MET 2009 Answer Key PDF

❒ PHYSICS

1. (c)	**2.** (c)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (b)	**10.** (b)
11. (c)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (a)	**16.** (a)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (b)
21. (c)	**22.** (a)	**23.** (a)	**24.** (d)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (c)	**30.** (b)
31. (c)	**32.** (d)	**33.** (a)	**34.** (b)	**35.** (d)	**36.** (b)	**37.** (c)	**38.** (c)	**39.** (a)	**40.** (a)
41. (b)	**42.** (c)	**43.** (d)	**44.** (a)	**45.** (a)	**46.** (b)	**47.** (c)	**48.** (d)	**49.** (a)	**50.** (b)
51. (c)	**52.** (d)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (d)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (a)

❒ CHEMISTRY

1. (d)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (a)
11. (b)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (a)	**19.** (b)	**20.** (c)
21. (d)	**22.** (a)	**23.** (b)	**24.** (c)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (d)	**30.** (a)
31. (b)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (d)	**40.** (c)
41. (d)	**42.** (a)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (a)	**47.** (b)	**48.** (c)	**49.** (d)	**50.** (a)
51. (b)	**52.** (c)	**53.** (d)	**54.** (a)	**55.** (b)	**56.** (c)	**57.** (d)	**58.** (a)	**59.** (b)	**60.** (d)

❒ MATHEMATICS

1. (b)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (d)	**10.** (d)
11. (c)	**12.** (b)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (d)	**19.** (c)	**20.** (c)
21. (c)	**22.** (d)	**23.** (b)	**24.** (a)	**25.** (c)	**26.** (d)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (c)
31. (a)	**32.** (c)	**33.** (b)	**34.** (a)	**35.** (a)	**36.** (a)	**37.** (c)	**38.** (d)	**39.** (c)	**40.** (c)
41. (a)	**42.** (d)	**43.** (d)	**44.** (b)	**45.** (d)	**46.** (b)	**47.** (c)	**48.** (c)	**49.** (a)	**50.** (b)
51. (c)	**52.** (d)	**53.** (b)	**54.** (b)	**55.** (c)	**56.** (a)	**57.** (b)	**58.** (d)	**59.** (d)	**60.** (c)
61. (c)	**62.** (a)	**63.** (c)	**64.** (d)	**65.** (b)	**66.** (c)	**67.** (b)	**68.** (c)	**69.** (d)	**70.** (c)
71. (d)	**72.** (a)	**73.** (c)	**74.** (a)	**75.** (c)	**76.** (b)	**77.** (d)	**78.** (b)	**79.** (b)	**80.** (d)

❒ GENERAL ENGLISH AND APTITUDE

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (d)	**18.** (d)	**19.** (b)	**20.** (b)
21. (d)	**22.** (d)	**23.** (d)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (d)	**28.** (d)	**29.** (a)	**30.** (d)
31. (a)	**32.** (b)	**33.** (c)	**34.** (a)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (b)	**39.** (c)	**40.** (a)

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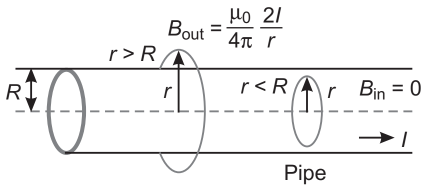
Solutions

PHYSICS

1. For a ruby laser, a crystal of ruby is formed into a cylinder. A fully reflecting mirror is placed on one end and a partially reflecting mirror on the other. A high-intensity lamp is spiraled around the ruby cylinder to provide a flash of white light that triggers the laser action. The green and blue wavelengths in the flash excite electrons in the chromium atoms to a higher energy level. Upon returning to their normal state, the electrons emit their characteristic ruby-red light. The mirrors reflect some of this light back and forth inside the ruby crystal, stimulating other excited chromium atoms to produce more red light, until the light pulse builds up to high power and drains the energy stored in the crystal. In ruby laser stimulated emission is due to transition from metastable state to ground state.

2. Required arrangement is shown in figure. According to Ampere's circuital law.



For an internal point, r < R

$$B_{internal} = \frac{\mu_0(0)}{2\pi r} = 0$$

For a point on the pipe, r = R

$$B = \frac{\mu_0 I}{2\pi r}$$

For an external point, r < R

$$B_{external} = \frac{\mu_0 I}{2\pi r}$$

Therefore, option (c) is correct.

3. Given, $f_a = 0.15$ m, $\mu_g = \frac{3}{2}, \mu_w = \frac{4}{3}$

According to Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{where } \mu = \frac{\mu_L}{\mu_M}$$

$$\frac{1}{f_a} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{(3/2)}{1} - 1\right)C \quad \text{where } C = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $$\frac{1}{f_a} = \frac{C}{2} \qquad \ldots\text{(i)}$$

Also, $$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{(3/2)}{(4/3)} - 1\right)C$$

or $$\frac{1}{f_w} = \frac{C}{8} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

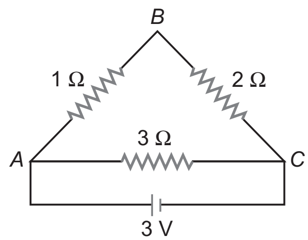
$$\frac{f_w}{f_a} = \frac{C}{2} \times \frac{8}{C} = 4$$

or $$f_w = 4f_a$$
$$= 4 \times 0.15 = 0.6 \text{ m}$$

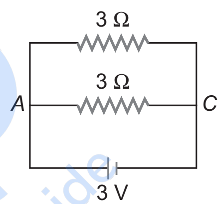
4. Phase difference $= \frac{2\pi}{\lambda} \times$ (path difference)

The phase difference is independent of time and depends only on the path difference $(x_2 - x_1)$. This holds only if the two sources are 'coherent', *ie*, they have a constant fixed phase difference between them.

5. Required arrangement is shown in figure.

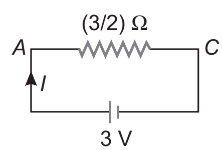


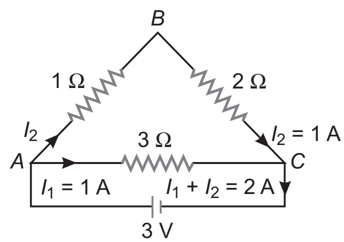
The equivalent circuit will look like (since the two resistances of 1 Ω and 2 Ω are in series, which form 3 Ω which is in parallel with 3 Ω resistance).



Therefore, the effective resistance is

$$\frac{(1+2) \times 3}{(1+2) + 3} = \frac{3}{2}\ \Omega$$





∴ Current in the circuit,

$$I = \frac{3}{(3/2)} = 2 \text{ A}$$

∴ Current in 3 Ω resistor $= \frac{I}{2} = 1$ A

6. In CE amplifier, the input signal is applied across base-emitter junction as shown in the figure below.

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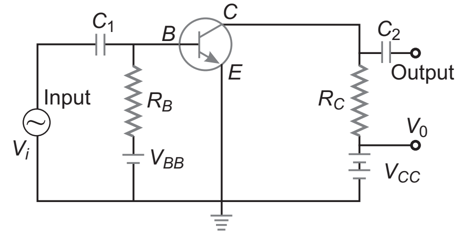


7. Let the initial number of atoms at time t = 0 be N_0.

Let *N* be the number of atoms at any instant *t*.

Mean life $\tau = \frac{1}{\lambda}$, where λ is disintegration constant.

Given, $t = \tau$

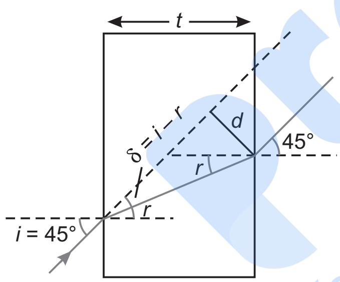
According to radioactive disintegration law,

$$N = N_0 e^{-\lambda t}$$

or $$N = N_0 e^{-\lambda \times \frac{1}{\lambda}} = \frac{N_0}{e}$$

or $$\frac{N_0}{N} = e$$

8. Here, angle of incidence i = 45°



$$\frac{\text{Lateral shift (d)}}{\text{Thickness of glass slab (t)}} = \frac{1}{\sqrt{3}}$$

Lateral shift $d = \frac{t \sin \delta}{\cos r} = \frac{t \sin (i - r)}{\cos r}$

$\Rightarrow$ $$\frac{d}{t} = \frac{\sin (i - r)}{\cos r}$$

or $$\frac{d}{t} = \frac{\sin i \cos r - \cos i \sin r}{\cos r}$$

or $$\frac{d}{t} = \frac{\sin 45° \cos r - \cos 45° \sin r}{\cos r}$$

$$= \frac{\cos r - \sin r}{\sqrt{2} \cos r}$$

or $$\frac{d}{t} = \frac{1}{\sqrt{2}}(1 - \tan r)$$

or $$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{2}}(1 - \tan r)$$

or $$\tan r = 1 - \frac{\sqrt{2}}{\sqrt{3}}$$

or $$r = \tan^{-1}\left(1 - \frac{\sqrt{2}}{\sqrt{3}}\right)$$

9. Ferromagnetic materials used in a transformer must have high permeability and low hysterisis loss.

10. According to Newton's corpuscular theory, speed of light in a rarer medium (like air) is lesser than that in a denser medium (like water, glass).

11. Phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta x$$

In a constructive interference,

$$\Delta\phi = 2n\pi \quad \text{where } n = 0, 1, 2, 3, \ldots$$

$\therefore$ $$2n\pi = \frac{2\pi}{\lambda} \Delta x$$

or $$\Delta x = n\lambda$$

12. In a potentiometer there is no current drawn from the cell whose emf is to be measured whereas a voltmeter always draws some current from the cell. Hence, the emf of a cell can be measured accurately using a potentiometer.

13. de-Broglie wavelength of an electron is given by

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

or $$\lambda \propto \frac{1}{\sqrt{K}}$$

$\therefore$ $$\frac{\lambda'}{\lambda} = \frac{1}{\sqrt{3K}} \frac{\sqrt{K}}{1} = \frac{1}{\sqrt{3}}$$

or $$\lambda' = \frac{\lambda}{\sqrt{3}}$$

Hence, de-Broglie wavelength will change by factor $\frac{1}{\sqrt{3}}$.

14. Pressure (p), volume (V) and temperature (T) are the thermodynamic coordinates whereas *R* is a universal gas constant valued at 8.314 J mol^{-1} K^{-1}.

15. In air, force of gravity acts on metals. Thus, these have their actual weight. Atomic weight of

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steel, *ie*, iron is 56 and that of aluminium is 27. Hence, it can be said that in air the weight of aluminium is half the weight of steel.

16. According to Einstein's mass energy equivalence relation

$$E = mc^2$$
$$= 10^{-6} \times 10^{-3} \times (3 \times 10^8)^2 \text{ J}$$
$$= 9 \times 10^7 \text{ J}$$
$$= \frac{9 \times 10^7}{3.6 \times 10^6} \text{ kWh}$$
$$(\because 1 \text{ kWh} = 3.6 \times 10^6 \text{ J})$$
$$= 25 \text{ kWh}$$

17. The number of significant figures in 4.8000×10^4 is 5 (zeros on right after decimal are counted while zeros in powers of 10 are not counted).

The number of significant figures in 48000.50 is 7 (all the zeros between two non-zero digits are significant).

18. β-emission takes place from a radioactive nucleus as

$$^{32}_{15}P \xrightarrow{-\beta} {}^{32}_{16}S + {}_{-1}e^0 + \bar{\nu}$$

where $\bar{\nu}$ is the anti-neutrino.

In β^+ decay a positron is emitted as

$$^{22}_{11}Na \longrightarrow {}^{22}_{10}Ne + {}_{+1}e^0 + \nu$$

where ν is the neutrino.

19. $I = \frac{P}{V} = \frac{550}{220} = 2.5 \text{ A}$

20. Let the total distance travelled by the body is 2*S*.

If t_1 is the time taken by the body to travel first half of the distance, then

$$t_1 = \frac{S}{2}$$

Let t_2 be the time taken by the body for each time interval for the remaining half journey.

$$\therefore \quad S = 3t_2 + 5t_2 = 8t_2$$

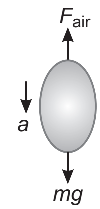
So, average speed $= \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$$= \frac{2S}{t_1 + 2t_2}$$
$$= \frac{2S}{\frac{S}{2} + \frac{S}{4}}$$
$$= \frac{8}{3} \text{ ms}^{-1}$$

21. Moment of inertia of a circular ring about a diameter

$$I = \frac{1}{2} Mr^2$$

22. From, Newton's laws of motion,



$$mg - F_{air} = ma$$

or
$$F_{air} = m(g - a)$$
$$= 0.05\,(9.8 - 9.5)$$
$$= 0.015 \text{ N}$$

23. Emulsion is a colloidal solution in which both the dispersed phase and dispersion medium are liquids. In most common emulsions, water is used as dispersed phase and oil is used as dispersion medium.

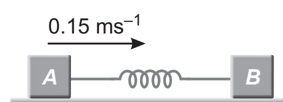
24. In fog, visible light is scattered more according to Rayleigh scattering, but scattering of infrared radiations is less due to high wavelengths, hence in fog, photographs of the objects taken with infrared radiations are more clear.

25. If we keep 1 N and 2 N forces act in same direction then these are balanced by 3 N force, but this is against statement of question.

Hence, options (c) is correct.

26. Sound waves transfer both energy and momentum.

27. As the block *A* moves with velocity 0.15 ms^{-1}, it compresses the spring which pushes *B* towards right. *A* goes on compressing the spring till the velocity acquired by *B* becomes equal to the velocity of *A*, *ie*, 0.15 ms^{-1}. Let this velocity be v. Now, spring is in a state of maximum compression. Let *x* be the maximum compression at this stage.



According to the law of conservation of linear momentum, we get

$$m_A u = (m_A + m_B)v$$

or
$$v = \frac{m_A u}{m_A + m_B}$$

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$$= \frac{2 \times 0.15}{2 + 3} = 0.06 \text{ ms}^{-1}$$

According to the law of conservation of energy.

$$\frac{1}{2} m_A u^2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2} m_A u^2 - \frac{1}{2}(m_A + m_B)v^2 = \frac{1}{2}kx^2$$

$$\frac{1}{2} \times 2 \times (0.15)^2 - \frac{1}{2}(2 + 3)(0.06)^2 = \frac{1}{2}kx^2$$

$$0.0225 - 0.009 = \frac{1}{2}kx^2 \quad \text{or} \quad 0.0135 = \frac{1}{2}kx^2$$

$$\text{or} \quad x = \sqrt{\frac{0.027}{k}} = \sqrt{\frac{0.027}{10.8}} = 0.05 \text{ m}$$

28. G P Thomson experimentally confirmed the existence of matter waves (de-Broglie's hypothesis) by demonstrating that electron beams are diffracted when they are scattered by the regular atomic arrays of crystals.

29. Given, $R_{300} = 0.3\ \Omega$, $R_t = 0.6\ \Omega$,

$$T = 300 \text{ K} = 27°\text{C}$$

Temperature coefficient of resistance,

$$\alpha = 1.5 \times 10^{-3} \text{ K}^{-1}$$

$$\therefore \quad R_{300} = R_0(1 + \alpha \times 27)$$

$$0.3 = R_0(1 + 1.5 \times 10^{-3} \times 27) \quad \ldots\text{(i)}$$

Again, $R_t = R_0(1 + \alpha t)$

$$0.6 = R_0(1 + 1.5 \times 10^{-3} \times t) \quad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{0.6}{0.3} = \frac{1 + 1.5 \times 10^{-3}\ t}{1 + 1.5 \times 10^{-3} \times 27}$$

$$\Rightarrow 2(1 + 1.5 \times 10^{-3} \times 27) = 1 + 1.5 \times 10^{-3} t$$

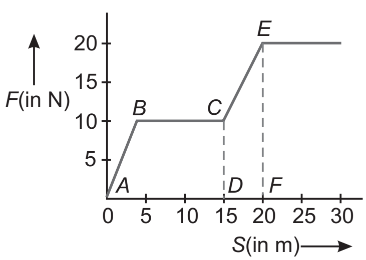
$$\Rightarrow 2 + 81 \times 10^{-3} = 1 + 1.5 \times 10^{-3} t$$

$$\Rightarrow 2 + 0.081 = 1 + 1.5 \times 10^{-3} t$$

$$\Rightarrow t = \frac{1.081}{1.5 \times 10^{-3}} = 720°\text{C} = 993 \text{ K}$$

30. Word done W = Area ABCEFDA

= Area ABCD + Area CEFD



$$= \frac{1}{2} \times (15 + 10) \times 10 + \frac{1}{2} \times (10 + 20) \times 5$$

$$= 125 + 75 = 200 \text{ J}$$

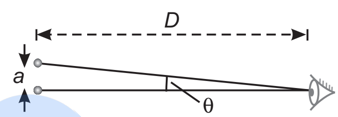
31. According to Rayleigh's criterion,

$$\theta = \frac{1.22\lambda}{d_e}$$

where λ = wavelength of light,

d_e = diameter of the pupil of the eye

$$\therefore \quad \theta = \frac{1.22 \times 500 \times 10^{-9}}{2.5 \times 10^{-3}} = 2.44 \times 10^{-4} \text{ radian}$$



But $\theta = \frac{a}{D}$

$\therefore$ Distance of separation,

$$a = D \times \theta = 10 \times 10^3 \times 2.44 \times 10^{-4}$$

$$= 2.44 \text{ m}$$

32. Here, torque $\tau = 1.6 \times 1 = 1.6$ N-m

So, when d = 0.4 m,

$$F = \frac{\tau}{d} = \frac{1.6}{0.4} = 4 \text{ N}$$

33. Let the final temperature of the mixture be *t*

Heat lost by water at 80°C

$$= ms\Delta t$$

$$= 0.1 \times 10^3 \times s_{water} \times (80° - t)$$

$$(\because m = V \times d = 0.1 \times 10^3 \text{ kg})$$

Heat gained by water at 60°C

$$= 0.3 \times 10^3 \times s_{water} \times (t - 60°)$$

According to principle of calorimetry

Heat lost = Heat gained

$$\therefore \quad 0.1 \times 10^3 \times s_{water} \times (80° - t) = 0.3 \times 10^3 \times s_{water} \times (t - 60°)$$

or $(80° - t) = 3 \times (t - 60°)$

or $4t = 260°$

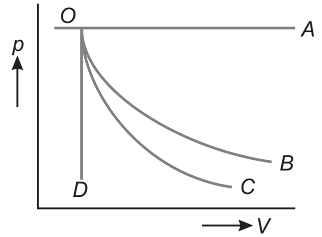
or $t = 65°\text{C}$

34.

Ultraviolet region	Lyman series
Visible region	Balmer series
Infrared region	Paschen series, Brackett sereis, Pfund series

From the above chart it is clear that Balmer series lies in the visible region of the electromagnetic spectrum.

35.



(i) Curve OA represents isobaric process (since pressure is constant)

Since, the slope of adiabatic process is more steeper than the isothermal process.

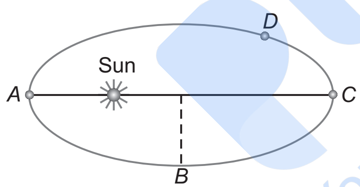
(ii) Curve OB represents isothermal process.

(iii) Curve OC represents adiabatic process.

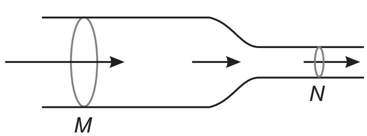
(iv) Curve OD represents isochoric process (since volume is constant).

36. Thermal radiation obeys the law of refraction and it travels along straight line like light. Hence, frequency does not change when travelling from one medium to another.

37. From Kepler's second law of planetary motion, the linear speed of a planet is maximum, when its distance from the sun is least, *ie*, at point *A*.



38. The velocity of flow will increase if cross-section decreases and *vice-versa*.

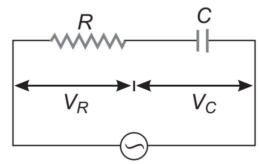


ie, $A_1 v_1 = A_2 v_2$

or $Av = \text{constant}$

Therefore, the rate of liquid flow will be greater at *N* than at *M*.

39. Let the applied voltage be *V* volt.



Here, $V_R = 12\text{ V}, V_C = 5\text{ V}$

$$\therefore \quad V = \sqrt{V_R^2 + V_C^2} = \sqrt{(12)^2 + (5)^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13\text{ V}$$

40. According to Stefan's law

$$E \propto T^4$$

$$\therefore \quad \frac{E'}{E} = \left(\frac{3T}{T}\right)^4 \text{ or } E' = 81E$$

41. For an equilateral prism,

angle of prism of refracting angle A = 60°

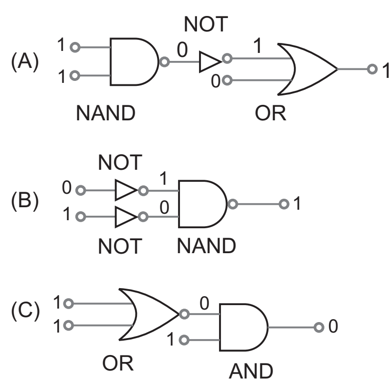
Here, $\delta_m = A = 60°$

∴Refractive index,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{60° + 60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$= \frac{\sin 60°}{\sin 30°} = \frac{\sin 60°}{\cos 60°} = \tan 60° = \sqrt{3}$$

42. The output gate circuit will be as shown below.



Hence, outputs of A, B and C are 1, 1 and 0 respectively.

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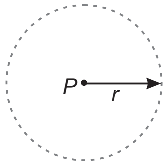
43. For an isotropic point source of power P, intensity I at a distance r from it is



$$I = \frac{P}{4\pi r^2}$$

Since power P remains the same,

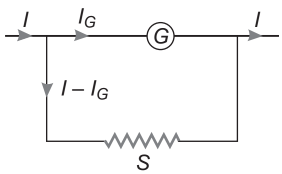
$$\therefore \qquad \frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{9}{4}\right)^2$$

$$\because \qquad I \propto A^2$$

where A is the amplitude of a wave

$$\therefore \qquad \frac{A_1}{A_2} = \sqrt{\frac{I_1}{I_2}} = \frac{9}{4}$$

44. Given, galvanometer resistance $G = 240\ \Omega$



Shunt resistance S = ?

$$I_G = \frac{4}{100} I$$

From figure voltage through the circuit.

$$(I - I_G)S = I_G G$$

or $$\left(1 - \frac{4I}{100}\right) S = \frac{4I}{100} \times 240$$

or $$S = \frac{4 \times 240}{96} = 10\ \Omega$$

45. The phenomena in which proton flips is nuclear magnetic resonance.

46. The given equation of a progressive wave is

$$y = 3 \sin \pi \left(\frac{t}{2} - \frac{x}{4}\right) = 3 \sin 2\pi \left(\frac{t}{4} - \frac{x}{8}\right)$$

The standard equation of a progressive wave is

$$y = y_0 \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Comparing these two equations, we get

$$T = 4\ s, \quad \lambda = 8\ m$$

∴ Velocity of wave,

$$v = \frac{\lambda}{T} = \frac{8}{4} = 2\ ms^{-1}$$

Distance travelled by wave in time *t* is

$$s = vt$$

or $$s = 2 \times 5 = 10\ m$$

47. Mesons and baryons are collectively known as hadrons. According to the quark model, it is possible to build all the hadrons using 3 quarks and 3 antiquarks.

48. Given, $m_\alpha = 6.4 \times 10^{-27}$ kg,

$q_\alpha = 3.2 \times 10^{-19}$ C, $E = 1.6 \times 10^5\ Vm^{-1}$

Force on α-particle

$$F = q_\alpha E = 3.2 \times 10^{-19} \times 1.6 \times 10^5$$
$$= 51.2 \times 10^{-15}\ N$$

Now, acceleration of the particle

$$\alpha = \frac{F}{m_\alpha} = \frac{51.2 \times 10^{-15}}{6.4 \times 10^{-27}}$$
$$= 0.8 \times 10^{13}\ ms^{-2}$$

∵ Initial velocity, $u = 0$

$$\therefore \qquad v^2 = 2\alpha S$$
$$= 2 \times 8 \times 10^{12} \times 2 \times 10^{-2}$$
$$= 32 \times 10^{10}$$

or $$v = 4\sqrt{2} \times 10^5\ ms^{-1}$$

49. Fundamental frequency of cylindrical open tube

$$n = \frac{v}{2L} = 390\ Hz$$

When it is immersed in water it becomes a closed tube of length $\frac{3}{4}$th of the initial length.

Therefore, its fundamental frequency is

$$n' = \frac{v}{4\left(\frac{3}{4}L\right)} = \frac{v}{3L} = \frac{2}{3}\left(\frac{v}{2L}\right)$$

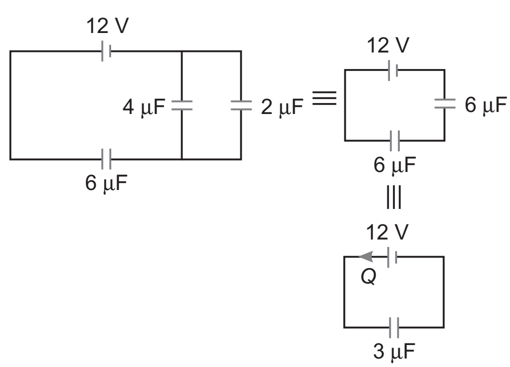
$$= \frac{2}{3} \times 390\ Hz = 260\ Hz$$

50. The surface temperature of the stars is determined using Wien's displacement law.

According to this law, $\lambda_m T = b$

where *b* is Wien's constant whose value is 2.898×10^{-3} mK.

51.

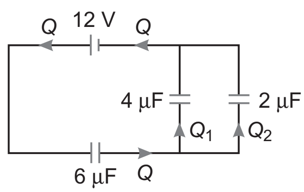


As the capacitors $4\,\mu F$ and $2\,\mu F$ are connected in parallel and are in series with $6\,\mu F$ capacitor, their equivalent capacitance is

$$\frac{(2+4)\times 6}{2+4+6} = 3\,\mu F$$

Charge in the circuit,

$$Q = 3\,\mu F \times 12\,V = 36\,\mu C$$



Since, the capacitors $4\,\mu F$ and $2\,\mu F$ are connected in parallel, therefore potential difference across them is same.

$$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{4}{2} \text{ or } Q_1 = 2Q_2$$

Also, $\quad Q = Q_1 + Q_2$

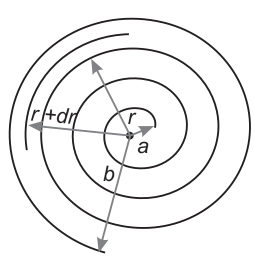
$$\therefore\ 36\mu C = 2Q_2 + Q_2 \quad \text{or} \quad Q_2 = \frac{36\,\mu C}{3} = 12\,\mu C$$

$$Q_1 = Q - Q_2 = 36\mu C - 12\mu C$$
$$= 24\,\mu C = 24 \times 10^{-6}\ C$$

52. Beam first converges and then diverges *ie* intensity of light first increases and then decreases.

53. Incandescent electric lamp produces continuous emission spectrum whereas mercury and sodium vapour give line emission spectrum. Polyatomic substances such as H_2, CO_2 and $KMnO_4$ produces band absorption spectrum.

54. Consider an element of thickness dr at a distance r from the centre of spiral coil.



Number of turns in coil = n

Number of turns per unit length

$$= \frac{n}{b-a}$$

Number of turns in element $dr = dn$

Number of turns per unit length in element dr

$$= \frac{ndr}{b-a}$$

ie, $$dn = \frac{ndr}{b-a}$$

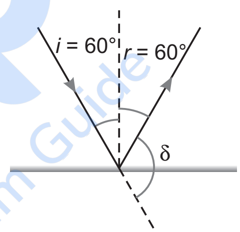
Magnetic field at its centre due to element dr is

$$dB = \frac{\mu_0 Idn}{2r} = \frac{\mu_0 I}{2}\,\frac{n}{(b-a)}\,\frac{dr}{r}$$

$$\therefore \quad B = \int_a^b \frac{\mu_0 Indr}{2(b-a)r} = \frac{\mu_0 In}{2(b-a)}\int_a^b \frac{dr}{r}$$

$$= \frac{\mu_0 In}{2(b-a)} \log_e\left(\frac{b}{a}\right)$$

55. Here, angle of incidence, $i = 60°$



$\therefore$ Angle of deviation

$$\delta = 180° - (i + r)$$
$$= 180° - 2i \qquad (\text{As } i = r)$$
$$= 180° - 2 \times 60° = 60°$$

56. Here, electric potential

$$V = 3x^2$$

Electric field,

$$E = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(3x^2) = -6x$$

$$\therefore \quad E_{(2,\,0,\,1)} = -12\ Vm^{-1}$$

57. When a this glass plate of thickness t and refractive index μ is introduced in one of the paths of the interfering waves then the path increases by $(\mu - 1)t$ and the whole pattern shifts by

$$y_0 = \frac{D}{d}(\mu - 1)t$$

Shifting is towards the side in which the plate is introduced without any change in fringe width. Therefore, when a glass plate of refractive index

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1.5 is kept in the path of light from one of the slits, only the fringes get shifted but the fringe width remains unchanged.

58. Since, electron is moving from left to right, the flux linked with loop will first increase and then decrease as the electron passes by. Therefore, induced current *I* in the loop will be first clockwise and then will move in anticlockwise direction as the electron passes by.

59. Here, $n_f = 1, \; n_i = n$

$$\therefore \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow \frac{1}{\lambda} = R\left(1 - \frac{1}{n^2}\right) \quad \ldots(i)$$

or $\frac{1}{\lambda R} = 1 - \frac{1}{n^2}$ or $\frac{1}{n^2} = 1 - \frac{1}{\lambda R}$

or $n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$

60. Magnetic dipole moment of a current is given by

$$M = NIA$$

where N = number of turns

I = current in a loop

A = area of the loop

From the above relation it is clear that magnetic dipole moment of a current loop is independent of the magnetic field in which it is lying.

CHEMISTRY

1. $H_2^+ : \sigma 1s^1, \overset{*}{\sigma} 1s^0$

$$\text{Bond order} = \frac{1}{2}$$

$H_2^- : \sigma 1s^2, \overset{*}{\sigma} 1s^1$

$$\text{Bond order} = \frac{2-1}{2} = \frac{1}{2}$$

The bond order of H_2^+ and H_2^- are same but H_2^+ is more stable than H_2^-. It is due to the presence of one electron in the antibonding molecular orbital in H_2^-.

2. Species : O_2^{2-} O_2^- O_2 O_2^+

Bond order : 1 1.5 2 2.5

3. $N = N_0 \left(\frac{1}{2}\right)^n$

Given, $N_0 = 2\,g$

$t_{1/2} = 15$ days

$T = 60$ days

$\Rightarrow \quad n = \frac{60}{15} = 4$

$\therefore \quad N = 2\left(\frac{1}{2}\right)^4$

or $\quad N = 0.125\,g$

4. $A \longrightarrow B$

$$\text{Rate} = k[A]^n$$

$(\text{Rate})_1 = k(0.05)^n = 2 \times 10^{-3} \quad \ldots(i)$

$(\text{Rate})_2 = k(0.1)^n = 1.6 \times 10^{-2} \quad \ldots(ii)$

Dividing the Eq. (ii) by Eq. (i).

$$\frac{(\text{Rate})_2}{(\text{Rate})_1} = \frac{k(0.1)^n}{k(0.05)^n} = \frac{1.6 \times 10^{-2}}{2 \times 10^{-3}}$$

$$(2)^n = 8 \quad \text{or} \quad (2)^n = 2^3$$

$\therefore \quad n = 3$

5. $AB \longrightarrow$ Product

$$\text{Rate} = k[AB]^n$$

$(\text{Rate})_1 = k[0.20]^n = 2.75 \times 10^{-8} \quad \ldots(i)$

$(\text{Rate})_2 = k[0.40]^n = 11.0 \times 10^{-8} \quad \ldots(ii)$

Dividing Eq. (ii) by Eq. (i),

$$\frac{(\text{Rate})_2}{(\text{Rate})_1} = \frac{k[0.40]^n}{k[0.20]^n} = \frac{11.0 \times 10^{-8}}{2.75 \times 10^{-8}}$$

$$2^n = 4$$

or $\quad 2^n = 2^2$

Hence, $\quad n = 2$

6. $A \longrightarrow B$

$\because \quad r = k[A]^0$

or $\quad r = k$

This is a zero order reaction.

$\therefore \quad t_{1/2} = \frac{a}{2k}$

7. Total milliequivalents of H^+

$$= 30 \times \frac{1}{3} + 20 \times \frac{1}{2} = 20$$

Total milliequivalents of OH^-

$$= 40 \times \frac{1}{4} = 10$$

Milli equivalence of H^+ left

$$= 20 - 10 = 10$$

$$\therefore \quad [H^+] = \frac{10}{1000} \text{ g ions/dm}^3 = 10^{-2}$$

$$\therefore \quad pH = 2$$

8. Weight of pure NaCl $= 6.5 \times 0.9 = 5.85$ g

No. of equivalence of NaCl $= \frac{5.85}{58.5} = 0.1$

No. of equivalence of NaOH obtained $= 0.1$

Volume of 1 M acetic acid required for the neutralisation of NaOH $= \frac{0.1 \times 1000}{1}$

$$= 100 \text{ cm}^3$$

9. $Zn^{2+} + 2e^- \longrightarrow Zn;\ E^\circ = -0.76$ V

$Fe^{2+} + 2e^- \longrightarrow Fe;\ E^\circ = -0.44$ V

Cell reaction is

$$Fe^{2+} + Zn \longrightarrow Zn^{2+} + Fe$$

$$E_{cell} = E_{cathode} - E_{anode}$$
$$= -0.44 - (-0.76)$$
$$= -0.44 + 0.76$$
$$= 0.32 \text{ V}$$

10. $[OH^-]$ in the diluted base $= \frac{10^{-6}}{10^2} = 10^{-8}$

Total $[OH^-] = 10^{-8} + [OH^-]$ of water

$$= (10^{-8} + 10^{-7}) \text{ M}$$
$$= 10^{-8}[1 + 10] \text{ M}$$
$$= 11 \times 10^{-8} \text{ M}$$

$$pOH = -\log 11 \times 10^{-8}$$
$$= -\log 11 + 8 \log 10$$
$$= 6.9586$$
$$pH = 14 - 6.9586$$
$$= 7.0414$$

11. No. of moles of $H_2 = \frac{1.12}{22400}$

No. of equivalence of hydrogen

$$= \frac{1.12 \times 2}{22400} = 10^{-4}$$

No. of Faradays required $= 10^{-4}$

$\therefore$ Current to be passed in one second

$$= 96500 \times 10^{-4}$$
$$= 9.65 \text{ A}$$

12. The number of ions per cc decreases with dilution and therefore, specific conductance decreases with dilution.

13. $p = p_A^\circ x_A + p_B^\circ x_B$

$$\Rightarrow \quad 84 = 70 \times 0.8 + p_B^\circ \times 0.2$$
$$84 = 56 + p_B^\circ \times 0.2$$
$$p_B^\circ = \frac{28}{0.2} = 140 \text{ mm}$$

14. Molarity of urea $= \dfrac{\frac{6}{60}}{\frac{100}{1000}} = 1$ M

Hence, 1 M solution of glucose is isotonic with 6% urea solution.

15. In countries nearer to polar region, the roads are sprinkled with $CaCl_2$ because $CaCl_2$ decreases the freezing point of ice and therefore, minimise the wear and tear of the roads.

16. $\Delta G = \Delta H - T \cdot \Delta S$

For the reaction,

$$H_2O(l) \rightleftharpoons H_2O(g)$$

$$\Delta G = 0 \qquad \text{(at equilibrium)}$$

$$\therefore \quad \Delta H = T \cdot \Delta S$$

17. *A* occupies corners, thus number of *A* atoms per unit cell $= 8 \times \frac{1}{8} = 1$

B occupies face centres, thus number of *B* atoms per unit cell $= 6 \times \frac{1}{2} = 3$

$\therefore$ The empirical formula of the compound is AB_3.

18. When nitro group is present in the benzene nucleus, it withdraws electrons from *o* and *p*-positions. Thus, the electron density at the *o* and *p*-positions decreases. *m*-positions become positions of comparatively higher electron density and therefore, electrophilic attack occurs at *m*-positions.

19. $$\underset{\text{acetic acid}}{CH_3COOH} \xrightarrow{LiAlH_4} \underset{(X)}{CH_3CH_2OH} \xrightarrow[300°C]{Cu} \underset{(Y)}{CH_3CHO}$$

$$\xrightarrow[\text{(aldol condensation)}]{\text{dil. NaOH}} \underset{\text{(Z) aldol}}{CH_3-\underset{\underset{OH}{|}}{CH}-CH_2CHO}$$

20. The best method for the conversion of an alcohol into an alkyl chloride is by treating the alcohol with $SOCl_2$ in the presence of pyridine.

$$ROH + SOCl_2 \longrightarrow RCl + HCl + SO_2$$

The other products being gases escape leaving behind pure alkyl halide.

21. The electrophile involved in the sulphonation of benzene is SO_3.

$$2H_2SO_4 \longrightarrow SO_3 + H_3O^+ + HSO_4^-$$

22. The carbon-carbon bond length in benzene (1.39 Å) is between that of C—C (1.54 Å) and C═C (1.34 Å) *ie*, in between that of C_2H_6 and C_2H_4.

23. Propanal is not formed during the dry distillation of a mixture of calcium formate and calcium acetate.

$$(HCOO)_2Ca \xrightarrow{\text{Dry distillation}} HCHO + CaCO_3$$

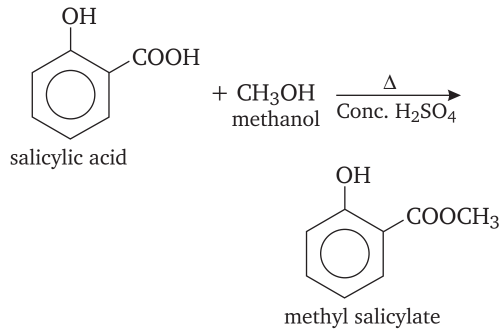
$$(CH_3COO)_2Ca \xrightarrow{\Delta} CH_3COCH_3 + CaCO_3$$

$$(HCOO)_2Ca + (CH_3COO)_2Ca \xrightarrow{\Delta} 2CH_3CHO + 2CaCO_3$$

24. $$\underset{\text{secondary alcohol}}{(H_3C)_2CHOH} + [O] \xrightarrow[K_2Cr_2O_7]{\text{Acidified}} \underset{\text{(X) acetone}}{(H_3C)_2C{=}O}$$

Ketone (*ie*, acetone) reacts with phenyl hydrazine but does not give silver mirror test.

25. Methanol reacts with salicylic acid in the presence of a few drops of conc H_2SO_4 to give methyl salicylate having the smell of oil of winter green.



26. $$\underset{\text{1, 1 - dichloroethane}}{CH_3-\underset{\underset{Cl}{|}}{\overset{\overset{Cl}{|}}{C}}-H} \xrightarrow{\text{Dil. NaOH}} \underset{\text{unstable}}{CH_3-C(H)(OH)(OH)} \longrightarrow \underset{\text{acetaldehyde}}{CH_3CHO} + H_2O$$

27. Aliphatic amines are more basic than NH_3 due to $+I$ effect of alkyl groups. In aqueous medium tertiary amine is less basic than secondary amine because the cation formed by protonation of tertiary amine is less solvated as compared to the cation formed by protonation of secondary amine. Hence, the increasing order of their basic strength is as:

$$NH_3 < (CH_3)_3N < CH_3NH_2 < (CH_3)_2NH$$

28. Ghee has least iodine value among the given options because it is the least unsaturated.

29. Sometimes the blood sugar level of diabetic patients decreases suddenly. So, diabetic patients generally carry a packet of glucose which can increase the blood sugar level almost instantaneously.

30. Naturally occurring amino acids are 20.

Hence, number of possible tripeptides

$$= 20^3 = 8000$$

31. Water boils at higher temperature inside the pressure cooker because pressure is high in the pressure cooker and therefore, cooling becomes fast.

32. Cinnabar (HgS) is a sulphide ore, hence it is concentrated by froth floatation process.

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33. K(19) : $1s^2, 2s^2 2p^6, 3s^2 3p^6, 4s^1$

$4s^1$ is the valence electron in potassium, hence the correct set of four quantum numbers for outermost electron is 4, 0, 0, $\frac{1}{2}$.

34. $\lambda = \frac{h}{mv}$

$= \frac{6.62 \times 10^{-34}}{6.62 \times 10^{-35} \times 100}$

$= 0.1$ kg

35. Ionisation energy generally increases from left to right in a period but ionisation energy of nitrogen is greater than oxygen due to stable p^3 configuration. Hence, the order is as

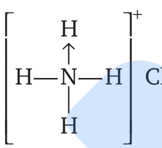
$$C < O < N < F$$

36. Na(11) : $1s^2, 2s^2 2p^6, 3s^1$

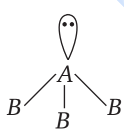
It is an alkali metal. Alkali metal oxides are basic in nature.

37. Alkali metals have low ionisation energy. They posses minimum value of ionisation energy in their period.

38. NH_4Cl contains ionic, covalent and coordinate linkage.



39. The pyramidal structure of covalent molecule AB_3 is as:



No. of lone pair = 1

No. of bond pair = 3

40. No. of millimoles of $Ca(OH)_2 = 50 \times 0.5 = 25$

No. of millimoles of $CaCO_3 = 25$

No. of milliequivalents of $CaCO_3 = 50$

$\therefore$ Volume of 0.1 N HCl $= \frac{50}{0.1} = 500 \text{ cm}^3$

41. Atomic mass of the metal $= 32 \times 2 = 64$

Formula of metal nitrate $= M(NO_3)_2$

$\therefore$ Molecular mass $= 64 + 28 + 96 = 188$

42. $v_{rms} = \sqrt{\frac{3p}{d}}$

$= \sqrt{\frac{3 \times 1.2 \times 10^5}{4}} = 300 \text{ ms}^{-1}$

43. Rate of diffusion $\propto \frac{1}{\sqrt{\text{molecular mass}}}$

$\therefore$ Order of diffusion : $H_2 > CH_4 > SO_2$

and amount left is in the order $SO_2 > CH_4 > H_2$

Hence, order of partial pressure is

$$pSO_2 > pCH_4 > pH_2$$

44. $2NH_3(g) \longrightarrow N_2(g) + 3H_2(g)$

$\Delta H_r = -(2 \times \text{enthalpy of formation of } NH_3)$

$= -(2 \times -46) = 92$ kJ

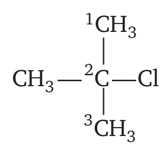
45.

	$2SO_2(g)$	+ $O_2(g)$	$\rightleftharpoons 2SO_3(g)$
Initial	5	5	0
At equilibrium	$(5-3)$ $= 2$	$(5-1.5)$ $= 3.5$	3

$\therefore \quad pO_2 = \frac{3.5 \times 1}{8.5} = 0.41$ atm

46. The value of equilibrium constant remains constant for a given reaction at constant temperature.

47. Physical adsorption decreases with increasing temperature or rate of physical adsorption increases with decreasing temperature.

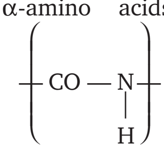
48. 

2-chloro-2-methylpropane

49. Lucas test is used to distinguish primary, secondary and tertiary alcohols.

50. Since, the compound on heating with CuO produced CO_2, it contains carbon. Again, it does not produce water, hence it does not contain hydrogen. So, the organic compound is carbon tetrachloride (CCl_4).

51. Proteins are the condensation polymers of α-amino acids. Proteins contain peptide

 linkages.

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52. The order of stability of metal oxides is as:

$$Fe_2O_3 < Cr_2O_3 < Al_2O_3 < MgO$$

53. The temperature of the slag zone in the metallurgy of iron using blast furnace is 800–1000°C.

54. The function of $Fe(OH)_3$ in the contact process is to remove arsenic impurity. $Fe(OH)_3$ is a positive sol, hence it removes arsenic impurity which is a negative sol.

55. Potassium tetraiodo mercurate (II) *ie*, $K_2[HgI_4]$ dissolves in KOH solution to give Nessler's reagent. Nessler's reagent is used to test NH_4^+ ions.

56. Argon is used in high temperature welding and other operations which require a non-oxidising atmosphere and the absence of nitrogen.

57. Chromyl chloride test is used for Cl^- ions. Chlorine is not liberated in this test.

$$4NaCl + K_2Cr_2O_7 + 3H_2SO_4 \longrightarrow K_2SO_4 + 2Na_2SO_4 + \underset{\substack{\text{chromyl chloride} \\ \text{(reddish brown vapour)}}}{2CrO_2Cl_2\uparrow} + 3H_2O$$

$$CrO_2Cl_2 + 4NaOH \longrightarrow 2NaCl + Na_2CrO_4 + 2H_2O$$

$$Na_2CrO_4 + (CH_3COO)_2Pb \longrightarrow 2CH_3COONa + \underset{\text{lead chromate}}{PbCrO_4}$$

58. $\mu = \sqrt{n(n+2)}$

$\Rightarrow \quad \sqrt{15} = \sqrt{n(n+2)}$

$\therefore \quad n = 3$

59. $[Co(NH_3)_5ONO]^{2+}$

Penta ammine nitrito cobalt (III) ion.

60. Let the oxidation state of Fe in $[Fe(H_2O)_5NO]SO_4$ is x.

$$[Fe(H_2O)_5NO]^{2+}$$

$\Rightarrow \quad x + 0 + 1 = 2$

$\therefore \quad x = +1$

Here NO exists as nitrosyl ion (NO^+).

MATHEMATICS

1. Given, $f(x) = x^3 + 3x - 2$

On differentiating w.r.t. x, we get

$$f'(x) = 3x^2 + 3$$

Put $f'(x) = 0 \Rightarrow 3x^2 + 3 = 0$

$\Rightarrow \quad x^2 = -1$

$\therefore$ f(x) is either increasing or decreasing.

At $x = 2$, $f(2) = 2^3 + 3(2) - 2 = 12$

At $x = 3$, $f(3) = 3^3 + 3(3) - 2 = 34$

$\therefore f(x) \in [12, 34]$.

2. Let $A = \left\{x \in R : \dfrac{2x - 1}{x^3 + 4x^2 + 3x}\right\}$

Now, $x^3 + 4x^2 + 3x = x(x^2 + 4x + 3)$

$$= x(x+3)(x+1)$$

$\therefore \quad A = R - \{0, -1, -3\}$

3. Given, $a_0 = 1$, $a_{n+1} = 3n^2 + n + a_n$

$\Rightarrow \quad a_1 = 3(0) + 0 + a_0 = 1$

and $a_2 = 3(1)^2 + 1 + a_1 = 3 + 1 + 1 = 5$

From option (b),

Let $\quad P(n) = n^3 - n^2 + 1$

$\therefore \quad P(0) = 0 - 0 + 1 = 1 = a_0$

$P(1) = 1^3 - 1^2 + 1 = 1 = a_1$

and $\quad P(2) = (2)^3 - (2)^2 + 1 = 5 = a_2$

Hence, option (b) is correct.

4. The total number of subsets of given set is $2^9 = 512$

Case I When selecting only one even number {2, 4, 6, 8}
Number of ways = ${}^4C_1 = 4$

Case II When selecting only two even numbers = ${}^4C_2 = 6$

Case III When selecting only three even numbers = ${}^4C_3 = 4$

Case IV When selecting only four even numbers = ${}^4C_4 = 1$

$\therefore$ Required number of ways

$$= 512 - (4 + 6 + 4 + 1) - 1$$
$$= 496$$

[Here, we subtract 1 for due to the null set]

5. Total number of points in a plane is 3p.

$\therefore$ Maximum number of triangles

$$= {}^{3p}C_3 - 3 \cdot {}^{p}C_3$$

[Here, we subtract those triangles which points are in a line]

$$= \frac{(3p)!}{(3p-3)!\,3!} - 3 \cdot \frac{p!}{(p-3)!\,3!}$$

$$= \frac{3p(3p-1)(3p-2)}{3 \times 2} - \frac{3 \times p(p-1)(p-2)}{3 \times 2}$$

$$= \frac{p}{2}[9p^2 - 9p + 2 - (p^2 - 3p + 2)]$$

$$= p^2(4p - 3)$$

6. The required number of ways = The even number of 0's *ie*, {0, 2, 4, 6, ...}

$$= \frac{n!}{n!} + \frac{n!}{2!(n-2)!} + \frac{n!}{4!(n-4)!}$$
$$= {}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = 2^{n-1}$$

7. Now, $(1 + x^2)^{12}(1 + x^{12} + x^{24} + x^{36})$

$$= [1 + {}^{12}C_1(x^2) + {}^{12}C_2(x^2)^2 + {}^{12}C_3(x^2)^3 + {}^{12}C_4(x^2)^4 + {}^{12}C_5(x^2)^5 + {}^{12}C_6(x^2)^6 + \ldots + {}^{12}C_{12}(x^2)^{12}] \times (1 + x^{12} + x^{24} + x^{36})$$

Coefficient of $x^{24} = {}^{12}C_6 + {}^{12}C_{12} + 1$

$$= {}^{12}C_6 + 2$$

8. $\left(1 + \frac{2x}{3}\right)^{3/2} (32 + 5x)^{-1/5}$

$$= \left[1 + \frac{3}{2}\left(\frac{2x}{3}\right)\right](32)^{-1/5}\left(1 + \frac{5}{32}x\right)^{-1/5}$$

(Neglect higher powers of *x*)

$$= [1 + x]2^{-1}\left[1 - \frac{1}{5}\left(\frac{5}{32}\right)x\right]$$

(Neglect higher powers of *x*)

$$= \frac{1}{2}(1 + x)\left(1 - \frac{x}{32}\right)$$

$$= \frac{(1 + x)(32 - x)}{64} = \frac{32 + 31x}{64}$$

(Neglect x^2 term)

9. $\frac{1}{(x-1)^2(x-2)} = \frac{1}{-2(1-x)^2\left(1 - \frac{x}{2}\right)}$

$$= -\frac{1}{2}\left[(1-x)^{-2}\left(1 - \frac{x}{2}\right)^{-1}\right]$$

$$= -\frac{1}{2}\left[(1 + 2x + \ldots)\left(1 + \frac{x}{2} + \ldots\right)\right]$$

$\therefore$ Coefficient of constant term is $-\frac{1}{2}$.

10. Given, $\frac{1}{e^{3x}}(e^x + e^{5x}) = a_0 + a_1x + a_2x^2 + \ldots$

$\Rightarrow \quad (e^{-2x} + e^{2x}) = a_0 + a_1x + a_2x^2 + \ldots$

$\Rightarrow \quad 2\left[1 + \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \ldots\right]$

$$= a_0 + a_1x + a_2x^2 + \ldots$$

$\Rightarrow \quad a_1 = a_3 = a_5 = \ldots = 0$

$\therefore \quad 2a_1 + 2^3a_3 + 2^5a_5 + \ldots = 0$

11. Given,

$$(x - a)(x - a - 1) + (x - a - 1)(x - a - 2) + (x - a)(x - a - 2) = 0$$

Let $x - a = t$, then

$$t(t-1) + (t-1)(t-2) + t(t-2) = 0$$

$\Rightarrow \quad t^2 - t + t^2 - 3t + 2 + t^2 - 2t = 0$

$\Rightarrow \quad 3t^2 - 6t + 2 = 0$

$\Rightarrow \quad t = \frac{6 \pm \sqrt{36 - 24}}{2(3)} = \frac{6 \pm 2\sqrt{3}}{2(3)}$

$\Rightarrow \quad x - a = \frac{3 \pm \sqrt{3}}{3}$

$\Rightarrow \quad x = a + \frac{3 \pm \sqrt{3}}{3}$

Hence, *x* is real and distinct.

12. Given, $f(x) = x^2 + ax + b$ has imaginary roots.

$\therefore$ Discriminant, $D < 0 \Rightarrow a^2 - 4b < 0$

Now, $\quad f'(x) = 2x + a$

$$f''(x) = 2$$

Also, $\quad f(x) + f'(x) + f''(x) = 0 \quad \ldots(i)$

$\Rightarrow \quad x^2 + ax + b + 2x + a + 2 = 0$

$\Rightarrow \quad x^2 + (a + 2)x + b + a + 2 = 0$

$\therefore \quad x = \frac{-(a + 2) \pm \sqrt{(a + 2)^2 - 4(a + b + 2)}}{2}$

$$= \frac{-(a + 2) \pm \sqrt{a^2 - 4b - 4}}{2}$$

Since, $a^2 - 4b < 0$

$\therefore$ $a^2 - 4b - 4 < 0$

Hence, Eq. (i) has imaginary roots.

13. Given, α, β and γ are the roots of $x^3 + 4x + 1 = 0$.

$\therefore$ $\alpha + \beta + \gamma = 0, \alpha\beta + \beta\gamma + \gamma\alpha = 4, \alpha\beta\gamma = -1$

Now, $\frac{\alpha^2}{\beta + \gamma} + \frac{\beta^2}{\gamma + \alpha} + \frac{\gamma^2}{\alpha + \beta} = \frac{\alpha^2}{-\alpha} + \frac{\beta^2}{-\beta} + \frac{\gamma^2}{-\gamma}$

$= -(\alpha + \beta + \gamma) = 0$

$\frac{\alpha^2\beta^2}{(\beta + \gamma)(\gamma + \alpha)} + \frac{\beta^2\gamma^2}{(\gamma + \alpha)(\alpha + \beta)} + \frac{\gamma^2\alpha^2}{(\beta + \gamma)(\alpha + \beta)}$

$= \alpha\beta + \beta\gamma + \gamma\alpha = 4$

and $\frac{\alpha^2\beta^2\gamma^2}{(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)} = -\alpha\beta\gamma = 1$

$(\because \alpha + \beta + \gamma = 0)$

$\therefore$ Required equation is

$x^3 + 4x - 1 = 0$

14. Given, $f(x) = 2x^4 - 13x^2 + ax + b$ is divisible by $(x - 2)(x - 1)$.

$\therefore$ $f(2) = 2(2)^4 - 13(2)^2 + a(2) + b = 0$

$\Rightarrow$ $2a + b = 20$ ...(i)

and $f(1) = 2(1)^4 - 13(1)^2 + a + b = 0$

$\Rightarrow$ $a + b = 11$ ...(ii)

On solving Eqs. (i) and (ii), we get

$a = 9, b = 2$

15. Given, $A = A', B = B'$

Now, $(AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

$\therefore$ $AB - BA$ is a skew-symmetric matrix.

16. Given, $\begin{vmatrix} 3 & 5 & x \\ 7 & x & 7 \\ x & 5 & 3 \end{vmatrix} = 0$

$\Rightarrow 3(3x - 35) - 5(21 - 7x) + x(35 - x^2) = 0$

$\Rightarrow 9x - 105 - 105 + 35x + 35x - x^3 = 0$

$\Rightarrow$ $x^3 - 79x + 210 = 0$

$\Rightarrow$ $(x + 10)(x - 3)(x - 7) = 0$

$\Rightarrow$ $x = -10, 3, 7$

17. Let *a* and *R* be the first term and common ratio of a GP.

$\therefore$ $T_p = aR^{p-1} = x$

$T_q = aR^{q-1} = y$

and $T_r = aR^{r-1} = z$

$\Rightarrow$ $\log x = \log a + (p - 1)\log R$

$\log y = \log a + (q - 1)\log R$

and $\log z = \log a + (r - 1)\log R$

$\therefore \begin{vmatrix} \log x & p & 1 \\ \log y & q & 1 \\ \log z & r & 1 \end{vmatrix} = \begin{vmatrix} \log a + (p-1)\log R & p & 1 \\ \log a + (q-1)\log R & q & 1 \\ \log a + (r-1)\log R & r & 1 \end{vmatrix}$

$= \begin{vmatrix} \log a & p & 1 \\ \log a & q & 1 \\ \log a & r & 1 \end{vmatrix} + \begin{vmatrix} (p-1)\log R & p & 1 \\ (q-1)\log R & q & 1 \\ (r-1)\log R & r & 1 \end{vmatrix}$

$= \log a \begin{vmatrix} 1 & p & 1 \\ 1 & q & 1 \\ 1 & r & 1 \end{vmatrix} + \log R \begin{vmatrix} p-1 & p-1 & 1 \\ q-1 & q-1 & 1 \\ r-1 & r-1 & 1 \end{vmatrix}$

$(C_2 \to C_2 - C_3)$

$= 0 + 0 = 0$ ($\because$ two columns are identical)

18. If matrix has no inverse it means the value of determinant should be zero.

$\therefore$ $\begin{vmatrix} 1 & -1 & x \\ 1 & x & 1 \\ x & -1 & 1 \end{vmatrix} = 0$

If we put $x = 1$, then column Ist and IIIrd are identical.

Hence, option (d) is correct.

19. Given, α, β are the roots of $x^2 - 2x + 4 = 0$

$\therefore$ $\alpha + \beta = 2$ ...(i)

and $\alpha\beta = 4$ ...(ii)

Now, $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

$= \sqrt{4 - 4 \times 4} = \sqrt{-12}$

$\Rightarrow$ $\alpha - \beta = 2\sqrt{3}\,i$ ...(iii)

On solving Eqs. (i) and (ii), we get

$\alpha = \frac{2 + 2\sqrt{3}\,i}{2} = -2\left(\frac{-1 - \sqrt{3}\,i}{2}\right) = -2\omega^2$

and $\beta = \frac{2 - 2\sqrt{3}\,i}{2} = -2\left(\frac{-1 + \sqrt{3}\,i}{2}\right) = -2\omega$

Now, $\alpha^6 + \beta^6 = (-2\omega^2)^6 + (-2\omega)^6$

$= 64(\omega^3)^4 + 64(\omega^3)^2$

$= 128$ $[\because \omega^3 = 1]$

20. Let $z = x + iy$

Given, $\left|\frac{z + 2i}{2z + i}\right| < 1$

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$$\Rightarrow \quad \frac{\sqrt{(x)^2+(y+2)^2}}{\sqrt{(2x)^2+(2y+1)^2}}<1$$

$\Rightarrow x^2+y^2+4+4y<4x^2+4y^2+1+4y$

$\Rightarrow \quad 3x^2+3y^2>3 \quad \Rightarrow \quad x^2+y^2>1$

21. Now, $(1+\sqrt{3}\,i)^n+(1-\sqrt{3}\,i)^n$

$$=\left[2\left(\frac{1+\sqrt{3}\,i}{2}\right)\right]^n+\left[2\left(\frac{1-\sqrt{3}\,i}{2}\right)\right]^n$$

$=(-2\omega^2)^n+(-2\omega)^n$

$=(-2)^n\,[(\omega^2)^{3r+1}+(\omega)^{3r+1}]$

[$\because$ $n=3r+1$, where r is an integer]

$=(-2)^n\,(\omega^2+\omega)$

$=-(-2)^n$

22. Let $f(x)=\sin^4 x+\cos^4 x$

$=(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x$

$=1-\frac{1}{4}\cdot 2(\sin 2x)^2$

$=1-\frac{1}{4}(1-\cos 4x)$

$=\frac{3}{4}+\frac{\cos 4x}{4}$

$\therefore$ Period of $f(x)=\frac{2\pi}{4}=\frac{\pi}{2}$

23. Now, $\tan(x-y)\tan y$

$$=\frac{\sin(x-y)\sin y}{\cos(x-y)\cos y}\times\frac{2}{2}$$

$$=\frac{\cos(x-2y)-\cos(x)}{\cos(x-2y)+\cos(x)}$$

$$=\frac{1-\dfrac{\cos x}{\cos(x-2y)}}{1+\dfrac{\cos(x)}{\cos(x-2y)}}$$

$$=\frac{1-\lambda}{1+\lambda}$$

$$\left(\text{Given, } \lambda=\frac{\cos x}{\cos(x-2y)}\right)$$

24. It is a standard result.

$\cos A\cos 2A\cos 2^2 A\ldots\cos 2^{n-1} A$

$$=\frac{\sin 2^n A}{2^n\sin A}$$

25. $\sin^2 x-\cos 2x=2-\sin 2x$

$\Rightarrow 1-\cos^2 x-(2\cos^2 x-1)$

$=2-2\sin x\cos x$

$\Rightarrow \quad -3\cos^2 x+2\sin x\cos x=0$

$\Rightarrow \quad \cos x\,(2\sin x-3\cos x)=0$

$\Rightarrow \quad \cos x=0,\quad (\because$ $2\sin x-3\cos x\neq 0)$

$\Rightarrow \quad x=2n\pi\pm\frac{\pi}{2}$

$\Rightarrow \quad x=(4n\pm 1)\frac{\pi}{2}$

26. $\cos^{-1}\left(-\frac{1}{2}\right)-2\sin^{-1}\left(\frac{1}{2}\right)+3\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$-4\tan^{-1}(-1)$

$=\pi-\cos^{-1}\left(\frac{1}{2}\right)-2\left(\frac{\pi}{6}\right)+3\left(\pi-\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

$+4\tan^{-1}(1)$

$=\pi-\frac{\pi}{3}-\frac{\pi}{3}+3\left(\pi-\frac{\pi}{4}\right)+4\cdot\frac{\pi}{4}$

$=\frac{\pi}{3}+3\cdot\frac{3\pi}{4}+\pi$

$=\frac{43\pi}{12}$

27. Given, $\sinh^{-1}2+\sinh^{-1}3=x$

$\Rightarrow \cosh(\sinh^{-1}2+\sinh^{-1}3)=\cosh x$

$\Rightarrow \quad \cosh(\sinh^{-1}2)\cosh(\sinh^{-1}3)$

$+\sinh(\sinh^{-1}2)\sinh(\sinh^{-1}3)$

$=\cosh x$

$\Rightarrow \quad \cosh x=\cosh(\cosh^{-1}\sqrt{1+2^2})\times$

$\cosh(\cosh^{-1}\sqrt{1+3^2})+2\times 3$

$\Rightarrow \quad \cosh x=(\sqrt{5}\sqrt{10}+6)\times\frac{2}{2}$

$=\frac{1}{2}(12+2\sqrt{50})$

28. $a(b\cos C-c\cos B)$

$=ab\cos C-ac\cos B$

$=\frac{a^2+b^2-c^2}{2}-\frac{a^2+c^2-b^2}{2}$

$=b^2-c^2$

29. We know that, $2s=a+b+c$

$$\therefore \frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$$

$$= \frac{2s(2s-2a)(2s-2b)(2s-2c)}{4b^2c^2}$$

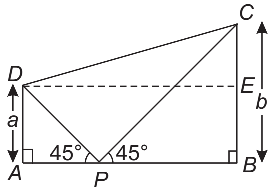
$$= 4\,\frac{s(s-a)}{bc} \times \frac{(s-b)(s-c)}{bc}$$

$$= 4\cos^2\frac{A}{2} \times \sin^2\frac{A}{2}$$

$$= \sin^2 A$$

30. In Δ APD,

$$\tan 45° = \frac{a}{AP} \quad \Rightarrow \quad AP = a$$



and in Δ BPC,

$$\tan 45° = \frac{b}{PB} \quad \Rightarrow \quad PB = b$$

$\therefore \quad DE = a + b \quad$ and $\quad CE = b - a$

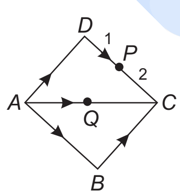
In Δ DEC,

$$DC^2 = DE^2 + EC^2$$
$$= (a+b)^2 + (b-a)^2$$
$$= 2(a^2 + b^2)$$

31. Now, $\overrightarrow{\mathbf{AB}} + 2\overrightarrow{\mathbf{AD}} + \overrightarrow{\mathbf{BC}} - 2\overrightarrow{\mathbf{DC}}$

$$= \overrightarrow{\mathbf{AC}} + 2\overrightarrow{\mathbf{AD}} - 2\overrightarrow{\mathbf{DC}}$$

$$= \overrightarrow{\mathbf{AC}} + 2(\overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CD}}) - 2\overrightarrow{\mathbf{DC}}$$



$$= 3\overrightarrow{\mathbf{AC}} - 4\overrightarrow{\mathbf{DC}}$$

$$= 3(2\overrightarrow{\mathbf{QC}}) - 4\left(\frac{3}{2}\overrightarrow{\mathbf{PC}}\right)$$

$$= 6\overrightarrow{\mathbf{QC}} - 6\overrightarrow{\mathbf{PC}} = 6(\overrightarrow{\mathbf{QC}} + \overrightarrow{\mathbf{CP}})$$

$\Rightarrow \quad k\overrightarrow{\mathbf{PQ}} = 6\overrightarrow{\mathbf{QP}} = -6\overrightarrow{\mathbf{PQ}}$ (given)

$\Rightarrow \quad k = -6$

32. Given, $l + m + n = 0, \quad \Rightarrow \quad l = -m - n$

and $\quad l^2 + m^2 - n^2 = 0$

$\therefore \quad (-m-n)^2 + m^2 - n^2 = 0$

$\Rightarrow \quad 2m^2 + 2mn = 0$

$\Rightarrow \quad 2m(m+n) = 0$

$\Rightarrow \quad m = 0$ or $m + n = 0$

If $m = 0$, then $l = -n$

$$\therefore \quad \frac{l_1}{-1} = \frac{m_1}{0} = \frac{n}{1}$$

and if $m + n = 0 \Rightarrow m = -n$, then $l = 0$

$$\therefore \quad \frac{l_2}{0} = \frac{m_2}{-1} = \frac{n_2}{1}$$

ie, $\quad (l_1, m_1, n_1) = (-1, 0, 1)$

and $\quad (l_2, m_2, n_2) = (0, -1, 1)$

$$\therefore \quad \cos\theta = \frac{0+0+1}{\sqrt{1+0+1}\,\sqrt{0+1+1}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

33. Now, $2\vec{\mathbf{a}} - \vec{\mathbf{c}} = 2(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) - (-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$

$$= \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

and $\quad \vec{\mathbf{a}} + \vec{\mathbf{b}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + 2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$

$$= \hat{\mathbf{i}} + \hat{\mathbf{k}}$$

Let θ be the angle between $2\vec{\mathbf{a}} - \vec{\mathbf{c}}$ and $\vec{\mathbf{a}} + \vec{\mathbf{b}}$.

$$\therefore \quad \cos\theta = \frac{(\hat{\mathbf{j}} + \hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} + \hat{\mathbf{k}})}{\sqrt{1^2+1^2}\,\sqrt{1^2+1^2}}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}\,\sqrt{2}} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

34. Given, $m_1 = |\vec{\mathbf{a}}_1| = \sqrt{2^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$$m_2 = |\vec{\mathbf{a}}_2| = \sqrt{3^2 + (-4)^2 + (-4)^2} = \sqrt{41}$$

$$m_3 = |\vec{\mathbf{a}}_3| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

and $\quad m_4 = |\vec{\mathbf{a}}_4| = \sqrt{(-1)^2 + (3)^2 + (1)^2} = \sqrt{11}$

$\therefore \quad m_3 < m_1 < m_4 < m_2$

35. Given, $\vec{\mathbf{a}} = \lambda\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \vec{\mathbf{b}} = \lambda\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\lambda\hat{\mathbf{k}}$

$$\therefore \quad \cos\theta = \frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}||\vec{\mathbf{b}}|}$$

$$= \frac{\lambda^2 - 7 + 6\lambda}{\sqrt{\lambda^2 + 49 + 9}\,\sqrt{\lambda^2 + 1 + 4\lambda^2}} < 0$$

$\Rightarrow \quad (\lambda+7)(\lambda-1)<0$

$\Rightarrow \quad -7<\lambda<1$

36. Let $\vec{\mathbf{a}}=\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}, \vec{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}$

and $\vec{\mathbf{c}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\lambda\hat{\mathbf{k}}$

Since, volume of tetrahedron $=\frac{1}{6}[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]$

$$\Rightarrow \frac{2}{3}=\frac{1}{6}\begin{vmatrix}1 & 2 & -1\\ 1 & 1 & 1\\ 1 & -1 & \lambda\end{vmatrix}$$

$$\Rightarrow \frac{2}{3}=\frac{1}{6}[1(\lambda+1)-2(\lambda-1)-1(-1-1)]$$

$\Rightarrow \quad 4=[-\lambda+5]$

$\Rightarrow \quad \lambda=1$

37. Given, $P(\overline{A}\cup\overline{B})=P(\overline{A\cap B})=\frac{7}{10}$

Since, $P(A\cap B)+P(\overline{A\cap B})=1$

$$\Rightarrow \quad P(A\cap B)=1-\frac{7}{10}=\frac{3}{10}$$

Also, $\quad P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$$\Rightarrow \quad \frac{4}{5}=P(A)+\frac{2}{5}-\frac{3}{10}$$

$$\Rightarrow \quad P(A)=\frac{4}{5}-\frac{2}{5}+\frac{3}{10}$$

$$=\frac{2}{5}+\frac{3}{10}=\frac{7}{10}$$

38. Given, $\quad x^2+4x+c=0$

For real roots,

$$D=b^2-4ac\ge 0$$

$$=16-4c\ge 0$$

$\Rightarrow$ $c=1, 2, 3, 4$ will satisfy the above inequality.

$\therefore$ Required probability $=\frac{4}{9}$

39. (A) Given, $P(E_1)=\frac{1}{4}, P\left(\frac{E_1}{E_2}\right)=\frac{1}{4}$

and $\quad P\left(\frac{E_2}{E_1}\right)=\frac{1}{2}$

$$\Rightarrow \quad \frac{P(E_2\cap E_1)}{P(E_1)}=\frac{1}{2}$$

$$\Rightarrow \quad P(E_2\cap E_1)=\frac{1}{8}$$

Also, $\quad P\left(\frac{E_1}{E_2}\right)=\frac{1}{4}$

$$\Rightarrow \quad \frac{P(E_1\cap E_2)}{P(E_2)}=\frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{8\,P(E_2)}=\frac{1}{4} \quad\Rightarrow\quad P(E_2)=\frac{1}{2}$$

(B) $P(E_1\cup E_2)=P(E_1)+P(E_2)-P(E_1\cap E_2)$

$$=\frac{1}{4}+\frac{1}{2}-\frac{1}{8}=\frac{5}{8}$$

(C) $P\left(\frac{\overline{E}_1}{\overline{E}_2}\right)=\frac{P(\overline{E}_1\cap\overline{E}_2)}{P(\overline{E}_2)}$

$$=\frac{1-P(E_1\cup E_2)}{1-P(E_2)}=\frac{1-\frac{5}{8}}{1-\frac{1}{2}}$$

$$=\frac{3}{4}$$

(D) $P\left(\frac{E_1}{\overline{E}_2}\right)=\frac{P(E_1\cap\overline{E}_2)}{P(\overline{E}_2)}$

$$=\frac{P(E_1)-P(E_1\cap E_2)}{1-P(E_2)}$$

$$=\frac{\frac{1}{4}-\frac{1}{8}}{1-\frac{1}{2}}=\frac{\frac{1}{8}}{\frac{1}{2}}=\frac{1}{4}$$

40. Given, distribution is

$X=x$	0	1	2	3
$P(X=x)$	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

$\therefore$ Mean, $m=\sum_{i=1}^{4} p_i x_i$

$$=0\times\frac{1}{3}+1\times\frac{1}{2}+2\times 0+3\times\frac{1}{6}$$

$$=0+\frac{1}{2}+0+\frac{1}{2}=1$$

Variance, $\sigma^2=\sum_{i=1}^{4} p_i(x_i-m)^2$

$$=\frac{1}{3}(0-1)^2+\frac{1}{2}(1-1)^2+0(2-1)^2+\frac{1}{6}(3-1)^2$$

$$=\frac{1}{3}+0+0+\frac{2}{3}=1$$

$\therefore \quad m=\sigma^2=1$

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41. Here, $n = 6$

According to the question

$$^6C_2p^2q^4 = 4 \cdot {^6C_4}p^4q^2$$

$\Rightarrow \quad q^2 = 4p^2$

$\Rightarrow \quad (1-p)^2 = 4p^2$

$\Rightarrow \quad 3p^2 + 2p - 1 = 0$

$\Rightarrow \quad (p+1)(3p-1) = 0$

$\Rightarrow \quad p = \frac{1}{3}$

($\because$ p cannot be negative)

42. Given equation is $x^2 + y^2 = r^2$.

After rotation

$x = X\cos 36° - Y\sin 36°$

and $\quad y = X\sin 36° + Y\cos 36°$

$\therefore \quad X^2(\cos^2 36° + \sin^2 36°) + Y^2(\sin^2 36° + \cos^2 36°) = r^2$

$\Rightarrow \quad X^2 + Y^2 = r^2$

43. Since, given lines are parallel.

$\therefore \quad d = \frac{15-5}{\sqrt{4^2+3^2}} = \frac{10}{5}$

$\Rightarrow \quad d = 2 =$ diameter of the circle

$\therefore$ Radius of circle $= 1$

$\therefore$ Area of circle $= \pi r^2 = \pi$ sq unit

44. Let point (x_1, y_1) be on the line $3x + 4y = 5$.

$\therefore \quad 3x_1 + 4y_1 = 5 \quad \dots(i)$

Also, $(x_1 - 1)^2 + (y_1 - 2)^2 = (x_1 - 3)^2 + (y_1 - 4)^2$

$\Rightarrow \quad x_1^2 + y_1^2 - 2x_1 - 4y_1 + 5 = x_1^2 + y_1^2 - 6x_1 - 8y_1 + 25$

$\Rightarrow \quad 4x_1 + 4y_1 = 20 \quad \dots(ii)$

On solving Eqs. (i) and (ii), we get

$x_1 = 15, y_1 = -10$

45. The point of intersection of lines $x + 3y - 1 = 0$ and $x - 2y + 4 = 0$ is $(-2, 1)$.

Let equation of line perpendicular to the given line is $2x - 3y + \lambda = 0$.

Since, it passes through $(-2, 1)$.

$\therefore \quad 2(-2) - 3(1) + \lambda = 0$

$\Rightarrow \quad \lambda = 7$

$\therefore$ Required line is $2x - 3y + 7 = 0$

46. Given equation is

$$2x^2 - 10xy + 12y^2 + 5x + \lambda y - 3 = 0$$

Here, $a = 2, h = -5, b = 12, g = \frac{5}{2}, f = \frac{\lambda}{2}, c = -3$

For pair of lines $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

$\Rightarrow \quad \begin{vmatrix} 2 & -5 & 5/2 \\ -5 & 12 & \lambda/2 \\ 5/2 & \lambda/2 & -3 \end{vmatrix} = 0$

$\Rightarrow \quad 2\left(-36 - \frac{\lambda^2}{4}\right) + 5\left(15 - \frac{5\lambda}{4}\right) + \frac{5}{2}\left(\frac{-5\lambda}{2} - 30\right) = 0$

$\Rightarrow -72 - \frac{\lambda^2}{2} + 75 - \frac{25\lambda}{4} - \frac{25\lambda}{4} - 75 = 0$

$\Rightarrow \quad \lambda^2 + 25\lambda + 144 = 0$

$\Rightarrow \quad (\lambda + 9)(\lambda + 16) = 0$

$\Rightarrow \quad \lambda = -9 \quad (\because |\lambda| < 16)$

47. Given, $x^2 - 2xy - xy + 2y^2 = 0$

$\Rightarrow \quad (x - 2y)(x - y) = 0$

$\Rightarrow \quad x = 2y, \quad x = y \quad \dots(i)$

Also, $\quad x + y + 1 = 0 \quad \dots(ii)$

On solving Eqs. (i) and (ii), we get

$A\left(-\frac{2}{3}, -\frac{1}{3}\right), B\left(-\frac{1}{2}, -\frac{1}{2}\right), C(0, 0)$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} -\frac{2}{3} & -\frac{1}{3} & 1 \\ -\frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[\frac{1}{3} - \frac{1}{6}\right] = \frac{1}{2}\left[\frac{1}{6}\right] = \frac{1}{12}$

48. Given pair of lines are $x^2 - 3xy + 2y^2 = 0$ and $x^2 - 3xy + 2y^2 + x - 2 = 0$.

$\therefore \quad (x - 2y)(x - y) = 0$

and $\quad (x - 2y + 2)(x - y - 1) = 0$

$\Rightarrow x - 2y = 0, \quad x - y = 0$ and $x - 2y + 2 = 0, \quad x - y - 1 = 0$

Since, the lines $x - 2y = 0$, $x - 2y + 2 = 0$ and $x - y = 0$, $x - y - 1 = 0$ are parallel.

Also, angle between $x - 2y = 0$ and $x - y = 0$ is not 90°.

$\therefore$ It is a parallelogram.

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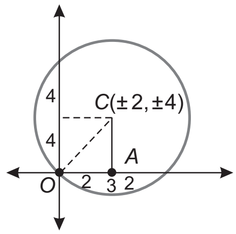
49. In ΔOAC, $OC^2 = 2^2 + 4^2 = 20$



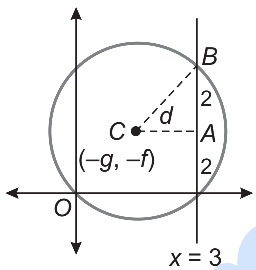
$\therefore$ Required equation of circle is

$$(x \pm 2)^2 + (y \pm 4)^2 = 20$$

$$\Rightarrow \quad x^2 + y^2 \pm 4x \pm 8y = 0$$

50. Let centre of circle be $C(-g, -f)$, then equation of circle passing through origin be

$$x^2 + y^2 + 2gx + 2fy = 0$$



$\therefore$ Distance, $d = |-g - 3| = g + 3$

In ΔABC, $(BC)^2 = AC^2 + BA^2$

$$\Rightarrow \quad g^2 + f^2 = (g+3)^2 + 2^2$$

$$\Rightarrow \quad g^2 + f^2 = g^2 + 6g + 9 + 4$$

$$\Rightarrow \quad f^2 = 6g + 13$$

Hence, required locus is $y^2 + 6x = 13$

51. The intersection point of diameter lines is (2, 3) which is the centre of circle.

Now, radius $= \sqrt{(5-2)^2 + (7-3)^2}$

$$= \sqrt{9 + 16} = 5$$

$\therefore$ Required equation of circle is

$$(x - 2)^2 + (y - 3)^2 = 5^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x - 6y - 12 = 0$$

52. Given circles are $x^2 + y^2 - 2x + 8y + 13 = 0$ and $x^2 + y^2 - 4x + 6y + 11 = 0$.

Here, $C_1 = (1, -4), C_2 = (2, -3)$,

$$\Rightarrow \quad r_1 = \sqrt{1 + 16 - 13} = 2$$

and $\quad r_2 = \sqrt{4 + 9 - 11} = \sqrt{2}$

Now, $d = C_1C_2 = \sqrt{(2-1)^2 + (-3+4)^2} = \sqrt{2}$

$$\therefore \cos\theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2} = \frac{2 - 4 - 2}{2 \times 2 \times \sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = 135^\circ$$

53. Let the required equation of circle be $x^2 + y^2 + 2gx + 2fy = 0$. Since, the above circle cuts the given circles orthogonally.

$$\therefore \quad 2(-3g) + 2f(0) = 8 \quad \Rightarrow \quad 2g = -\frac{8}{3}$$

and $\quad -2g - 2f = -7$

$$\Rightarrow \quad 2f = +7 + \frac{8}{3} = \frac{29}{3}$$

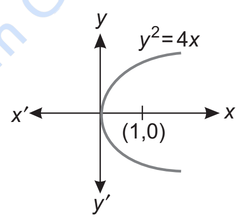
$\therefore$ Required equation of circle is

$$x^2 + y^2 - \frac{8}{3}x + \frac{29}{3}y = 0$$

or $\quad 3x^2 + 3y^2 - 8x + 29y = 0$

54. Given curve is $y^2 = 4x$.

Also, point (1, 0) is the focus of the parabola. It is clear from the graph that only one normal is possible.



55. Here, $2ae = 6$ and $2b = 8 \Rightarrow b = 4$

$$\Rightarrow \quad b^2 = 16 \quad \Rightarrow \quad a^2(1 - e^2) = 16$$

$$\Rightarrow \frac{9}{e^2}(1 - e^2) = 16 \quad \Rightarrow \quad 9 - 9e^2 = 16e^2$$

$$\Rightarrow \quad 25e^2 = 9 \quad \Rightarrow \quad e = \frac{3}{5}$$

56. Given, $\quad x^2y^2 = c^4$

$$\Rightarrow \quad y^2(a^2 - y^2) = c^4$$

$$\Rightarrow \quad y^4 - a^2y^2 + c^4 = 0$$

Let y_1, y_2, y_3 and y_4 are the roots.

$$\therefore \quad y_1 + y_2 + y_3 + y_4 = 0$$

57. Given, $4x - 3y = 5$ and $2x^2 - 3y^2 = 12$

$$\therefore \quad 2\left(\frac{5 + 3y}{4}\right)^2 - 3y^2 = 12$$

$$\Rightarrow \quad \frac{(25 + 9y^2 + 30y)}{8} - 3y^2 = 12$$

$\Rightarrow \quad 15y^2 - 30y + 71 = 0$

$\Rightarrow \quad y = \dfrac{30 \pm \sqrt{900 - 4260}}{30}$

$= 1 \pm \dfrac{\sqrt{-3360}}{30}$

Also, $\quad 2x^2 - 3\left(\dfrac{4x-5}{3}\right)^2 = 12$

$\Rightarrow \quad 10x^2 - 40x + 61 = 0$

$\Rightarrow \quad x = \dfrac{40 \pm \sqrt{1600 - 4 \times 10 \times 61}}{2 \times 10}$

$= \dfrac{40 \pm \sqrt{-840}}{20} = 2 \pm \dfrac{\sqrt{-840}}{20}$

$\therefore$ Points are $A\left(2 + \dfrac{\sqrt{-840}}{20}, 1 + \dfrac{\sqrt{-3360}}{30}\right)$ and $B\left(2 - \dfrac{\sqrt{-840}}{20}, 1 - \dfrac{\sqrt{-3360}}{30}\right)$.

$\therefore$ Mid point of *AB* is (2, 1).

58. Given, $\dfrac{5}{r} = 2 + 3\cos\theta + 4\sin\theta$

$\Rightarrow \quad \dfrac{5}{r} = 2 + 5\left(\dfrac{3}{5}\cos\theta + \dfrac{4}{5}\sin\theta\right)$

$\Rightarrow \quad \dfrac{5/2}{r} = 1 + \dfrac{5}{2}(\cos\phi\cos\theta + \sin\phi\sin\theta)$

$\left(\text{put } \cos\phi = \dfrac{3}{5}, \text{ then } \sin\phi = \dfrac{4}{5}\right)$

$\Rightarrow \quad \dfrac{5/2}{r} = 1 + \dfrac{5}{2}\cos(\theta - \phi)$

It is of the form $\dfrac{1}{r} = 1 + e\cos\theta$

$\therefore \quad e = \dfrac{5}{2}$

59. Let A = (1, 0, 0), B = (0, 1, 0) and C = (0, 0, 1)

Now, $AB = \sqrt{(0-1)^2 + (1-0)^2 + 0^2} = \sqrt{2}$

$BC = \sqrt{0^2 + (0-1)^2 + (1-0)^2} = \sqrt{2}$

and $CA = \sqrt{(1-0)^2 + 0^2 + (0-1)^2} = \sqrt{2}$

$\therefore$ Perimeter of triangle = AB + BC + CA

$= \sqrt{2} + \sqrt{2} + \sqrt{2}$

$= 3\sqrt{2}$

60. $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= (\cos^2\alpha - \sin^2\alpha) + (\cos^2\beta - \sin^2\beta) + (\cos^2\gamma - \sin^2\gamma) + \sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$= \cos^2\alpha + \cos^2\beta + \cos^2\gamma$

$= 1$

61. We know that image (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax + by + cz + d = 0$ is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

Here, point is (3, 2, 1) and plane is $2x - y + 3z = 7$.

$\therefore \quad \dfrac{x-3}{2} = \dfrac{y-2}{-1} = \dfrac{z-1}{3} = \dfrac{-2[2(3) - (2) + 3(1) - 7]}{2^2 + 1^2 + 3^2}$

$\Rightarrow \quad \dfrac{x-3}{2} = \dfrac{y-2}{-1} = \dfrac{z-1}{3} = -2(0)$

$\Rightarrow \quad x = 3, \; y = 2, \; z = 1$

62. Given equation of sphere is

$$x^2 + y^2 + z^2 - 12x - 4y - 3z = 0$$

$\therefore$ Centre of sphere is $\left(6, 2, \dfrac{3}{2}\right)$.

$\therefore$ Radius of sphere $= \sqrt{(6)^2 + (2)^2 + \left(\dfrac{3}{2}\right)^2}$

$= \sqrt{36 + 4 + \dfrac{9}{4}} = \sqrt{\dfrac{169}{4}}$

$= \dfrac{13}{2}$

63. $\lim\limits_{x \to \infty} \left(\dfrac{x+5}{x+2}\right)^{x+3} = \lim\limits_{x \to \infty} \left(1 + \dfrac{3}{x+2}\right)^{x+3}$

$= \lim\limits_{x \to \infty} \left[\left(1 + \dfrac{3}{x+2}\right)^{\frac{x+2}{3}}\right]^{\frac{3(x+3)}{x+2}}$

$= e^{\lim\limits_{x \to \infty} 3\left(\frac{1 + \frac{3}{x}}{1 + \frac{2}{x}}\right)} = e^3$

64. Given, $f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$

Now, $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{2 \sin x - \sin 2x}{2x \cos x}$

$\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{2 \cos x - 2 \cos 2x}{2(\cos x - x \sin x)}$

$= \lim_{x \to 0} \frac{2-2}{2(1-0)} = 0$

Since, $f(x)$ is continuous at $x = 0$

$f(0) = \lim_{x \to 0} f(x) \Rightarrow a = 0$

65. Given, $\frac{x}{1} = \frac{1-\sqrt{y}}{1+\sqrt{y}}$

Applying componendo and dividendo, we get

$$\frac{1+x}{1-x} = \frac{(1+\sqrt{y}) + (1-\sqrt{y})}{(1+\sqrt{y}) - (1-\sqrt{y})}$$

$$\Rightarrow \quad \frac{1+x}{1-x} = \frac{2}{2\sqrt{y}}$$

$$\Rightarrow \quad y = \left(\frac{1-x}{1+x}\right)^2$$

On differentiating w.r.t. *x*, we get

$$\frac{dy}{dx} = \frac{-2(1+x)^2(1-x) - (1-x)^2 \cdot 2(1+x)}{(1+x)^4}$$

$$= \frac{(1-x)(1+x)(-2-2x-2+2x)}{(1+x)^4}$$

$$= \frac{4(x-1)}{(x+1)^3}$$

66. Given, $x = \cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$

and $y = \sin^{-1}\left(\frac{t}{\sqrt{1+t^2}}\right)$

$\Rightarrow \quad x = \tan^{-1} t,$

and $\quad y = \tan^{-1} t$

$\therefore \quad y = x \Rightarrow \frac{dy}{dx} = 1$

67. Given, $\frac{d}{dx}\left[a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)\right] = \frac{1}{x^4 - 1}$

On integrating both sides, we get

$$a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)$$

$$= \frac{1}{2}\int\left[\frac{1}{x^2-1} - \frac{1}{x^2+1}\right] dx$$

$$\Rightarrow \quad a \tan^{-1} x + b \log\left(\frac{x-1}{x+1}\right)$$

$$= \frac{1}{4}\log\left(\frac{x-1}{x+1}\right) - \frac{1}{2}\tan^{-1} x$$

$$\Rightarrow \quad a = -\frac{1}{2}, \quad b = \frac{1}{4}$$

$$\therefore \quad a - 2b = -\frac{1}{2} - 2\left(\frac{1}{4}\right) = -1$$

68. Given, $y = e^{a \sin^{-1} x}$

On differentiating w.r.t. *x*, we get

$$y_1 = e^{a \sin^{-1} x} \, a \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad y_1\sqrt{1-x^2} = ay$$

$$\Rightarrow \quad (1-x^2)y_1^2 = a^2y^2$$

Again, differentiating w.r.t. *x*, we get

$$(1-x^2)2y_1y_2 - 2xy_1^2 = a^2 2yy_1$$

$$\Rightarrow \quad (1-x^2)y_2 - xy_1 - a^2y = 0$$

Using Leibnitz's rule,

$$(1-x^2)y_{n+2} + {}^nC_1 y_{n+1}(-2x) + {}^nC_2 y_n(-2)$$

$$-xy_{n+1} - {}^nC_1 y_n - a^2y_n = 0$$

$$\Rightarrow \quad (1-x^2)y_{n+2} + xy_{n+1}(-2n-1)$$

$$+ y_n[-n(n-1) - n - a^2] = 0$$

$$\Rightarrow \quad (1-x^2)y_{n+2} - (2n+1)xy_{n+1}$$

$$= (n^2 + a^2)y_n$$

69. Given, error in diameter $= \pm 0.04$

$\therefore$ Error in radius, $dr = \pm 0.02$

$\therefore$ Per cent error in the volume of sphere

$$= \frac{dV}{V} \times 100 = \frac{d\left(\frac{4}{3}\pi r^3\right)}{\frac{4}{3}\pi r^3} \times 100$$

$$= \frac{3dr}{r} \times 100$$

$$= \frac{3 \times (\pm 0.02)}{10} \times 100 = \pm 0.6$$

70. Given, $f(x) = x^3 + ax^2 + bx + c, \; a^2 \leq 3b.$

On differentiating w.r.t. *x*, we get

$$f'(x) = 3x^2 + 2ax + b$$

Put $f'(x) = 0$

$\Rightarrow$ $3x^2 + 2ax + b = 0$

$$\Rightarrow \quad x = \frac{-2a \pm \sqrt{4a^2 - 12b}}{2 \times 3}$$

$$= \frac{-2a \pm 2\sqrt{a^2 - 3b}}{3}$$

Since, $a^2 \le 3b$,

$\therefore$ x has an imaginary value.

Hence, no extreme value of x exist.

71. Let $y = \dfrac{\log x}{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \log x}{x^2} = \frac{1 - \log x}{x^2}$$

Put $\dfrac{dy}{dx} = 0 \Rightarrow \log x = 1$

$\Rightarrow$ $x = e$

$$\text{Now, } \frac{d^2y}{dx^2} = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \log x)\,2x}{(x^2)^2}$$

$$= -\frac{(3 - 2\log x)}{x^3}$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)_{(x = e)} = \frac{-(3-2)}{e^3} = -\frac{1}{e^3} < 0, \text{ maxima}$$

Hence, maximum value at $x = e$ is $\dfrac{1}{e}$.

72. Given, $z = \tan(y + ax) + \sqrt{y - ax}$

$$\Rightarrow z_x = \sec^2(y + ax)\, a + \frac{1}{2\sqrt{y - ax}}(-a)$$

$$\Rightarrow z_{xx} = 2\sec^2(y + ax)\tan(y + ax)a^2 + \frac{1\,(-a^2)}{4(y - ax)^{3/2}}$$

and $z_y = \sec^2(y + ax) + \dfrac{1}{2\sqrt{y - ax}}$

$$\Rightarrow z_{yy} = 2\sec^2(y + ax)\tan(y + ax) - \frac{1}{4(y - ax)^{3/2}}$$

$$\therefore \quad z_{xx} - a^2 z_{yy} = 0$$

73. Let $I = \displaystyle\int \frac{dx}{(x + 1)\sqrt{4x + 3}}$

Put $4x + 3 = t^2 \Rightarrow 4\,dx = 2t\,dt$

$$\therefore \quad I = \frac{1}{2}\int \frac{t\,dt}{\left(\frac{t^2 - 3}{4} + 1\right)t} = 2\int \frac{dt}{1 + t^2}$$

$$= 2\tan^{-1} t + c = 2\tan^{-1}\sqrt{4x + 3} + c$$

74. Let $I = \displaystyle\int \left(\frac{2 - \sin 2x}{1 - \cos 2x}\right) e^x\,dx$

$$= \int \left(\frac{2 - 2\sin x\cos x}{2\sin^2 x}\right) e^x\,dx$$

$$= \int \underset{\text{II}}{\operatorname{cosec}^2 x}\ \underset{\text{I}}{e^x}\,dx - \int \cot x\, e^x\,dx$$

$$= -\cot x\, e^x - \int (-\cot x)\, e^x\,dx - \int \cot x\, e^x\,dx + c$$

$$= -\cot x\, e^x + c$$

75. We know that, if

$I_n = \displaystyle\int \sin^n x\,dx$, then

$$I_n = -\frac{\sin^{n-1} x\cos x}{n} + \frac{n-1}{n} I_{n-2}$$

where n is a positive integer.

$\Rightarrow$ $nI_n - (n - 1)I_{n-2} = -\sin^{n-1} x\cos x$

76. Let $I = \displaystyle\int_0^{\pi} \frac{1}{1 + \sin x}\,dx$

$$= \int_0^{\pi} \frac{1}{1 + \dfrac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}}\,dx$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{\left(1 + \tan\frac{x}{2}\right)^2}\,dx$$

Put $\tan\dfrac{x}{2} = t \Rightarrow \dfrac{1}{2}\sec^2\dfrac{x}{2}\,dx = dt$

$$\therefore \quad I = \int_0^{\infty} \frac{2\,dt}{(1 + t)^2} = \left[-\frac{2}{1 + t}\right]_0^{\infty} = 2$$

77. Area, $A_1 = \displaystyle\int_0^{\pi/4} \sin x\,dx$

$$= -[\cos x]_0^{\pi/4} = 1 - \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2} - 1}{\sqrt{2}}$$

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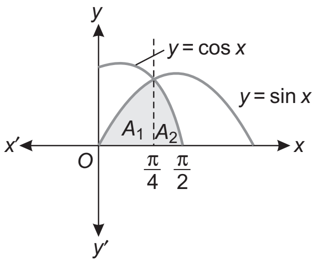

and area, $A_2 = \int_{\pi/4}^{\pi/2} \cos x \, dx$

$$= [\sin x]_{\pi/4}^{\pi/2} = \left[1 - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{\sqrt{2}-1}{\sqrt{2}}$$

$\therefore \quad A_1 : A_2 = \frac{\sqrt{2}-1}{\sqrt{2}} : \frac{\sqrt{2}-1}{\sqrt{2}} = 1 : 1$

78. Given table is

t	0	2	4	6	8	10
v	0	12	16	20	35	60

Here, $\quad h = \frac{10-0}{5} = 2$

$\therefore$ Total distance

$$= \frac{h}{2}[f(x_0) + 2\{f(x_1) + f(x_2) + f(x_3) + f(x_4)\} + f(x_5)]$$

$$= \frac{2}{2}[0 + 2(12 + 16 + 20 + 35) + 60]$$

$$= 166 + 60 = 226$$

79. Given, $\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1$

Put $x + y = z \quad \Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$

$\therefore \quad \frac{dz}{dx} - 1 = \sin z \tan z - 1$

$\Rightarrow \quad \int \frac{\cos z}{\sin^2 z} dz = \int dx$

Put $\quad \sin z = t$

$\Rightarrow \quad \cos z \, dz = dt$

$\therefore \quad \int \frac{1}{t^2} dt = x - c \quad \Rightarrow \quad -\frac{1}{t} = x - c$

$\Rightarrow \quad -\operatorname{cosec} z = x - c$

$\Rightarrow \quad x + \operatorname{cosec}(x+y) = c$

80. Given, $y = ae^x + bx\,e^x + cx^2e^x$ ...(i)

On differentiating w.r.t. x, we get

$y' = ae^x + b(xe^x + e^x) + c(x^2e^x + 2xe^x)$

$\Rightarrow \quad y' = ae^x + bxe^x + cx^2e^x + be^x + 2cxe^x$

$\Rightarrow \quad y' = y + be^x + 2cxe^x$ ...(ii)

Again differentiating w.r.t. x, we get

$y'' = y' + be^x + 2c(xe^x + e^x)$

$\Rightarrow \quad y'' = y' + be^x + 2cxe^x + 2ce^x$

$\Rightarrow \quad y'' = 2y' - y + 2ce^x$ ...(iii)

[from Eq. (ii)]

Again differentiating w.r.t. x, we get

$y''' = 2y'' - y' + 2ce^x$

$\Rightarrow \quad y''' = 2y'' - y' + (y'' - 2y' + y)$

[from Eq. (iii)]

$\Rightarrow \quad y''' - 3y'' + 3y' - y = 0$