MET 2008 Answer Key PDF

Physics

1. Surface tension (T) of a liquid is equal to the work (W) required to increase the surface area (ΔA) of the liquid film by unity at constant temperature.

$$W = T\Delta A = \text{surface energy}$$

Also, volume of big drop $= 27 \times$ volume of small drop

ie, $$V' = 27V$$

where V' is volume of big drop of diameter D and V the volume of small drop of diameter d.

$$\therefore \quad \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = 27 \times \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$\Rightarrow \quad \frac{D}{2} = 3 \times \frac{d}{2}$$

$$\Rightarrow \quad d = \frac{D}{3}$$

Radius of small drop, $r = \dfrac{d}{2} = \dfrac{D}{6}$.

$\therefore$ Change in surface energy $= T(A_2 - A_1)$

$$= T[27 \cdot 4\pi r^2 - 4\pi R^2]$$

$$= T\ 4\pi\left[27\left(\frac{D}{6}\right)^2 - \left(\frac{D}{2}\right)^2\right]$$

$$= 4\pi T\left[\frac{3D^2}{4} - \frac{D^2}{4}\right] = 2\pi D^2 T$$

2. From Kepler's third law of planetary motion. The square of the period of revolution (T) of any planet around the sun is directly proportional to the cube of the semi major axis (R) of its elliptical orbit.

ie, $$T^2 \propto R^3$$

Given, $$T_p = 27T_e,$$

$$\frac{T_e^2}{T_p^2} = \frac{R_e^3}{R_p^3}$$

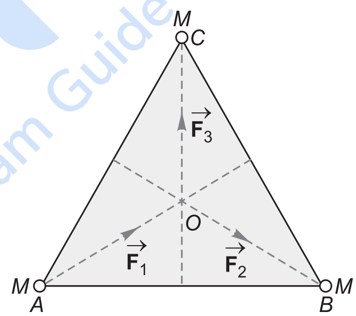
$$\frac{T_e^2}{(27T_e)^2} = \frac{R_e^3}{R_p^3}$$

$$\frac{R_p}{R_e} = (27)^{2/3}$$

$$\frac{R_p}{R_e} = 3^2$$

$$R_p = 9R_e$$

3. The net force acting on a unit mass placed at O due to three equal masses M at vertices A, B and C is the gravitational field intensity at point O. The gravitational force on the particle placed at the point of intersection of three medians.



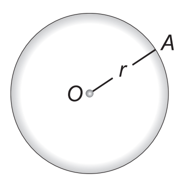
$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 = 0$$

Since, the resultant of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ is equal and opposite to $\vec{\mathbf{F}}_3$.

4. In a turbulent flow, the velocity of the liquid, in contact with the walls of the tube is equal to critical velocity.

5. Potential due to charge (q) at point (r) is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$



Since, charge Q is rotated in a circle of radius r, hence its potential remains same at all points on the path, hence $\Delta V = 0$.

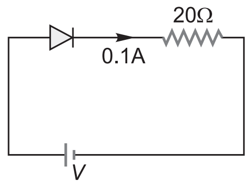
Also, work done $= q\Delta V$

where q is charge and $\Delta V = 0$.

$\therefore$ Work done $= 0$.

6. The circuit diagram of the given situation is shown. Since, it is a closed mesh, the voltage is

$$V = V' + IR$$



Given, $V' = 0.5$ volt, $I = 0.1$ A, $R = 20\,\Omega$

$\therefore \quad V = 0.5 + 0.1 \times 20$

$V = 2.5$ volt

7. A dipole placed in an external electric field is acted upon by a torque which tends to align the dipole in the direction of the field. Therefore, work must be done to change the orientation of the dipole against the torque.

If dipole be rotated from an initial orientation $\theta = \theta_1$ to final orientation $\theta = \theta_2$, the total work required is

$$W = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta$$

$$W = pE\,[-\cos\theta]_{\theta_1}^{\theta_2}$$

where p is dipole moment and E the electric field.

In first case,

$$W = pE\,(1 - \cos 60^\circ)$$

$$W = pE\left(1 - \frac{1}{2}\right) = \frac{pE}{2}$$

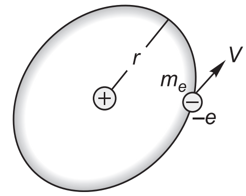
$\Rightarrow \quad pE = 2W$

In second case,

$$W_2 = pE\,(1 - \cos 180^\circ)$$

$$W_2 = 2W\,(1 + 1) = 4W$$

8. Magnetic moment or magnetic dipole moment is a measure of the strength of a magnetic source. It is given by



$$\mu = IA \qquad \text{...(i)}$$

where I is current and A the area.

The effective current (I) is $I = \dfrac{e}{T}$

where e is electron charge and T the time period.

Also, $\nu = \dfrac{1}{T}$ = frequency

and area $A = \pi r^2$, where r is radius of circular path.

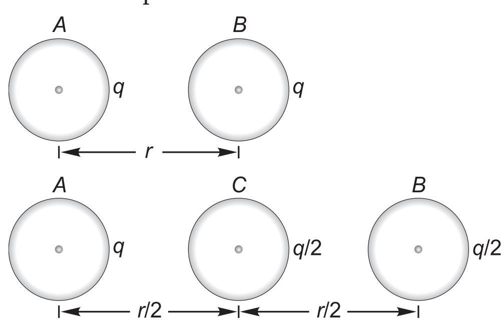
Putting these values in Eq. (i), we get

$$\mu = e\nu \,.\, \pi r^2$$

Note : Magnetic moment can also be expressed terms of angular momentum (L)

$\mu = IA = -\dfrac{2}{2M_e}$ L. where M_e is mass of electron.

9. From Coulomb's law, the force of attraction between two charged particles (q), kept at distance r apart is



$$F = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{r^2}$$

when two identical spheres are brought in contact, charge on them is equalised, hence total charge on C is equally shared when brought in contact with sphere B having a charge q. Therefore, charge on B and C is $\dfrac{q}{2}$.

From Coulomb's law, the force on C is

$$F_C = \frac{q \times q/2}{4\pi\varepsilon_0 (r/2)^2} - \frac{(q/2)(q/2)}{4\pi\varepsilon_0 (r/2)^2}$$

$$= \frac{qq}{4\pi\varepsilon_0 r^2}(2-1) = F$$

Note : The force will be opposite because A and B spheres will repel the third sphere.

10. The standard equation of wave is

$$y = a \sin(\omega t - kx) \quad \text{...(i)}$$

where a is amplitude, ω the angular velocity and x the displacement at instant t.

Given equation is

$$y = 0.1 \sin(100\pi t - kx) \quad \text{...(ii)}$$

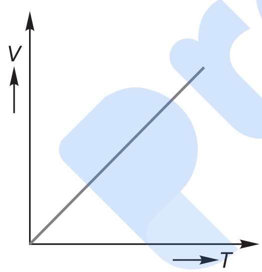
Comparing Eq. (i) with Eq. (ii), we get

$$\omega = 100\pi$$

$\therefore$ Wave number $= \dfrac{\omega}{v} = \dfrac{100\pi}{100} = \pi \text{ m}^{-1}$

11. At constant pressure, the volume of a given mass of a gas is directly proportional to its absolute temperature (T).

ie, $\dfrac{V}{T} = $ constant.



This is another form of Charles' law. Hence, variation of volume with temperature is as follows.

Hence, correct graph will be (c).

12. Optical rotation or optical activity is the rotation of linearly polarized light as it travels through certain materials. It occurs in solutions of chiral molecules (*eg*, sugar), solids with rotated crystal planes (*eg*, quartz) and spin polarized gases of atoms or molecules. Any linear polarization of light can be written as an equal combination of right hand (RHC) and left hand (LHC) circularly polarized light.

$$E_{\theta_D} = E_{\text{RHC}} + e^{i2\theta_D} E_{\text{LHC}}$$

where E is the electric field of light.

13. The average power or simply power is the average amount of work done or energy transferred per unit time.

The instantaneous power is then the limiting value of the average power as the time interval Δt approaches zero.

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

$$\therefore \quad W = \int P dt$$

Given, $\quad P = 3t^2 - 2t + 1$

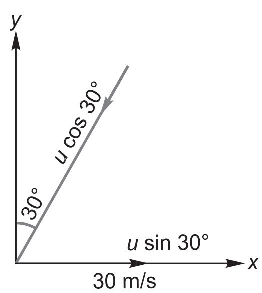
$$\therefore \quad W = \int_2^4 (3t^2 - 2t + 1)\, dt$$

Using $\int x^n\, dx = \dfrac{x^{n+1}}{n+1}$, we have

$$W = [t^3 - t^2 + t]_2^4 = 56 - 12 + 2$$

$$\Rightarrow \quad W = 46 \text{ J}$$

14. If a constant force $\vec{\mathbf{F}}$ is applied on a body for a short interval of time Δt, then the impulse of this force is $F \times \Delta t$.



Since, impulse = change in momentum (Δp)

$$\therefore \quad F \times \Delta t = \Delta p$$

$$\Rightarrow \quad F = \frac{\Delta p}{\Delta t}$$

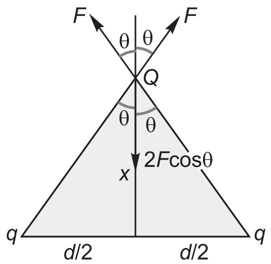
Change in x-direction

$$F = \frac{m\,[30 - (-15 \sin 30°)]}{0.01}$$

$$F = \frac{0.1 \times 37.5}{0.01} = 375 \text{ N}$$

Note : Unit of impulse is same as that of momentum.

15. From Coulomb's law, the force of attraction/repulsion acting between two stationary point charges (q_1, q_2) separated by a distance (r) is



$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Taking the net force, we have

$$F_{\text{net}} = 2F\cos\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{Qqx}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}\right]$$

For maximum,

$$\frac{dF_{\text{net}}}{dx} = 0$$

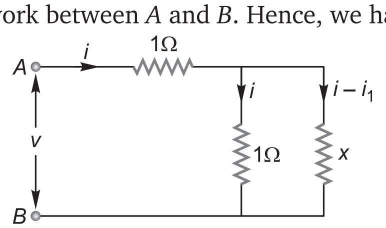
$$\therefore \quad \left(x^2 + \frac{d^2}{4}\right)^{3/2} - \frac{3}{2}x\left[x^2 + \frac{d^2}{4}\right]^{1/2}(2x) = 0$$

$$\therefore \quad \left(x^2 + \frac{d^2}{4}\right)^{1/2}\left(x^2 + \frac{d^2}{4} - 3x^2\right) = 0$$

$$\text{or} \quad 2x^2 = \frac{d^2}{4}$$

$$\Rightarrow \quad x = \frac{d}{2\sqrt{2}}$$

16. Let x be the equivalent resistance of entire network between A and B. Hence, we have



$R_{AB} = 1 +$ resistance of parallel combination of $1\,\Omega$ and $x\,\Omega$

$$\therefore \quad R_{AB} = 1 + \frac{x}{1+x}$$

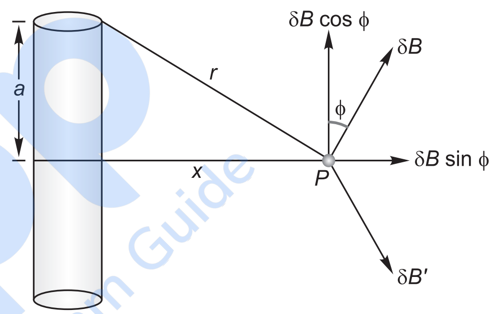
$$\therefore \quad x = 1 + \frac{x}{1+x}$$

$$\Rightarrow \quad x + x^2 = 1 + x + x$$

$$\Rightarrow \quad x^2 - x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1+\sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}\,\Omega$$

17. For a circular coil of radius a carrying a current i, the magnetic field at point P, distance x from coil is given by



$$B = \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}}\ \text{NA}^{-1}\ \text{m}^{-1} \qquad \text{...(i)}$$

At the centre of coil $x = 0$

$$\therefore \quad B' = \frac{\mu_0 i}{2a}\ \text{NA}^{-1}\ \text{m}^{-1} \qquad \text{...(ii)}$$

$$\text{Given,} \quad B = \frac{1}{8}B'$$

$$\therefore \quad \frac{\mu_0 i a^2}{2(a^2 + x^2)^{3/2}} = \frac{1}{8}\left(\frac{\mu_0 i}{2a}\right)$$

$$\Rightarrow \quad \frac{a^2}{(a^2 + x^2)^{3/2}} = \frac{1}{8a}$$

$$\Rightarrow \quad 8a^3 = (a^2 + x^2)^{3/2}$$

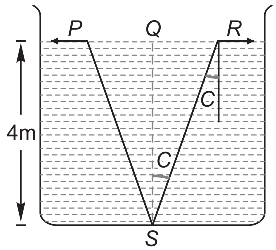
$$\Rightarrow \quad a^2 + x^2 = 4a^2$$

$$\Rightarrow \quad x = \sqrt{3}\cdot a$$

Given, $a = R$

$$\therefore \quad x = \sqrt{3}\,R$$

18. Let S be the light source. If light falls on the surface at critical angle C, it grazes along the surface as shown.



$$\sin C = \frac{1}{n} = \frac{3}{5}$$

From ΔQSR, we have

$$\tan C = \frac{QR}{QS} = \frac{r}{4}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{r}{4}$$

$$\Rightarrow \quad r = 3$$

Hence, diameter $= 2r = 2 \times 3 = 6$ m

19. Critical angle (θ_C) is the angle of incidence (i) in the denser medium for which the angle of refraction in the rarer medium is 90°. For total internal reflection to take place

$$i > \theta_C$$

Taking sine on both sides, we get

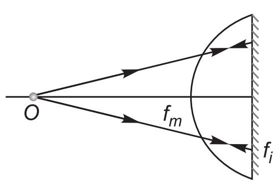
$$\sin i > \sin \theta_C$$

[as angle i at both faces will be 45°]

$$\Rightarrow \quad \frac{1}{\sqrt{2}} > \frac{1}{\mu}$$

$$\Rightarrow \quad \mu > \sqrt{2}$$

20. When an object is placed in front of such a lens, the rays are first of all refracted from the convex surface, then reflected from the polished plane surface and again refracted from convex surface.



Let f_l, f_m be focal lengths of convex surface and mirror (plane polished surface) respectively, then effective focal length is

$$\frac{1}{F} = \frac{1}{f_l} + \frac{1}{f_m} + \frac{1}{f_l} = \frac{2}{f_l} + \frac{1}{f_m}$$

Since, $f_m = \dfrac{R}{2} = \infty$

$$\therefore \quad \frac{1}{F} = \frac{2}{f_l}$$

From lens formula

$$\frac{1}{f_l} = (\mu - 1)\left(\frac{1}{R}\right)$$

$$\therefore \quad \frac{1}{F} = \frac{2(\mu - 1)}{R}$$

$$\Rightarrow \quad F = \frac{R}{2(\mu - 1)}$$

$$\text{or} \quad R_{eq} = 2F = \frac{R}{(\mu - 1)}$$

21. From Kirchhoff's second law

$$V = \Sigma ir \quad \text{(for closed mesh)}$$

where V is potential difference, i the current and r the resistance.

$$\therefore \quad E + E = Ir + Ir = 2Ir$$

$$\text{or} \quad I = \frac{E}{r} \quad \text{...(i)}$$

$$V_x - V_y = E - Ir$$

Putting the value of I from Eq. (i), we get

$$V_x - V_y = E - \frac{E}{r} \times V = 0.$$

22. Induced emf is given by

$$e = -\frac{d\phi}{dt}$$

If the radius of loop is r at a time t, then the instantaneous magnetic flux is given by

$$\phi = \pi r^2 B$$

$$\therefore \quad e = -\frac{d}{dt}(\pi r^2 B)$$

$$e = -\pi B\left(\frac{2r\,dr}{dt}\right)$$

$$e = -2\pi Br\frac{dr}{dt}$$

Numerically, $e = 2\pi Br\left(\dfrac{dr}{dt}\right)$

23. The minimum energy needed to ionise an atom is called ionisation energy. The potential difference through which an electron should be accelerated to acquire this much energy is called ionisation potential.

$$(E_2)_H - (E_1)_H = 10.2 \text{ eV}$$

or $$\frac{(E_1)_H}{4} - (E_1)_H = 10.2 \text{ eV}$$

$\therefore$ $$(E_1)_H = -13.6 \text{ eV}$$

Hence, ionisation potential energy is

$$= (E_\infty)_H - (E_1)_H = 13.6 \text{ eV}$$

$\therefore$ Ionisational potential $= 13.6$ V

24. Since, train (source) is moving towards pedestrain (observer), the perceived frequency will be higher than the original.

$$f' = f\left(\frac{v + v_o}{v - v_s}\right)$$

Here, $v_o = 0$ (as observer is stationary)

$v_s = 25$ m/s (velocity of source)

$v = 350$ m/s (velocity of sound)

and $f = 1$ kHz (original frequency)

Hence, $$f' = 1000\left(\frac{350 + 0}{350 - 25}\right)$$

$$= 1000 \times \frac{350}{325}$$

$$= 1077 \text{ Hz}$$

25. For two coherent sources, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

For maximum intensity, $\cos \phi = +1$

$\therefore$ $$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$= (\sqrt{I_1} + \sqrt{I_2})^2$$

For minimum intensity, $\cos \phi = -1$

$\therefore$ $$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$= (\sqrt{I_1} - \sqrt{I_2})^2$$

Hence, $$I_{\max} = (\sqrt{9I} + \sqrt{I})^2$$

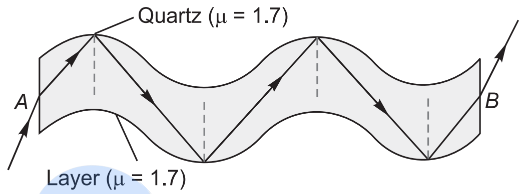
$$= (3\sqrt{I} + \sqrt{I})^2 = 16I$$

and $$I_{\min} = (\sqrt{9I} - \sqrt{I})^2$$

$$= (3\sqrt{I} - \sqrt{I})^2 = 4I$$

Note : In an interference pattern, maximum intensity is obtained for constructive interference and minimum intensity is obtained for destructive interference.

26. An optical fibre is a device based on total internal reflection by which a light signal can be transferred from one place to the other with a negligible loss of energy.



It consists of a very long and thin fibre of quartz glass (refractive index $= 1.7$). Each fibre is of thickness nearly 10^{-4} cm. The fibre is coated all around by a thin layer of a material (refractive index $= 1.5$) rarer than the quartz glass.

When a light ray is incident at one end A of the fibre making a small angle of incidence. It suffers refraction from air to quartz and strikes the fibre-layer interface at an angle of incidence greater than the critical angle. It therefore, suffers total internal reflection and strikes its opposite interface. At this interface, it again suffers total internal reflection. Thus, the ray reaches the other end B of the fibre.

27. Bernoulli's theorem for unit mass of liquid

$$\frac{p}{\rho} + \frac{1}{2}v^2 = \text{constant}$$

As the liquid starts flowing, it pressure energy decreases

$$\frac{1}{2}v^2 = \frac{p_1 - p_2}{\rho} \Rightarrow \frac{1}{2}v^2 = \frac{3.5 \times 10^5 - 3 \times 10^5}{10^3}$$

$$\Rightarrow v^2 = \frac{2 \times 0.5 \times 10^5}{10^3} \Rightarrow v^2 = 100$$

$$\Rightarrow \quad v = 10 \text{ m/s}$$

28. $$a_1 v_1 = a_2 v_2$$

$$\Rightarrow \quad 4.20 \times 5.18 = 7.60 \times v_2$$

$$\Rightarrow \quad v_2 = 2.86 \text{ m/s}$$

29. de-Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{mv} \quad ...(i)$$

where h is Planck's constant.

If kinetic energy of particle of mass m is v, then

$$K = \frac{1}{2} mv^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2K}{m}} \quad ...(ii)$$

Combining Eqs. (i) and (ii), we get

$$\lambda = \frac{h}{m\sqrt{\frac{2K}{m}}} = \frac{h}{\sqrt{2mK}} \quad ...(iii)$$

Given : $m = 9.1 \times 10^{-31}$ kg,

$K = 10 \text{ keV} = 10 \times 10^3 \times 1.6 \times 10^{-19}$ J

$h = 6.6 \times 10^{-34}$ J-s

Substituting the above values in Eq. (iii), we get

$$\lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$= 1.22 \times 10^{-11}$$

$$\approx 0.12 \text{ Å}$$

Note : If an electron is accelerated through a potential difference of V volt, then Eq. (iii) takes the form

$$\lambda = \frac{h}{\sqrt{2meV}}$$

After putting the numerical values for electrons, we get

$$\lambda = \sqrt{\frac{150}{V}} \text{ Å}$$

30. The given equations of waves be written as

$$y_1 = 0.25 \sin (310t) \quad ...(i)$$

and $\quad y_2 = 0.25 \sin (316t) \quad ...(ii)$

Comparing Eqs. (i) and (ii) with the standard wave equation, written as

$$y = a \sin (\omega t) \quad ...(iii)$$

We have,

$$\omega_1 = 310$$

$$\Rightarrow \quad f_1 = \frac{310}{2\pi} \text{ unit}$$

and $\quad \omega_2 = 316$

$$\Rightarrow \quad f_2 = \frac{316}{2\pi} \text{ unit}$$

Hence, beat frequency

$$= f_2 - f_1$$

$$= \frac{316}{2\pi} - \frac{310}{2\pi}$$

$$= \frac{3}{\pi} \text{ unit}$$

31. The activity or decay rate R of a radioactive substance is the number of decays per second.

$$\therefore \quad R = \lambda N$$

$$\text{or} \quad R = \lambda N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$\text{or} \quad R = R_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

where $R_0 = \lambda N_0$ is the activity of the radioactive substance at time $t = 0$.

According to question

$$\frac{R}{R_0} = 1 - \frac{75}{100} = 25\%$$

$$\therefore \quad \frac{25}{100} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

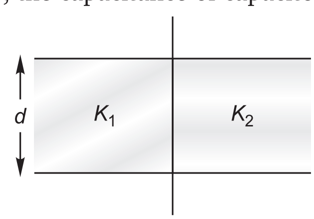
$$\text{or} \quad \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$$\text{or} \quad \frac{t}{T_{1/2}} = 2$$

$$\therefore \quad t = 2T_{1/2} = 2 \times 3.20 = 6.40 \text{ h}$$

$$\text{or} \quad t \approx 6.38 \text{ h}$$

32. Initially, the capacitance of capacitor



$$C = \frac{\varepsilon_0 A}{d}$$

$$\therefore \qquad \frac{\varepsilon_0 A}{d} = 1\,\mu F \qquad ...(i)$$

When it is filled with dielectrics of dielectric constants K_1 and K_2 as shown, then there are two capacitors connected in parallel. So,

$$C' = \frac{K_1 \varepsilon_0 (A/2)}{d} + \frac{K_2 \varepsilon_0 (A/2)}{d}$$

(as area becomes half)

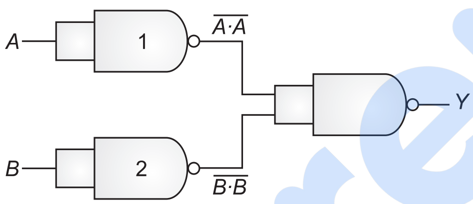
$$C' = \frac{4\varepsilon_0 A}{2d} + \frac{6\varepsilon_0 A}{2d}$$

$$= \frac{2\varepsilon_0 A}{d} + \frac{3\varepsilon_0 A}{d}$$

Using Eq. (i), we obtain

$$C' = 2 \times 1 + 3 \times 1 = 5\,\mu F$$

33. The given combination can be represented as



All the gates in the combination are NAND gates.

Output of gate-1, $Y_1 = \overline{A \cdot A}$

Output of gate-2, $Y_2 = \overline{B \cdot B}$

Output of gate-3, $Y = \overline{Y_1 \cdot Y_2}$

$$\therefore \qquad Y = \overline{\overline{A \cdot A} \cdot \overline{B \cdot B}} \qquad ...(i)$$

By Demorgan's theorem, we have

$$\overline{A \cdot A} = \overline{A}$$

and

$$\overline{B \cdot B} = \overline{B}$$

Therefore, Eq. (i) becomes,

$$Y = \overline{\overline{A} \cdot \overline{B}}$$

Again from Demorgan's theorem

$$\overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}}$$

$$\therefore \qquad Y = \overline{\overline{A}} + \overline{\overline{B}} = A + B$$

(as $\overline{\overline{A}} = A$)

Hence, the combination behaves as OR gate.

34. In a junction transistor, both the electron and hole play role, hence they are called the bi-polar devices or the bi-polar transistors and they are abbreviated as BJT in short form.

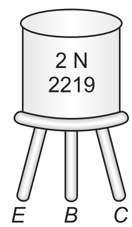
There are three parts in a transistor; namely emitter, base and collector. In the emitter part of the transistor the doping is more and it is less in collector part.

The doping is very less in the base part. So, emitter current is actually the sum of base and collector current.

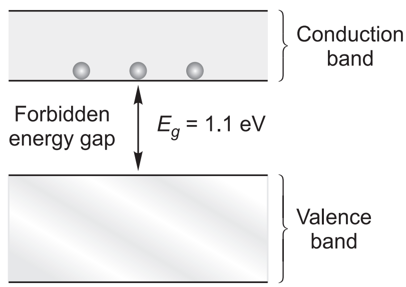
ie, $\qquad I_E = I_B + I_C$

Hence, maximum current flows in emitter region.

Note : Figure shown, represents the transistor available in the market. For identifying the electrodes, the collector terminal is generally kept far separated as compared to the remaining two electrodes or a red dot is etched near the collector terminal.



35. The energy band scheme of semiconductors is shown here.



In semiconductors, valence band and conduction band are separated by an energy gap called the forbidden energy gap. It is very small. At room temperature some electrons in valence band acquire thermal energy. This energy is more than forbidden energy gap E_g, thus they jump into the conduction band and leaves their vacancy in the valence band which act as holes.

Hence, at room temperature valence band is partially empty and conduction band is partially filled.

36. When coil is open, there is no current in it, hence no flux associated with it, *ie*, $\phi = 0$.

Also, we know that flux linked with the coil is directly proportional to the current in the coil, *ie*, $\phi \propto i$

or $\qquad \phi = Li$

where L is proportionality constant known as self-inductance.

$$\therefore \qquad L = \frac{\phi}{i} = 0$$

Again since $i = 0$, hence, $R = \infty$.

37. If a current i is passed in the circuit and it is changed with a rate $\frac{di}{dt}$, the induced emf e produced in the circuit is directly proportional to the rate of change of current.

Thus,

$$e \propto \frac{di}{dt}$$

When the proportionality constant is removed, the constant L comes here.

$$\therefore \qquad e = -L\frac{di}{dt} \qquad \text{...(i)}$$

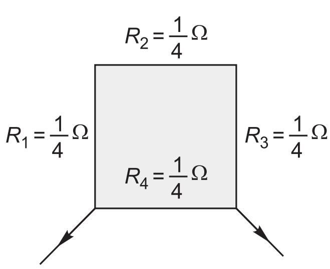
The minus sign here is a reflection of Lenz's law.

Here, $\quad di = (2 - 10)\text{A} = -8\text{A}, \quad dt = 0.1$ s, $e = 3.28$ V.

$$\therefore \qquad 3.28 = -\frac{L(-8)}{0.1}$$

$$\therefore \qquad L = \frac{3.28 \times 0.1}{8} = 0.04 \text{ H}$$

38. When rod is bent in the form of square, then each side has resistance of $\frac{1}{4}\Omega$. As shown R_1, R_2 and R_3 are connected in series, so their equivalent resistance



$$R' = R_1 + R_2 + R_3$$
$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}\Omega$$

Now, R' and R_4 are connected in parallel, so equivalent resistance of the circuit is

$$R = \frac{R' \times R_4}{R' + R_4}$$
$$= \frac{(3/4)(1/4)}{(3/4) + (1/4)}$$
$$= \frac{(3/16)}{1} = \frac{3}{16}\Omega$$

39. $\tan\phi = \dfrac{\omega L - \dfrac{1}{\omega C}}{R}$

ϕ being the angle by which the current leads the voltage.

Given, $\qquad \phi = 45°$

$$\therefore \qquad \tan 45° = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\Rightarrow \qquad 1 = \frac{\omega L - \frac{1}{\omega C}}{R}$$

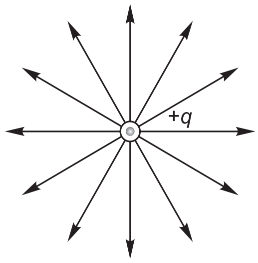
$$\Rightarrow \qquad R = \omega L - \frac{1}{\omega C}$$

$$\Rightarrow \qquad C = \frac{1}{\omega(\omega L - R)}$$

$$= \frac{1}{2\pi f(2\pi f L - R)}$$

Note : In series resonant L-C-R circuit, $\frac{1}{\omega CR}$ is greater than unity.

40. The imaginary surface joining the points of same potential in an electric field is called the equipotential surface, *ie*, the potential difference between any two points on an equipotential surface is zero. Hence, if a charge is moved on an equipotential surface from one point to the other, no work is needed to be done. But this is possible only when the charge is moved perpendicular to the electric field (*ie*, perpendicular to the lines of force). It means that the electric lines of force at each point of an equipotential surface are normal to the surface.



Hence, the angle between electric field and equipotential surface is 90°.

Alternative : Potential gradient along equipotential surface is zero.

ie, $$E\cos\theta = -\frac{dV}{dr} = 0$$

$\therefore$ $$\theta = 90^\circ$$

41. According to Newton's second law of motion, force acting on a body is equal to the rate of change of momentum during impact.

$$F = \frac{\Delta p}{\Delta t}$$

Also, $$F = ma$$

$\therefore$ $$ma = \frac{p_2 - p_1}{\Delta t}$$

or $$a = \frac{mv_2 - (-mv_1)}{m\,\Delta t}$$

or $$a = \frac{v_2 + v_1}{\Delta t}$$

$\therefore$ $$a = \frac{\sqrt{2\times 10\times 20} + \sqrt{2\times 10\times 5}}{0.02}$$

or $$a = \frac{20+10}{0.02} = 1500\ \text{m/s}^2$$

42. Volume remains constant after coalescing. Thus,

$$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3$$

where R is radius of bigger drop and r is radius of each smaller drop.

$\therefore$ $$R = 2^{1/3} r$$

Now, surface energy per unit surface area is the surface tension.

So, surface energy, $W = T\,\Delta A$

or $$W = 4\pi R^2 T$$

Therefore, surface energy of bigger drop

$$W_1 = 4\pi(2^{1/3} r)^2 T$$
$$= (2^{2/3})4\pi r^2 T$$

Surface energy of smaller drop

$$W_2 = 4\pi r^2 T$$

Hence, required ratio

$$\frac{W_1}{W_2} = 2^{2/3} : 1$$

43. In fission process, when a parent nucleus breaks into daughter products, then some mass is lost in the form of energy. Thus, mass of fission products < mass of parent nucleus

$$\Rightarrow \frac{\text{Mass of fission products}}{\text{Mass of parent nucleus}} < 1$$

44. Width of central maximum is given by

$$w = \frac{2f\lambda}{a} \quad \text{...(i)}$$

where f is focal length of lens, a is width of slit and λ is wavelength of light used.

From Eq. (i), it is clear that fringe width

$$w \propto \lambda$$

So, when blue light is used in the experiment instead of red light, the fringes will become narrower.

45. If object in a denser medium is seen from a rarer medium then image of object will appear at a lesser distance. The distance between object and its image, called as normal shift is given by

$$x = t\left[1 - \frac{1}{\mu}\right]$$

Here, $t = 6$ cm, $\mu = 1.5$

$\therefore$ $$x = 6\left[1 - \frac{1}{1.5}\right]$$
$$= 6\left[\frac{0.5}{1.5}\right]$$
$$= 2\ \text{cm}$$

46. When plates of capacitor are separated by a dielectric medium of dielectric constant K, its capacity

$$C_m = \frac{K\varepsilon_0 A}{d} = KC_0$$

ie, $C_m = KC_0$

Here, $C_0 = C$

$\therefore$ $C_m = KC$

Now, two capacitors of capacities KC and C are in series, their effective capacitance

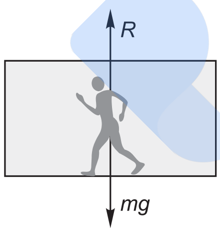
$$\frac{1}{C'} = \frac{1}{KC} + \frac{1}{C}$$

or $$\frac{1}{C'} = \frac{1+K}{KC}$$

$\therefore$ $$C' = \frac{KC}{K+1}$$

47. When a person of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the person is mg.

This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. This reaction exerted by the surface of contact on the person is the apparent weight of the person.



When lift is accelerating upwards at the rate of a, then reaction

$$R - mg = ma$$

$\therefore$ $$R = m(g+a)$$

Hence, apparent weight is greater than actual weight.

48. Due to high resistance of voltmeter, connected in series, the effective resistance of circuit will increase and hence the current in circuit will decrease. Due to which the ammeter and voltmeter will not be damaged.

49. $T = 100\,\mu s = 10^{-4}$ s

$$\lambda = \frac{0.6931}{T} = \frac{0.6931}{10^{-4}}$$

$$= 0.6931 \times 10^4 \text{ s}^{-1}$$

Number of atoms in 215 mg

$$= \frac{6.023 \times 10^{23}}{215} \times 215 \times 10^{-3}$$

$\therefore$ $N = 6.023 \times 10^{20}$

Activity, $$\frac{dN}{dt} = \lambda N$$

$$= 0.6931 \times 10^4 \times 6.023 \times 10^{20}$$

$$= 4.17 \times 10^{24} \text{ Bq}$$

50. For maximum wavelength of Balmer series

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{R \times 5}{36} \quad \text{...(i)}$$

For minimum wavelength of Balmer series,

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R}{4} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$\therefore$ $$\frac{\lambda_{min}}{\lambda_{max}} = \frac{R \times 5}{36} \times \frac{4}{R} = \frac{5}{9}$$

51. When the ball is released from the top of tower then ratio of distances covered by the ball in first, second and third second

$h_I : h_{II} : h_{III} = 1 : 3 : 5$ [because $h_n \propto (2n-1)$]

$\therefore$ Ratio of work done

$$mgh_I : mgh_{II} : mgh_{III} = 1 : 3 : 5$$

52. $$m_1 v_1 - m_2 v_2 = (m_1 + m_2)v$$

$\Rightarrow$ $2 \times 3 - 1 \times 4 = (2+1)v$

$\Rightarrow$ $$v = \frac{2}{3} \text{ m/s}$$

53. Total energy radiated from a body

$$Q = A\varepsilon\sigma T^4 t$$

$\Rightarrow$ $$Q \propto AT^4 \propto r^2 T^4 \quad (\because A = 4\pi r^2)$$

$\Rightarrow$ $$\frac{Q_P}{Q_Q} = \left(\frac{r_P}{r_Q}\right)^2 \left(\frac{T_P}{T_Q}\right)^4$$

$$= \left(\frac{8}{2}\right)^2 \left[\frac{(273+127)}{(273+527)}\right]^4 = 1$$

54. According to Newton's law of cooling t_1 will be less than t_2.

55. If an electron and a photon propagates in the form of waves having the same wavelength, it implies that they have same momentum. This is according to de-Broglie equation

$$p \propto \frac{1}{\lambda}$$

56. $\lambda_0 = \frac{hc}{W_0} = \frac{12400}{4}$

$= 3100$ Å

$= 310$ nm

57. Threshold wavelength

$$\lambda_0 = \frac{12375}{2.1} = 5892.8 \text{ Å}$$

58. Electric flux

$$\phi = \frac{\text{pulse power}}{\text{area}} = \frac{10^{12}}{10^{-4}} = 10^{16} \text{ W/cm}^2$$

59. A laser device produces amplification in the ultraviolet or visible region.

60. $\frac{Q}{t} \propto \frac{r^2}{l}$, from the given options,

option (b) has higher value of $\frac{r^2}{l}$.

Chemistry

1. Molecules or ions having no unpaired electrons are diamagnetic, *eg*,

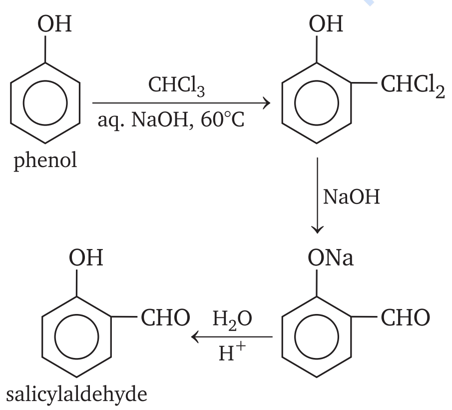
$$Li_2 = 6e^- = \sigma 1s^2, \overset{*}{\sigma} 1s^2, \sigma 2s^2$$

2. NO_2^- can participate in linkage isomerism because it may be bonded to metal either through nitrogen or through oxygen.

$[(NH_3)_5CoNO_2]Cl_2$
pentaaminenitro cobalt (III) chloride

and $[(NH_3)_5CoONO]Cl_2$
pentaaminenitrito cobalt (III) chloride

3. Reimer-Tiemann reaction : In this reaction phenol reacts with chloroform and alkali to form salicylaldehyde.



4. Benzoic acid in benzene exist as a dimer. So, number of molecules decreases and hence, osmotic pressure decreases.

5. Mohr's salt is $(NH_4)_2SO_4 \cdot FeSO_4 \cdot 6H_2O$.

The equation is

$$5Fe^{2+} + MnO_4^- + 8H^+ \longrightarrow 5Fe^{3+} + Mn^{2+} + 4H_2O$$

Total change in oxidation number of iron

$$= (+3) - (+2) \quad = +1$$

So, equivalent wt. of Mohr's salt

$$= \frac{\text{mol. wt. of Mohr's salt}}{\text{change in oxidation number}}$$

$$= \frac{392}{1} = 392$$

6. A salt is precipitated only when the product of ionic concentration is more than its solubility product.

$$K_{sp} = 1 \times 10^{-8}$$

$$[A^+] = 10^{-3} \text{ M}$$

$$[B^-] = \frac{1 \times 10^{-8}}{10^{-3}} = 10^{-5} \text{ M}$$

So, *AB* will be precipitated only when the concentration of B^- is more than 10^{-5} M.

7. Weight of 11.2 dm^3 of CO_2 gas at STP

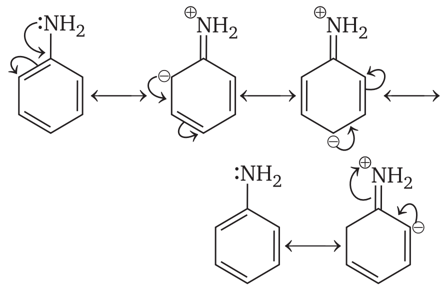
$$= 44 / 2 = 22 \text{ g}$$

$$\underset{56\,g}{KOH} + \underset{44\,g}{CO_2} \longrightarrow KHCO_3$$

$\because$ KOH required for complete neutralisation of CO_2 (44 g) = 56 g

$\therefore$ KOH required for neutralisation of 22 g of $CO_2 = \frac{56}{44} \times 22 = 28$ g

8. —NH_2 has $+R$ effect, it donates electrons to the benzene ring. As a result, the lone pair of electrons on the N-atom get delocalized over the benzene ring and thus less readily available for protonation. Hence, aniline is a weaker base than cyclohexylamine.



resonance structures of aniline

9. Fats are also known as triglycerides. These triglycerides are the triesters of fatty acid with glycerol. So, the characteristic feature of fat is ester group.

10. Compounds having asymmetric C-atom ($\overset{*}{C}$) is optically active, *eg*,

$$CH_3-CH_2-\overset{\overset{\large H}{|}}{\underset{\underset{\large Br}{|}}{{}^{*}C}}-CHO$$

The C-atom whose four valencies are satisfied by four different groups is asymmetric C-atom.

11. In cyclopropane the angle strain is maximum. Hence, it is highly strained molecule and consequently most unstable. The angle strain in cyclobutane is less than cyclopropane. Hence, cyclobutane is more stable. This stability increases upto 6 membered rings, then decreases from 7 to 11 membered rings and from the 12 membered rings onwards attain the stability of 6 membered ring. Heat of combustion is a method of measuring chemical stability. Hence, cyclohexane has the lowest heat of combustion.

12. The physical states of dispersing phase and dispersion medium in colloid like pesticide spray are liquid and gas respectively.

13. Potassium dichromate is an oxidising agent it oxidises ferrous ions into ferric ions in acid medium.

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$$

$$[Fe^{2+} \longrightarrow Fe^{3+} + e^-] \times 6$$

$$Cr_2O_7^{2-} + 6Fe^{2+} + 14H^+ \longrightarrow 6Fe^{3+} + 2Cr^{3+} + 7H_2O$$

14. Sucrose is obtained from cane sugar. On hydrolysis, it gives equimolar mixture of D (+) glucose and D (–) fructose.

$$\text{Sucrose} \xrightarrow[\text{or invertase}]{H^+} \text{D (+) glucose} + \text{D (–)fructose}$$

In sucrose, glucose is present in the pyranose form and fructose in the furanose form. Further since $C_1-\alpha$ of glucose is connected to $C_2-\beta$ of fructose, therefore sucrose is a non-reducing sugar.

15. During inductive effect shifting of σ electrons takes place due to which partial charges are developed on the atoms.

$$\overset{+\delta''}{C}-\overset{+\delta'}{C}-\overset{+\delta}{C}-\overset{-\delta}{Cl}$$

16. Electronic configuration of

$$_{28}Ni = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^8, 4s^2$$

$$Ni^{2+} = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^8, 4s^0$$

$$_{29}Cu = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^1$$

$$Cu^+ = 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^0$$

So, the given configuration is of Cu^+.

17. (i) Electronic configuration of Co^{3+} ion

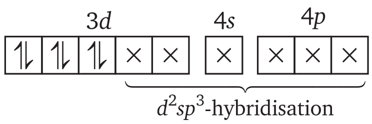
$$= [Ar] \overset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}$$

$$\underset{\text{ion}}{[CoF_6]^{3-}} \overset{3d}{\boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}$$

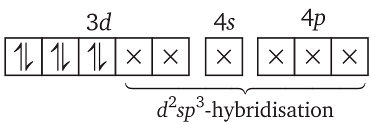
$$\underbrace{\overset{4s}{\boxed{\times}}\ \overset{4p}{\boxed{\times}\boxed{\times}\boxed{\times}}\ \boxed{\times}\boxed{\times}}_{sp^3d^2\text{-hybridisation}}\overset{4d}{\boxed{}\boxed{}\boxed{}}$$

F^- is a weak ligand. It cannot pair up electrons within d-subshell and forms outer orbital octahedral complex.

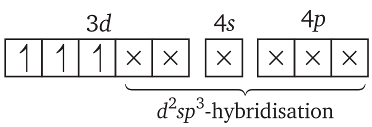
(ii) $[Co(NH_3)_6]^{3+}$ ion



(iii) $[Fe(CN)_6]^{3-}$ ion



(iv) $[Cr(NH_3)_6]^{3+}$ ion



NH_3 and CN^- are strong ligands. So, they form inner orbital complex.

18. The first ionisation energy of xenon is quite close to that of oxygen and the molecular diameter of xenon and oxygen are almost identical.

Based on the above facts it is suggested that as oxygen combines with PtF_6, so xenon should also form similar compound with PtF_6.

19. We know that,

$$pV = nRT$$

or $$pV = \frac{w}{M} RT$$

or $$M = \frac{w}{V} \frac{RT}{p}$$

or $$M = d \frac{RT}{p}$$

$$d = 1.964 \text{ g/dm}^3 = 1.964 \times 10^{-3} \text{ g / cc.}$$

$$p = 76 \text{ cm} = 1 \text{ atm}$$

$$R = 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1}$$

$$= 82.1 \text{ cc atm K}^{-1} \text{ mol}^{-1}$$

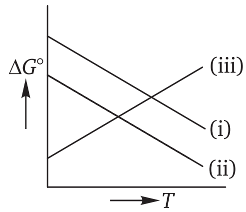
$$T = 273 \text{ K}$$

$$M = \frac{1.964 \times 10^{-3} \times 82.1 \times 273}{1} = 44.$$

The molecular weight of CO_2 is 44.

So, the gas is CO_2.

20.



Reaction (i) $M(s) + \frac{1}{2}O_2 \longrightarrow MO(s)$

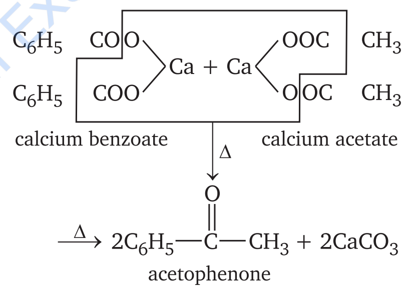
[where, M = highly reactive metal]

Reaction (ii) $CO(g) + \frac{1}{2}O_2 \longrightarrow CO_2(g)$

Reaction (iii) $C(s) + \frac{1}{2}O_2(g) \longrightarrow CO_2(g)$

ΔG° *vs* T plot in the Ellingham's diagram slopes downwards for the reaction (I) and reaction (II). For reaction (III) ΔG° *vs* T in the diagram slopes upwards.

21.



22. According to electron sea model, in a metallic crystal a lattice of positive ions, called kernels, remains immersed in a sea of mobile valence electrons. These electrons move freely within the boundaries of metallic crystal.

23. The electronic configuration of peroxide ion (O_2^{2-}) is as :

$$O_2^{2-} : KK\, (\sigma 2s)^2 (\overset{*}{\sigma} 2s)^2 (\sigma 2p_z)^2\, (\pi 2p_x)^2$$
$$(\pi 2p_y)^2 (\pi^* 2p_x)^2 (\pi^* 2p_y)^2$$

$$\text{Bond order} = \frac{8-6}{2} = 1$$

Since, all the electrons in molecular orbitals are paired, it is diamagnetic.

24. Insulin regulates the metabolism of carbohydrates (glucose).

25. Flocculation value $\propto \dfrac{1}{\text{coagulating power}}$

$Fe(OH)_3$ is a positively charged sol.

To coagulate $Fe(OH)_3$, negatively charged electrolyte is used and greater the value of negative charge, coagulating power will be strong. Among the given electrolytes, NaCl has lowest coagulating power, so its flocculation value will be maximum.

26.
$$k = \frac{2.303}{t} \log \frac{A_0}{A}$$
$$= \frac{2.303}{40} \log \frac{0.1}{0.005}$$
$$= \frac{2.303}{40} \log 20 = 0.075$$

Rate of reaction when concentration of X is 0.01 M will be

$0.075 \times 0.01 = 7.5 \times 10^{-4}$ M min^{-1}

27. At pH = 4, an amphoteric Zwitter ion structure changes into cation when an acid is added to it.

$$R - \underset{\underset{\oplus NH_3}{|}}{CH} - COO^- \xrightarrow{+H^+} R - \underset{\underset{\oplus NH_3}{|}}{CH} - COOH$$

28. Bond breaking processes or decomposition processes are endothermic process.

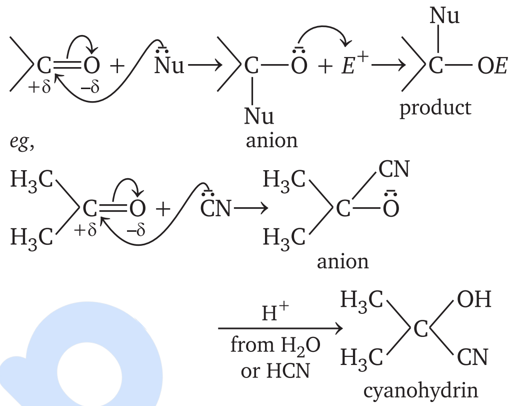
29. $E^\circ_{Cu^{2+}/Cu} - E^\circ_{Ag^+/Ag} = -0.46$ V ...(i)

$E^\circ_{Cu^{2+}/Cu} - E^\circ_{Zn^{2+}/Zn} = 1.10$ V ...(ii)

On subtracting Eq (i) from equation (ii), we get

$E^\circ_{Ag^+/Ag} - E^\circ_{Zn^{2+}/Zn} = +1.56$ V

30. Ketone undergoes nucleophilic addition reaction because if nucleophiles attack first followed by electrophiles, anion is formed as intermediate and if electrophiles attack first followed by nucleophiles, cation, formed as intermediate. The stability of anion is more than cation, so nucleophiles attack first followed by electrophiles.



So, probable mechanism will be

31. $CH_3COOH \xrightarrow{NH_3} \underset{(A)}{CH_3 - \overset{\overset{O}{||}}{C} - \overset{-}{O}\overset{\oplus}{N}H_4}$

$\xrightarrow[-H_2O]{\text{Heat}} \underset{\text{acetamide}}{\underset{(B)}{CH_3 - \overset{\overset{O}{||}}{C} - NH_2}} \xrightarrow[\Delta]{P_4O_{10}} \underset{\text{acetonitrile or methyl cyanide}}{\underset{(C)}{CH_3 - C \equiv N}}$

32. The presence of unpaired electron in phosphorus atom is explained by Hund's rule, which states that "Pairing of electrons in d and f-orbitals can't occur until each orbital of a given subshell contains one electron or is singly occupied"

$_{15}P : 1s^2, 2s^2, 2p^6, 3s^2, 3p_x^1, 3p_y^1, 3p_z^1$

$$\overset{3p}{\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}}$$

33. Given, $m = 200$ g, $v = 3 \times 10^3$ cm/s,

$h = 6.624 \times 10^{-27}$ erg s, $\lambda = ?$

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.624 \times 10^{-27}}{200 \times 3 \times 10^{3}}$$

$\therefore \quad \lambda = 1.104 \times 10^{-32}$ cm

34. Cubic close stacking is *a–b–c–a–b–c* ...pattern of stacking spheres. NaCl has face centred cubic arrangement which is also called cubic close stacking arrangement.

35. Equivalent weight of bivalent metal = 37.2

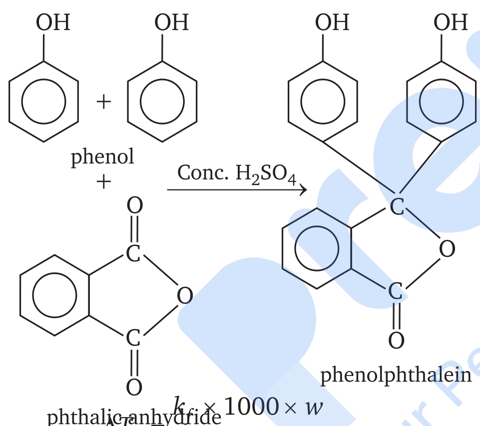
$\therefore$Atomic weight of metal = $37.2 \times 2 = 74.4$

$\therefore$Formula of its chloride is MCl_2

Hence, molecular weight of MCl_2

$$= 74.4 + 2 \times 35.5 = 145.4$$

36.



37. $$\Delta T_f = \frac{k_f \times 1000 \times w}{M \times W}$$

$$w = \frac{\Delta T_f \times M \times W}{k_f \times 1000}$$

$\Delta T_f = 0 - (-0.6) = 0.6°C$

$M = 60$ g / mol

$k_f = 1.5°C$ kg mol^{-1}

$W = 3 \times 10^3$ g

$$w = \frac{0.6 \times 60 \times 3 \times 10^3}{1.5 \times 1000}$$

$= 72$ g

38. If $K < 1$, $\Delta G° > 0$, hence the value of $\Delta G°$ is positive if $K < 1$.

39. Molecular weight of $(COOH)_2 \cdot 2H_2O = 126$ g

$\therefore$Equivalent weight $= \frac{126}{2} = 63$

[$\because$ basicity= 2]

Weight of $(COOH)_2 \cdot 2H_2O$ in solution

= 32.5 g

$$\text{Normality} = \frac{\text{number of gram equivalent}}{\text{volume (in litre)}}$$

$$= \frac{32.5 / 63}{0.5}$$

$= 1$ N

40. In acidic medium phenolphthalein is colourless and in alkaline medium it is pink. Therefore, in the titration of strong base (KOH) *vs* weak acid (oxalic acid) phenolphthalein is used.

41. $\frac{r_+}{r_-} = \frac{95}{181} = 0.524$

$\because$ The radius ratio lies between 0.414 to 0.732.

$\therefore$The coordination number of Na^+ is 6.

42. van der Waals' equation for one mole of a gas is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

where, b = volume correction, which arises due to finite size of molecules.

43. The α-hydrogen atoms of acetaldehyde due to $-E$ effect of $\rangle C = O$ group is slightly acidic in nature. In crossed aldol condensation between formaldehyde and acetaldehyde, in the first step OH^- ion (from the base added) abstracts one of these acidic α-hydrogen to form carbanion or enolate ion which is stabilised by resonance.

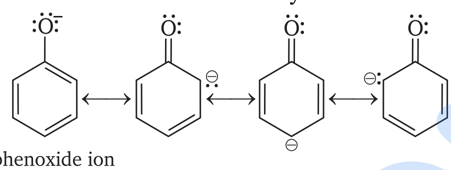
$$HO^- + H - CH_2 - \overset{\overset{\displaystyle O}{\|}}{C} - H \xrightleftharpoons{\text{Slow}} H_2O + \overset{-}{:}CH_2 - \overset{\overset{\displaystyle O}{\|}}{C} - H$$

44. $CH_3-CCl_2-CH_3 \xrightarrow{\text{Hydrolysis}}$
2, 2 - dichloro propane

$CH_3-C(OH)_2-CH_3 \xrightarrow[-H_2O]{}$
unstable

$CH_3-\underset{\underset{O}{\|}}{C}-CH_3$
acetone

45. Phenol is more acidic than alcohol because phenoxide ion is stabilised by resonance.



phenoxide ion

resonance structure of phenoxide ion of phenol

46. Citric acid is found in lemon, therefore lemon gives sour taste.

47. Due to common ion effect, rate of ionisation of NH_4OH decreases, so lower $[OH^-]$ is obtained. Hence, pH value decreases.

48. The electronic configuration of $_{24}Cr$ is $=[Ar]_{18}\ 3d^5, 4s^1$ Chromium after loss of one electron gain stable configuration due to presence of half filled d orbitals, therefore its second ionisation enthalpy is highest.

49. The metals having higher negative value of standard reduction potential are placed above hydrogen in electrochemical series. The metals placed above hydrogen has a great tendency to donate electrons or to undergo oxidation. The metals having great power to undergo oxidation are strongest reducing agent. Zn has highest negative value of standard reduction potential. Therefore, it is the strongest reducing agent.

50. $(CH_3)_3CBr + H_2O \longrightarrow (CH_3)_3C-OH + HBr$

Br is substituted by $—OH^-$ (nucleophile)

S_N1 (unimolecular nucleophilic substitution reaction)

51. Rate of a reaction $= \dfrac{\text{change in concentration}}{\text{time}}$

$\therefore$ units of rate $= \dfrac{\text{mol / L}}{\text{s}} = \text{mol L}^{-1}\ \text{s}^{-1}$

52. $(CH_3)_2CH\cdot NH_2 + 2CH_3COCl \xrightarrow[-HCl]{}$ $(CH_3)_2CH-N(COCH_3)_2$
iso - propyl amine (*pri* amine) ... *ter.* amine

53. Fehling solution is only reduced by aldehydes but not by ketones, therefore it is used to distinguish between aldehyde (C_2H_5CHO) and ketone $(CH_3)_2CO$.

$$C_2H_5CHO + \underset{\text{Fehling solution}}{Cu(OH)_2} \longrightarrow C_2H_5COOH + \underset{\text{red ppt.}}{Cu_2O} + H_2O$$

$$(CH_3)_2C{=}O + \underset{\text{Fehling solution}}{Cu(OH)_2} \longrightarrow \text{No reaction}$$

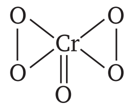
54. $\underset{\text{glucose}}{CH{=}O-CHOH-(CHOH)_3-CH_2OH} + \underset{\text{phenyl hydrazine}}{H_2NNHC_6H_5} \longrightarrow CH{=}N\cdot NHC_6H_5-CHOH-(CHOH)_3-CH_2OH$

$\xrightarrow[-NH_3,\ -C_6H_5NH_2]{H_2N\cdot NHC_6H_5} CH{=}N\cdot NHC_6H_5-C{=}O-(CHOH)_3-CH_2OH$

$\xrightarrow{C_6H_5NHNH_2} \underset{\text{glucosazone}}{CH{=}N\cdot NHC_6H_5-C{=}N\cdot NHC_6H_5-(CHOH)_3-CH_2OH}$

$\therefore$ Three molecules of phenyl hydrazine are utilised in the formation of glucosazone.

55. The structure of CrO_5 is as



The structure shows that four oxygen atom in CrO_5 are involve in the formation of peroxide linkage (OS of oxygen = – 1) while one is present as oxide (OS of oxygen = – 2). Therefore,

CrO_5

$$\underset{\text{for Cr}}{x} + \underset{\text{for O}}{1 \times (-2)} + \underset{\text{for (O – O)}}{4(-1)} = 0$$

$$x - 2 - 4 = 0$$

$$x = +6$$

56. Liquor ammonia bottles are opened only after cooling because it has high vapour pressure and it is mild explosive.

57. According to Faraday's second law of electrolysis "when the same quantity of electricity is passed through solutions of different electrolytes connected in series, the weights of substances produced at the electrodes are directly proportional to their equivalent weights".

If M stands for atomic weight of substances

Equivalent weight of Cu $= \frac{M}{2}$

Equivalent weight of Ni $= \frac{M}{2}$

Equivalent weight of Ag $= \frac{M}{1}$

So, the proportion of moles of metals

$$\frac{M}{2} : \frac{M}{2} : \frac{M}{1}$$

$\therefore$ $1 : 1 : 2$

58. According to question, the rate of reaction is doubled on increasing temperature from T_1 K to T_2 K.

At $T_1, k_1 = k$ (say)

and $T_2, k_2 = 2k$

On applying,

$$\log_{10} \frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log_{10} \frac{2k}{k} = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log_{10} 2 = \frac{E_a}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

59. $X(g) + 3Y(g) \rightleftharpoons 2Z(g)$

$$\Delta S^\circ = 2S^\circ(Z) - \{S^\circ(X) + 3S^\circ(Y)\}$$

$$= 2 \times 50 - \{60 + 3 \times 40\}$$

$$= 100 - 180 = -80 \text{ J K}^{-1} \text{ mol}^{-1}$$

$$\Delta G = \Delta H - T\Delta S$$

At equilibrium,

$$\Delta G = 0$$

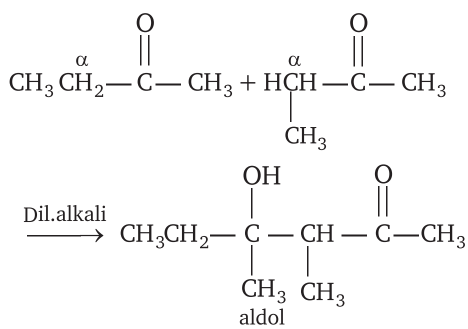
$\therefore$ $\Delta H = T\Delta S$

$$T = \frac{\Delta H}{\Delta S}$$

$$= \frac{-40 \times 10^3 \text{ J}}{-80 \text{ JK}^{-1} \text{ mol}^{-1}}$$

$$= 500 \text{ K}$$

60. Those aldehydes or ketones which have α-hydrogen atoms undergo aldol condensation reaction.



Mathematics

1. Given, $f(x) = x^2 + \dfrac{1}{x^2+1}$

$\therefore \quad f(x) = x^2 + \left(\dfrac{1}{x^2+1} - 1\right) + 1$

$= (x^2+1) - \left(\dfrac{x^2}{x^2+1}\right)$

$= 1 + x^2\left(1 - \dfrac{1}{x^2+1}\right)$

$\geq 1, \forall x \in R$

Hence, range of $f(x)$ is $[1, \infty)$.

2. Given,

$$f(x) = \begin{cases} ax^2 + b, & b \neq 0, x \leq 1 \\ bx^2 + ax + c, & x > 1 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2ax, & b \neq 0, x \leq 1 \\ 2bx + a, & x > 1 \end{cases}$$

Since, $f(x)$ is continuous at $x = 1$

$\therefore \quad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$\Rightarrow \quad a + b = b + a + c$

$\Rightarrow \quad c = 0$

Also, $f(x)$ is differentiable at $x = 1$.

$\therefore$(LHD at $x = 1$) = (RHD at $x = 1$

$\Rightarrow \quad 2a = 2b(1) + a$

$\Rightarrow \quad a = 2b$

3. Let the equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Since, this passes through (1, 2).

$\therefore \quad 1^2 + 2^2 + 2g(1) + 2f(2) + c = 0$

$\Rightarrow \quad 5 + 2g + 4f + c = 0 \quad \ldots$ (i)

Also, the circle $x^2 + y^2 = 4$ intersects the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ orthogonally.

$\therefore \quad 2(g \cdot 0 + f \cdot 0) = c - 4$

$\Rightarrow \quad c = 4$

On putting the value of c in Eq. (i), we get

$$2g + 4f + 9 = 0$$

Hence, the locus of centre $(-g, -f)$ is

$$-2x - 4y + 9 = 0$$

or $\quad 2x + 4y - 9 = 0$

4. Given,

$$\int f(x) \sin x \cos x \, dx = \frac{1}{2(b^2 - a^2)} \log[f(x)] + c$$

On differentiating both sides, we get

$$f(x) \sin x \cos x = \frac{d}{dx}\left(\frac{\log[f(x)]}{2(b^2 - a^2)} + c\right)$$

$$\Rightarrow f(x) \sin x \cos x = \frac{1}{2(b^2 - a^2)} \cdot \frac{1}{f(x)} f'(x)$$

$$\Rightarrow 2(b^2 - a^2) \sin x \cos x = \frac{f'(x)}{[f(x)]^2}$$

On integrating both sides, we get

$$\int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x)\, dx = \int \frac{f'(x)}{[f(x)]^2} dx$$

$$\Rightarrow \quad -b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\therefore \quad f(x) = \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$$

5. Given , $|z - 3| = |z - 5|$

On squaring both sides, we get

$$(z-3)(\bar{z}-3) = (z-5)(\bar{z}-5)$$

$\Rightarrow z\bar{z} - 3\bar{z} - 3z + 9 = z\bar{z} - 5\bar{z} - 5z + 25$

$\Rightarrow \quad 2\bar{z} + 2z = 16 \Rightarrow z + \bar{z} = 8$

$\Rightarrow \quad 2x = 8 \Rightarrow x = 4$

(putting $z = x + iy$)

Hence, locus of z is a straight line parallel to y-axis.

6. Let the roots of the equation be p, q.

Let $\quad S = p^2 + q^2$

$= (p+q)^2 - 2pq \quad \ldots$(i)

Given equation is

$$x^2 - (\sin \alpha - 2)\, x - (1 + \sin \alpha) = 0$$

$\therefore \quad p+q=(\sin\alpha-2),\ pq=-(1+\sin\alpha)$

From Eq. (i),

$$S=(\sin\alpha-2)^2+2(1+\sin\alpha)$$
$$=\sin^2\alpha-4\sin\alpha+4+2+2\sin\alpha$$
$$=\sin^2\alpha-2\sin\alpha+6$$
$$\Rightarrow \quad S=(\sin\alpha-1)^2+5$$

This is least when $\sin\alpha-1=0$

$$\Rightarrow \quad \alpha=\frac{\pi}{2}$$

7. Let $P(n): 10^{n-2}\ge 81n$

For $n=4$, $\quad 10^2 \not\ge 81\times 4$

For $n=5$, $\quad 10^3\ge 81\times 5$

Hence, by mathematical induction for $n\ge 5$, the proposition is true.

8. General term of $(3+2x)^{74}$ is

$$T_{r+1}={}^{74}C_r(3)^{74-r}\,2^r\,x^r$$

Let two consecutive terms are T_{r+1}th and T_{r+2}th terms.

According to the given condition,

Coefficient of T_{r+1} = Coefficient of T_{r+2}

$$\Rightarrow \quad {}^{74}C_r 3^{74-r}2^r={}^{74}C_{r+1}3^{74-(r+1)}2^{r+1}$$

$$\Rightarrow \quad \frac{{}^{74}C_{r+1}}{{}^{74}C_r}=\frac{3}{2}$$

$$\Rightarrow \quad \frac{74-r}{r+1}=\frac{3}{2}$$

$$\Rightarrow \quad 148-2r=3r+3$$

$$\Rightarrow \quad r=29$$

Hence, two consecutive terms are 30 and 31.

9. Now, $2.\overline{357}=2+0.357357357\ldots$

$$\Rightarrow \quad 2.\overline{357}=2+0.357+0.000357+\ldots$$

$$\Rightarrow \quad 2.\overline{357}=2+\frac{357}{10^3}+\frac{357}{10^6}+\ldots$$

$$\Rightarrow \quad 2.\overline{357}=2+\frac{\frac{357}{10^3}}{1-\frac{1}{10^3}}$$

$$=2+\frac{357}{999}$$

$$\Rightarrow \quad 2.\overline{357}=\frac{2355}{999}$$

10. Given,

$$S_n=\frac{1}{1^3}+\frac{1+2}{1^3+2^3}+\ldots+\frac{1+2+3+\ldots+n}{1^3+2^3+3^3+\ldots+n^3}$$

Now, $\quad T_n=\dfrac{1+2+3+\ldots+n}{1^3+2^3+3^3+\ldots+n^3}=\dfrac{\Sigma n}{\Sigma n^3}$

$$=\frac{n(n+1)/2}{\{n(n+1)/2\}^2}$$

$$=\frac{2}{n(n+1)}$$

$$=2\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$\therefore \quad S_n=T_1+T_2+\ldots+T_n$

$$=2\left(\frac{1}{1}-\frac{1}{2}\right)+2\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots+2\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$=2\left(1-\frac{1}{n+1}\right)$$

$$=2-\frac{2}{n+1}\le 2 \qquad \left(\because \frac{2}{n+1}\le 1\right)$$

11. Given, $E(\theta)=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$

$$E(\theta)E(\phi)=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}\times\begin{bmatrix}\cos^2\phi & \cos\phi\sin\phi\\ \cos\phi\sin\phi & \sin^2\phi\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta\cos^2\phi+\cos\theta\sin\theta\cos\phi\sin\phi & \cos^2\theta\cos\phi\sin\phi+\cos\theta\sin\theta\sin^2\phi\\ \cos\theta\sin\theta\cos^2\phi+\sin^2\theta\cos\phi\sin\phi & \cos\theta\sin\theta\cos\phi\sin\phi+\sin^2\theta\sin^2\phi\end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi\cos(\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\phi\cos(2n+1)\frac{\pi}{2} & \cos\theta\sin\phi\cos(2n+1)\frac{\pi}{2} \\ \cos\phi\sin\theta\cos(2n+1)\frac{\pi}{2} & \sin\theta\sin\phi\cos(2n+1)\frac{\pi}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad \left[\because \cos(2n+1)\frac{\pi}{2} = 0\right]$$

12. Given equation can be rewritten as

$$(y-2)^2 = 12x$$

Here, vertex and foci are (0, 2) and (3, 2).

$\therefore$ Vertex of the required parabola is (3, 2) and focus is (3, 4). The axis of symmetry is $x = 3$ and latusrectum $= 4 \cdot 2 = 8$

Hence, required equation is

$$(x-3)^2 = 8(y-2)$$

or $\qquad x^2 - 6x - 8y + 25 = 0$

13. Tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$ is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1 \qquad \text{... (i)}$$

$\therefore$ $p =$ perpendicular distance from focus $(ae, 0)$ to the line (i)

$$= \frac{\left|\frac{ae}{a}\cos\theta + 0 - 1\right|}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}$$

$$= \frac{1 - e\cos\theta}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} \qquad \text{...(i)}$$

Also, $p' =$ perpendicular distance from centre (0, 0) to the line (i)

$$= \frac{1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} \qquad \text{...(ii)}$$

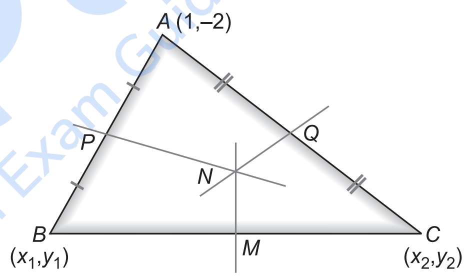
Again, $\quad r = SP = a(1 - e\cos\theta)$

$$\therefore \qquad ap = \frac{a - ae\cos\theta}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}} = rp'$$

14. Let $B(x_1, y_1)$ and $C(x_2, y_2)$ be two vertices and $P\left(\frac{x_1+1}{2}, \frac{y_1-2}{2}\right)$ lies on perpendicular bisector $x - y + 5 = 0$

$$\therefore \qquad \frac{x_1+1}{2} - \frac{y_1-2}{2} = -5$$

$$\Rightarrow \qquad x_1 - y_1 = -13 \qquad \text{... (i)}$$



Also, PN is perpendicular to AB

$$\therefore \qquad \frac{y_1+2}{x_1-1} \times 1 = -1$$

$$\Rightarrow \qquad y_1 + 2 = -x_1 + 1$$

$$\Rightarrow \qquad x_1 + y_1 = -1 \qquad \text{... (ii)}$$

On solving Eqs. (i) and (ii), we get

$$x_1 = -7, y_1 = 6$$

$\therefore$ The coordinates of B are (–7,6). Similarly, the coordinates of C are $\left(\frac{11}{5}, \frac{2}{5}\right)$.

Hence, the equation of BC is

$$y - 6 = \frac{\frac{2}{5} - 6}{\frac{11}{5} + 7}(x + 7)$$

$$\Rightarrow \quad y - 6 = \frac{-14}{23}(x+7)$$

$$\Rightarrow \quad 14x + 23y - 40 = 0$$

15. Given, $\cos\theta = -\frac{\sqrt{3}}{2} < 0$ and θ does not lie in third quadrant.

$\therefore \theta$ must be lying in 2nd quadrant.

$$\Rightarrow \quad \tan\theta = -\frac{1}{\sqrt{3}}$$

and $\quad \cot\theta = -\sqrt{3} \quad$ ... (i)

Also, α lies in 3rd quadrant and $\sin\alpha = -\frac{3}{5}$

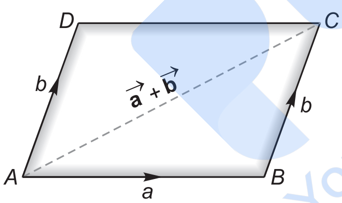
$\therefore \quad \tan\alpha = \frac{3}{4}$ and $\cos\alpha = -\frac{4}{5} \quad$ ... (ii)

$$\therefore \frac{2\tan\alpha + \sqrt{3}\tan\theta}{\cot^2\theta + \cos\alpha} = \frac{2\cdot\frac{3}{4} - \sqrt{3}\cdot\frac{1}{\sqrt{3}}}{3 - \frac{4}{5}}$$

$$= \frac{\frac{3}{2} - 1}{3 - \frac{4}{5}} = \frac{5}{22}$$

16. Diagonal $\quad \overrightarrow{\mathbf{AC}} = \vec{\mathbf{a}} + \vec{\mathbf{b}}$

$$\Rightarrow \quad |\overrightarrow{\mathbf{AC}}| = |\vec{\mathbf{a}} + \vec{\mathbf{b}}|$$



$$\Rightarrow \quad |\overrightarrow{\mathbf{AC}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 + 2\,\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}$$

$$\Rightarrow \quad |\overrightarrow{\mathbf{AC}}|^2 = \{|3\vec{\alpha} - \vec{\beta}|^2 + |\vec{\alpha} + 3\vec{\beta}|^2 + 2(3\vec{\alpha} - \vec{\beta})\cdot(\vec{\alpha} + 3\vec{\beta})\}$$

$$= 9\vec{\alpha}^2 + \vec{\beta}^2 - 6\vec{\alpha}\cdot\vec{\beta} + \vec{\alpha}^2 + 9\vec{\beta}^2 + 6\vec{\alpha}\cdot\vec{\beta} + 6\vec{\alpha}^2 - 6\vec{\beta}^2 + 16\vec{\alpha}\cdot\vec{\beta}$$

$$\Rightarrow |\overrightarrow{\mathbf{AC}}|^2 = 16\vec{\alpha}^2 + 4\vec{\beta}^2 + 16\vec{\alpha}\cdot\vec{\beta}$$

$$\Rightarrow |\overrightarrow{\mathbf{AC}}|^2 = 64 + 16 + 16|\vec{\alpha}||\vec{\beta}|\cos\frac{\pi}{3}$$

$$|\overrightarrow{\mathbf{AC}}|^2 = 80 + 16 \times 4 \times \frac{1}{2} = 112$$

$$\Rightarrow \quad |\overrightarrow{\mathbf{AC}}| = \sqrt{112} = 4\sqrt{7}$$

Other diagonal is $|\overrightarrow{\mathbf{BD}}| = |\vec{\mathbf{a}} - \vec{\mathbf{b}}|$

$$\Rightarrow \quad |\overrightarrow{\mathbf{BD}}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}|\cdot|\vec{\mathbf{b}}|$$

$$= 64 + 16 - 16 \times 4 \times \frac{1}{2}$$

$$= 80 - 32$$

$$= 48$$

$$\Rightarrow \quad |\overrightarrow{\mathbf{BD}}| = \sqrt{48} = 4\sqrt{3}$$

17. For an obtuse angle

$(cx\,\hat{\mathbf{i}} - 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\cdot(x\,\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2cx\,\hat{\mathbf{k}}) < 0$

$$\Rightarrow \quad cx^2 - 12 + 6cx < 0$$

$$\Rightarrow \quad cx^2 + 6cx - 12 < 0$$

We know that, if

$$ax^2 + bx + c > 0 \text{ or } < 0, \forall x$$

Then, $\quad b^2 - 4ac < 0$

$$\therefore \quad (6c)^2 - 4c(-12) < 0$$

$$\Rightarrow \quad 3c^2 + 4c < 0$$

$$\Rightarrow \quad 3c\left(c + \frac{4}{3}\right) < 0$$

$$\Rightarrow \quad -\frac{4}{3} < c < 0$$

18. Given,

$$y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}(a + x) = y - ay^2$$

$$\Rightarrow \quad \int \frac{dy}{y(1 - ay)} = \int \frac{dx}{a + x}$$

$$\Rightarrow \quad \int\left(\frac{1}{y} + \frac{a}{1 - ay}\right)dy = \int \frac{dx}{a + x}$$

$$\Rightarrow \quad \log y - \log(1 - ay) = \log(a + x) + \log c$$

$$\Rightarrow \quad \log y = \log(1 - ay)(a + x)c$$

$$\Rightarrow \quad y = c(1 - ay)(a + x)$$

19. Given,

$$y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$$

$$\Rightarrow \quad y = (c_1 \cos c_3 + c_2 \cos c_3)\cos x$$
$$- (c_1 \sin c_3 + c_2 \sin c_3)\sin x - c_4 e^{c_5} e^x$$

$$\Rightarrow \quad y = A\cos x - B\sin x + Ce^x$$

where, $A = c_1 \cos c_3 + c_2 \cos c_3$

$B = c_1 \sin c_3 + c_2 \sin c_3$

and $C = -c_4 e^{c_5}$

Which is an equation containing three arbitrary constant.

Hence, the order of the differential equation is 3.

20. Given, $f(x) = x^3 + bx^2 + cx + d$

$$\Rightarrow \quad f'(x) = 3x^2 + 2bx + c$$

(As we know, if $ax^2 + bx + c > 0$ for all $x \Rightarrow a > 0$ and $D < 0$)

Now, $D = 4b^2 - 12c = 4(b^2 - c) - 8c$

(where $b^2 - c < 0$ and $c > 0$)

$\therefore \quad D = (-\text{ve})$ *ie*, $D < 0$

$$\Rightarrow \quad f'(x) = 3x^2 + 2bx + c > 0$$

for all $x \in (-\infty, \infty)$.

(as $D < 0$ and $a > 0$)

Hence, $f(x)$ is strictly increasing function.

21. $\dfrac{d}{dx}\{\sin^{-1}(x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2})\}$

$$= \frac{d}{dx}\sin^{-1}\{(x\sqrt{1-(\sqrt{x})^2} + \sqrt{x}\sqrt{1-x^2})\}$$

$$\left[\begin{array}{l} \because \quad \sin^{-1}A + \sin^{-1}B \\ = \sin^{-1}(A\sqrt{1-B^2} + B\sqrt{1-A^2}) \end{array}\right]$$

$$\Rightarrow \frac{d}{dx}\{\sin^{-1}(x\sqrt{1-(\sqrt{x})^2} + \sqrt{x}\sqrt{1-x^2})\}$$

$$= \frac{d}{dx}(\sin^{-1}x + \sin^{-1}\sqrt{x})$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

22. Let $I = \displaystyle\int \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}\,dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\therefore \quad I = \int \frac{\tan\theta\tan^{-1}(\tan\theta)}{(1+\tan^2\theta)^{3/2}}\sec^2\theta\, d\theta$$

$$\Rightarrow \quad I = \int \frac{\theta\tan\theta\sec^2\theta}{\sec^3\theta}\, d\theta$$

$$\Rightarrow \quad I = \int \theta\sin\theta\, d\theta$$

$$\Rightarrow \quad I = \theta(-\cos\theta) + \int\cos\theta\, d\theta$$

$$\Rightarrow \quad I = -\theta\cos\theta + \sin\theta + c$$

$$\Rightarrow \quad I = -\tan^{-1}x\,\frac{1}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

$$\Rightarrow \quad I = \frac{(x - \tan^{-1}x)}{\sqrt{1+x^2}} + c$$

23. $$|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

$$\Rightarrow \quad |A| = 2(1 + \sin^2\theta)$$

Now, $0 \le \sin^2\theta \le 1$ for all $\theta \in [0, 2\pi]$.

$$\Rightarrow \quad 2 \le 2 + 2\sin^2\theta \le 4 \text{ for all } \theta \in [0, 2\pi].$$

$\therefore$ The range of $|A|$ is $[2, 4]$.

24. In the neighbourhood for $x = \dfrac{2\pi}{3}$, we have

$\cos x < 0$ and $\sin x > 0$

$$\therefore \quad y = -\cos x + \sin x$$

$$\Rightarrow \quad \frac{dy}{dx} = \sin x + \cos x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\left(x = \frac{2\pi}{3}\right)} = \sin\frac{2\pi}{3} + \cos\frac{2\pi}{3}$$

$$= \frac{\sqrt{3} - 1}{2}$$

25. $\displaystyle\lim_{n\to\infty}\left(\frac{1}{1-n^2} + \frac{2}{1-n^2} + \ldots \frac{n}{1-n^2}\right)$

$$= \lim_{n\to\infty}\left(\frac{1 + 2 + \ldots + n}{1 - n^2}\right)$$

$$= \lim_{n \to \infty} \frac{n(n+1)}{2(1-n^2)}$$

$$= \lim_{n \to \infty} \frac{\left(1 + \frac{1}{n}\right)}{2\left(\frac{1}{n^2} - 1\right)} = -\frac{1}{2}$$

26. $$g[f(x)] = \begin{cases} [f(x)]^2 + 1, & f(x) \neq 2 \\ 3, & f(x) = 2 \end{cases}$$

$$\Rightarrow \quad g[f(x)] = \begin{cases} \sin^2 x + 1, & x \neq n\pi \\ 3, & x = n\pi \end{cases}$$

$$\therefore \quad \text{RHL} = \lim_{h \to 0} g[f(0+h)]$$

$$= \lim_{h \to 0} (\sin^2 h + 1) = 1$$

and $$\text{LHL} = \lim_{h \to 0} g[f(0-h)]$$

$$= \lim_{h \to 0} (\sin^2 h + 1) = 1$$

$$\therefore \quad \lim_{x \to 0} g[f(x)] = 1$$

27. Given, $y = \int_0^x |t|\, dt$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = |x|$$

Since, the tangent is parallel to the line $y = 2x$

$$\therefore \quad |x| = 2 \Rightarrow x = \pm 2$$

When $x = 2$, $y = \int_0^2 |t|\, dt = \int_0^2 t\, dt$

$$= \left[\frac{t^2}{2}\right]_0^2 = 2$$

When $x = -2$,

$$y = \int_0^{-2} |t|\, dt = \int_{-2}^{0} t\, dt$$

$$= \left[\frac{t^2}{2}\right]_{-2}^{0} = -2$$

Then, tangent at (2, 2) is

$$y - 2 = 2(x - 2)$$

$$\Rightarrow \quad 2x - y - 2 = 0$$

For x-intercept, put $y = 0$, then $x = 1$

Also, tangent at $(-2, -2)$ is

$$y + 2 = 2(x + 2)$$

$$\Rightarrow \quad 2x - y + 2 = 0$$

For x-intercept, put $y = 0$, then $x = -1$.

28. Given,

$$P = x^3 - \frac{1}{x^3}, Q = x - \frac{1}{x}$$

$$\therefore \quad \frac{P}{Q^2} = \frac{\left(x - \frac{1}{x}\right)\left(x^2 + 1 + \frac{1}{x^2}\right)}{\left(x - \frac{1}{x}\right)^2}$$

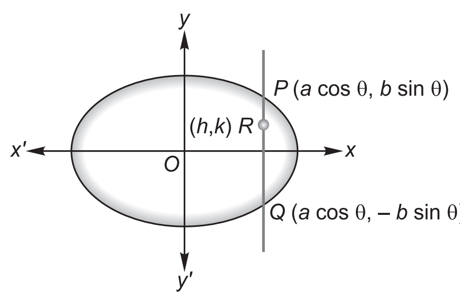
$$= \frac{\left(x - \frac{1}{x}\right)^2 + 3}{\left(x - \frac{1}{x}\right)}$$

$$= \left(x - \frac{1}{x}\right) + \frac{3}{\left(x - \frac{1}{x}\right)}$$

Clearly, the minimum does not exist.

29. Let $P(a \cos \theta, b \sin \theta), Q(a \cos \theta, -b \sin \theta)$

Given, $\quad PR : RQ = 1 : 2$



Let a point $R(h, k)$ divides the line joining the points P and Q internally in the ratio $1 : 2$.

$$\therefore \quad h = a \cos \theta \Rightarrow \cos \theta = \frac{h}{a} \quad \text{... (i)}$$

and $$k = \frac{b}{3} \sin \theta \Rightarrow \sin \theta = \frac{3k}{b} \quad \text{... (ii)}$$

On squaring and adding Eqs. (i) and (ii), we get

$$\frac{h^2}{a^2}+\frac{9k^2}{b^2}=1$$

Hence, locus of R is

$$\frac{x^2}{a^2}+\frac{9y^2}{b^2}=1$$

30. Let $P(x_1, y_1)$ be a point on the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, \text{ then } \frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}=1$$

The chord of contact of tangents from P to the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2 \text{ is } \frac{x\,x_1}{a^2}-\frac{yy_1}{b^2}=2 \qquad \dots \text{(i)}$$

The equation of the asymptotes are

$$\frac{x}{a}-\frac{y}{b}=0 \text{ and } \frac{x}{a}+\frac{y}{b}=0$$

The point of intersection of Eq. (i) with the two asymptotes are given by

$$X_1=\frac{2a}{\frac{x_1}{a}-\frac{y_1}{b}}, Y_1=\frac{2b}{\frac{x_1}{a}-\frac{y_1}{b}}$$

and $$X_2=\frac{2a}{\frac{x_1}{a}+\frac{y_1}{b}}, Y_2=\frac{-2b}{\frac{x_1}{a}+\frac{y_1}{b}}$$

$\therefore$Area of triangle

$$=\frac{1}{2}(X_1Y_2-X_2Y_1)$$

$$=\frac{1}{2}\left(\frac{4ab\times 2}{\frac{x_1^2}{a^2}-\frac{y_1^2}{b^2}}\right)=4ab$$

31. Let S denotes the sum of the series. The general term of the given series is $T_r=(-1)^r(3+5r)^nC_r$ (nth term of AP)

$$\therefore \quad S=\sum_{r=0}^{n}(-1)^r\,(3+5r)\,{}^nC_r$$

$$=3\sum_{r=0}^{n}(-1)^r\,{}^nC_r+5\sum_{r=0}^{n}(-1)^r\,r\,{}^nC_r$$

$$=3(C_0-C_1+C_2-C_3+C_4-\dots +(-1)^n\cdot C_n)+5(-C_1+2C_2-3C_3+4C_4-\dots+(-1)^n\cdot n\cdot C_n)$$

$$\Rightarrow \quad S=0+0=0$$

32. SInce, diagonal element of a skew-symmetric matrix are all zero.

$$\therefore \qquad \text{tr}(A)=\sum_{i=1}^{n}a_{ii}=0$$

33. Let $\Delta=\begin{vmatrix}1 & \alpha & \alpha^2\\ \cos(p-d)a & \cos pa & \cos(p-d)a\\ \sin(p-d)a & \sin pa & \sin(p-d)a\end{vmatrix}$

Applying $C_3\to C_3-C_1$, we get

$$\Rightarrow \quad \Delta=\begin{vmatrix}1 & \alpha & \alpha^2-1\\ \cos(p-d)a & \cos pa & 0\\ \sin(p-d)a & \sin pa & 0\end{vmatrix}$$

$$=(\alpha^2-1)\{-\cos pa\sin(p-d)a+\sin pa\cos(p-d)a\}$$

$$=(\alpha^2-1)\sin\{-(p-d)a+pa\}$$

$$\Rightarrow \quad \Delta=(\alpha^2-1)\sin da$$

which is independent of p.

34. Let S_n be the sum of first n terms of the series

$$\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\dots$$

$$\therefore \quad S_n=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\dots+\left(1-\frac{1}{2^n}\right)$$

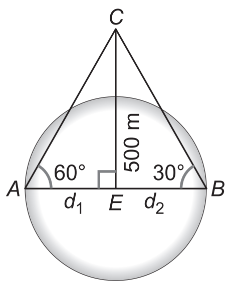
$$=n-\frac{1}{2}\frac{\left(1-\frac{1}{2^n}\right)}{\left(1-\frac{1}{2}\right)}$$

$$=n-1+\frac{1}{2^n}=n-1+2^{-n}$$

35. In ΔAEC,

$$\tan 60^\circ=\frac{500}{d_1}$$

$$\Rightarrow \qquad d_1=\frac{500}{\sqrt{3}}\text{ m} \qquad \dots \text{(i)}$$



and in ΔBEC,

$$\tan 30° = \frac{500}{d_2}$$

$$\Rightarrow \quad d_2 = 500\sqrt{3} \text{ m} \quad \dots \text{(ii)}$$

$\therefore$ Required diameter,

$$AB = d_1 + d_2$$

$$= \frac{500}{\sqrt{3}} + 500\sqrt{3} = \frac{2000}{\sqrt{3}} \text{ m}$$

36. Given,

$$\sec x - 1 = (\sqrt{2} - 1) \tan x$$

$$\Rightarrow \quad 1 - \cos x = (\sqrt{2} - 1) \sin x$$

$$\Rightarrow \quad \sin \frac{x}{2} \left(\sin \frac{x}{2} - (\sqrt{2} - 1) \cos \frac{x}{2} \right) = 0$$

$$\Rightarrow \quad \sin \frac{x}{2} = 0$$

$$\text{or} \quad \tan \frac{x}{2} = \sqrt{2} - 1 = \tan \frac{45°}{2}$$

$$\Rightarrow \quad \frac{x}{2} = n\pi \quad \text{or} \quad \frac{x}{2} = n\pi + \frac{\pi}{8}$$

$$\Rightarrow \quad x = 2n\pi, 2n\pi + \frac{\pi}{4}$$

37. Now, $\cos\left(x + \frac{\pi}{6}\right) + \sin\left(x + \frac{\pi}{6}\right)$

$$= \sqrt{2}\left[\frac{1}{\sqrt{2}} \cos\left(x + \frac{\pi}{6}\right) + \frac{1}{\sqrt{2}} \sin\left(x + \frac{\pi}{6}\right)\right]$$

$$= \sqrt{2} \cos\left(x + \frac{\pi}{6} - \frac{\pi}{4}\right)$$

$$= \sqrt{2} \cos\left(x - \frac{\pi}{12}\right)$$

For maximum value $x = \frac{\pi}{12}$

38. Given,

$$z_r = \cos \frac{r\alpha}{n^2} + i \sin \frac{r\alpha}{n^2}$$

(where $r = 1, 2, 3, \dots, n$)

$$z_1 = \cos \frac{\alpha}{n^2} + i \sin \frac{\alpha}{n^2}$$

$$z_2 = \cos \frac{2\alpha}{n^2} + i \sin \frac{2\alpha}{n^2}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$z_n = \cos \frac{n\alpha}{n^2} + i \sin \frac{n\alpha}{n^2}$$

$$\therefore \lim_{n\to\infty} (z_1 z_2 \dots z_n) = \lim_{n\to\infty} \left(\cos \frac{\alpha}{n^2} + i \sin \frac{\alpha}{n^2} \right)$$

$$\times \left(\cos \frac{2\alpha}{n^2} + i \sin \frac{2\alpha}{n^2} \right) \dots \left(\cos \frac{n\alpha}{n^2} + i \sin \frac{n\alpha}{n^2} \right)$$

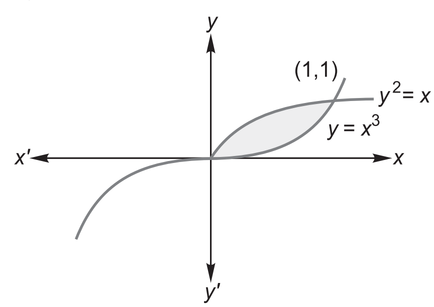
$$= \lim_{n\to\infty} \left[\cos \left\{ \frac{\alpha}{n^2} (1 + 2 + 3 + \dots + n) \right\} + i \sin \left\{ \frac{\alpha}{n^2} (1 + 2 + 3 + \dots + n) \right\} \right]$$

$$= \lim_{n\to\infty} \left[\cos \frac{\alpha n(n+1)}{2n^2} + i \sin \frac{\alpha n(n+1)}{2n^2} \right]$$

$$= \cos \frac{\alpha}{2} + i \sin \frac{\alpha}{2} = e^{\frac{i\alpha}{2}}$$

39. Let p : Paris is in France
and q : London is in England
Given, $p \wedge q$
Its negation is $\sim (p \wedge q) = \sim p \vee \sim q$
Hence, Paris is not in France or London is not in England.

40. Solving the given curves $y = \sqrt{x}$ or $y^2 = x$ and $y = x^3$



we get, the points of intersection (0, 0) and (1, 1).

$\therefore$ Required area $= \int_0^1 (\sqrt{x} - x^3)\, dx$

$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{5}{12} \text{ sq unit}$$

41. Given equations of lines are $3x - 4y + 7 = 0$

and $\quad -12x - 5y + 2 = 0$

$$a_1a_2 + b_1b_2 = 3 \times (-12) + (-4)(-5)$$
$$= -36 + 20 = -16$$
$$\Rightarrow \quad a_1a_2 + b_1b_2 \le 0$$

$\therefore$ Obtuse angle bisector is

$$\frac{3x - 4y + 7}{\sqrt{3^2 + (-4)^2}} = -\frac{-12x - 5y + 2}{\sqrt{(-12)^2 + (-5)^2}}$$

$$\Rightarrow \quad 13(3x - 4y + 7) = -5(-12x - 5y + 2)$$

$$\Rightarrow \quad 21x + 77y - 101 = 0$$

42. Given, $\dfrac{dy}{dx} = \dfrac{y}{x} - \cos^2\left(\dfrac{y}{x}\right)$

$$\Rightarrow \quad \frac{x\,dy - y\,dx}{x} = -\left(\cos^2\frac{y}{x}\right)dx$$

$$\Rightarrow \quad \sec^2\left(\frac{y}{x}\right)\left(\frac{x\,dy - y\,dx}{x^2}\right) = -\frac{dx}{x}$$

$$\Rightarrow \quad \sec^2\frac{y}{x}\cdot d\left(\frac{y}{x}\right) = -\frac{dx}{x}$$

$$\Rightarrow \quad \tan\frac{y}{x} = -\log x + c$$

When $\quad x = 1,\ y = \dfrac{\pi}{4},$

$$c = 1$$

$$\therefore \quad \tan\left(\frac{y}{x}\right) = 1 - \log x$$

$$\Rightarrow \quad \log x = 1 - \tan\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad x = e^{1 - \tan\left(\frac{y}{x}\right)}$$

43. Here, $P(1) = 2$ and from the equation

$$P(k) = k(k+1) + 2$$

$$\Rightarrow \quad P(1) = 4$$

So, $P(1)$ is not true.

Hence, mathematical induction is not applicable.

44. The general equation of all non-vertical lines in a plane is $ax + by = 1$, where $b \ne 0$.

On differentiating, we get $a + b\dfrac{dy}{dx} = 0$

On again differentiating, we get

$$b\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 0$$

45. Let the required vector be $\vec{\mathbf{c}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{k}}$

As it is a unit vector, therefore

$$|\vec{\mathbf{c}}| = 1 \quad \Rightarrow \quad x^2 + y^2 = 1 \qquad \text{...(i)}$$

$\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ are inclined at the angle 45°.

$$\therefore \quad \cos 45° = \frac{2x - y}{\sqrt{4 + 4 + 1}}$$

$$\Rightarrow \quad 2x - y = \frac{3}{\sqrt{2}} \qquad \text{...(ii)}$$

$\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are inclined at an angle 60°.

$$\therefore \quad -\frac{y}{\sqrt{2}} = \cos 60° \quad \Rightarrow \quad y = -\frac{1}{\sqrt{2}} \qquad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get $x = \dfrac{1}{\sqrt{2}}$

Hence, the required vector is $\dfrac{1}{\sqrt{2}}\hat{\mathbf{i}} - \dfrac{1}{\sqrt{2}}\hat{\mathbf{k}}$.

46. Given,

$$\int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right)dx = a + \frac{b}{\log 2}$$

Put $\log x = z \quad \Rightarrow \quad x = e^z \quad \Rightarrow \quad dx = e^z\,dz$

$$\therefore \quad \int_2^e \left(\frac{1}{\log x} - \frac{1}{(\log x)^2}\right)dx$$

$$= \int_{\log 2}^1 \left(\frac{1}{z} - \frac{1}{z^2}\right)e^z\,dz$$

$$= \int_{\log 2}^{1} e^z \left(\frac{1}{z} + d\left(\frac{1}{z}\right)\right) dz$$

$$= \left[e^z \cdot \frac{1}{z}\right]_{\log 2}^{1} = e - \frac{2}{\log 2}$$

$\therefore \quad a = e \quad \text{and} \quad b = -2$

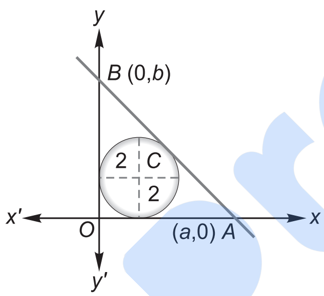
47. Let the equation of AB be $\frac{x}{a} + \frac{y}{b} = 1$.

Since, the line AB touches the circle

$$x^2 + y^2 - 4x - 4y + 4 = 0$$

$$\therefore \quad \frac{\left|\frac{2}{a} + \frac{2}{b} - 1\right|}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} = 2$$

[Since, O (0, 0) and C (2, 2) lie on the same side of AB, therefore $\frac{2}{a} + \frac{2}{b} - 1 < 0$]



$$\Rightarrow \quad \frac{-(2b + 2a - ab)}{\sqrt{a^2 + b^2}} = 2$$

$$\Rightarrow \quad 2a + 2b - ab + 2\sqrt{a^2 + b^2} = 0 \quad \text{...(i)}$$

Since, $\Delta\, OAB$ is a right angled triangle. So, its circumcentre is the mid point of AB.

$$\therefore \quad h = \frac{a}{2} \quad \text{and} \quad k = \frac{b}{2} \quad \text{...(ii)}$$

$$\Rightarrow \quad a = 2h \text{ and } b = 2k$$

From Eqs. (i) and (ii), we get

$$4h + 4k - 4hk + 2\sqrt{4h^2 + 4k^2} = 0$$

$$\Rightarrow \quad h + k - hk + \sqrt{h^2 + k^2} = 0$$

So, the locus of $P(h, k)$ is

$$x + y - xy + \sqrt{x^2 + y^2} = 0$$

But the locus of the circumcentre is given to be

$$x + y - xy + k\sqrt{x^2 + y^2} = 0$$

$$\therefore \quad k = 1$$

48. Let $f(x) = \tan x$

The point of discontinuity of $f(x)$ are those points where $\tan x$ is infinite.

ie, $\quad \tan x = \tan \infty$

$$\Rightarrow \quad x = (2n + 1)\frac{\pi}{2}, n \in I$$

49. Given curves are

$$x^3 - 3xy^2 + 2 = 0$$

and $\quad 3x^2y - y^3 - 2 = 0$

On differentiating w.r.t. x, we get

$$\left(\frac{dy}{dx}\right)_{c_1} = \frac{x^2 - y^2}{2xy}$$

and $\quad \left(\frac{dy}{dx}\right)_{c_2} = \frac{-2xy}{x^2 - y^2}$

Now, $\quad \left(\frac{dy}{dx}\right)_{c_1} \times \left(\frac{dy}{dx}\right)_{c_2} = -1$

Hence, the two curves cut at right angles.

50. Given,

$$f(x) = \frac{2 \sin 8x \cos x - 2 \sin 6x \cos 3x}{2 \cos 2x \cos x - 2 \sin 3x \sin 4x}$$

$$= \frac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) + (\cos 7x - \cos x)}$$

$$= \frac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x} = \frac{2 \sin 2x \cos 5x}{2 \cos 5x \cos 2x}$$

$$= \tan 2x$$

$\therefore$ Period of $f(x) = \frac{\pi}{2}$

51. Let $y = f(\tan x)$ and $u = g(\sec x)$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = f'(\tan x) \sec^2 x$$

and $\quad \frac{du}{dx} = g'(\sec x) \cdot \sec x \tan x$

$$\therefore \quad \frac{dy}{du} = \frac{dy/dx}{du/dx} = \frac{f'(\tan x)\sec^2 x}{g'(\sec x)\sec x \tan x}$$

$$\therefore \left(\frac{dy}{du}\right)_{x=\pi/4} = \frac{f'\left(\tan\frac{\pi}{4}\right)}{g'\left(\sec\frac{\pi}{4}\right)\sin\frac{\pi}{4}}$$

$$= \frac{f'(1)\cdot\sqrt{2}}{g'(\sqrt{2})} = \frac{2\cdot\sqrt{2}}{4} = \frac{1}{\sqrt{2}}$$

52. Area of $\Delta = \frac{1}{2}\left\|\begin{matrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ a\cos\theta & -b\sin\theta & 1 \end{matrix}\right\|$

$$\Rightarrow \quad \Delta = \frac{1}{2}|[1(-ab\sin\theta\cos\theta$$
$$-ab\sin\theta\cos\theta)|$$
$$= \frac{ab\sin 2\theta}{2}$$

Since, maximum value of $\sin 2\theta$ is 1, when $\theta = \frac{\pi}{4}$

$$\therefore \quad \Delta_{\max} = \frac{ab}{2}$$

53. The angle between the two curves is the angle between their tangents at their point of contact.

$\therefore$ The point of intersection of two curves is $a^x = b^x$

For distinct values of a and b, the equality is true, if $x = 0$

Now, if $x = 0$, then $y = a^0 = 1$

$\therefore$ The two curves intersect at (0, 1).

Let m_1 and m_2 be the slope of the tangent to the curve $y = a^x$ and $y = b^x$ respectively at $P(0, 1)$.

$$\therefore m_1 = \left(\frac{dy}{dx}\right)_P = (a^x \log a)_{(0,1)} = \log a$$

$$m_2 = \left(\frac{dy}{dx}\right)_P = (b^x \log b)_{(0,1)} = \log b$$

If α be the angle of intersection of the tangents.

Then, $\quad \tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\Rightarrow \quad \tan\alpha = \left|\frac{\log a - \log b}{1 + \log a \log b}\right|$$

54. Given, $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$

For domain of equation $x \neq 1$

But here $x = 1$

Which is not possible.

Hence, no solution exist.

55. We know that, when we multiply a complex number by i, it rotates the vector for z by an angle of $\frac{\pi}{2}$ in the anticlockwise direction.

$\therefore$ Multiplying by i^2 will rotate the vector for z by π.

$$\therefore \quad z' = i^2(-4 + 5i) = -(-4 + 5i)$$

Stretching $\frac{3}{2}$ times

$$\therefore \quad z'' = \frac{3}{2}(4 - 5i)$$
$$= 6 - \frac{15}{2}i$$

56. By definition, Any row of a matrix/determinant A when multiplied by its corresponding cofactors results in the value of $|A|$.

57. Since, $f(x)$ is a continuous decreasing function on $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$.

$\therefore f(x)$ attains every value between $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ its minimum value

ie, $\quad f\left(\frac{\pi}{3}\right) = \frac{1}{2} - \frac{\pi}{3}\left(1 + \frac{\pi}{3}\right)$

and maximum value is

$$f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}\left(1 + \frac{\pi}{6}\right)$$

58. Given, $(p \vee q) \Rightarrow r$

$$\equiv \sim r \Rightarrow \sim(p \vee q)$$
$$\equiv \sim r \Rightarrow (\sim p \wedge \sim q)$$

59. By Rolle's theorem,

$$f(1) = f(3)$$

$$\Rightarrow \quad a + b + 11 - 6 = 27a + 9b + 33 - 6$$

$$\Rightarrow \quad 26a + 8b + 22 = 0$$

$$\Rightarrow \quad 13a + 4b + 11 = 0 \quad \ldots(\text{i})$$

Now, $\quad f'(x) = 3ax^2 + 2bx + 11$

$$\Rightarrow \quad f'\left(2 + \frac{1}{\sqrt{3}}\right)$$

$$= 3a\left(2 + \frac{1}{\sqrt{3}}\right)^2 + 2b\left(2 + \frac{1}{\sqrt{3}}\right) + 11$$

$$\Rightarrow 0 = 3a\left(4 + \frac{1}{3} + \frac{4}{\sqrt{3}}\right) + 4b + \frac{2b}{\sqrt{3}} + 11$$

$$\Rightarrow \quad 13a + 4b + \frac{12a}{\sqrt{3}} + \frac{2b}{\sqrt{3}} + 11 = 0$$

$$\Rightarrow \quad -11 + \frac{12a}{\sqrt{3}} + \frac{2b}{\sqrt{3}} + 11 = 0$$

[from Eq. (i)]

$$\Rightarrow \quad 6a + b = 0 \quad \ldots(\text{ii})$$

On solving Eqs. (i) and (ii), we get

$$a = 1, b = -6$$

60. Now, $f\left(\frac{1}{x}\right) f\left(\frac{1}{y}\right) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$

$$= \cos\left(\log\frac{1}{x}\right)\cos\left(\log\frac{1}{y}\right)$$

$$-\frac{1}{2}\left[\cos\left(\log\frac{x}{y}\right) + \cos[\log(xy)]\right]$$

$$= \cos(-\log x)\cos(-\log y)$$

$$-\frac{1}{2}[\cos(\log x - \log y) + \cos(\log x + \log y)]$$

$$= \cos(\log x)\cos(\log y)$$

$$-\frac{1}{2}[2\cos(\log x)\cos(\log y)]$$

$$= \cos(\log x)\cos(\log y)$$

$$- \cos(\log x)\cos(\log y) = 0$$

61. Now, $\alpha + \beta = \sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2}$

$$+ \sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}$$

$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

Also, $\quad \alpha = \frac{\pi}{3} + \sin^{-1}\frac{1}{3} < \frac{\pi}{3} + \sin^{-1}\frac{1}{2}$

As $\sin\theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$.

$$\therefore \quad \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\Rightarrow \quad \beta > \frac{\pi}{2} > \alpha$$

$$\Rightarrow \quad \alpha < \beta$$

62. Given,

$$1 + \sin\theta + \sin^2\theta + \ldots \infty = 4 + 2\sqrt{3}$$

$$\Rightarrow \quad \frac{1}{1 - \sin\theta} = 4 + 2\sqrt{3} \quad (\because 0 < \sin\theta < 1)$$

$$\Rightarrow \quad 1 - \sin\theta = \frac{1}{4 + 2\sqrt{3}} \times \frac{4 - 2\sqrt{3}}{4 - 2\sqrt{3}}$$

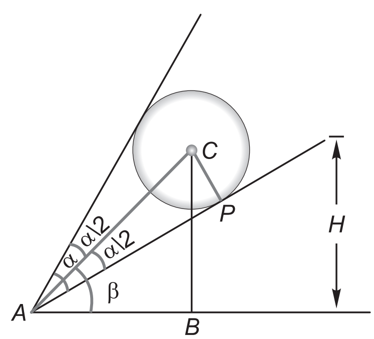
$$\Rightarrow \quad 1 - \sin\theta = \frac{4 - 2\sqrt{3}}{16 - 12} = 1 - \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3}$$

$$\theta = \frac{\pi}{3}$$

or $\quad \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

63. In ΔAPC, $\sin(\angle PAC) = \frac{CP}{AC}$



$$\Rightarrow \quad AC = \frac{r}{\sin\frac{\alpha}{2}} = r\,\text{cosec}\,\frac{\alpha}{2} \quad \ldots(\text{i})$$

Again in ΔABC,

$$\sin\beta = \frac{BC}{AC}$$

$$\Rightarrow \quad BC = AC \sin\beta$$

$$\Rightarrow \quad H = r \sin\beta \operatorname{cosec}\left(\frac{\alpha}{2}\right)$$

[from Eq. (i)]

64. $(a-b)^2 \cos^2 \frac{C}{2} + (a+b)^2 \sin^2 \frac{C}{2}$

$$= (a^2 + b^2 - 2ab)\cos^2 \frac{C}{2} + (a^2 + b^2 + 2ab)\sin^2 \frac{C}{2}$$

$$= a^2 + b^2 + 2ab\left(\sin^2 \frac{C}{2} - \cos^2 \frac{C}{2}\right)$$

$$= a^2 + b^2 - 2ab\cos C$$

$$= a^2 + b^2 - (a^2 + b^2 - c^2)$$

$$= c^2$$

65. Given, $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2$

$$\therefore \quad \left(1 - \frac{s-b}{s-a}\right)\left(1 - \frac{s-c}{s-a}\right) = 2$$

$$\Rightarrow \quad \frac{(b-a)(c-a)}{(s-a)^2} = 2$$

$$\Rightarrow \quad a^2 = b^2 + c^2$$

Hence, triangle is a right angled triangle.

66. Given, $a + b + c = 0$

Let the roots of the equation $4ax^2 + 3bx + 2c = 0$ are α and β.

Now, $D = b^2 - 4ac$

$$= 9b^2 - 4(4a)(2c)$$

$$= 9b^2 - 32ac$$

$$= 9(a+c)^2 - 32ac$$

Hence, roots are real.

67. The given graph is a connected graph.

68. Given, $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

$$\Rightarrow \quad \tan^{-1} x + 2\tan^{-1}\frac{1}{x} = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1} x + \tan^{-1}\left(\frac{2\left(\frac{1}{x}\right)}{1 - \left(\frac{1}{x}\right)^2}\right) = \frac{2\pi}{3}$$

$$\left[\because 2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$\Rightarrow \quad \tan^{-1} x + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{x + \frac{2x}{x^2-1}}{1 - \frac{2x^2}{x^2-1}}\right) = \frac{2\pi}{3}$$

$$\Rightarrow \quad \frac{x^3 - x + 2x}{x^2 - 1 - 2x^2} = \tan\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \quad \frac{x^3 + x}{-1 - x^2} = \tan\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow \quad \frac{x(x^2+1)}{-1(x^2+1)} = -\sqrt{3}$$

$$\Rightarrow \quad x = \sqrt{3}$$

69. Given complex number is $\frac{(1+i)^2}{1-i}$

$$= \frac{(1 + i^2 + 2i)}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{2i(1+i)}{1-i^2}$$

$$= \frac{2i + 2i^2}{1+1}$$

$$= \frac{2i - 2}{2}$$

$$= i - 1$$

$\therefore$ Required conjugate is $-i-1$.

70. Since, the number of vertices of odd degree is always even and number of even degree is always even. So, $m + n$ is an even number.

71. We have, $\sin\left[2\cos^{-1}\frac{\sqrt{5}}{3}\right]$

$$= \sin\left[\cos^{-1}\left(2\cdot\left(\frac{\sqrt{5}}{3}\right)^2 - 1\right)\right]$$

$$[\because 2\cos^{-1}x = \cos^{-1}(2x^2 - 1)]$$

$$= \sin\left[\cos^{-1}\left(\frac{1}{9}\right)\right]$$

$$= \sin\left[\sin^{-1}\sqrt{1 - \left(\frac{1}{9}\right)^2}\right]$$

$$[\because \cos^{-1}x = \sin^{-1}(\sqrt{1 - x^2})]$$

$$= \sin\left[\sin^{-1}\sqrt{\frac{80}{81}}\right]$$

$$= \frac{\sqrt{80}}{9}$$

$$= \frac{4\sqrt{5}}{9}$$

72. Given, $G = \{3, 6, 9, 12\}$

The composition table is

$\times_{15}$	3	6	9	12
3	9	3	12	6
6	3	6	9	12
9	12	9	6	3
12	6	12	3	9

From the table 6 is the identity element.

73. Since, identity element is its own inverse. So, minimum number of element is 1.

74. We have, $\dfrac{3x^2 + 1}{x^2 - 6x + 8}$

On dividing, we get

$$\frac{3x^2 + 1}{x^2 - 6x + 8} = 3 + \frac{18x - 23}{x^2 - 6x + 8} \quad \text{...(i)}$$

Now, $\dfrac{18x - 23}{(x-2)(x-4)} = \dfrac{A}{x-2} + \dfrac{B}{x-4}$

$$\Rightarrow 18x - 23 = A(x - 4) + B(x - 2)$$

$$\Rightarrow 18x - 23 = (A + B)x - 4A - 2B$$

Equating the coefficient of x and constant term, we get

$$A + B = 18$$

$$-4A - 2B = -23$$

On solving these equations, we get

$$A = -\frac{13}{2}, B = \frac{49}{2}$$

$$\therefore \frac{18x - 23}{(x-2)(x-4)} = -\frac{13}{2(x-2)} + \frac{49}{2(x-4)}$$

Then, from Eq. (i), we get

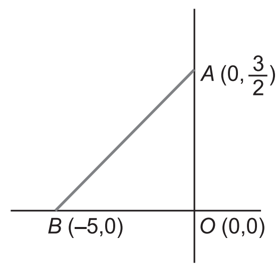
$$\frac{3x^2 + 1}{x^2 - 6x + 8} = 3 - \frac{13}{2(x-2)} + \frac{49}{2(x-4)}$$

75. ΔAOB is the given triangle.

$$\text{Slope of } AB = \frac{\frac{3}{2} - 0}{0 + 5} = \frac{3}{10}$$

$$\text{Slope of } BO = \frac{0 - 0}{0 + 5} = 0$$

The equation of line passing through A and perpendicular to BO is $y - 0 = -0\left(x - \frac{3}{2}\right)$



$$\Rightarrow \quad y = 0 \quad \text{...(i)}$$

and equation of line passing through 0 and perpendicular to AB is $y - 0 = -\frac{10}{3}(x - 0)$

$$\Rightarrow \quad y = -\frac{10}{3}x \quad \text{...(ii)}$$

The intersection point of Eqs. (i) and (ii) is (0, 0), which is the required orthocentre.

76. Given, $y = -x^2 + 6x - 3$

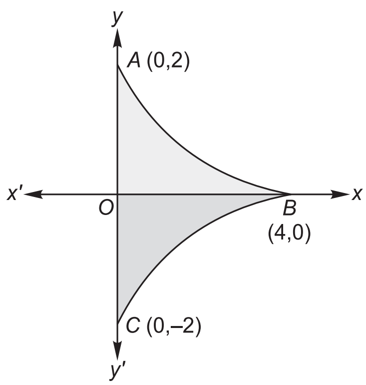
$$\Rightarrow \quad \frac{dy}{dx} = -2x + 6 = y'$$

y is increasing, if $y' > 0$

$$\Rightarrow \quad -2x + 6 > 0 \quad \Rightarrow \quad -2x > -6$$

$$\Rightarrow \quad 2x < 6 \quad \Rightarrow \quad x < 3$$

77. Given, curve $x = 4 - y^2$ and y-axis.



The required area is $ABCOA$

$$= 2 \times \text{area of } ABOA$$

$$= 2 \times \int_0^4 \sqrt{4 - x}\, dx$$

$$= 2\left[-\frac{(4-x)^{3/2}}{3/2}\right]_0^4$$

$$= 2\left[-\frac{2}{3} \times 0 + \frac{2}{3}(4)^{3/2}\right]$$

$$= \frac{4}{3} \times 8$$

$$= \frac{32}{3} \text{ sq unit}$$

78. We know that, if $a = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \ldots$

Then, the total number of positive divisors of a is

$$T(a) = (\alpha_1 + 1)(\alpha_2 + 1)\ldots..$$

Given, $252 = 2^2 \times 3^2 \times 7^1$

Here, $\alpha_1 = 2, \alpha_2 = 2, \alpha_3 = 1$

$$\therefore \quad T(a) = (2+1)(2+1)(1+1)$$

$$= 3 \cdot 3 \cdot 2$$

$$= 18$$

79. We have, $5^{124} = (5^3)^{41} \cdot 5$

Now, $\quad 5^3 \equiv 1 \pmod{124}$

$$\therefore \quad (5^3)^{41} \equiv 1 \pmod{124}$$

$$(5^3)^{41} \cdot 5 \equiv 1 \cdot 5 \pmod{124}$$

$$\Rightarrow \quad 5^{124} \equiv 5 \pmod{124}$$

80. The set of odd integers under addition is not a group. Since, addition of two odd numbers is always an even number. *ie*, closure property does not hold.