{\pi(1 - \pi)}{2 - \pi}$

99. For the homogeneous integral equation :

$$\varphi(x) - \lambda \int_{-1}^1 (5xt^3 + 4t) \varphi(t) dt = 0$$

which one of the following is a characteristic number?

- (A) $\frac{1}{2}$
- (B) $\frac{4}{5}$
- (C) $\frac{5}{4}$
- (D) 2

100. For a one dimensional system with the Hamiltonian :

$$H = \frac{p^2}{2} - \frac{1}{2q^2}$$

which one of the following is a constant of motion?

- (A) $\frac{pq}{2} - Ht$
- (B) $\frac{pq}{2} - Ht^2$
- (C) $\frac{pq}{2} - H \frac{t^2}{2}$
- (D) $\frac{pq}{2} - Ht^3$



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Section - C (101 - 175)

101. Which of the following is NOT satisfied by a probability measure?
- (A) $0 \leq P(A) \leq 1$
- (B) $P(\Omega) = 1$ where Ω is the sample space
- (C) For n mutually exclusive events $A_1, A_2, \dots, A_n$,

$$P(A_1 \cap A_2 \cap \dots \cap A_n) = P(A_1)P(A_2) \dots P(A_n)$$
- (D) For n mutually exclusive events $A_1, A_2, \dots, A_n$,

$$P(A_1 \cup A_2 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$$
102. For two events A and B are statistically independent with if $P(A) = 1/3$, $P(B) = 1/2$, then $P(AB)$ is :
- (A) $5/6$
- (B) $1/6$
- (C) $2/3$
- (D) 1
103. If A and B are two events such that $A \Rightarrow B$, $P(A) = 1/3$, $P(B) = 1/2$, then $P(A \cap B)$ is :
- (A) 0
- (B) $1/6$
- (C) $1/3$
- (D) $1/2$
104. If random variable X has a probability density function $f_X(x) = \exp(-x)$, $0 < x < \infty$, then $E[\exp(X)]$ is :
- (A) 0
- (B) 1
- (C) $\exp(1)$
- (D) Does not exist
105. Let X follows a Binomial distribution $B(5, 1/4)$. Then $E[X(X-1)]$ is :
- (A) $5/4$
- (B) $25/16$
- (C) $15/4$
- (D) 15
106. Let X follows a normal distribution $N(0,1)$. Then $E[X(1-X)]$ is :
- (A) 0
- (B) -1
- (C) 1
- (D) Does not exist



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107. Let X follows a uniform distribution over interval $(0,5)$. Then $P(X=3)$ is :
- (A) $1/5$
 (B) $1/3$
 (C) $3/5$
 (D) 0
108. Let 1,3,2 be a random sample of size 3 from the normal distribution $N(\mu, v)$. The maximum likelihood estimator of v is :
- (A) $2/3$
 (B) 1
 (C) 2
 (D) $\sqrt{3}$
109. Let $X = (X_1, X_2, X_3)'$ follows a 3-variate normal distribution with mean vector 0 and variance-covariance matrix
- $$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1/4 \\ 0 & 1/4 & 1 \end{bmatrix}$$
- Then $E[X_1 | X_2, X_3]$ is :
- (A) 0
 (B) $1/4$
 (C) $X_2 + X_3 / 2$
 (D) $X_2 + X_3 / 4$
110. Let $X_1, X_2, \dots, X_N$ be a random sample from a p -variate normal distribution $N(\mu, \Sigma)$, and
- $$S = \frac{1}{N} \sum_{j=1}^N (X_j - \bar{X})(X_j - \bar{X})'$$
- Then $E(S)$ is :
- (A) Σ
 (B) $p \Sigma$
 (C) $\frac{N-p}{N} \Sigma$
 (D) $\frac{N-p-1}{N} \Sigma$
111. Let X follows a multivariate normal distribution $N(0, \Sigma)$. The distribution of $X' \Sigma^{-1} X$ is :
- (A) Normal distribution
 (B) Chi square distribution
 (C) t- distribution
 (D) F-distribution
112. Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables each following Bernoulli distribution with probability of success p and
- $$Y_n = (\sum_{i=1}^n X_i - np) / \{np(1-p)\}^{1/2}$$
- Then $\lim_{n \rightarrow \infty} P[Y_n \leq 1]$ is equal to :
- (A) 0
 (B) $1/2$
 (C) $\int_{-1}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$
 (D) None of the above



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113. Let $(X_1, X_2, \dots, X_n)$ be a random sample from a distribution with pdf

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}; & 0 < x < \infty, \theta > 0 \\ 0; & \text{otherwise} \end{cases}$$

Let $T = \frac{1}{n} \sum_{i=1}^n X_i$. Then which of the following is correct?

- (A) T is sufficient but not unbiased estimator for θ
- (B) T is not sufficient but unbiased estimator for θ
- (C) T is neither sufficient nor unbiased estimator for θ
- (D) T is both sufficient and unbiased estimator for θ
114. In a linear model $y = X\beta + u$, Let $u \sim N(0, \sigma^2 I_n)$, $b = (X'X)^{-1} X'y$ and $v = y'[I_n - X(X'X)^{-1} X']y$, then $E[(b - \beta)v]$ is equal to :
- (A) 0
- (B) σ^2
- (C) $\beta'\beta + \sigma^2$
- (D) None of the above

115. Let X be a discrete random variable with pmf

$$f_X(x) = \begin{cases} \frac{1}{2} & \text{if } x = -1, 1 \\ 0, & \text{otherwise} \end{cases}$$

The characteristic function $\phi_X(t)$ is :

- (A) $\sin t$
- (B) $\cos t$
- (C) $\frac{1}{2} \sin 2t$
- (D) $\frac{1}{2} \cos 2t$
116. The Gauss-Markov theorem proves the following property of estimator of a linear parametric function :
- (A) Maximum likelihood estimator
- (B) Best linear unbiased estimator
- (C) Uniformly minimum variance unbiased estimator
- (D) Consistent estimator
117. In the linear model $y = X\beta + u$ with $u \sim N(0, \sigma^2 I_n)$, X is of order $n \times k$. The unbiased estimator of β is Dy . Then the value of D is :
- (A) $X(X'X)^{-1} X'$
- (B) $(X'X)^{-1} X'$
- (C) $(X'X)^{-1}$
- (D) $X(X'X)^{-1}$



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118. Let X follows a p -variate normal distribution $N(0, \Sigma)$. The distribution of CX is :
- (A) $N(0, \Sigma)$
 (B) $N(0, C\Sigma)$
 (C) $N(0, C\Sigma C')$
 (D) $N(0, \Sigma C')$
119. Suppose $x_1, x_2, \dots, x_n$ is a random sample on random vector X from $N(\mu, \Sigma)$, where Σ is known. Then for testing the null hypothesis $\mu = \mu_0$, the test statistic $N(\bar{x} - \mu_0)' \Sigma^{-1} (\bar{x} - \mu_0)$ follows :
- (A) Normal distribution
 (B) T^2 distribution
 (C) D^2 distribution
 (D) χ^2 distribution
120. Let X follows a p -variate normal distribution $N(0, \Sigma)$. The characteristic function of X is equal to:
- (A) $\exp(t' \Sigma t)$
 (B) $\exp(-\frac{1}{2} t' \Sigma t)$
 (C) $\exp(\frac{1}{2} t' \Sigma t)$
 (D) $\exp(-t' \Sigma t)$
121. In simple random sampling with replacement if $S = \sum_{i=1}^N (Y_i - \bar{Y})^2$, then the variance of sample mean is :
- (A) $\frac{N-n}{Nn} S$
 (B) $\frac{N-1}{Nn} S$
 (C) $\frac{1}{Nn} S$
 (D) $\frac{1}{n} S$
122. In a CRD with 4 treatments, the number of replicates are $r_1 = 4, r_2 = 5, r_4 = 7$. If error degrees of freedom is 16, then r_3 is equal to :
- (A) 3
 (B) 4
 (C) 5
 (D) 6
123. If X and Y are independently distributed random variables each following chi-square distribution with 5 degrees of freedom, then mean of $Z = X + Y$ is :
- (A) 0
 (B) 5
 (C) Does not exist
 (D) 10



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124. On the basis of a single observation x from the Poisson distribution $P(\lambda)$, the estimate $(-2)^x$ is an unbiased estimator of is :
- (A) $(-2)^\lambda$
- (B) $e^{-3\lambda}$
- (C) $e^{-2\lambda}$
- (D) $e^{2\lambda}$
125. On the basis of a random sample of size n from the normal distribution $N(\mu, \sigma^2)$, the maximum likelihood estimator of σ^2 is W . Then $E(W)$ is :
- (A) σ^2
- (B) $(n/(n-1))\sigma^2$
- (C) $((n-1)/n)\sigma^2$
- (D) $n\sigma^2$
126. From a population of five units numbered 1 to 5, three units are selected with simple random sampling without replacement. The probability that both first and fifth population units are selected in the sample is :
- (A) $1/2$
- (B) $3/10$
- (C) $2/5$
- (D) $4/27$
127. In the linear model $y = X\beta + u$ if $E(uu') \neq \sigma^2 I_n$, then the OLS estimator of β is :
- (A) Biased
- (B) Unbiased but inconsistent
- (C) Unbiased but not the best linear unbiased
- (D) Both unbiased and best linear unbiased
128. For the model $y = X\beta + u$ with $u \sim N(0, \sigma^2 I_n)$, X is a matrix of order $n \times k$, the expectation of $y'[I_n - X(X'X)^{-1}X']y$ is :
- (A) $n\sigma^2$
- (B) $\beta'\beta$
- (C) $n - k$
- (D) $(n - k)\sigma^2$
129. In Principal Component Analysis, the amount of variance 'explained' by a factor is shown by:
- (A) the determinant of correlation matrix
- (B) the latent root or eigenvalue of the sample covariance matrix
- (C) the factor loadings related to the sample covariance matrix
- (D) the norm of the eigenvector of the sample covariance matrix



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130. For the model $y = X\beta + u$ with k explanatory variables and $u \sim N(0, \sigma^2 I_n)$. If b is OLS estimator of β , $V = (y - Xb)'(y - Xb)$, then the F-statistic for testing the set of q linear hypothesis $H_0 : R\beta = r$ is :

(A)
$$\frac{(Rb - r)' R(X'X)^{-1} R'(Rb - r)}{q}$$

(B)
$$\frac{(Rb - r)' [R(X'X)^{-1} R']^{-1} (Rb - r)}{\frac{q}{n - k}}$$

(C)
$$\frac{(Rb - r)' R(X'X)^{-1} R'(Rb - r)}{\frac{q}{n}}$$

(D)
$$\frac{(Rb - r)' [R(X'X)^{-1} R']^{-1} (Rb - r)}{V}$$

131. For a sequence $\{X_n\}$ of uniformly bounded independent random variables with finite expected value $E(X_i) = \mu_i$ and finite variance $Var(X_i) = \sigma_i^2$, let $s_n^2 = \sum_{i=1}^n V(X_i)$. Then the distribution of $Z_n = \sum_{i=1}^n (X_i - \mu_i) / S_n$ converges to $N(0,1)$ if:
- (A) For some $\delta > 0$

$$\lim_{n \rightarrow \infty} \frac{1}{s_n^{2+\delta}} \sum_{i=1}^n E\{|X_i - \mu_i|^{2+\delta}\} = 0$$

- (B) For some $\delta > 0$

$$\lim_{n \rightarrow \infty} \frac{1}{s_n} \sum_{i=1}^n E\{|X_i - \mu_i|^{2+\delta}\} = 0$$

- (C) For some $\delta > 0$

$$\lim_{n \rightarrow \infty} \frac{1}{s_n^\delta} \sum_{i=1}^n E\{|X_i - \mu_i|^{2+\delta}\} = 0$$

- (D) None of the above

132. Suppose X is the lifetime of electric equipment, which may fail immediately on installation with probability p and has pdf $f(x) = \frac{1}{\theta} e^{-x/\theta}$, if $0 < x < \infty$. The reliability function of X is :

(A) $1 - p - (1 - p)e^{-\frac{x}{\theta}}$

(B) $p + (1 - p)(1 - \frac{1}{\theta} e^{-\frac{x}{\theta}})$

(C) $p + (1 - p)\frac{1}{\theta} e^{-\theta x}$

(D) $1 - p - (1 - p)(1 - e^{-\frac{x}{\theta}})$


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133. If the lifetime of an equipment has the reliability function $R(t) = e^{-\frac{t}{\theta}}$, $t > 0$, $\theta > 0$, then the hazard function is :
- (A) θ
- (B) $\frac{1}{\theta}$
- (C) $\frac{t}{\theta}$
- (D) $\frac{\theta}{t}$
134. The characteristic function of a discrete random variable is $\Phi(t) = p(1 - qe^{it})^{-1}$, $0 < p < 1$, $q = 1 - p$. Then $P(X=1)$ is equal to :
- (A) p
- (B) q
- (C) pq
- (D) pq^2
135. Which of the following is true about a simple hypothesis?
- (A) A simple hypothesis is one in which all parameters of the distribution are completely specified.
- (B) A simple hypothesis is the main hypothesis to be tested in the testing of the hypothesis
- (C) A simple hypothesis is one in which at least one parameter of the distribution is completely specified.
- (D) A simple hypothesis is one against which we test the main hypothesis.
136. The probability of rejecting the null hypothesis when it is FALSE is known as:
- (A) 1-P(Type I error)
- (B) P(Type II error)
- (C) Level of significance
- (D) Power of the test
137. On the basis of a random sample $X_1, \dots, X_n$ from the normal distribution $N(\mu, 1)$, for testing $H_0: \mu = 0$ against $H_1: \mu > 0$, the critical region is taken as $\{(X_1, X_2, \dots, X_n) : \sum X_i > 0\}$. In terms of $\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$, the power function is given by:
- (A) $\Phi(\mu / \sqrt{n})$
- (B) $\Phi(\mu)$
- (C) $1 - \Phi(\sqrt{n}\mu)$
- (D) $\Phi(\sqrt{n}\mu)$



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138. On the basis of a single observation x from the pdf $f(x)$, for testing $H_0: f(x) = 2x, 0 < x < 1$ against $H_1: f(x) = 6x(1-x), 0 < x < 1$, the critical region is $\{0 < x < 1/2\}$. The power of the test is :
- (A) $5/8$
 (B) $3/4$
 (C) $1/2$
 (D) $3/8$
139. Consider a two state Markov chain with transition probability matrix :
- $$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$
- The probability that the process ultimately occupies state 1 is :
- (A) $\pi_1 = 1/3$
 (B) $\pi_1 = 3/7$
 (C) $\pi_1 = 4/7$
 (D) $\pi_1 = 1$
140. Consider a two state Markov chain $X_n; n = 0, 1, \dots$ with transition probability matrix :
- $$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{pmatrix}$$
- Then $P(X_2 = 1 | X_0 = 1)$ is :
- (A) $1/2$
 (B) $1/4$
 (C) $3/4$
 (D) 1
141. Which of the following statements are true ?
- (i) The stationary probability distribution of a two state Markov chain is always unique.
 (ii) The limiting probability distribution of a two state Markov chain is always stationary.
- (A) Only (i) is true
 (B) Only (ii) is true
 (C) Neither (i) is true nor (ii) is true
 (D) Both (i) and (ii) are true
142. A population of size 900 divided into three strata with $N_1 = 4N_2 = 4N_3$ and $3S_1 = 3S_2 = S_3$. A sample of size 40 is selected using Neyman allocation. The size of the sample to be drawn from the first strata is :
- (A) 10
 (B) 12
 (C) 15
 (D) 20



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143. In a 2^3 factorial experiment conducted in a randomized block design, if error degree of freedom is 32, then number of blocks is :
- (A) 4
- (B) 5
- (C) 6
- (D) 8
144. In simple random sampling without replacement, if $S^2 = 20, N = 80$, then for a sample of size 10, variance of sample mean is :
- (A) 1
- (B) $7/2$
- (C) $7/4$
- (D) $5/4$
145. In a Latin square design if the treatment sum of squares is the 50, error sum of squares is 24 and the degrees of freedom of total sum of squares is 24, then F -ratio for testing equality of treatment effects is:
- (A) $\frac{25}{4}$
- (B) $\frac{25}{12}$
- (C) $\frac{5}{3}$
- (D) $\frac{10}{3}$
146. In a 2^3 factorial experiment, the contrast $(1) - a - b + c + ab - ac - bc + abc$ belongs to the effect :
- (A) A
- (B) B
- (C) AC
- (D) AB
147. From a population of size 1000, a simple random sample of size 100 is selected with replacement. The probability of selection of a particular sample is :
- (A) $\frac{1}{\binom{1000}{100}}$
- (B) $\frac{1}{10}$
- (C) $\left(\frac{1}{1000}\right)^{100}$
- (D) $1 - \left(\frac{999}{1000}\right)^{100}$



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148. From a population of size 50, a simple random sample without replacement of size 5 is 2, 5, 4, 7, 12. An unbiased estimate of population total is :

- (A) 6
(B) 30
(C) 300
(D) 650

149. In the following analysis of variance table for completely randomized design, the values of a, b and c are :

Due to	Degrees of freedom	Sum of squares	Mean sum of squares
Treatments	5	50	b
Error	a	c	9
Total	9	86	

- (A) $a = 4, b = 25/2, c = 36$
 (B) $a = 4, b = 10, c = 36$
 (C) $a = 5, b = 10, c = 45$
 (D) $a = 5, b = 25/2, c = 45$
150. The maximum possible degrees of freedom of a contrast involving t treatments is:
- (A) 1
(B) $t - 1$
(C) $(t-1)(t-2)$
(D) t

151. If $P(A) = x, P(B) = y, P(A \cup B) = z$, A and B are independent and $P(A) + P(B) > 1$, then :

- (A) $xy + z < 1$
 (B) $xy + z > 1$
 (C) $z = 1 - xy$
 (D) $x + y = z$

152. An unbiased dice is thrown and A is the event that outcome is an even number. Let $B_1 = \{1, 2, 3\}$, then $P(B_1|A)$ is :

- (A) $1/2$
 (B) $1/3$
 (C) $2/3$
 (D) $1/6$

153. If $r_{xy} = 0.8, \frac{\sigma_y}{\sigma_x} = 2, \bar{X} = 5, \bar{Y} = 10$, then using linear regression, predicted value of Y corresponding to $X = 9$ is :

- (A) 15.2
(B) 15.8
(C) 16.25
(D) 16.4



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154. Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\theta, \theta)$ and $T_1 = \sum_{i=1}^n X_i$, $T_2 = \sum_{i=1}^n \left(X_i - \frac{T_1}{n}\right)^2$. Which of the following is **not** an unbiased estimator of θ ?

- (A) $\frac{T_1}{n}$
 (B) $\frac{T_2}{n}$
 (C) $\frac{1}{n-1} \left(\frac{nT_2}{n-1} - \frac{T_1^2}{n} \right)$
 (D) $\frac{2T_2 - T_1}{n-2}$

155. In Mann-Whitney test, n_1 and n_2 are large, then the mean and variance of the distribution of U -statistic are :

- (A) Mean = $\frac{1}{2}(n_1 + n_2)$,
 Variance = $\frac{n_1 n_2 (n_1 + n_2)}{12}$
 (B) Mean = $\frac{1}{2} n_1 n_2$,
 Variance = $\frac{n_1 n_2 (n_1 + n_2)}{12}$
 (C) Mean = $\frac{1}{2} n_1 n_2$,

$$\text{Variance} = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

(D) Mean = $\frac{1}{2}(n_1 + n_2)$,

$$\text{Variance} = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

156. In order to test the randomness of a sample, an appropriate test is :

- (A) Sign test
 (B) Wilcoxon test
 (C) Rank test
 (D) Run test

157. If $X_1, \dots, X_n$ are n identically independently distributed random variables, the correlation coefficient between X_i and $\bar{X}$, ($i = 1, 2, \dots, n$) is:

- (A) 0
 (B) $1/n$
 (C) $1/\sqrt{n}$
 (D) 1

158. The mean and variance of a binomial distribution are 8 and 4 respectively. The value of $P(X=1)$ is :

- (A) $\frac{1}{2^8}$
 (B) $\frac{1}{2^{16}}$
 (C) $\frac{1}{2^4}$
 (D) $\frac{1}{2^{12}}$



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159. Let $\hat{R}$ be the ratio of sample means of study and auxiliary variables in the context of the ratio method of estimation. Let $\bar{X}_N$ and $\bar{x}_n$ be the population and sample means of auxiliary variables. The bias of ratio estimator of $\bar{X}$ can be expressed in a compact form as :
- (A) $-Cov(\hat{R}, \bar{X}_N)$
 (B) $-Cov(\hat{R}, \bar{x}_n)$
 (C) $Cov(\hat{R}, \bar{X}_N)$
 (D) $Cov(\hat{R}, \bar{x}_n)$
160. For a Latin square design of order p , the number of different Latin squares which can be formed is :
- (A) $p!$
 (B) $p!(p-1)!$
 (C) $p(p-1)!$
 (D) p^2
161. Consider a BIBD with parameters (v, b, r, k, λ) , where v denotes the number of treatments, b denotes the number of blocks, r denotes the number of replications, k denotes the number of plots in a block, and λ denotes the number of times a pair of treatments occurs in the blocks. Then, under usual notations, which of the following are the correct statements:
- (i) $bk = vr$
 (ii) $\lambda(v-1) = r(k-1)$
 (iii) $b \geq v$
 (A) Only (i) and (iii)
- (B) Only (ii) and (iii)
 (C) Only (i) and (ii)
 (D) (i), (ii) and (iii)
162. In simple random sampling without replacement, let X and Y be the auxiliary and main variables. If v_R is the variance of regression estimate of population mean of Y , upto the second order of approximation based on Taylor series, v_S is the variance of the sample mean of Y and the correlation coefficient between X and Y is 0.8, then v_R/v_S is equal to :
- (A) 0.20
 (B) 0.36
 (C) 0.64
 (D) 0.80
163. A population is divided into three strata and a sample of size 600 has to be drawn. If strata sizes are in ratio $N_1:N_2:N_3 :: 1:2:3$ and their standard deviations are in ratio $S_1:S_2:S_3 :: 3:2:1$, then under the Neyman allocation the sample sizes for the three strata are respectively:
- (A) 100, 200, 300
 (B) 300, 200, 100
 (C) 180, 240, 180
 (D) 200, 200, 200



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164. A total of 28 percent of students choose biology, 7 percent choose mathematics, and 5 percent choose both biology and mathematics. What percentage of students choose neither biology nor mathematics?
- (A) 0.7
(B) 0.3
(C) 0.35
(D) 0.21
165. A fair dice is tossed two times. The probability that the maximum of two numbers obtained is 4, is:
- (A) $1/36$
(B) $1/6$
(C) $5/36$
(D) $7/36$
166. For a $m \times m$ Latin square design, if the error degrees of freedom is 12, then the degrees of freedom associated with total sum of squares is :
- (A) 15
(B) 18
(C) 20
(D) 24
167. For two random variables X and Y , if the regression lines are $2X - 4 = 0$ and $4Y - 5 = 0$, then the correlation coefficient between X and Y is:
- (A) $\frac{1}{2\sqrt{2}}$
(B) $-\frac{1}{2\sqrt{2}}$
(C) 0
(D) 1
168. For n observations $x_1, x_2, \dots, x_n$, if $A = \frac{1}{n} \sum_{i=1}^n x_i$, $B = \text{median}\{x_1, x_2, \dots, x_n\}$, $C = \sum_{i=1}^n |x_i - A|$, $D = \sum_{i=1}^n |x_i - B|$ then which of the following holds :
- (A) $A + B = -(C + D)$
(B) $A + B \geq C + D$
(C) $C \leq D$
(D) $C \geq D$
169. If $\bar{X} = 10$, $\bar{Y} = 20$ and the regression line of Y on X is $2X - 3Y = c$, then the value of c is :
- (A) -10
(B) -40
(C) 10
(D) 40

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170. Based on a single observation x from a uniform distribution $U(-\theta, \theta)$, the critical region for testing $H_0: \theta = 1$ against $H_1: \theta = 2$ is $C = x: |x| > 0.9$. The level of significance is :
- (A) 0.05
(B) 0.01
(C) 0.10
(D) 0.50
171. In linear programming problem, if the objective function line is parallel to one of the constrain lines, then the LPP has :
- (A) Unique optimum solution
(B) Unbounded optimum solution
(C) No feasible solution
(D) Multiple optimum solutions
172. In LPP, the basic feasible solution is a solution with :
- (A) A minimal set of zero variables
(B) Only zero variables
(C) Only non-zero variables
(D) A minimal set of non-zero variables
173. The customers arrive at a service counter at an inter arrival rate of 10 minutes and reception takes on average 6 minutes to provide service. The average queue length at steady state is :
- (A) 3/10
(B) 5/10
(C) 7/10
(D) 9/10
174. Cars arrive at a service station according to Poisson distribution with a mean rate of 5 cars per hour. The service time per car is an exponential distribution with a mean of 10 minutes. At a steady state, the average waiting time in the queue is :
- (A) 15 minutes
(B) 20 minutes
(C) 40 minutes
(D) 50 minutes
175. In queueing process, if the number of customers arriving in a fixed time interval follows Poisson distribution with an average of 30 customers per hour, the expected time interval between two successive arrivals is :
- (A) 1 minute
(B) 2 minutes
(C) 6 minutes
(D) 1 hour



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