A1	ELIGIBILITY EXAMINATION-2025 PAPER-2 Total Number of Questions : 100 Maximum Marks : 200 Subject : MATHEMATICAL SCIENCES Subject Code : 26	Serial Number :	
VERSION CODE	MENTION YOUR REGISTER NUMBER	Paper Code	MAT21125A

INSTRUCTIONS FOR CANDIDATES

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2. This question booklet contains **100** questions and each question will have one statement and four different options / responses & out of which you have to choose one correct answer.
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				⊗	②	③	④	①	②	③	④	①	●	●	④
①	●	③	④	●	②	③	④	①	●	③	④	①	②	③	④

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1. For the set $A=\left\{\frac{1}{n}-\frac{1}{m}: n,m \in N\right\}$, inf A and sup A are ______ respectively

(1) 0 and 1
(2) –1 and 1
(3) 0 and ∞
(4) $-\infty$ and 0

2. If $x_n=\sqrt{n+1}-\sqrt{n}$ for $n \in N$, then $\lim_{n\to\infty}\left(\sqrt{n}\,x_n\right)=$ ______

(1) 3
(2) $\frac{1}{2}$
(3) 2
(4) ∞

3. The set of all real algebraic numbers is

(1) Countable set
(2) Finite set
(3) Uncountable set
(4) Empty set

4. $\lim_{n\to\infty}\left(1+\frac{1}{2n}\right)^n=$ ________

(1) ∞
(2) 1
(3) $e^{1/2}$
(4) e^2

5. The value of $\int_0^3 [x]\,dx$ is ______ where [x] denotes greatest integer not greater than x.

(1) $\frac{1}{3}$
(2) 1
(3) 3
(4) $\frac{2}{3}$

6. Given a system of two non-degenerate linear equations in x and y

$a_1x+b_1y=c_1$
$a_2x+b_2y=c_2$. Then we have

(1) $a_1b_2-a_2b_1\neq 0$ implies the system has many solutions

(2) $a_1b_2-a_2b_1\neq 0$ implies the system has exactly one solution

(3) $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ implies the system has infinitely many solutions

(4) $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ implies the system has exactly one solution

7. Using mean value theorem, which of the following is true for all $x, y \in R$?

(1) $|\sin x - \sin y| = |\sin xy|$

(2) $|\sin x - \sin y| > |x - y|$

(3) $|\sin x - \sin y| \leq |x - y|$

(4) $|\sin x - \sin y| = |xy|$

8. Let $g: [a,b] \to \mathbb{R}$ be continuous on $[a, b]$ and differentiable in (a, b) with $\lim\limits_{x \to a} g'(x) = c$. Then

(1) $g'(a)$ does not exist at all

(2) $g'(a)$ exists but not equal to c

(3) both $g'(a)$ and $g'(b)$ do not exist

(4) $g'(a)$ exists and $g'(a) = c$

9. Let $f(x) = x$ and $g(x) = x - 1$ defined on $[0, 1]$. Which of the following is true?

(1) All the three functions f, g and fg are increasing on $[0, 1]$

(2) f and g are not increasing but fg is increasing on $[0, 1]$

(3) All the three functions f, g and fg are non-increasing on $[0, 1]$

(4) f and g are strictly increasing but fg is not increasing on $[0, 1]$

10. Let $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0,0) \\ 0 & \text{if } (x, y) = (0,0) \end{cases}$

Then which of the following is true?

(1) $f(x, y)$ is continuous at $(0, 0)$

(2) Partial derivatives of $f(x, y)$ do not exist

(3) $f(x, y)$ is differentiable at $(0, 0)$

(4) $f(x, y)$ is not continuous at $(0, 0)$ but partial derivatives exist at $(0, 0)$

11. Which of the following set is countable?

(1) The set of real numbers $\mathbb{R}$

(2) The collection of all mappings of natural numbers N into N

(3) Cantor ternary set

(4) The Cartesian product of a finite collection of countable sets

12. Which of the following statement is true for a monotonic function f on (a, b)?

(1) Discontinuities of f are always isolated

(2) f has discontinuities of second kind

(3) The set of discontinuities of f in (a, b) is uncountable

(4) The set of points of (a, b) at which f is discontinuous is at most countable

13. Which of the following sets has outer measure different from zero?

(1) The set of irrational numbers in [0, 1]

(2) The set of all algebraic numbers

(3) The Cantor's ternary set

(4) A countable set in R

14. Which of the following statement is not true for a metric space (X, d)?

(1) Two disjoint closed sets in (X, d) are separated

(2) Closure of a connected set in (X, d) is connected

(3) If (X, d) is connected with at least two points, then it is countable

(4) The intervals [0, 1] and [1, 2] are not separated

15. Which of the following statement is not true?

(1) A function of bounded variation is bounded

(2) A function of bounded variation is always continuous

(3) A bounded monotone function is a function of bounded variation

(4) A continuous function need not be of bounded variation

16. A function f: R→R need not be Lebesgue measurable if

(1) f is monotone

(2) $\{x \in R: f(x) \geq \alpha\}$ is measurable for each $\alpha \in Q$

(3) $\{x \in R: f(x) \geq \alpha\}$ is measurable for each $\alpha \in R$

(4) For each open set G in R, $f^{-1}(G)$ is measurable

17. Let $f: R^2 \to R$ be defined by $f(x,y) = x^3y + e^{xy^2}$, then which one of the following is not correct?

(1) $f_x = 3x^2y + y^2 e^{xy^2}$

(2) $f_y = x^3 + 2xy\, e^{xy^2}$

(3) $f_{xy} = 3x^2 + 2xe^{xy^2} + 2xy^3e^{xy^2}$

(4) $f_{yx} = 3x^2 + 2ye^{xy^2} + 2xy^3e^{xy^2}$

18. Let X and Y be metric spaces and let f: X → Y be a continuous map. For any subset S of X, which one of the following statement is true?

(1) If S is open, then f(S) is open

(2) If S is connected, then f(S) is connected

(3) If S is closed, then f(S) is closed

(4) If S is bounded, then f(S) is bounded

19. The series $\sum \frac{1}{n^p}$ is

(1) Convergent if $p > 1$ and divergent if $p \leq 1$

(2) Convergent if $p < 1$ and divergent if $p > 1$

(3) Convergent if $p = 0$ and divergent if $p > 1$

(4) Convergent if $p < 1$ and divergent $p \geq 2$

20. Let $S = \{x \in X : ||x||_\infty \leq 1\}$. Then

(1) S is convex and compact

(2) S is not convex but compact

(3) S is convex but not compact

(4) S is neither convex nor compact

21. The dimension of the subspace $W = \{((a_{ij})) : a_{ij} = 0, \text{ i is even}\}$ of all 10×10 real matrices is

(1) 25 (2) 50

(3) 75 (4) 100

22. Which of the following statement is not true?

(1) The intersection of any two subspaces of a vector space is a subspace

(2) The intersection of an arbitrary collection of subspaces of a vector space is a subspace

(3) The union of two subspaces of a vector space is necessarily a subspace

(4) The union of two subspaces is a subspace if and only if one is contained in the other

23. The condition for a, b, c such that the vector $\alpha = (a, b, c)$ is a linear combination of vectors $(1, -3, 2)$ and $(2, -1, 1)$ is

(1) $a - b + c = 0$ (2) $a - 3b - 5c = 0$

(3) $2a - b + c = 0$ (4) $a + b + c = 0$

24. If w_1 and w_2 are two finite dimensional subspaces of a vector space V(F), then $\dim w_1 + \dim w_2 =$

(1) $\dim(w_1 \cap w_2) + \dim(w_1 + w_2)$ (2) $\dim(w_1 \cup w_2) + \dim(w_1 + w_2)$

(3) $\dim(w_1 \cap w_2) - \dim(w_1 + w_2)$ (4) $\dim(w_1 \cap w_2) - \dim(w_1 \cup w_2)$

25. Let T : U → V be a linear transformation between finite dimensional vector spaces, then

(1) rank (T) + nullity (T) = dim U + dim V
(2) rank (T) – nullity (T) = dim U
(3) rank (T) – nullity (T) = dim V
(4) rank (T) + nullity (T) = dim U

26. The rank of matrix $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ is

(1) 1
(2) 2
(3) 4
(4) 3

27. The solution of the system of equations

$x + 2y - z = 3$

$3x - y + 2z = 1$

$2x - 2y + 3z = 2$

$x - y + z = -1$ is

(1) $x = 2, y = 2, z = 1$
(2) $x = 2, y = -4, z = 3$
(3) $x = 1, y = 1, z = 0$
(4) $x = -1, y = 4, z = 4$

28. Which of the following statement is not true?

(1) The eigenvalues of a real symmetric matrix are all real
(2) The eigenvalues of a real Skew-symmetric matrix are either purely imaginary or zero
(3) The eigenvalues of a Hermitian matrix are imaginary
(4) The eigenvalues of a Skew-Hermitian matrix are either purely imaginary or zero

29. Which of the following statement is true?

(1) If '0' is the eigenvalue of a matrix A, then A is non-singular
(2) If '0' is the eigenvalue of A, then A is invertible
(3) Similar matrices have different eigenvalues
(4) If λ is an eigenvalue of non-singular matrix A, then $\frac{1}{\lambda}$ is the eigenvalue of A^{-1}

30. Eigen vectors of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ are

(1) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
(2) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$
(3) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3/2 \end{bmatrix}$
(4) $\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

31. Let T : $\mathbb{R}^2 \to \mathbb{R}^2$ and $T(x, y) = (2y, 3x - y)$, $\forall (x, y) \in \mathbb{R}^2$. Then the matrix representation of T relative to the basis $\{(1, 3), (2, 5)\}$ is

(1) $\begin{pmatrix} 30 & 48 \\ 18 & 29 \end{pmatrix}$ (2) $\begin{pmatrix} -30 & -48 \\ -18 & -29 \end{pmatrix}$

(3) $\begin{pmatrix} -30 & -48 \\ 18 & 29 \end{pmatrix}$ (4) $\begin{pmatrix} -3 & -4 \\ 8 & 9 \end{pmatrix}$

32. If –1, 2, 2 are the eigenvalues of a matrix A of order 3 × 3, then eigenvalues of A^3 are

(1) –1, 8, 8 (2) –3, 6, 6

(3) $-1, \frac{1}{8}, \frac{1}{8}$ (4) $-1, \frac{1}{2}, \frac{-1}{2}$

33. If the square matrix A of order 3 has eigenvalues 2, 2, 8, then trace of A is

(1) 4 (2) 12

(3) 16 (4) 32

34. The minimal polynomial of the matrix $\begin{pmatrix} 3 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 2 \end{pmatrix}$ is

(1) $(t-2)^2 (t-3)$ (2) $-(t-2)^2 (t-3)$

(3) $(t-2)(t-3)$ (4) $(t-2)(t-3)^2$

35. If x and y are orthogonal vectors in the inner product space, then $||x + y||^2 =$

(1) $||x||^2 + ||y||^2$ (2) $||x + y|| + ||x - y||$

(3) $||x|| - ||y||$ (4) $2||x||\,||y||$

36. The quadratic form Q(x, y) corresponding to the symmetric matrix $\begin{pmatrix} 5 & -3 \\ -3 & 8 \end{pmatrix}$ is

(1) $5x^2 + 6xy + 8y^2$ (2) $5x^2 - 6xy + 8y^2$

(3) $5x^2 - 3xy + 8y^2$ (4) $5x^2 + 3xy + 8y^2$

37. The orthonormal basis from the vectors (1, 1) and (2, 1) in $\mathbb{R}^2$ are

(1) $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{5}}\right), \left(\frac{-1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$

(2) $\left(\frac{-1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right), \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(3) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

(4) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), (1,1)$

38. A real quadratic form $Q(x) = X^T A X$ is negative definite if and only if

(1) all the eigenvalues of A are non-negative

(2) all the eigenvalues of A are negative

(3) all the eigenvalues of A are positive

(4) A has positive as well as negative eigenvalues

39. For what value of m, the vector (m, 3, 1) is a linear combination of vectors $e_1 = (3, 2, 1)$, $e_2 = (2, 1, 0)$.

(1) 5

(2) 3

(3) 2

(4) 1

40. Which of the following are Jordan canonical form J of a linear operator $T: V \to V$ whose characteristic polynomial $\Delta t = (t-2)^5$ and whose minimal polynomial $m(t) = (t-2)^2$?

(1) $J = \text{diag}\left(\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}, [2]\right)$

(2) $J = \text{diag}\left(\begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}, [3]\right)$

(3) $J = \text{diag}\left(\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, [1]\right)$

(4) $J = \text{diag}\left(\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}, [1]\right)$

41. Assume f has a pole of order m at $z = a$. Which one of the following is false?

(1) $(z-a)^m f(z)$ is bounded in a deleted neighbourhood of a

(2) $\lim\limits_{z \to a} (z-a)^m f(z) \neq 0$ & $\lim\limits_{z \to a} (z-a)^{m+1} f(z) = 0$

(3) $\frac{1}{f}$ has zero of order m

(4) There exists a holomorphic function g from U to $\mathbb{C}$ & a + ve integer m such that $f(z) = \frac{g(z)}{(z-a)^m}$ in a deleted disk centered at a

42. Let $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$. Then the value of $\int_\gamma \frac{e^z}{z^3} dz$ equals

(1) $2\pi i$
(2) πi
(3) 0
(4) 2π

43. The value of the integral

$$\frac{1}{2\pi i}\int_r \left(\sin\left(\cos z^2\right) + \left(z\bar{z}\right)^2 + \frac{1}{z} + \frac{z+1}{z-1} + \frac{\cos z}{z-4} \right) dz \text{ is}$$

(1) 3
(2) 0
(3) 5
(4) 6

44. The number of roots of the polynomial $2z^5 + 8z - 1$ has in the region $|z| \ge 2$ is

(1) 3
(2) 2
(3) 1
(4) 0

45. For which pair of paths γ & f: $[\gamma] \to \mathbb{C}$ given below in (I) – (IV), $\int_\gamma f = 0$?

(I) $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$ and $f(z) = \frac{e^z}{z^2 + 4}$

(II) $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$ and $f(z) = \frac{e^z - 1}{z}$

(III) $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$ and $f(z) = \frac{1}{z^2} - \frac{3}{z^4} + \frac{1}{z^{2025}}$

(IV) γ is any path closed & iying completely inside the half plane $\{z \in \mathbb{C}.\ \text{Re}\ z > 0\}$ and $f(z) = \frac{1}{z}$

(1) All of (I), (II), (III) & (IV)
(2) Only (I), (II) & (III)
(3) (I) & (II) only
(4) (II) only

46. Let $w \in \mathbb{C}$ and $A_\omega := \{z \in \mathbb{C} : \exp(z) = \omega\}$ pick out the correct statement.

(1) For every $\omega \in \mathbb{C}$, A_ω is infinite
(2) For every $\omega \in \mathbb{C}$, A_ω is finite
(3) For every $\omega \in \mathbb{C}^\alpha$, A_ω is finite
(4) For every $\omega \in \mathbb{C}^\alpha$, A_ω is infinite

47. The relation $x^2 \equiv x \bmod (1155)$ for $0 \leq x < 1155$ has how many solutions?

(1) 2
(2) 8
(3) 16
(4) 1155

48. In $\mathbb{Z}$ and $\mathbb{Z}[i]$, which of the following is correct?

(1) Integer 2 is irreducible in both $\mathbb{Z}$ and $\mathbb{Z}[i]$

(2) Integer 2 is reducible in $\mathbb{Z}$ but irreducible in $\mathbb{Z}[i]$

(3) Integer 2 is irreducible in $\mathbb{Z}$ but reducible in $\mathbb{Z}[i]$

(4) Integer 2 is reducible in both $\mathbb{Z}$ and $\mathbb{Z}[i]$

49. Let G denote the group of all automorphisms of the field $F_{3^{100}}$ that consists of 3^{100} elements. Then the number of distinct subgroups of G is equal to

(1) 4
(2) 100
(3) 3
(4) 9

50. Let $F \subseteq \mathbb{C}$ be the splitting field of $x^7 - 2$ over Q and $z = e^{2\pi i/7}$ a primitive 7^{th} root of unity. Let $[F: Q(z)] = a$ and $[F: Q\left(\sqrt[7]{2}\right)] = b$. Then which of the following is correct?

(1) $a = b = 7$
(2) $a > b$
(3) $a = b = 6$
(4) $a < b$

51. Let $\mathbb{C}$ be the field of complex numbers and $\mathbb{C}^*$ be the group of non-zero complex numbers under multiplication. Then which of the following is true?

(1) $\mathbb{C}^*$ is cyclic

(2) Every finite subgroup of $\mathbb{C}^*$ is cyclic

(3) $\mathbb{C}^*$ has finitely many finite subgroups

(4) Every proper subgroup of $\mathbb{C}^*$ is cyclic

52. Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to

(1) The cyclic group of order 6

(2) The permutation group on {1, 2, 3}

(3) The permutation group on {1, 2, 3, 4, 5, 6}

(4) The permutation group on {1}

53. In which of the following topologies on $\mathbb{R}$, the set $\mathbb{R}$ is connected?

a. Indiscrete topology

b. Co-countable topology

c. Cofinite topology

d. Usual metric topology

(1) Only a, c and d

(2) All of a, b, c and d

(3) Only c and d

(4) Only a and c

54. Let $L: = \{(x,0) : 0 < x \leq 1\}$

And $A_n : = \{(1/n, y) : 0 \leq y \leq 1\}$ for $n \in \mathbb{N}$

Let $P' = \{(0,1)\}$

Let $X = L \cup \left(\bigcup_{n \in \mathbb{N}} A_n\right) \cup \{P\}$

Consider the following statements

(I) X is connected

(II) X is path connected

Then,

(1) (II) is true therefore (I) is true

(2) (II) is true but (I) is not true

(3) (I) is false therefore (II) is also false

(4) (I) is true but (II) is not true

55. Let X be a topological space such that every continuous function $f: X \to \mathbb{R}$ is bounded.

Pick out the statement which is NOT true.

(1) If $f: X \to \mathbb{R}$ is continuous & $\lambda \notin f(x)$, then $g(x) = \dfrac{1}{\lambda - f(x)}$ is continuous

(2) Every continuous function $f: X \to \mathbb{R}$ attains its bounds

(3) There exists a continuous function $f: X \to \mathbb{R}$ which does not attain its supremum bound

(4) Every $f: X \to \mathbb{C}$ is bounded

56. Let y be the solution of $y' + y = |x + 1|\ \forall\ x \in \mathbb{R}$, $y(-2) = 0$, then y(2) equals to

(1) $2 + 2e^{-3} - 2e^{-4}$
(2) $2 - 2e^{-3} + 2e^{-4}$
(3) $2 + 2e^{3} - 2e^{4}$
(4) $-2 + 2e^{-3} - 2e^{-4}$

57. Let (x(t), y(t)) satisfy the system of ordinary differential equations

$$\frac{dx}{dt} = -x + ty$$

$$\frac{dy}{dt} = tx - y$$

If $(x_1(t), y_1(t))$ and $(x_2(t), y_2(t))$ are two solutions and $\phi(t) = x_1(t)\, y_2(t) - x_2(t)\, y_1(t)$ then $\frac{d\phi}{dt}$ is equal to

(1) -2ϕ
(2) 2ϕ
(3) $-\phi$
(4) ϕ

58. Let W be the Wronskian of two linearly independent solutions of $2y'' + y' + x^2 y = 0\ \forall\ x \in \mathbb{R}$. Then for all x, there exists a constant $C \in \mathbb{R}$ such that W(x) is

(1) Ce^{-x}
(2) $Ce^{-x/2}$
(3) Ce^{2x}
(4) Ce^{-2x}

59. The function $u(r, \theta)$ satisfying the Laplace equation $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \quad e < r < e^2$ subjected to the conditions $u(e, \theta) = 1$, $u(e^2, \theta) = 0$ is

(1) $\log(e/r)$
(2) $\log(e/r^2)$
(3) $\log(e^2/r)$
(4) $\sum_{n=1}^{\infty} \left(\frac{r - e^2}{e - e^2}\right) \sin n\theta$

60. Nature of the second order partial differential equation $3U_{xx} + U_{yy} + 4U_{yz} + 4U_{zz} = 0$ is

(1) Parabolic
(2) Hyperbolic
(3) Elliptic
(4) Circle

61. Let f: $[0, 2] \to R$ be a twice continuously differentiable function. If $\int_0^2 f(x)\, dx \approx 2f(1)$, then the error in the approximation is

(1) $\frac{f'(\xi)}{12}$ for some $\xi \in (0, 2)$
(2) $f'(\xi)/2$ for some $\xi \in (0, 2)$
(3) $f''(\xi)/3$ for some $\xi \in (0, 2)$
(4) $f''(\xi)/6$ for some $\xi \in (0, 2)$

62. Let $l_K(x)$, K = 0,1,2,.....,n denote the Lagrange's fundamental polynomials of degree n for the nodes $x_0, x_1, \ldots, x_n$, then the value of $\sum_{K=0}^{n} l_K(x)$ is

(1) 0
(2) 1
(3) $x^n + 1$
(4) $x^n - 1$

63. The number of roots of the equation $x^2 - \cos x = 0$ in the interval $[-\pi/2, \pi/2]$ is equal to

(1) 1
(2) 2
(3) 3
(4) Infinitely many

64. The order of linear multistep method $u_{j+1} = (1-a)\, u_j + a u_{j-1} + \frac{h}{4}\{(a+3)\, u'_{j+1} + (3a+1)\, u'_{j-1}\}$ for solving $u' = f(x,u)$ is

(1) 2 if a = –1
(2) 2 if a = 2
(3) 3 if a = –1
(4) 3 if a = –2

65. Suppose any points $P(x_0, y_0)$, $Q(x_1, y_1)$ and a function $F(x, y, y')$ of three independent variables are given where $y' \equiv \frac{dy}{dx}$. In order to find among all curves $y = y(x)$ joining P & Q that one which furnishes for the definite integral $I(y) = \int_{x_0}^{x_1} F(x, y(x), y'(x))\, dx$ the smallest value which of the following assumption suffices?

(1) F is of class C^1
(2) F is of class C^2 for all values x, y(x) & y'(x) furnished by all of the admissible functions
(3) F is of class C^3 for all values x, y(x) & y'(x) furnished by all of the admissible functions
(4) It is enough to treat y(x) and F to be of class C^1 only with respect to their arguments

66. The curve extremizing a functional $I(y) = \int_1^2 \frac{\sqrt{1+(y'(x))^2}}{x}\, dx$, $y(1) = 0$, $y(2) = 1$ is

(1) an ellipse
(2) a parabola
(3) a circle
(4) a straight line

67. The solution Volterra integral equation $U(x) = 1 + x + \int_0^x (x-t)\, u(t)\, dt$ is

(1) $U(x) = e^x$
(2) $U(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\infty$
(3) $U(x) = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \ldots\infty$
(4) $U(x) = \cos x$

68. The differential equation equivalent to the integral equation $y(x) = \frac{1}{2}x^2 + \int_0^x t(t-x)\,y(t)\,dt$ is

(1) $y'' - xy = 0,\ y(0) = 0,\ y'(0) = 0$

(2) $y'' + xy = 0,\ y(0) = 0,\ y'(0) = 0$

(3) $y'' + xy = 1,\ y(0) = 0,\ y'(0) = 0$

(4) $y'' + x^2y = 1,\ y(0) = 1,\ y'(0) = 0$

69. A rigid body having one point O fixed and no external torque about O has equal principal moments of inertia. Then the body must rotate with

(1) Angular velocity of variable magnitude

(2) Angular velocity with constant magnitude

(3) Constant regular momentum but varying angular velocity

(4) Varying angular momentum with varying angular velocity

70. A particle starts from the origin and the components of its velocity parallel to the axes of coordinates at time t are $2t + 3$ and $4t$. Then the path is

(1) $x^2 + y^2 - xy - y = 0$

(2) $2x^2 + y^2 - 2xy - 6y = 0$

(3) $x^2 - y^2 - xy - y = 0$

(4) $4x^2 + y^2 - 4xy - 18y = 0$

71. The average of n observations $x_1, x_2, \ldots, x_n$ is M. If x_1 is replaced by x^1, then the new average is

(1) $M - x_1 + x^1$

(2) $\frac{M - x_1 + x^1}{n}$

(3) $\frac{(n-1)\,M - x_1 + x^1}{n}$

(4) $\frac{nM - x_1 + x^1}{n}$

72. Sum of squares of the deviations is minimum when the deviations are taken from

(1) Mean

(2) Median

(3) Mode

(4) An arbitrary value

73. The probability density function corresponding to the characteristic function $\phi(t) = \frac{1}{1 + 4t^2}$ is

(1) e^{-2x} for $x > 0$, 0 elsewhere

(2) $\frac{1}{4} e^{-\frac{|x|}{2}}$ for $-\infty < x < \infty$

(3) $\frac{1}{2\pi}\frac{1}{(1 + x^2)}$ for $-\infty < x < \infty$

(4) $\frac{1}{\sqrt{2\pi}} e^{-\frac{(x-2)^2}{2}}$ for $-\infty < x < \infty$

74. Let X_1 and X_2 be independent, identically distributed continuous random variables with the probability density function,

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 < x < 1 \\ 0 & , \text{ otherwise} \end{cases}$$

Using Chebyschev's Inequality, the lower bound of $P(|x_1 + x_2 - 1| \leq \frac{1}{2})$ is

(1) $\frac{5}{6}$
(2) $\frac{4}{5}$
(3) $\frac{3}{5}$
(4) $\frac{1}{3}$

75. In testing of hypotheses for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$ $(\theta_1 > \theta_0)$ NP lemma is used to obtain _______

(1) Most powerful test procedure
(2) Power of test procedure
(3) Size of the test procedure
(4) Unbiased test procedure

76. Suppose A and B are events with $P(A) = 0.5$ and $P(B) = 0.4$ and $P(A \cap B^C) = 0.2$. Then $P(B^C | A \cup B)$ is equal to

(1) $\frac{1}{2}$
(2) $\frac{1}{3}$
(3) $\frac{1}{4}$
(4) 0

77. Let X be a discrete random variable with the moment generating function $M_x(t) = e^{0.5(e^t - 1)}$, $t \in \mathbb{R}$. Then $P(X \leq 1)$ equals

(1) $e^{\frac{1}{2}}$
(2) $\frac{3}{2} e^{-\frac{1}{2}}$
(3) $\frac{1}{2} e^{\frac{1}{2}}$
(4) $e^{-(e-1)/2}$

78. If arrivals are according to Poisson process, then distribution of interarrival time is

(1) Gamma
(2) Chi-square
(3) Exponential
(4) Normal

79. If $\{N_1(t)\}$ and $\{N_2(t)\}$ are two independent Poisson processes with rates λ_1 and λ_2 respectively, then $N_1(t) - N_2(t)$ is

(1) Poisson process with rate $\lambda_1 + \lambda_2$

(2) Poisson process with rate $\lambda_1 - \lambda_2$

(3) Poisson process with rate $\frac{\lambda_1}{\lambda_2}$

(4) Not a Poisson process

80. Let X denote the number of times heads occur in n tosses of a fair coin. If $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in Arithmetic Progression (A.P.), then the value of n is

(1) 7 or 14

(2) 5 or 10

(3) 6 or 12

(4) 11 or 22

81. Suppose that a book of 600 pages contains 40 printing mistakes. Assume that these errors are randomly distributed throughout the book and X, the number of errors per page has a Poisson distribution. What is the probability that 10 pages selected at random will be free of errors?

(1) $\frac{1}{3}e^{-1}$

(2) $2e^{-\frac{1}{3}}$

(3) $e^{-2/3}$

(4) $\frac{1}{3}e^{-2}$

82. Following are two statements about the estimator at a parameter θ of exponential family

(I) The maximum likelihood estimator of θ is CAN estimator of θ

(II) The moment estimator of θ based on a sufficient statistic is CAN estimator of θ

Which of the following statements is/are correct?

(1) Both (I) and (II) are TRUE

(2) Both (I) and (II) are FALSE

(3) (I) is TRUE but (II) is FALSE

(4) (I) is FALSE but (II) is TRUE

83. Let $x_1 = 1.1$, $x_2 = 0.5$, $x_3 = 1.4$ and $x_4 = 1.2$ be the observed values of a random sample of size four from a distribution with probability density function, $f(x \mid \theta) = \begin{cases} e^{\theta - x}, & \text{if } x \geq \theta, \theta \in (-\infty, \infty) \\ 0, & \text{otherwise} \end{cases}$

Then the maximum likelihood estimate of θ^2 is

(1) 0.5

(2) 0.25

(3) 1.21

(4) 1.44

84. Suppose $X_1, X_2, ..., X_n$ is a random sample from $U(-\theta, \theta)$ distribution. Then the maximum likelihood estimator of θ is

Let $X_{(K)}$ – K^{th} order statistics of $(X_1, X_2, ..., X_n)$, $K = 1, 2, ..., n$.

(1) $X_{(n)}$

(2) Max $\{|X_{(1)}|, |X_{(n)}|\}$

(3) $-X_{(n)}$

(4) Max $\{-X_{(1)}, X_{(n)}\}$

85. For a monotone sequence of r. V's $\{X_n\}$, $n \geq 1$, which of the following statement(s) is/are correct?

(1) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{q.s} X$

(2) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{q.m} X$

(3) $X_n \xrightarrow{L} X \Rightarrow X_n \xrightarrow{P} X$

(4) $X_n \xrightarrow{q.m} X \Rightarrow X_n \xrightarrow{q.s} X$

86. Let the correlation coefficient between two variables X and Y be unity. Then the relation between the regression coefficients β_{YX} and β_{XY} that always holds is

(1) $\beta_{YX} > \beta_{XY}$

(2) $\beta_{YX} < \beta_{XY}$

(3) $\beta_{YX} = \beta_{XY}$

(4) $\beta_{YX} \cdot \beta_{XY} = 1$

87. If two components connected in series, are connected to a third component in parallel, with respective lifetimes X_1, X_2 and X_3, what is the system lifetime?

(1) max. {min. $\{X_1, X_2\}$, X_3}

(2) min. {max. $\{X_1, X_2\}$, X_3}

(3) min. {min. $\{X_1, X_2\}$, X_3}

(4) max. {max. $\{X_1, X_2\}$, X_3}

88. If independent random variables X and Y have Poisson distribution with parameter 1, then which of the following is not true?

(1) $E(X + Y)^2 = 6$

(2) $E(X - Y)^2 = 2$

(3) $V(X + Y) = 2$

(4) $V(X + Y) = 4$

89. If X is a geometric random variable taking values 0, 1, 2... with parameter $\frac{1}{2}$, then which of the following is not true?

(1) Square of coefficient of variation of X is 2

(2) $E(X^2) = 3$

(3) Square of coefficient of variation of X is 4

(4) $V(X) = 2$

90. Given that random variable X and its reciprocal have the same distribution with possibly different parameters, which of the following can be the distribution of X?

(1) Normal (2) Cauchy

(3) F (4) t

91. Given that random variable X takes values 0 and 1 with $P(X = 1) = \frac{1}{2}$, match the following:

List-I	List-II
(a) E(X)	(i) 1
(b) Coefficient of variation of X	(ii) $\frac{3}{4}$
(c) $E(X^2) + \frac{E(X)}{2}$	(iii) $\frac{1}{4}$
(d) V(X)	(iv) $\frac{1}{2}$

(1) (a) → (iv), (b) → (iii), (c) → (i), (d) → (ii)

(2) (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)

(3) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

(4) (a) → (iv), (b) → (i), (c) → (ii), (d) → (iii)

92. If P(X) is the probability generating function of a random variable X taking values 0,1,2,..., then which of the following is not true?

(1) $P(0) < P(1)$ (2) $P(1) = 1$

(3) $P'(1) = E(X)$ (4) $P'(0) = E(X)$

93. In a CRD model with 5 treatments, which of the following gives the maximum number of linearly independent parametric functions that are estimable?

(1) 5 (2) 4

(3) 3 (4) 6

94. In a randomised block design with 5 treatments and 4 blocks, what is the maximum number of linear parametric functions of treatments alone that are estimable?

(1) 4 (2) 9

(3) 7 (4) 3

95. Given a BIBD with 5 treatments and 6 blocks, which of the following is the determinant of NN', N being the incidence matrix of the BIBD?

(1) 6 (2) 5

(3) 10 (4) 11

96. Given that random variable X has probability generating function $\frac{1}{3} + \frac{2}{3}s$, $0 \le s \le 1$, which of the following is not true?

(1) $P(X = 1) = E(X)$ (2) $P(X = 1) = E(X^2)$

(3) $P(X = 1) = 3V(X)$ (4) $P(X = 1) = \frac{1}{3}V(X)$

97. If characteristic function φ of random variable X is real valued, then which of the following is not true?

(1) X and –X have the same distribution

(2) Characteristic function of –X is the complex conjugate of φ

(3) φ minus characteristic function of –X is equal to zero

(4) X and –X do not have the same distribution

98. Which of the following is not true with reference to a 2^3 full factorial model?

(1) There are 7 factorial effects

(2) There are 4 factorial effects

(3) There are 3 main effects

(4) There are 3 two-factor interaction effects

99. The moment generating function of a random variable X is $M(t) = \frac{2}{5}e^{t} + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$. Then its mean and variance are respectively:

(1) 2 and 0.5 (2) 1 and 0.8

(3) 2 and 0.8 (4) 1 and 0.5

100. Test procedure based on which of the following is not consistent for testing $\theta = 0$ against $\theta \neq 0$, given a random sample of size n from Cauchy $(\theta, 1)$ distribution?

(1) Sample mean

(2) Sample median

(3) Sample first quartile

(4) Sample third quartile

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