WARNING	Any malpractice or any attempt to commit any kind of malpractice in the Examination will DISQUALIFY THE CANDIDATE.

PAPER – II MATHEMATICS-2016

Version Code	B2	Question Booklet Serial Number :	6237916
Time : 150 Minutes	Number of Questions : 120		Maximum Marks : 480
Name of Candidate			
Roll Number			
Signature of Candidate			

INSTRUCTIONS TO THE CANDIDATE

1. **Please ensure that the VERSION CODE shown at the top of this Question Booklet is the same as that shown in the OMR Answer Sheet issued to you.** If you have received a Question Booklet with a different Version Code, please get it replaced with a Question Booklet with the same Version Code as that of the OMR Answer Sheet from the Invigilator. **THIS IS VERY IMPORTANT.**
2. Please fill the items such as Name, Roll Number and Signature in the columns given above. Please also write Question Booklet Serial No. given at the top of this page against item 3 in the OMR Answer Sheet.
3. This Question Booklet contains 120 questions. For each question, five answers are suggested and given against (A), (B), (C), (D) and (E) of which only one will be the **Most Appropriate Answer**. Mark the bubble containing the letter corresponding to the 'Most Appropriate Answer' in the OMR Answer Sheet, by using either **Blue or Black ball-point pen only**.
4. **Negative Marking:** In order to discourage wild guessing, the score will be subjected to penalization formula based on the number of right answers actually marked and the number of wrong answers marked. Each correct answer will be awarded FOUR marks. ONE mark will be deducted for each incorrect answer. More than one answer marked against a question will be deemed as incorrect answer and will be negatively marked.
5. Please read the instructions given in the OMR Answer Sheet for marking answers. Candidates are advised to strictly follow the instructions contained in the OMR Answer Sheet.

IMMEDIATELY AFTER OPENING THIS QUESTION BOOKLET, THE CANDIDATE SHOULD VERIFY WHETHER THE QUESTION BOOKLET ISSUED CONTAINS ALL THE 120 QUESTIONS IN SERIAL ORDER. IF NOT, REQUEST FOR REPLACEMENT.

DO NOT OPEN THE SEAL UNTIL THE INVIGILATOR ASKS YOU TO DO SO.

SEAL

PLEASE ENSURE THAT THIS QUESTION BOOKLET CONTAINS 120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120.
PRINTED PAGES : 32

1. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$, then the values of x are

(A) 1, 5 (B) $-1, -5$ (C) 1, 6 (D) $-1, -6$ (E) 3, 3

2. If $A = \begin{vmatrix} 8 & 27 & 125 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{vmatrix}$, then the value of A^2 is equal to

(A) 0 (B) 36 (C) 64 (D) 2400 (E) 3600

3. If $A = \begin{bmatrix} x & 1 & -x \\ 0 & 1 & -1 \\ x & 0 & 7 \end{bmatrix}$ and $\det(A) = \begin{vmatrix} 3 & 0 & 1 \\ 2 & -1 & 2 \\ 0 & 0 & 3 \end{vmatrix}$, then the value of x is

(A) -3 (B) 3 (C) 2 (D) -8 (E) -2

4. The coefficient of x^2 in the expansion of the determinant

$\begin{vmatrix} x^2 & x^3+1 & x^5+2 \\ x^3+3 & x^2+x & x^3+x^4 \\ x+4 & x^3+x^5 & 2^3 \end{vmatrix}$ is

(A) -10 (B) -8 (C) -2 (D) -6 (E) 8

Space for rough work

5. Let $A=\begin{bmatrix} 1 & \frac{-1-i\sqrt{3}}{2} \\ \frac{-1+i\sqrt{3}}{2} & 1 \end{bmatrix}$. Then $A^{100}=$

(A) $2^{100}A$ (B) $2^{99}A$ (C) $2^{98}A$ (D) A (E) A^2

6. The least integer satisfying $\frac{396}{10}-\frac{19-x}{10}<\frac{376}{10}-\frac{19-9x}{10}$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. If $|x-1|+|x-3|\le 8$, then the values of x lie in the interval

(A) $(-\infty,-2]$ (B) $[-2, 6]$ (C) $(-3,7)$

(D) $(-2,\infty)$ (E) $[6, \infty)$

8. Let p: 57 is an odd prime number,
q: 4 is a divisor of 12,
r: 15 is the LCM of 3 and 5
be three simple logical statements. Which one of the following is true?

(A) $p\vee(\sim q\wedge r)$ (B) $\sim p\vee(q\wedge r)$ (C) $(p\wedge q)\vee\sim r$

(D) $(p\vee q)\wedge\sim r$ (E) $\sim p\wedge(\sim q\wedge r)$

Space for rough work

9. Let p, q, r be three simple statements. Then $\sim(p \vee q) \vee \sim(p \vee r) \equiv$

(A) $(\sim p) \wedge (\sim q \vee \sim r)$ (B) $(\sim p) \wedge (q \vee r)$ (C) $p \wedge (q \vee r)$
(D) $p \vee (q \wedge r)$ (E) $(p \vee q) \wedge r$

10. If p : 3 is a prime number and q : one plus one is three, then the compound statement "It is not that 3 is a prime number or it is not that one plus one is three" is

(A) $\sim p \vee q$ (B) $\sim(p \vee q)$ (C) $p \wedge \sim q$
(D) $\sim p \vee \sim q$ (E) $p \vee \sim q$

11. The value of $\sin^2\frac{\pi}{8} + \sin^2\frac{3\pi}{8} + \sin^2\frac{5\pi}{8} + \sin^2\frac{7\pi}{8}$ is equal to

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 2

12. The value of $\frac{\sqrt{3}}{\sin 15^\circ} - \frac{1}{\cos 15^\circ}$ is equal to

(A) $4\sqrt{2}$ (B) $2\sqrt{2}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$ (E) $\frac{\sqrt{3}}{2}$

Space for rough work

13. If $\sin x + \cos x = \sqrt{2}$, then $\sin x \cos x =$

(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) $\sqrt{2}$ (E) $\frac{1}{\sqrt{2}}$

14. If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then $\tan(2\theta + \phi) =$

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) 3 (E) $\frac{1}{2}$

15. The value of x satisfying the equation $\tan^{-1} x + \tan^{-1}\left(\frac{2}{3}\right) = \tan^{-1}\left(\frac{7}{4}\right)$ is equal to

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $-\frac{1}{3}$ (E) $\frac{1}{3}$

16. If $\tan A - \tan B = x$ and $\cot B - \cot A = y$, then $\cot(A - B)$ is

(A) $\frac{1}{x-y}$ (B) $\frac{1}{x+y}$ (C) $\frac{1}{x} + y$ (D) $\frac{1}{x} - \frac{1}{y}$ (E) $\frac{1}{x} + \frac{1}{y}$

17. If $\tan^{-1} x + \tan^{-1} y = \frac{2\pi}{3}$, then $\cot^{-1} x + \cot^{-1} y$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) π

Space for rough work

al to

18. If the orthocenter, centroid, incentre and circumcentre coincide in a triangle ABC, and if the length of side AB is $\sqrt{75}$ units, then the length of the altitude of the triangle through the vertex A is

(A) $\sqrt{3}$ units (B) 3 units (C) $\frac{\sqrt{15}}{2}$ units

(D) $\frac{15}{2}$ units (E) $\frac{\sqrt{5}}{2}$ units

19. If $A(2,4)$ and $B(6,10)$ are two fixed points and if a point P moves so that $\angle APB$ is always a right angle, then the locus of P is

(A) $x^2+y^2+8x+14y+52=0$ (B) $x^2+y^2-8x+14y-52=0$

(C) $x^2+y^2+8x-14y+52=0$ (D) $x^2+y^2-8x-14y-52=0$

(E) $x^2+y^2-8x-14y+52=0$

20. The points (–1, 0) and (–2, 1) are the two extremities of a diagonal of a parallelogram. If (–6, 5) is the third vertex, then the fourth vertex of the parallelogram is

(A) (2, –6) (B) (2, –5) (C) (3, –4)

(D) (–3, 4) (E) (3, –5)

21. The slope of the straight line $\frac{x}{10}-\frac{y}{4}=3$ is

(A) $\frac{5}{2}$ (B) $\frac{-5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{-2}{5}$ (E) $\frac{3}{4}$

Space for rough work

22. If y-intercept of the line $4x - ay = 8$ is thrice its x-intercept, then the value of a is equal to

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $-\frac{3}{4}$ (D) $-\frac{4}{3}$ (E) $-\frac{2}{3}$

23. The equation of one of the straight lines passing through the point (0, 1) and is at a distance of $\frac{3}{5}$ units from the origin is

(A) $4x+3y=3$ (B) $-x+y=1$ (C) $x+y=1$
(D) $5x+4y=4$ (E) $-5x+4y=4$

24. The nearest point on the line $x+y-3=0$ from the point $(3, -2)$ is

(A) $(3, 5)$ (B) $(4, 1)$ (C) $(3, -5)$
(D) $(4, -1)$ (E) $(5, -1)$

25. The image of the origin with respect to the line $4x+3y=25$, is

(A) (4, 3) (B) (3, 4) (C) (6, 8) (D) (4, 6) (E) (8, 6)

26. If the area of the circle $4x^2+4y^2+8x-16y+\lambda=0$ is 9π sq. units, then the value of λ is

(A) 4 (B) -4 (C) 16 (D) -16 (E) -8

Space for rough work

27. The radius of the circle passing through the points $(2,3)$, $(2,7)$ and $(5,3)$ is

(A) 5 (B) 4 (C) $\frac{5}{2}$ (D) 2 (E) $\sqrt{5}$

28. If a diameter of the circle $x^2+y^2-2x-6y+6=0$ is a chord of another circle C having centre (2, 1), then the radius of the circle C is

(A) 2 (B) $\sqrt{3}$ (C) 3 (D) $\sqrt{5}$ (E) 5

29. In the family of concentric circles $2(x^2+y^2)=k$, the radius of the circle passing through (1, 1) is

(A) $\sqrt{2}$ (B) 4 (C) $2\sqrt{2}$ (D) 1 (E) $3\sqrt{2}$

30. Let P be a point on an ellipse at a distance of 8 units from a focus. If the eccentricity is $\frac{4}{5}$, then the distance of the point P from the directrix is

(A) $\frac{5}{8}$ (B) $\frac{8}{5}$ (C) 5 (D) 8 (E) 10

31. If (–3, 0) is the vertex and y-axis is the directrix of a parabola, then its focus is at the point

(A) (0, –6) (B) (–6, 0) (C) (6, 0)
(D) (0, 0) (E) (3, 0)

Space for rough work

32. The foci of the ellipse $4x^2+9y^2=1$ are

(A) $\left(\pm\frac{\sqrt{3}}{2},0\right)$ (B) $\left(\pm\frac{\sqrt{5}}{2},0\right)$ (C) $\left(\pm\frac{\sqrt{5}}{3},0\right)$

(D) $\left(\pm\frac{\sqrt{5}}{6},0\right)$ (E) $\left(\pm\frac{\sqrt{5}}{4},0\right)$

33. The directrix of a parabola is $x+8=0$ and its focus is at $(4,3)$. Then the length of the latus-rectum of the parabola is

(A) 5 (B) 9 (C) 10 (D) 12 (E) 24

34. If the eccentricity of the ellipse $ax^2+4y^2=4a$, $(a<4)$ is $\frac{1}{\sqrt{2}}$, then its semi-minor axis is equal to

(A) 2 (B) $\sqrt{2}$ (C) 1 (D) $\sqrt{3}$ (E) 3

35. The hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through the point $\left(\sqrt{6},3\right)$ and the length of the latus rectum is $\frac{18}{5}$. Then the length of the transverse axis is equal to

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

Space for rough work

36. The angle between $\vec{a}$ and $\vec{b}$ is $\frac{5\pi}{6}$ and the projection of $\vec{a}$ on $\vec{b}$ is $\frac{-9}{\sqrt{3}}$, then $|\vec{a}|$ is equal to

(A) 12 (B) 8 (C) 10 (D) 4 (E) 6

37. The direction cosines of the straight line given by the planes $x=0$ and $z=0$ are

(A) 1, 0, 0 (B) 0, 0, 1 (C) 1, 1, 0 (D) 0, 1, 0 (E) 0, 1, 1

38. If $\vec{a}=2\hat{i}-\hat{j}-m\hat{k}$ and $\vec{b}=\frac{4}{7}\hat{i}-\frac{2}{7}\hat{j}+2\hat{k}$ are collinear, then the value of m is equal to

(A) –7 (B) –1 (C) 2 (D) 7 (E) –2

39. Let $\vec{a}=2\hat{i}+5\hat{j}-7\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}+5\hat{k}$. Then $(3\vec{a}-5\vec{b})\cdot(4\vec{a}\times5\vec{b})=$

(A) –7 (B) 0 (C) –13 (D) 1 (E) –8

Space for rough work

40. If $\vec{a}+2\vec{b}-\vec{c}=\vec{0}$ and $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\lambda\vec{a}\times\vec{b}$, then the value of λ is equal to

(A) 5 (B) 4 (C) 2 (D) -2 (E) -4

41. If $\vec{a}\cdot\vec{b}=0$ and $\vec{a}+\vec{b}$ makes an angle of $60°$ with $\vec{b}$, then $|\vec{a}|$ is equal to

(A) 0 (B) $\frac{1}{\sqrt{3}}|\vec{b}|$ (C) $\frac{1}{|\vec{b}|}$ (D) $|\vec{b}|$ (E) $\sqrt{3}|\vec{b}|$

42. If $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular and $\vec{b}=3\hat{i}-4\hat{j}+2\hat{k}$, then $|\vec{a}|$ is equal to

(A) $\sqrt{41}$ (B) $\sqrt{39}$ (C) $\sqrt{19}$ (D) $\sqrt{29}$ (E) $\sqrt{31}$

43. The straight line $\vec{r}=(\hat{i}+\hat{j}+\hat{k})+\alpha(2\hat{i}-\hat{j}+4\hat{k})$ meets the xy plane at the point

(A) $(2,\ -1,\ 0)$ (B) $(3,\ 4,\ 0)$ (C) $\left(\frac{1}{2},\ \frac{3}{4},\ 0\right)$

(D) $\left(\frac{1}{2},\ \frac{7}{4},\ 0\right)$ (E) $\left(\frac{1}{2},\ \frac{5}{4},\ 0\right)$

Space for rough work

44. The equation of the plane passing through $(-1, 5, -7)$ and parallel to the plane $2x-5y+7z+11=0$, is

(A) $\vec{r}\cdot(2\hat{i}-5\hat{j}-7\hat{k})+76=0$ (B) $\vec{r}\cdot(2\hat{i}-5\hat{j}+7\hat{k})+76=0$

(C) $\vec{r}\cdot(2\hat{i}-5\hat{j}+7\hat{k})+75=0$ (D) $\vec{r}\cdot(2\hat{i}-5\hat{j}+7\hat{k})+65=0$

(E) $\vec{r}\cdot(2\hat{i}-5\hat{j}-7\hat{k})+55=0$

45. The angle subtended at the point $(1, 2, 3)$ by the points $P(2, 4, 5)$ and $Q(3, 3, 1)$, is

(A) 90° (B) 60° (C) 30° (D) 0° (E) 45°

46. If the two lines $\frac{x-1}{2}=\frac{1-y}{-a}=\frac{z}{4}$ and $\frac{x-3}{1}=\frac{2y-3}{4}=\frac{z-2}{2}$ are perpendicular, then the value of a is equal to

(A) -4 (B) 5 (C) -5 (D) 4 (E) -2

47. If the line $\frac{x+1}{2}=\frac{y+1}{3}=\frac{z+1}{4}$ meets the plane $x+2y+3z=14$ at P, then the distance between P and the origin is

(A) $\sqrt{14}$ (B) $\sqrt{15}$ (C) $\sqrt{13}$ (D) $\sqrt{12}$ (E) $\sqrt{17}$

Space for rough work

48. The point of intersection of the straight lines

$\vec{r} = (3\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(-\hat{i} - 2\hat{j} + 2\hat{k})$ and $\frac{3-x}{-1} = \frac{y+4}{2} = \frac{z-5}{7}$ is

(A) $(-3, -4, -5)$ (B) $(-3, 4, 5)$ (C) $(-3, 4, -5)$

(D) $(-3, -4, 5)$ (E) $(3, -4, 5)$

49. The vector equation of the straight line $\frac{x-2}{1} = \frac{y}{-3} = \frac{1-z}{2}$ is

(A) $\vec{r} = 2\hat{i} + \hat{k} + t(\hat{i} + 3\hat{j} + 2\hat{k})$ (B) $\vec{r} = 2\hat{i} - \hat{k} + t(\hat{i} - 3\hat{j} - 2\hat{k})$

(C) $\vec{r} = 2\hat{i} + \hat{k} + t(\hat{i} - 3\hat{j} + 2\hat{k})$ (D) $\vec{r} = 2\hat{i} - \hat{j} + t(\hat{i} - 3\hat{j} - 2\hat{k})$

(E) $\vec{r} = 2\hat{i} + \hat{k} + t(\hat{i} - 3\hat{j} - 2\hat{k})$

50. The straight line $\vec{r} = (\hat{i} + \hat{j} + 2\hat{k}) + t(2\hat{i} + 5\hat{j} + 3\hat{k})$ is parallel to the plane $\vec{r} \cdot (2\hat{i} + \hat{j} - 3\hat{k}) = 5$. Then the distance between the straight line and the plane is

(A) $\frac{9}{\sqrt{14}}$ (B) $\frac{8}{\sqrt{14}}$ (C) $\frac{7}{\sqrt{14}}$ (D) $\frac{6}{\sqrt{14}}$ (E) $\frac{5}{\sqrt{14}}$

51. Two fair dice are rolled. Then the probability of getting a composite number as the sum of face values is equal to

(A) $\frac{7}{12}$ (B) $\frac{5}{12}$ (C) $\frac{1}{12}$ (D) $\frac{3}{4}$ (E) $\frac{2}{3}$

Space for rough work

52. If the mean of the numbers $a, b, 8, 5, 10$ is 6 and their variance is 6.8, then ab is equal to

(A) 6 (B) 7 (C) 12 (D) 14 (E) 25

53. In a class, in an examination in Mathematics, 10 students scored 100 marks each, 2 students scored zero and the average of the remaining students is 72 marks. If the class average is 76, then the number of students in the class is

(A) 44 (B) 40 (C) 38 (D) 34 (E) 32

54. A bag contains 3 red, 4 white and 5 blue balls. If two balls are drawn at random, then the probability that they are of different colours is

(A) $\frac{47}{66}$ (B) $\frac{23}{33}$ (C) $\frac{47}{132}$ (D) $\frac{47}{33}$ (E) $\frac{70}{33}$

55. There are 5 positive numbers and 6 negative numbers. Three numbers are chosen at random and multiplied. The probability that the product being a negative number is

(A) $\frac{11}{34}$ (B) $\frac{17}{33}$ (C) $\frac{16}{35}$ (D) $\frac{15}{34}$ (E) $\frac{16}{33}$

Space for rough work

56. The value of $\lim_{x\to 0}\dfrac{\cot 4x}{\operatorname{cosec} 3x}$ is equal to

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{2}$ (E) 0

57. Let $f(x)=\begin{cases}\cos x & \text{if} \quad x\geq 0\\ -\cos x & \text{if} \quad x<0\end{cases}$.

Which one of the following statements is **not true**?

(A) $f(x)$ is continuous at $x=1$
(B) $f(x)$ is continuous at $x=-1$
(C) $f(x)$ is continuous at $x=2$
(D) $f(x)$ is continuous at $x=-2$
(E) $f(x)$ is continuous at $x=0$

58. The value of $\lim_{n\to+\infty}\dfrac{{}^{n}C_{3}-{}^{n}P_{3}}{n^{3}}$ is equal to

(A) $\frac{-5}{6}$ (B) $\frac{5}{6}$ (C) $\frac{1}{6}$ (D) $-\frac{1}{6}$ (E) $\frac{2}{3}$

59. If $f(x)=3x+5$ and $g(x)=x^2-1$, then $(f\circ g)(x^2-1)$ is equal to

(A) $3x^4-3x+5$
(B) $3x^4-6x^2+5$
(C) $6x^4+3x^2+5$
(D) $6x^4-6x+5$
(E) $3x^2+6x+4$

Space for rough work

60. The period of the function $f(x)=\tan(4x-1)$ is

(A) π (B) $\frac{\pi}{2}$ (C) 2π (D) $\frac{\pi}{4}$ (E) $\frac{3\pi}{4}$

61. If $2^x+2^y=2^{x+y}$, then the value of $\frac{dy}{dx}$ at (1, 1) is equal to

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

62. If $f(x)=\frac{\sin^{-1}x}{\sqrt{1-x^2}}$, then the value of $\left(1-x^2\right)f'(x)-xf(x)$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

63. If $f(x)=\left(\frac{x}{2}\right)^{10}$, then $f(1)+\frac{f'(1)}{\lfloor 1}+\frac{f''(1)}{\lfloor 2}+\frac{f'''(1)}{\lfloor 3}+....+\frac{f^{(10)}(1)}{\lfloor 10}$ is equal to

(A) 1 (B) 10 (C) 11 (D) 512 (E) 1024

Space for rough work

64. If $f'(4)=5$, $g'(4)=12$, $f(4)\,g(4)=2$ and $g(4)=6$, then $\left(\dfrac{f}{g}\right)'(4)=$

(A) $\dfrac{5}{36}$ (B) $\dfrac{11}{18}$ (C) $\dfrac{23}{36}$ (D) $\dfrac{13}{18}$ (E) $\dfrac{19}{36}$

65. If the derivative of $(ax-5)e^{3x}$ at $x=0$ is -13, then the value of a is equal to

(A) 8 (B) -5 (C) 5 (D) -2 (E) 2

66. Let $y=\tan^{-1}(\sec x+\tan x)$. Then $\dfrac{dy}{dx}=$

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{\sec x+\tan x}$

(D) $\dfrac{1}{\sec^2 x}$ (E) $\dfrac{1}{\tan x}$

67. If $s=\sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$ and $t=\sqrt{1-x^2}$, then $\dfrac{ds}{dt}$ at $x=\dfrac{1}{2}$ is

(A) 1 (B) 2 (C) -2 (D) 4 (E) -4

Space for rough work

68. The minimum value of $2x^3 - 9x^2 + 12x + 4$ is deleted

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

69. The slope of the curve $y = e^x \cos x,\ x \in (-\pi, \pi)$ is maximum at

(A) $x = \frac{\pi}{2}$ (B) $x = -\frac{\pi}{2}$ (C) $x = \frac{\pi}{4}$ (D) $x = 0$ (E) $x = \frac{\pi}{3}$

70. If $y = f(x)$ is continuous on $[0, 6]$, differentiable on $(0, 6)$, $f(0) = -2$ and $f(6) = 16$, then at some point between $x = 0$ and $x = 6$, $f'(x)$ must be equal to

(A) -18 (B) -3 (C) 3 (D) 14 (E) 18

71. The equation of the tangent to the curve $y = x^3 - 6x + 5$ at (2, 1) is

101 (A) $6x - y - 11 = 0$ (B) $6x - y - 13 = 0$ (C) $6x + y + 11 = 0$

(D) $6x - y + 11 = 0$ (E) $x - 6y - 11 = 0$

Space for rough work

72. Let $f(x)=2x^3-5x^2-4x+3,\ \frac{1}{2}\le x\le 3$. The point at which the tangent to the curve is parallel to the x-axis, is

(A) $(1, -4)$ (B) $(2, -9)$ (C) $(2, -4)$

(D) $(2, -1)$ (E) $(2, -5)$

73. Two sides of a triangle are 8 m and 5 m in length. The angle between them is increasing at the rate 0.08 rad/sec. When the angle between the sides of fixed length is $\frac{\pi}{3}$, the rate at which the area of the triangle is increasing is,

(A) $0.4\ m^2/sec$ (B) $0.8\ m^2/sec$ (C) $0.6\ m^2/sec$

(D) $0.04\ m^2/sec$ (E) $0.08\ m^2/sec$

74. If $y=8x^3-60x^2+144x+27$ is a strictly decreasing function in the interval

(A) $(-5, 6)$ (B) $(-\infty, 2)$ (C) $(5, 6)$ (D) $(3, \infty)$ (E) $(2, 3)$

75. $\int(\sec x)^m\left(\tan^3 x+\tan x\right)dx$ is equal to

(A) $\sec^{m+2} x+C$ (B) $\tan^{m+2} x+C$ (C) $\frac{\sec^{m+2} x}{m+2}+C$

(D) $\frac{\tan^{m+2} x}{m+2}+C$ (E) $\frac{\sec^{m+1} x}{m+1}+C$

Space for rough work

76. $\int \frac{1}{7}\sin\left(\frac{x}{7}+10\right)dx$ is equal to

(A) $\frac{1}{7}\cos\left(\frac{x}{7}+10\right)+C$ (B) $-\frac{1}{7}\cos\left(\frac{x}{7}+10\right)+C$ (C) $-\cos\left(\frac{x}{7}+10\right)+C$

(D) $-7\cos\left(\frac{x}{7}+10\right)+C$ (E) $\cos(x+70)+C$

77. $\int\left(\frac{x-a}{x}-\frac{x}{x+a}\right)dx$ is equal to

(A) $\log\left|\frac{x+a}{x}\right|+C$ (B) $a\log\left|\frac{x+a}{x}\right|+C$ (C) $a\log\left|\frac{x}{x+a}\right|+C$

(D) $\log\left|\frac{x}{x+a}\right|+C$ (E) $a\log\left|\frac{x-a}{x+a}\right|+C$

78. $\int x^4 e^{x^5}\cos\left(e^{x^5}\right)dx$ is equal to

(A) $\frac{1}{3}\sin\left(e^{x^5}\right)+C$ (B) $\frac{1}{4}\sin\left(e^{x^5}\right)+C$ (C) $\frac{1}{5}\sin\left(e^{x^5}\right)+C$

(D) $\sin\left(e^{x^5}\right)+C$ (E) $2\sin\left(e^{x^5}\right)+C$

Space for rough work

79. $\int \frac{2x+\sin 2x}{1+\cos 2x}\,dx$ is equal to

(A) $x+\log|\tan x|+C$ (B) $x\log|\tan x|+C$ (C) $x\tan x+C$

(D) $x+\tan x+C$ (E) $x\sec x+C$

80. $\int \frac{1}{\sin x\cos x}\,dx$ is equal to

(A) $\log|\tan x|+C$ (B) $\log|\sin 2x|+C$ (C) $\log|\sec x|+C$

(D) $\log|\cos x|+C$ (E) $\log|\sin x|+C$

81. $\int \frac{1}{8\sin^2 x+1}\,dx$ is equal to

(A) $\sin^{-1}(\tan x)+C$ (B) $\frac{1}{3}\sin^{-1}(\tan x)+C$ (C) $\frac{1}{3}\tan^{-1}(3\tan x)+C$

(D) $\tan^{-1}(3\tan x)+C$ (E) $\sin^{-1}(3\tan x)+C$

82. $\int_0^{\pi/2} \log\left(\frac{\cos x}{\sin x}\right)dx$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) π (D) 2π (E) 0

Space for rough work

83. The value of $\int_{-1}^{2} 4x^2 |x| \, dx$ is equal to

113

(A) 17 (B) 16 (C) 15 (D) 14 (E) 13

84. The area of the region bounded by $y^2 = 16 - x^2$, $y = 0$, $x = 0$ in the first quadrant is (in square units)

(A) 8π (B) 6π (C) 2π (D) 4π (E) $\frac{\pi}{2}$

85. The value of $\int_{2}^{4} (x-2)(x-3)(x-4) \, dx$ is equal to

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) $\frac{1}{3}$ (E) 0

86. The area bounded by the lines $y - 2x = 2$, $y = 4$ and the y-axis is equal to (in square units)

(A) 1 (B) 4 (C) 0 (D) 3 (E) 2

87. The general solution of the differential equation $(x+y+3)\frac{dy}{dx} = 1$ is

(A) $x+y+3 = Ce^{y}$ (B) $x+y+4 = Ce^{y}$ (C) $x+y+3 = Ce^{-y}$

(D) $x+y+4 = Ce^{-y}$ (E) $x+y+4e^{y} = C$

Space for rough work

88. The differential equation representing the family of curves $y^2 = a(ax+b)$ where a and b are arbitrary constants, is of

(A) order 1, degree 1 (B) order 1, degree 3 (C) order 2, degree 3

(D) order 1, degree 4 (E) order 2, degree 1

89. The solution of the differential equation $\dfrac{x\frac{dy}{dx}-y}{\sqrt{x^2-y^2}} = 10x^2$ is

(A) $\sin^{-1}\left(\dfrac{y}{x}\right)-5x^2=C$ (B) $\sin^{-1}\left(\dfrac{y}{x}\right)=10x^2+C$ (C) $\dfrac{y}{x}=5x^2+C$

(D) $\sin^{-1}\left(\dfrac{y}{x}\right)=10x^2+Cx$ (E) $\sin^{-1}\left(\dfrac{y}{x}\right)+5x^2=C$

90. The general solution of the differential equation $x\,dy - y\,dx = y^2dx$ is

(A) $y=\dfrac{x}{C-x}$ (B) $x=\dfrac{2y}{C+x}$ (C) $y=(C+x)\,(2x)$

(D) $y=\dfrac{2x}{C+x}$ (E) $x=\dfrac{y}{C-x}$

Space for rough work

91. If $*$ is the operation defined by $a*b=a^b$ for $a,b\in\mathbb{N}$, then $(2*3)*2$ is equal to

(A) 81 (B) 512 (C) 216 (D) 64 (E) 243

92. The domain of the function $f(x)=\begin{cases}(x^2-9)/(x-3), & \text{if } x\neq 3\\ 6, & \text{if } x=3\end{cases}$ is

(A) $(0,3)$ (B) $(-\infty,3)$ (C) $(-\infty,\infty)$ (D) $(3,\infty)$ (E) $(-3,3)$

93. Let $f(x)=x^3$ and $g(x)=3^x$. The values of a such that $g(f(a))=f(g(a))$ are

(A) 0, 2 (B) 1, 3 (C) $0,\pm 3$ (D) $1,\pm 2$ (E) $0,\ \pm\sqrt{3}$

94. If $f\left(\frac{x+1}{2x-1}\right)=2x$, $x\in\mathbb{N}$, then the value of $f(2)$ is equal to

A

(A) 1 (B) 4 (C) 3 (D) 2 (E) 5

Space for rough work

95. If $A \setminus B = \{a,b\}$, $B \setminus A = \{c,d\}$ and $A \cap B = \{e,f\}$, then the set B is equal to

(A) $\{a,b,c,d\}$ (B) $\{e,f,c,d\}$ (C) $\{a,b,e,f\}$

(D) $\{c,d,a,e\}$ (E) $\{d,e,a,b\}$

96. The function $f: A \to B$ given by $f(x) = x$, $x \in A$, is one to one but not onto. Then

(A) $B \subset A$ (B) $A = B$ (C) $A' \subset B'$ (D) $A \subset B$ (E) $A \cap B = \varnothing$

97. The principal argument of the complex number $z = \dfrac{1+\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}}{1+\sin\frac{\pi}{3}-i\cos\frac{\pi}{3}}$ is

(A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{6}$ (C) $\dfrac{2\pi}{3}$ (D) $\dfrac{\pi}{2}$ (E) $\dfrac{\pi}{4}$

98. If $\dfrac{(1+i)(2+3i)(3-4i)}{(2-3i)(1-i)(3+4i)} = a+ib$, then $a^2+b^2 =$

(A) 132 (B) 25 (C) 144 (D) 128 (E) 1

Space for rough work

99. Let z, w be two nonzero complex numbers. If $\overline{z + i\,w} = 0$ and $\arg(zw) = \pi$, then $\arg z =$

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{8}$

100. If $z = \frac{2-i}{i}$, then $\text{Re}(z^2) + \text{Im}(z^2)$ is equal to

(A) 1 (B) –1 (C) 2 (D) –2 (E) 3

101. If $|z+1| < |z-1|$, then z lies

(A) on the x-axis (B) on the y-axis (C) in the region $x < 0$
(D) in the region $y > 0$ (E) in the region $x > y$

102. If $\left|z - \frac{3}{z}\right| = 2$, then the greatest value of $|z|$ is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Space for rough work

103. If the roots of the quadratic equation $mx^2 - nx + k = 0$ are $\tan 33°$ and $\tan 12°$, then the value of $\frac{2m+n+k}{m}$ is equal to

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

104. If α and β are the roots of $4x^2 + 2x - 1 = 0$, then $\beta =$

(A) $-\frac{1}{4\alpha}$ (B) $-\frac{1}{2\alpha}$ (C) $-\frac{1}{\alpha}$ (D) $-\frac{1}{3\alpha}$ (E) $\frac{1}{\alpha}$

105. If α and α^2 are the roots of the equation $x^2 - 6x + c = 0$, then the positive value of c is

(A) 2 (B) 3 (C) 4 (D) 9 (E) 8

106. If one of the roots of the quadratic equation $ax^2 - bx + a = 0$ is 6, then value of $\frac{b}{a}$ is equal to

(A) $\frac{1}{6}$ (B) $\frac{11}{6}$ (C) $\frac{37}{6}$ (D) $\frac{6}{11}$ (E) $\frac{6}{37}$

Space for rough work

107. If the equation $2x^2 + (a+3)x + 8 = 0$ has equal roots, then one of the values of a is

(A) –9 (B) –5 (C) –11 (D) 11 (E) 9

108. If 6^{th} term of a G.P. is 2, then the product of first 11 terms of the G.P. is equal to

(A) 512 (B) 1024 (C) 2048 (D) 256 (E) 32

109. If the produce of five consecutive terms of a G.P. is $\frac{243}{32}$, then the middle term is

(A) $\frac{2}{3}$ (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) $\frac{3}{4}$ (E) 1

110. If a_1, a_2, a_3, a_4 are in A.P., then $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \frac{1}{\sqrt{a_3}+\sqrt{a_4}} =$

(A) $\frac{\sqrt{a_4}-\sqrt{a_1}}{a_3-a_2}$ (B) $\frac{a_4-a_1}{a_3-a_2}$ (C) $\frac{a_3-a_2}{\sqrt{a_4}-\sqrt{a_1}}$

(D) $\frac{a_1-a_4}{a_3-a_1}$ (E) $\frac{a_5-a_0}{a_1-a_4}$

Space for rough work

111. If $a_1, a_2, a_3, \cdots, a_{20}$ are in A.P. and $a_1 + a_{20} = 45$, then $a_1 + a_2 + a_3 + \cdots + a_{20}$ is equal to

(A) 90 (B) 900 (C) 350 (D) 450 (E) 730

112. Sum of the series $1(1) + 2(1+3) + 3(1+3+5) + 4(1+3+5+7) + \cdots + 10(1+3+5+7+\cdots+19)$ is equal to

(A) 385 (B) 1025 (C) 1125 (D) 2025 (E) 3025

113. In an A.P., the 6^{th} term is 52 and the 11^{th} term is 112. Then the common difference is equal to

(A) 4 (B) 20 (C) 12 (D) 8 (E) 6

114. If the coefficients of x^3 and x^4 in the expansion of $(3+kx)^9$ are equal, then the value of k is

(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$ (E) 1

115. The total number of 7 digit positive integral numbers with distinct digits that can be formed using the digits 4, 3, 7, 2, 1, 0, 5 is

(A) 4320 (B) 4340 (C) 4310 (D) 4230 (E) 4220

Space for rough work

116. If ${}^{n}P_4 = 5\left({}^{n}P_3\right)$, then the value of n is equal to

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

117. The remainder when 2^{2016} is divided by 63, is

(A) 1 (B) 8 (C) 17 (D) 32 (E) 61

118. If ${}^{n}C_2 + {}^{n}C_3 = {}^{6}C_3$ and ${}^{n}C_x = {}^{n}C_3, x \neq 3$, then the value of x is equal to

(A) 5 (B) 4 (C) 2 (D) 6 (E) 1

119. If $\sum_{k=0}^{18} \frac{k}{{}^{18}C_k} = a \sum_{k=0}^{18} \frac{1}{{}^{18}C_k}$, then the value of a is equal to

(A) 3 (B) 9 (C) 6 (D) 18 (E) 36

120. If the square of the matrix $\begin{pmatrix} a & b \\ a & -a \end{pmatrix}$ is the unit matrix, then b is equal to

(A) $\frac{a}{1+a^2}$ (B) $\frac{1-a^2}{a}$ (C) $\frac{1+a^2}{a}$ (D) $\frac{a}{1-a^2}$ (E) $1+a^2$

Space for rough work