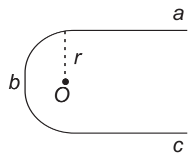
# MET 2010 Answer Key PDF

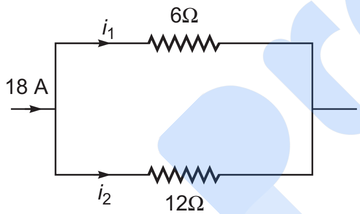
## Physics

**1.** Field due to a straight wire of infinite length is $\frac{\mu_0 i}{4\pi r}$ if the point is on a line perpendicular to its length while at the centre of a semicircular coil is $\frac{\mu_0 \pi i}{4\pi r}$.



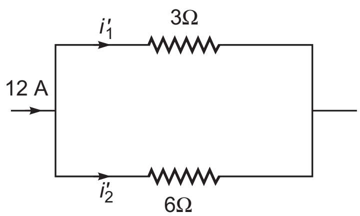

$$\therefore \quad B = B_a + B_b + B_c$$
$$= \frac{\mu_0}{4\pi}\frac{i}{r} + \frac{\mu_0}{4\pi}\frac{\pi i}{r} + \frac{\mu_0}{4\pi}\frac{i}{r}$$
$$= \frac{\mu_0}{4\pi}\frac{i}{r}(\pi + 2) \text{ out of the page}$$

**2.** As $3\,\Omega$ and $6\,\Omega$ resistances are in parallel their equivalent resistance will be $2\,\Omega$. Here $2\,\Omega$ and $4\,\Omega$ are in series, their equivalent resistance will be $6\,\Omega$.




From current distribution law

$$i_1 = \frac{12 \times 18}{18} = 12\text{ A}$$
$$i_2 = \frac{6 \times 18}{18} = 6\text{ A}$$




Now 12 A current is entering in parallel combination of $3\,\Omega$ and $6\,\Omega$ again from current distribution law

$$i_1' = \frac{6 \times 12}{9} = 8\text{ A}$$
$$i_2' = \frac{3 \times 12}{9} = 4\text{ A}$$

$\therefore$ Potential difference across $3\,\Omega$ resistance

$$= 8 \times 3 = 24\text{ V}$$

**3.** Let the temperature of junction be $\theta$. In equilibrium, rate of flow of heat through rod 1 is equal to sum of rate of flow of heat through rods 2 and 3, *ie*,

$$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2 + \left(\frac{dQ}{dt}\right)_3$$

$$\therefore \quad \frac{KA(\theta - 20)}{l} = \frac{KA(60 - \theta)}{l} + \frac{KA(70 - \theta)}{l}$$

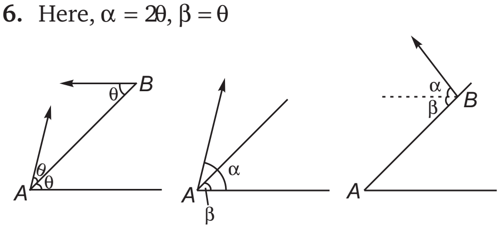
$$\Rightarrow \quad \theta - 20 = 130 - 2\theta$$
$$\text{or} \quad 3\theta = 150$$
$$\therefore \quad \theta = 50°\text{C}$$

**4.** The surface tension of liquids decreases with increase of temperature. For small temperature difference it decreases almost linearly. The surface tension of a liquid becomes zero at a particular temperature, called the critical temperature of that liquid.

**5.** Current amplification factor, $\beta = \frac{i_C}{i_B}$

$$\therefore \quad i_C = i_B \times \beta$$
$$= 250 \times 80\,\mu\text{A}$$

**6.** Here, $\alpha = 2\theta$, $\beta = \theta$




Time of flight of *A* is,

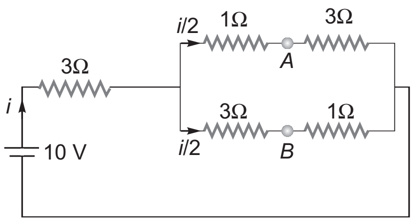
$$T_1 = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$
$$= \frac{2u\sin(2\theta - \theta)}{g\cos\theta}$$
$$= \frac{2u}{g}\tan\theta$$

Time of flight of B is, $T_2 = \frac{2u\sin\theta}{g\cos\theta}$

$$= \frac{2u}{g}\tan\theta$$

So, $T_1 = T_2$. The acceleration of both the particles is $g$ downwards. Therefore, relative acceleration between the two is zero or relative motion between the two is uniform. The relative velocity of $A$ w.r.t. $B$ is towards AB, therefore collision will take place between the two in mid air.

7. $R_{eq} = 5\Omega$, current $i = \frac{10}{5} = 2A$ and current in each branch = 1 A. Potential difference between $C$ and $A$,




$$V_C - V_A = 1 \times 1 = 1\text{ V} \qquad \text{...(i)}$$

Potential difference between $C$ and $B$

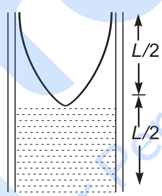
$$V_C - V_B = 1 \times 3 = 3\text{V} \qquad \text{...(ii)}$$

On solving Eqs. (i) and (ii) we get

$$V_A - V_B = 2\text{V}$$

8. Solar radiations are transverse electromagnetic waves. The central core of the sun emits a continuous electromagnetic spectrum.

9. If an open pipe is half submerged in water, it will become a closed organ pipe of length half that of open pipe as shown in figure. So, its frequency will become,




$$f_c = \frac{v}{4\left(\frac{L}{2}\right)} = \frac{v}{2L} = f_o$$

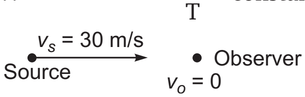
*ie,* equal to that of open pipe *ie,* frequency will remain unchanged.

10. Rms velocity is given by

$$v = \sqrt{\frac{3kT}{m}} \quad \text{or} \quad v^2 = \frac{3kT}{m}$$

For a gas, $k$ and $m$ are constants.

$$\therefore \qquad \frac{v^2}{T} = \text{constant}$$

11. 


Here, n = 500 Hz, $v_o = 0$

$v_s = 30$ m/s, v = 330 m/s

$$\therefore \text{ From,} \qquad n' = n\left(\frac{v - v_o}{v - v_s}\right)$$

$$= 500\left(\frac{330}{330 - 30}\right)$$

$$= 550 \text{ Hz}$$

12. The value of acceleration due to gravity at height $h$ (when $h$ is not negligible as compared to $R$)

$$g' = g\frac{R^2}{(R+h)^2}$$

Here, $$g' = \frac{g}{2}$$

$$\therefore \qquad \frac{g}{2} = g\frac{R^2}{(R+h)^2}$$

or $$\frac{1}{2} = \frac{R^2}{(R+h)^2}$$

or $$\sqrt{\frac{1}{2}} = \frac{R}{R+h}$$

or $$R + h = \sqrt{2}R$$

$$\therefore \qquad h = (\sqrt{2} - 1)R$$

13. For working safely, the activity must reduce to $\frac{1}{64}$.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \frac{1}{64}$$

$$\therefore \qquad n = 6$$

Thus, $$t = nT = 6 \times 2 = 12\text{ h}$$

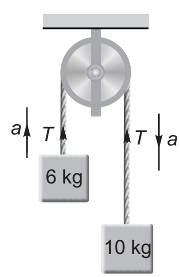
14. Potential difference across diode

$$= 3.2 - 3 = 0.2\text{ V}$$

$\therefore$ Current through diode

$$i = \frac{0.2}{100} = 2 \times 10^{-3}\text{A}$$

15.




$$10g - T = 10a \qquad \text{...(i)}$$

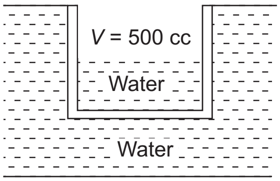
$$T - 6g = 6a \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$T = 75\text{N}$$

**16.** As the flask floats in water when less than half filled with water, it will float just fully submerged when half filled. In this situation, mass of flask + mass of water in it = $V\sigma$

*ie,* $390 + 250 = V \times 1 \text{ (as } \sigma = 1 \text{ g/cc)}$




*ie,* outer volume of flask

$$V = 640 \text{ cc}$$

Now as inner volume of flask is given to be 500 cc, so the volume of the material of flask $= 640 - 500 = 140$ cc. But as mass of flask is 390 g, so density of material of flask

$$\rho = \frac{m}{V} = \frac{390}{140} = 2.8 \text{ g/cc}$$

**17.** $A = 60°, \delta_m = 30°$

As $$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$\therefore$ $$\mu = \frac{\sin\frac{(60° + 30°)}{2}}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

**18.** Let $n_1$ and $n_2$ be the frequencies of the two notes of wavelengths $\lambda_1$ and $\lambda_2$.

Here, $\lambda_1 = 1 \text{ m}, \lambda_2 = 1.01 \text{ m}$

If $v$ is velocity of sound, then

$$n_1 = \frac{v}{1} \quad \text{and} \quad n_2 = \frac{v}{1.01}$$

If the number of beats produced per second is $b$, then

$$b = n_1 - n_2$$

$$4 = \frac{v}{1} - \frac{v}{1.01}$$

$\therefore$ $$v = \frac{4.04}{0.01} = 404 \text{ m/s}$$

**19.** Work done = area of trapezium

$$= \frac{1}{2} \times (8 \times 10^5 + 4 \times 10^5) \times 0.2$$

$$= 1.2 \times 10^5 \text{ J}$$

**20.** As $F \propto v^2$

or $$F = kv^2$$

$\therefore$ $$k = \frac{F}{v^2}$$

$$[k] = \frac{[F]}{[v^2]} = \frac{[MLT^{-2}]}{[L^2T^{-2}]}$$

$$= [ML^{-1}T^0]$$

**21.** If temperature of surrounding is considered, then net loss of energy of a body by radiation

$$Q = Ae\sigma(T^4 - T_0^4) \Rightarrow Q \propto (T^4 - T_0^4)$$

$\therefore$ $$\frac{Q_1}{Q_2} = \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4}$$

$$= \frac{(273 + 327)^4 - (273 + 27)^4}{(273 + 427)^4 - (273 + 27)^4}$$

$$= \frac{(600)^4 - (300)^4}{(700)^4 - (300)^4} = 0.52$$

**22.** Resistance of 40 W bulb $= \frac{240 \times 240}{40}$

$$= 1440 \, \Omega$$

Its safe current $= \frac{240}{1440} = 0.167$ A

Resistance of 60 W bulb $= \frac{240 \times 240}{60}$

$$= 960 \, \Omega$$

Its safe current $= \frac{240}{960} = 0.25$ A

When connected in series to 420 V supply, then the current

$$i = \frac{420}{1440 + 960} = \frac{420}{2400}$$

$$= 0.175 \text{ A}$$

Thus, current is greater for 40 W bulb, so it will fuse.

**23.** Angular velocity $\omega$ is constant.

$\therefore$ $$v \propto r \quad \text{or} \quad \frac{v_A}{v_B} = \frac{r_A}{r_B}$$

**24.** According to equation of motion

$$v = u + at$$

$\Rightarrow$ $$-2 = 10 + a \times 4$$

$\therefore$ $$a = -3 \text{ m/s}^2$$

**25.** According to Brewster's law

$$\mu = \tan i_p = \tan 60° = \sqrt{3}$$

**26.** Young's modulus, $Y = \frac{mgl}{a_1 l_1}$

$\therefore$ $$l_1 = \frac{mgl}{Y\pi r^2} \qquad \ldots(i)$$

and $$Y = \frac{mg(2l)}{a_2 l_2} = \frac{mg(2l)}{\pi(2r)^2 l_2}$$

or $$l_2 = \frac{mgl}{2Y\pi r^2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$\therefore$ $$l_1 + l_2 = \frac{mgl}{Y\pi r^2} + \frac{mgl}{2Y\pi r^2} = \frac{3}{2}\frac{mgl}{Y\pi r^2}$$

**27.** Given, $x = 0.20$ m, $y = 0.20$ m, $u = 1.8$ m/s.
Let the ball strikes the *n*th step of stairs,
Vertical distance travelled

$$= ny = n \times 0.20 = \frac{1}{2}gt^2$$

Horizontal distance travelled, $nx = ut$

or $$t = \frac{nx}{u}$$

$\therefore$ $$ny = \frac{1}{2}g \times \frac{n^2x^2}{u^2}$$

or $$n = \frac{2u^2}{g}\frac{y}{x^2} = \frac{2 \times (1.8)^2 \times 0.20}{9.8 \times (0.20)^2} = 3.3 \approx 4$$

**28.** $E = E_0 \cos \omega t = 10 \cos(2\pi \times ft)$

$$= 10 \cos\left(2\pi \times 50 \times \frac{1}{600}\right)$$

$$= 10 \cos\left(\frac{\pi}{6}\right) = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ V}$$

**29.** Induced emf produced in coil

$$e = \frac{-d\phi}{dt} = \frac{-d}{dt}(BA)$$

$\therefore$ $$|e| = A\frac{dB}{dt} = 0.01 \times \frac{1}{1 \times 10^{-3}}$$

$$|e| = 10 \text{ V}$$

Current produced in coil,

$$i = \frac{|e|}{R} = \frac{10}{2} = 5 \text{ A}$$

Heat evolved $= i^2Rt = (5)^2 \times (2) \times 1 \times 10^{-3}$

$$= 0.05 \text{ J}$$

**30.** From displacement method size of object, $O = \sqrt{I_1 I_2}$

Here, $O = 3$ cm, $I_1 = 9$ cm,

$\therefore$ $$3 = \sqrt{9I_2}$$

or $$I_2 = 1 \text{ cm}$$

**31.** As gas is suddenly expanded so it is an adiabatic process,

*ie,* $$pV^{\gamma} = \text{constant}$$

or $$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

Given $V_2 = 3V_1$, $C_V = 2R$

$\therefore$ $$C_p = 2R + R = 3R$$

$\Rightarrow$ $$\gamma = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

$\therefore$ $$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} = (3)^{1.5} = 5.1 \approx 5$$

**32.** When bigger pendulum of time period $\frac{5T}{4}$ completes one vibration, the smaller pendulum will complete $\left(\frac{5}{4}\right)$ vibrations. It means the smaller pendulum will be leaded the bigger pendulum by phase $\frac{T}{4}$ sec $= \frac{\pi}{2}$ rad $= 90°$.

**33.** Given, $u = 72$ km/h $= 20$ m/s

$a = \mu g = 0.5 \times 10$ m/s$^2$

From $$v^2 = u^2 - 2as$$

$\therefore$ $$(0)^2 = (20)^2 - 2 \times 0.5 \times 10 \times s$$

$\therefore$ $$s = \frac{20 \times 20}{2 \times 0.5 \times 10}$$

or $$s = 40 \text{ m}$$

**34.** Given, $s = 2$ m, $u = 80$ m/s, $v = 0$

From $$v^2 = u^2 - 2as$$

$\therefore$ $$(0)^2 = (80)^2 - 2 \times a \times 2$$

or $$a = \frac{80 \times 80}{4} = 1600 \text{ m/s}^2$$

**35.** Net induced emf due to disc is

$$\varepsilon = \frac{1}{2}B\omega l^2$$

$$= \frac{1}{2} \times 0.1 \times 2\pi \times 10 \times 0.1 \times 0.1$$

$$= \pi \times 10^{-2} \text{V}$$

**36.** According to first law of thermodynamics

$$\Delta Q = \Delta U + \Delta W$$

$\therefore$ $$\Delta U = \Delta Q - \Delta W$$

$$= 540 - \frac{p(V_2 - V_1)}{J}$$

$$= 540 - \frac{1.013 \times 10^5 \times [(1671 - 1) \times 10^{-6}]}{4.2}$$

$$= 540 - 40 = 500 \text{ cal}$$

**37.** Frequency of vibration is given by

$$n = \frac{p}{2l}\sqrt{\frac{T}{m}} \quad \left(\text{where } v = \sqrt{\frac{T}{m}}\right)$$

$\therefore$ $$n = \frac{pv}{2l} = \frac{5 \times 20}{2 \times 10} = 5 \text{ Hz}$$

**38.** $v_{max} = A\omega$

when $A$ becomes twice $v_{max}$ is also doubled.

**39.** From Kepler's third law of planetary motion

$$T^2 \propto R^3$$

$$\therefore \quad \frac{T_2^2}{T_1^2} = \frac{R_2^3}{R_1^3}$$

$$\text{or} \quad \frac{T_2^2}{(24)^2} = \left(\frac{6400 + 6400}{36000 + 6400}\right)^3$$

$$\text{or} \quad T_2^2 = (24)^2 \times \left(\frac{16}{53}\right)^3$$

$$\Rightarrow \quad T_2 = 4\,\text{h}$$

**40.** From the law of conservation of momentum

$$3 \times 16 + 6 \times v = 9 \times 0$$

$$\text{or} \quad v = -8\,\text{m/s}$$

$$\Rightarrow \quad v = 8\,\text{m/s} \quad \text{(numerically)}$$

Therefore, its kinetic energy

$$K = \frac{1}{2} \times 6 \times (8)^2 = 192\,\text{J}$$

**41.** In the reaction

$${}_7N^{14} + \alpha \longrightarrow {}_8X^{17} + {}_1p^1$$

8 is the atomic number of oxygen molecule.

So, here $X$ is oxygen $(O_2)$ molecule.

**42.** Induced emf in the coil is given by

$$e = \frac{L\,di}{dt}$$

$$\text{or} \quad e = 10 \times \frac{(10-5)}{0.2}$$

$$\Rightarrow \quad e = 250\,\text{V}$$

**43.** New force $F = \dfrac{1}{4\pi\varepsilon_0} \dfrac{(6-2)(2-2)}{r^2} \times 10^{-12}$

$$= 0$$

**44.** Given, $N_P = 20$, $N_S = 10$, $e_P = 220\,\text{V}$

$\therefore$ Transformation ratio, $k$

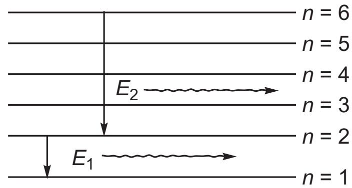
$$\frac{e_S}{e_P} = \frac{N_S}{N_P}$$

$$\text{or} \quad e_S = \frac{N_S}{N_P} \times e_P$$

$$= \frac{10}{20} \times 220 = 110\,\text{V}$$

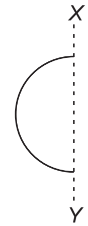
**45.** As $\quad E_1 > E_2$

$\therefore \quad \nu_1 > \nu_2$




*ie*, photons of higher frequency will be emitted if transition takes place from n = 2 to n = 1.

**46.** Here,




$$L = \pi R \quad \text{or} \quad R = \frac{L}{\pi}$$

$\therefore$ Moment of inertia about XY

$$= \frac{1}{2}\left(\frac{1}{2} MR^2\right)$$

$$= \frac{1}{4} \frac{ML^2}{\pi^2}$$

**47.** Let a be the acceleration of each block. Then,

$$T_3 = (m_1 + m_2 + m_3)a \quad \ldots\text{(i)}$$

$$\text{and} \quad T_2 = (m_1 + m_2)a \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$T_2 = \left(\frac{m_1 + m_2}{m_1 + m_2 + m_3}\right) \times T_3$$

$$= \left(\frac{10 + 6}{10 + 6 + 4}\right) \times 40 = 32\,\text{N}$$

**48.** $H = \dfrac{u^2 \sin^2\theta}{2g}$ and $T = \dfrac{2u\sin\theta}{g}$

$$\Rightarrow \quad T^2 = \frac{4u^2\sin^2\theta}{g^2}$$

$$\therefore \quad \frac{T^2}{H} = \frac{8}{g}$$

$$\Rightarrow \quad T = \sqrt{\frac{8H}{g}} = 2\sqrt{\frac{2H}{g}}$$

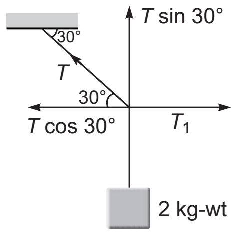
**49.** The net capacitance $C = \dfrac{\varepsilon_0 A}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}}$

$$= \frac{\varepsilon_0 A}{\dfrac{d}{2}\left(\dfrac{1}{1} + \dfrac{1}{2}\right)} = \frac{4\varepsilon_0 A}{3d}$$

**50.** Effective spring constant of parallel combination

$$k_e = k_1 + k_2$$

**51.** $T \sin 30° = 2\,\text{kg-wt}$




$$\Rightarrow \quad T = 4\,\text{kg-wt}$$
$$T_1 = T\cos 30°$$
$$= 4\cos 30° = 2\sqrt{3}$$

**52.** $KE = \dfrac{p^2}{2m} \Rightarrow KE_2 = KE_1\left(\dfrac{p_2}{p_1}\right)^2 = KE_1\left(\dfrac{2p}{p}\right)^2$

$$\Rightarrow \quad KE_2 = 4KE_1$$
$$= KE_1 + 3KE_1 = KE_1 + 300\%\text{ of } KE_1$$

**53.** Angular width $\theta = \dfrac{\beta}{D}$

$$= \frac{\lambda}{d} = \frac{4800\times10^{-10}}{0.6\times10^{-3}}$$
$$= 8\times10^{-4}\text{ rad}$$

**54.** Potential difference $V = \dfrac{hc}{e\lambda}$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times2\times10^{-10}}$$
$$= 6200\text{ volt}$$

**55.** Loss in kinetic energy

$$= \frac{1}{2}\frac{m_1 m_2 (u_1 - u_2)^2}{(m_1 + m_2)}$$
$$= \frac{1}{2}\frac{m\cdot m(u_1 - u_2)^2}{(m+m)}$$
$$= \frac{m}{4}(u_1 - u_2)^2$$

**56.** Torque $\vec{\tau} = \dfrac{d\vec{L}}{dt} = \dfrac{L_2 - L_1}{\Delta t} = \dfrac{4A_0 - A_0}{4} = \dfrac{3A_0}{4}$

**57.** When a number of small droplets coalesce to form a bigger drop surface energy is released because its surface area decreases.

**58.** If man slides down with some acceleration, then its apparent weight decreases. For critical condition rope can bear only 2/3 of his weight. If $a$ is the minimum acceleration, then

tension in the rope $= m(g - a)$

$= $ breaking strength

$$\Rightarrow \quad m(g-a) = \frac{2}{3}mg$$
$$\Rightarrow \quad a = g - \frac{2g}{3} = \frac{g}{3}$$

**59.** Resulting amplitude of two interfering waves

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$$

Here, $\phi = \pi/3$, $a_1 = 4$, $a_2 = 3$

$$\Rightarrow \quad A \simeq 6$$

**60.** The saturation vapour pressure is the static pressure of a vapour when the vapour pressure of some material is in equilibrium with the liquid phase of that material. The saturation vapour pressure of any material is solely dependent on the temperature of that material. Since, temperature change does not occur in the given case, hence pressure will be the same.

# Chemistry

**1.** $$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$\therefore \quad v_{rms} \propto \sqrt{T}$

$\therefore$ At two different temperatures,

$$\frac{v_{rms}}{v'_{rms}} = \sqrt{\frac{T}{T'}}$$

Given, $\quad v'_{rms} = 2v_{rms}$

$$\frac{1}{2} = \sqrt{\frac{T}{T'}}$$

or $$\frac{1}{4} = \frac{T}{T'}$$

$\therefore \quad T' = 4T$

$\therefore$ $v_{rms}$ gets doubled, when the temperature is increased four times.

**2.** All the reaction proceed by stable ions. After the lose of $H^+$ ion, phenol forms phenoxide ion. The phenoxide ion is resonance stabilized, thus makes the phenol more acidic.

**3.** $O^{2-}$ and $N^{3-}$ both are isoelectronic but differ in the charge possessed by them. As the negative charge increases, the electrons are held less and less tightly by the nucleus, therefore ionic radii increases. Hence, ionic radii of $N^{3-}$ is greater than $O^{2-}$.

In a period from left to right atomic radii decreases but in a group on moving downwards it increases.

**4.** According to Bohr model,
radius of hydrogen atom

$$(r_n) = \frac{0.529 \times n^2}{Z} \text{ Å}$$

(where, n = number of orbit, Z = atomic number)

$$r_3 = \frac{0.529 \times (3)^2}{1} = 4.761 \text{ Å}$$

**5.** $Co^{3+}$ and $Pt^{4+}$ have 6 coordination number.
$CoCl_3 \cdot 6NH_3$ and $PtCl_4 \cdot 5NH_3$

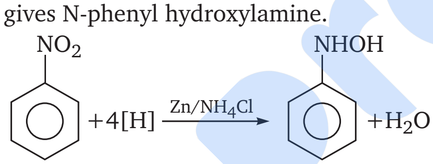
$$[Co(NH_3)_6]Cl_3 \xrightarrow{\text{In solution}} [Co(NH_3)_6]^{3+} + 3Cl^-$$

$$[PtCl(NH_3)_5]Cl_3 \xrightarrow{\text{In solution}} [PtCl(NH_3)_5]^{3+} + 3Cl^-$$

Number of ionic species are same in the solution of both complexes, therefore their equimolar solutions will show same conductance.

**6.** Entropy is the measure of randomness. In liquids randomness is more than solids.
$\because$ When ice melts, randomness increases, (solid $\longrightarrow$ liquid)
$\therefore$ Entropy increases.

**7.** Reduction of nitrobenzene by Zn and $NH_4Cl$ gives N-phenyl hydroxylamine.



N-phenyl hydroxylamine

**8.** Proteins are the polymer of amino acids and contain polypeptide bonds. DNA, however, are the polymers of nucleotide and contain no polypeptide bond. Hence, DNA is not a protein.

**9.** D-glucose, D-fructose and D-mannose form the same osazone when treated with excess of phenyl hydrazine because they differ only in Ist and 2nd carbon atoms which are transformed to the same form.

| D-glucose | D-fructose | D-mannose |
|---|---|---|
| CHO | $CH_2OH$ | CHO |
| H—C—OH | C=O | HO—C—H |
| HO—C—H | HO—C—H | HO—C—H |
| H—C—OH | H—C—OH | H—C—OH |
| H—C—OH | H—C—OH | H—C—OH |
| $CH_2OH$ | $CH_2OH$ | $CH_2OH$ |

They form the following osazone

$CH{=}NNHC_6H_5$
|
$C{=}NNHC_6H_5$
|
$(CHOH)_3$
|
$CH_2OH$

**10.** $$Ag^+ + e^- \longrightarrow Ag$$

$\because$ 96500 C are required to deposite Ag $= 108$ g

$\therefore$ 965 C are required to deposite Ag

$$= \frac{108}{96500} \times 965$$
$$= 1.08 \text{ g}$$

**11.** $$E^\circ_{cell} = \frac{2.303\,RT}{nF} \log K_{eq}$$

$$E^\circ_{cell} = \frac{0.0591}{n} \log K_{eq} \quad \text{(At 298 K)}$$

$$0.591 = \frac{0.0591}{1} \log K_{eq}$$

$\therefore$ $\log K_{eq} = 10$
$\therefore$ $K_{eq} = 1 \times 10^{10}$

**12.** Higher the negative value of E°, more is the reducing power. The order of E° values (negative value) is

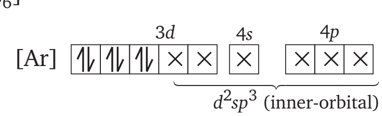
$$\underset{(Mg)}{-2.37} > \underset{(Zn)}{-0.76} > \underset{(Fe)}{-0.44}$$

$\therefore$ Mg can reduce both $Zn^{2+}$ and $Fe^{2+}$. Zn can reduce $Fe^{2+}$, but not $Mg^{2+}$. Fe cannot reduce Mg and Zn but can oxidise them.

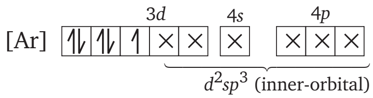
**13.** In diamond,
the maximum proportion of available volume that can be filled by hard spheres $= \frac{\pi\sqrt{3}}{16}$
$= 0.34$

**14.** On adding a pentavalent impurity with germanium, we get n-type of semiconductors because excess of electrons is responsible for conduction.

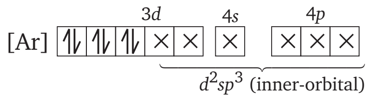
**15.** The complex in which nd orbitals are used in hybridisation, are called outer orbital complex.
(a) $[Fe(CN)_6]^{4-}$ =



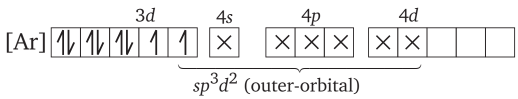
(b) $[Mn(CN)_6]^{4-}$ =




(c) $[Co(NH_3)_6]^{3+}$ =



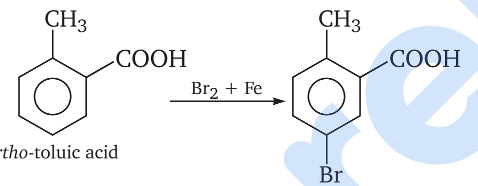

(d) $[Ni(NH_3)_6]^{2+}$ =




**16.** $C_2H_5Br + AgNO_2$ (alc.) $\rightarrow \underset{\text{nitro ethane}}{C_2H_5NO_2} + AgBr$

**17.** *Iso*-propyl chloride is a 2° halide and 2° halides can undergo hydrolysis either by $S_N1$ or $S_N2$ mechanism depending upon the nature of solvent used.

**18.**




*ortho*-toluic acid

($\because$ In the product, —Br is *para* to —$CH_3$ and *meta* to —COOH.)

**19.** —$NO_2$ group at any position shows electron withdrawing effect, thus acid strength is increased. But o-nitro benzoic acid believed to have *ortho* effect. As a result, resonance gets prevented. Hence, its acid strength is maximum, thus, the order of acid strength (II) < (III) > (IV) > (I)

(The effect is more at *para* position than *meta*.)

**20.** Iodoform test is given by the compounds containing either $CH_3CO$—group or $CH_3\overset{|}{C}HOH$ group.

The structures of the given compounds are as

(a) $CH_3CH_2CH_2CH_2OH$

(b) $CH_3COC_6H_5$

(c) $CH_3CHO$

(d) $CH_3COC_2H_5$

$\therefore$ n butyl alcohol does not give iodoform test because it does not possess the $CH_3CO$—or $CH_3\overset{|}{C}HOH$ group.

**21.** A + NaOH $\longrightarrow$ alcohol + acid

Thus, it is Cannizaro reaction.

A is thus aldehyde without H at α-carbon.

(like $C_6H_5CHO$, HCHO)

$\underset{\text{benzaldehyde}}{2C_6H_5CHO} + NaOH \longrightarrow C_6H_5CH_2OH + C_6H_5COONa$

**22.** $\overset{5}{C}H_3—\overset{4}{C}H(OH)—\overset{3}{C}H=\overset{2}{C}(CH_3)—\overset{1}{C}HO$

4-hydroxy-2-methylpent-2-en-1-al

**23.** $E_a$ (A → B) = 80 kJ mol$^{-1}$

Heat of reaction (A → B) = 200 kJ mol$^{-1}$

For (B → A) backward reaction,

$E_a$ (B → A) = $E_a$ (A → B) + heat of reaction

$= 80 + 200 = 280$ kJ mol$^{-1}$

**24.** $Fe(OH)_3$ is a positively charged sol, thus coagulated by negative ion (anion). Smaller the charge on anion, smaller is its coagulating power or higher is its flocculation value.

(a) $KBr \longrightarrow K^+ + Br^-$

(b) $K_2SO_4 \longrightarrow 2K^+ + SO_4^{2-}$

(c) $K_2CrO_4 \longrightarrow 2K^+ + CrO_4^{2-}$

(d) $K_4[Fe(CN)_6] \longrightarrow 4K^+ + [Fe(CN)_6]^{4-}$

$\because Br^-$ has smaller charge.

$\therefore$ KBr is least effective in coagulating $Fe(OH)_3$ sol.

**25.** Maximum oxidation state exhibited by d block elements (O.S) = no. of ns electrons + no. of (n − 1) d electrons.

(a) O. S. = 2 + 2 = 4 (b) O. S. = 5 + 1 = 6

(c) O. S. = 3 + 2 = 5 (d) O. S. = 5 + 2 = 7

$\therefore$ $(n-1)d^5ns^2$ configuration will achieve the highest oxidation state.

**26.** Relative lowering of vapour pressure = mole fraction of solute **(Raoult's law)**

$$\frac{p - p_S}{p} = \chi_2$$

$$\frac{p - p_S}{p} = \frac{wM}{mW}$$

$$0.0125 = \frac{wM}{mW}$$

or $$\frac{w}{mW} = \frac{0.0125}{18} = 0.00070$$

Hence, $$\text{molality} = \frac{w}{mW} \times 1000$$

$$= 0.0007 \times 1000 = 0.70$$

**27.** $H_2O_2$ oxidises the acidified potassium dichromate solution into blue peroxide of chromium, $CrO_5$.

$$Cr_2O_7^{2-} + 2H^+ + 4H_2O_2 \longrightarrow 2CrO_5 + 5H_2O$$

**28.** $NaOH + S \longrightarrow \underset{\text{'A'}}{Na_2SO_3} + Na_2S + H_2O$

**29.** On igniting at 1400°C, $Fe_2O_3$ get reduced to metallic Fe.

$$3Fe_2O_3 + CO \longrightarrow 2Fe_3O_4 + CO_2$$
$$Fe_3O_4 + CO \longrightarrow 3FeO + CO_2$$
$$FeO + CO \longrightarrow Fe + CO_2$$

**30.** $H_2SO_4$ forms hydrate with water. That's why it has great affinity towards water.

**31.** In blood, He is much less soluble than nitrogen, hence He—$O_2$ mixture is used by deep sea divers in preference to $N_2$—$O_2$ mixture.

**32.** One mole $= 6.02 \times 10^{23}$ molecules/atoms/ions.

$\because$ $CO_2$ molecule has one C and two oxygen atoms.

$\therefore$ $CO_2$ contains $6.023 \times 10^{23}$ atoms of C and $6.023 \times 10^{23}$ molecules of $O_2$.

**33.** In acidic medium following reaction takes place.

$$8H^+ + 5e^- + MnO_4^- \longrightarrow Mn^{2+} + 4H_2O$$

$\therefore$ Equivalent weight of $KMnO_4$ in acidic medium $= \dfrac{\text{molecular weight of } KMnO_4}{5}$

$$= \frac{158}{5}$$
$$= 31.6$$

**34.** (a) Density of water $= 1 \text{ g cm}^{-3}$

Mass of water $= 1 \text{ m}^3 = 10^6 \text{ cm}^{-3}$

$$\text{Mass} = \text{volume} \times \text{density}$$
$$= 10^6 \text{ cm}^{-3} \times 1 \text{ g cm}^{-3}$$
$$= 10^6 \text{ g}$$
$$= \frac{10^6}{10^3} \text{ kg}$$
$$= 1000 \text{ kg}$$

(b) Mass of normal adult man $= 65$ kg

(c) Density of Hg $= 13.6 \text{ g cm}^{-3}$

Volume of Hg $= 10 \text{ L} = 10 \times 1000 \text{ cm}^{-3}$

$\therefore$ Mass of Hg $= 13.6 \times 10 \times 1000$

$= 136000$ g

$= 13.6$ kg

$\therefore$ Mass of 1 $m^3$ water is highest.

**35.** e/m ratio for $He^{2+} = \dfrac{2}{4}$

e/m ratio for $H^+ = \dfrac{1}{1}$

e/m ratio for $He^+ = \dfrac{1}{4}$

e/m ratio for $D^+ = \dfrac{1}{2}$

$\therefore$ The e/m is highest for hydrogen.

**36.** The ionisation potential decreases down the group (due to increase in size of atom) and increases in a period from left to right.

$\therefore$ Out of the given choices Li > K > Cs is correct.

**37.** Let the oxidation state of Fe in $Fe_3O_4 = x$

$\therefore$ $3x + 4 \times (-2) = 0$

or $3x - 8 = 0$

$\therefore$ $x = \dfrac{8}{3}$

**38.** $T_{50} = 15$ min

$$k = \frac{2.303 \log 2}{T_{50}} = \frac{2.303 \log 2}{15}$$

$a = 0.1$ M

$(a - x) = 0.025$ M

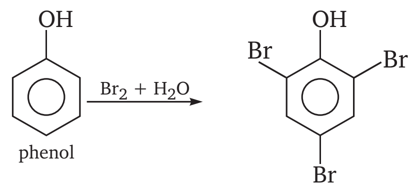
For first order reaction,

$$k = \frac{2.303}{t} \log\left(\frac{a}{a-x}\right)$$
$$\frac{2.303 \log 2}{15} = \frac{2 \times 2.303}{t} \log \frac{0.1}{0.025}$$
$$= \frac{2.303}{t} \log 4$$

$\therefore$ $\dfrac{2.303 \log 2}{15} = \dfrac{2 \times 2.303 \log 2}{t}$

$\therefore$ $t = 30$ min

**39.**




phenol

2,4,6-tribromophenol

**40.** $CH_3COOH \xrightarrow[\Delta]{Cl_2/\text{Red P}} \underset{\alpha\text{-chloroacetic acid}}{CH_2ClCOOH}$

This reaction is called Hell-Volhard-Zelinsky reaction.

**41.** $\underset{1^\circ \text{ amine}}{CH_3NH_2} + \underset{\text{Hinsberg's reagent}}{C_6H_5SO_2Cl} \longrightarrow \underset{N\text{-methyl benzene sulphonamide}}{C_6H_5SO_2NHCH_3}$

$\xrightarrow[\text{(alkali)}]{\text{NaOH}} \underset{\text{soluble sodium salt}}{C_6H_5SO_2N(Na)CH_3}$

**42.** Angle strain, $\alpha = \frac{1}{2}[109°28' - \theta]$

In case of cyclopropane,

$$\theta = 60°$$

$$\therefore \quad \alpha = \frac{1}{2}(109°28' - 60°)$$

$$= 24°44'$$

**43.** An species or group of atoms can act as ligand only when it carries an unshared pair, *ie,* lone pair of electrons.

**44.** Bond order $\propto \frac{1}{\text{bond length}}$

$$\text{BO of NO} < \text{BO of } NO^+$$

$\therefore$ Bond length of NO is greater than the bond length of $NO^+$.

**45.** $\pi\, 2p_x$ and $\pi\, 2p_y$ or $\overset{*}{\pi}\, 2p_x$ and $\overset{*}{\pi}\, 2p_y$ orbitals have nearly equal energy and thus, are called degenerate orbitals.

**46.** pH varies inversely with temperature. Thus, if the pH of neutral water is 6.5, the temperature of water is more than 25°C.

**47.** Electron acceptors are Lewis acids. They are electron deficient compounds. $BF_3$ is Lewis acid because B has only 6 electrons in its valence shell and it can accept electrons. $NF_3$, $Cl^-$ and $H_2O$ have lone pair of electrons. Thus, they are electron donors and Lewis bases.

**48.** Given, concentration of NaOH $= 10^{-10}$ M

$$\underset{10^{-10}}{NaOH} \longrightarrow \underset{10^{-10}}{Na^+} + \underset{10^{-10}}{OH^-}$$

$\therefore \quad [OH]^-$ from NaOH $= 10^{-10}$

$\therefore \quad$ Total $[OH^-] = 10^{-7} + 10^{-10}$

$$= 10^{-7}(1 + 0.001)$$

$$= 10^{-7}\left(\frac{1001}{1000}\right)$$

$$= 10^{-10} \times 1001$$

$$pOH = -\log[OH^-]$$

$$= -\log(1001 \times 10^{-10})$$

$$= -3.004 + 10$$

$$= 6.9996$$

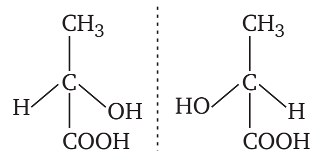
$\because \quad pH + pOH = 14$

$\therefore \quad pH = 14 - 6.9996$

$$= 7.0004 \approx 7$$

**49.** Carbanions contain even number of valence electrons and thus, show diamagnetic behaviour.

**50.** Enantiomers are non-superimposable mirror images. *eg,* lactic acid




Diastereomers are non-superimposable and are not the mirror images of each other. Moreover, *meso* form has plane of symmetry.

**51.** Inductive effect involves only displacement (and not delocalisation) of σ-electrons.

**52.** Given, concentration of NaCl = 0.2 M

$$K_{sp}(AgCl) = 1.20 \times 10^{-10}$$

Let the solubility of AgCl in NaCl = x

$$\underset{x}{AgCl} \longrightarrow \underset{x}{Ag^+} + \underset{x}{Cl^-}$$

Solubility $\quad \underset{0.2}{NaCl} \longrightarrow \underset{0.2}{Na^+} + \underset{0.2}{Cl^-}$

$\therefore \quad [Ag^+] = x$ and $[Cl^-] = (x + 0.2)$

$\therefore \quad K_{sp}(AgCl) = [Ag^+][Cl^-]$

$$= x(x + 0.2)$$

$$= x^2 + 0.2x$$

$\therefore \quad K_{sp} = 0.2x \; (x^2 << 1)$

or $\quad 1.2 \times 10^{-10} = 0.2x$

$\therefore \quad x = 6 \times 10^{-10}$

**53.** Naturally occurring fats are called lipids.

**54.** Pepsin hydrolyses proteins into amino acids as

$$\text{Proteins} \xrightarrow{\text{Pepsin}} \text{amino acids.}$$

**55.** The enthalpy of formation of $Al_2O_3$ is very high and hence, it is not possible to reduce it by carbon.

**56.** $\underset{\text{oxalic acid}}{\begin{array}{c} COOH \\ | \\ COOH \end{array}} + \text{conc. } H_2SO_4 \longrightarrow CO + CO_2 + H_2O$

Concentrated $H_2SO_4$ is a strong dehydrating agent.

**57.** Osmotic pressure is a colligative property *ie,* depends only upon the number of particles or ions in solution. More the number of ions in solution, more will be the osmotic pressure of solution.

(i) 0.1 M urea and 0.1 M glucose will have same number of molecules in solution as they don't ionise.

(ii) $KCl \longrightarrow K^+ + Cl^-$ (2 ions)

(iii) $CaCl_2 \longrightarrow Ca^{2+} + 2Cl^-$ (3 ions)

$\therefore CaCl_2$ produces maximum number of ions.

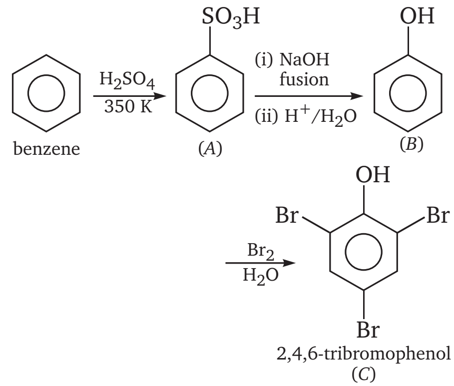
$\therefore$ It will have highest osmotic pressure.

**58.** Quick lime, CaO is used to dry ammonia, as with other given dehydrating agents ammonia reacts.

$$4NH_3 + CaCl_2 \longrightarrow CaCl_2 \cdot 4NH_3$$
$$4NH_3 + 2P_2O_5 \longrightarrow 4NH_4PO_3$$

$Ca(OH)_2$ is never used as dehydrating agent.

**59.**




**60.** Collin's reagent is used to convert

$$—CH_2OH \longrightarrow —CHO.$$

# Mathematics

**1.** f(x) is defined, if $3x^2 - 4x + 5 > 0$

$$\Rightarrow \quad 3\left[x^2 - \frac{4}{3}x + \frac{5}{3}\right] > 0$$

$$\Rightarrow \quad 3\left[\left(x - \frac{2}{3}\right)^2 + \frac{11}{9}\right] > 0$$

which is true for all real *x*.

$\therefore$ Domain of $f = (-\infty, \infty)$

Let $y = \log_e(3x^2 - 4x + 5) \Rightarrow e^y = 3x^2 - 4x + 5$

$\Rightarrow 3x^2 - 4x + (5 - e^y) = 0$

For *x* to be real,

$$16 - 12(5 - e^y) \geq 0$$

$$\Rightarrow \quad 12e^y \geq 44$$

$$\Rightarrow \quad e^y \geq \frac{11}{3}$$

$$\Rightarrow \quad y \geq \log_e \frac{11}{3}$$

Range of $f = \left[\log_e \frac{11}{3}, \infty\right)$

**2.** $\sqrt{9 - x^2}$ is defined for

$$9 - x^2 \geq 0 \Rightarrow (3 - x)(3 + x) \geq 0$$

$$\Rightarrow \quad (x - 3)(x + 3) \leq 0 \quad \ldots(i)$$

$$\Rightarrow \quad -3 \leq x \leq 3$$

$\sin^{-1}(3 - x)$ defined for

$$-1 \leq 3 - x \leq 1$$

$$\Rightarrow \quad -4 \leq -x \leq -2$$

$$\Rightarrow \quad 2 \leq x \leq 4 \quad \ldots(ii)$$

Also, $\sin^{-1}(3 - x) \neq 0$

$$\Rightarrow \quad 3 - x \neq 0 \text{ or } x \neq 3 \quad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we get

The domain of

$$f = ([-3, 3] \cap [2, 4]) - \{3\} = [2, 3)$$

**3.** Given, $a_0 = 1$

and $\quad a_{n+1} = 3n^2 + n + a_n$

$$\Rightarrow \quad a_n = 3(n - 1)^2 + (n - 1) + a_{n-1}$$

Now, put $n = 1, 2, 3, \ldots, n$

We get,

$$a_1 = 0 + a_0 = 0 + 1 = 1$$

$$\Rightarrow \quad a_2 = 4 + a_1$$

$$\Rightarrow \quad a_3 = 14 + a_2$$

$$\Rightarrow \quad a_4 = 30 + a_3$$

$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$

$$\Rightarrow \quad a_{n-1} = 3(n - 2)^2 + (n - 2) + a_{n-2}$$

$$\Rightarrow \quad a_n = 3(n - 1)^2 + (n - 1) + a_{n-1}$$

On adding, we get

$$a_n = (1 + 4 + 14 + 30 + \ldots + 3(n - 1)^2 + (n - 1))$$

$$\Rightarrow \quad a_n = \Sigma\, 3(n^2 + 1 - 2n) + (n - 1)$$

$$\Rightarrow \quad a_n = \Sigma(3n^2 + 3 - 6n + n - 1)$$

$$\Rightarrow \quad a_n = \Sigma(3n^2 - 5n + 2)$$

$$\Rightarrow \quad a_n = 3\Sigma n^2 - 5\Sigma_n + 2\Sigma 1$$

$$\Rightarrow \quad a_n = \frac{3n(n+1)(2n+1)}{6} - \frac{5n(n+1)}{2} + 2n$$

$$\Rightarrow \quad a_n = \frac{n}{2}(n+1)\{2n+1-5\} + 2n$$

$$\Rightarrow \quad a_n = \frac{n}{2}(n+1)(2n-4) + 2n$$

$$= n(n+1)(n-2) + 2n$$

$$\Rightarrow \quad a_n = (n^2+n)(n-2) + 2n$$

$$= n^3 - 2n^2 + n^2 - 2n + 2n$$

$$\Rightarrow \quad a_n = n^3 - n^2 \qquad (\because n \geq 0)$$

**4.** Required number of ways

$$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 2^5 - 1 = 31$$

**5.** Number of selections of any number of copies of a book $= P + 1$, (because copies of the same book are identical things). Similarly, for each book.

$\therefore$ Total number of selections

$$= (P+1)(P+1)\ldots \text{ to } n \text{ factors}$$

$$= (P+1)^n$$

But this includes a selections which is empty *ie,* zero copy of each is selected. Excluding this, the required number of non-empty selections

$$= (P+1)^n - 1$$

**6.** The number of straight lines

$$= {}^{18}C_2 - ({}^5C_2 - 1) = 144$$

**7.** The general term is

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\sqrt{\frac{3}{2x^2}}\right)^r$$

$$= {}^{10}C_r \left(\frac{1}{3}\right)^{5-(r/2)} \left(\frac{3}{2}\right)^{r/2} \cdot x^{5-(3r/2)}$$

for term independent of $x$.

$5 - \frac{3r}{2} = 0 \Rightarrow r = \frac{10}{3}$, which is not a positive integer. Hence, there is no term independent of $x$.

**8.** $$\frac{\text{The coefficient of } t_{r+1}}{\text{The coefficient of } t_r} = \frac{{}^{2n}C_r}{{}^{2n}C_{r-1}}$$

$$= \frac{2n-r+1}{r}$$

The coefficient of $t_{r+1} \geq$ The coefficient of $t_r$, provided

$$\frac{2n-r+1}{r} \geq 1 \text{ or } 2n+1 \geq 2r$$

or $$r \leq \frac{2n+1}{2} \text{ or } r \leq n + \frac{1}{2}$$

Hence, the greatest coefficient

= The coefficient of $(n+1)$th term

$$= {}^{2n}C_n = \frac{(2n)!}{(n!)^2}$$

**9.** The general term in the expansion of $(\sqrt{5} + 4\sqrt{11})^{124}$ is

$$T_{r+1} = {}^{124}C_r (\sqrt{5})^{124-r} (\sqrt[4]{11})^r$$

$$= {}^{124}C_r \cdot 5^{(124-r)/2} \cdot 11^{r/4}$$

For integer terms $r = 0, 4, 8, 12, \ldots, 124$

$\therefore$ Number of terms which are integers = 32

**10.** We have,

$$(1+x)^m \left(1 + \frac{1}{x}\right)^n = (1+x)^m \left(\frac{x+1}{x}\right)^n$$

$$= \frac{(1+x)^{m+n}}{x^n} = x^{-n}(1+x)^{m+n}$$

$\therefore$ Required term independent of $x$

= Coefficient of $x^0$ in $x^{-n}(1+x)^{m+n}$

= Coefficient of $x^n$ in $(1+x)^{m+n}$

$$= {}^{m+n}C_n$$

**11.** Let $\alpha, \beta$ are the roots of the equation $(1-a)x^2 + 3ax - 1 = 0$, then $\alpha + \beta = \frac{3a}{a-1}$,

$$\alpha\beta = \frac{1}{a-1}$$

As $a > 1$, $\alpha + \beta > 0$ and $\alpha\beta > 0$ and $D = 9a^2 + 4(1-a)$

$= 9\left(a^2 - \frac{4}{9}a + \frac{4}{9}\right) = 9\left\{a\left(a - \frac{4}{9}\right) + \frac{4}{9}\right\} > 0$, as $a > 1$

$\therefore$ The equation has real and positive roots.

**12.** $a \cos 2x + b \sin^2 x + c = 0$

$$\Rightarrow \quad a(1 - 2\sin^2 x) + b \sin^2 x + c = 0$$

ie, $$(b - 2a)\sin^2 x + (a + c) = 0$$

It is an identity, if $b - 2a = 0$, $a + c = 0$, so

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{-1}$$

Thus, number of triplet $(a, b, c)$ are infinite.

**13.** We have,

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\Rightarrow \quad 4 = 6 + 2(\alpha\beta + \beta\gamma + \gamma\alpha)$$

$$\Rightarrow \quad \beta\gamma + \gamma\alpha + \alpha\beta = -1 \qquad \ldots(i)$$

Also, $$\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma$$

$$= (\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \alpha\beta - \beta\gamma - \gamma\alpha)$$

$\Rightarrow \quad 8 - 3\alpha\beta\gamma = 2(6+1)$

$\Rightarrow \quad 3\alpha\beta\gamma = 8 - 14 = -6$

or $\quad \alpha\beta\gamma = -2 \quad \ldots$(ii)

Now, $\quad (\alpha^2 + \beta^2 + \gamma^2)^2 = \alpha^4 + \beta^4 + \gamma^4 + 2(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2)$

$= (\alpha^4 + \beta^4 + \gamma^4) + 2[(\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)]$

$\Rightarrow \quad (\alpha^4 + \beta^4 + \gamma^4) = 36 - 2[(-1)^2 - 2(-2)(2)] = 18$

**14.** $ax^2 + 2b|x| - c = 0$

$\Rightarrow \quad a|x|^2 + 2b|x| - c = 0$

$\therefore \quad |x| = \dfrac{-2b \pm \sqrt{4b^2 + 4ac}}{2} = -b \pm \sqrt{b^2 + ac}$

Since, a, b, c are positive. So,

$|x| = -b + \sqrt{b^2 + ac}$

$\therefore$ x has two real values, neglecting

$|x| = -b - \sqrt{b^2 + ac}$, as $|x| > 0$

**15.** Since, the diagonal elements of a skew-symmetric matrix are always zero.

$\therefore$ Trace of A = 0

**16.** Here, the rank of $A$ is 3.

Therefore, the minor of order 3 of $A \neq 0$

$$\Rightarrow \quad \begin{vmatrix} y+a & b & c \\ a & y+b & c \\ a & b & y+c \end{vmatrix} \neq 0$$

[Applying $C_1 \to C_1 + C_2 + C_3$ and taking $(y + a + b + c)$ common from $C_1$]

$$\Rightarrow \quad (y+a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & y+b & c \\ 1 & b & y+c \end{vmatrix} \neq 0$$

[Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$]

$$\Rightarrow \quad (y+a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & y & 0 \\ 0 & 0 & y \end{vmatrix} \neq 0$$

Expanding along $C_1$

$\Rightarrow \quad (y+a+b+c)(y^2) \neq 0$

$\Rightarrow \quad y \neq 0$ and $y \neq -(a+b+c)$

**17.** We have, $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$

$$= \begin{vmatrix} 1 & \dfrac{\log y}{\log x} & \dfrac{\log z}{\log x} \\ \dfrac{\log x}{\log y} & 1 & \dfrac{\log z}{\log y} \\ \dfrac{\log x}{\log z} & \dfrac{\log y}{\log z} & 1 \end{vmatrix}$$

$$= \frac{1}{\log x \cdot \log y \cdot \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

[$\because$ all rows are identical]

**18.** Since, $A$ is an orthogonal matrix, therefore,

$AA' = I \Rightarrow |A \cdot A'| = |I| \Rightarrow |A||A'| = 1$

$\Rightarrow \quad |A||A| = 1 \quad (\because |I| = 1)$

$\Rightarrow \quad |A|^2 = 1 \quad (\because |A'| = |A|)$

$\Rightarrow \quad |A| = \pm 1$

**19.** Dividing the equation,

$a^3x^2 + abcx + c^3 = 0$ by $c^2$, we get

$$a\left(\frac{ax}{c}\right)^2 + b\left(\frac{ax}{c}\right) + c = 0$$

$\Rightarrow \quad \dfrac{ax}{c} = \alpha, \beta$ are roots.

$\Rightarrow \quad x = \dfrac{c}{a}\alpha$ and $x = \dfrac{c}{a}\beta$

$\Rightarrow \quad x = \alpha^2\beta$ and $x = \alpha\beta^2$

[$\because \alpha\beta = \dfrac{c}{a}$ as $\alpha, \beta$ are the roots of $ax^2 + bx + c = 0$]

are roots of above equation.

**20.** Expression can be written as

$= (1+i)^{n_1} + (1-i)^{n_1} + (1+i)^{n_2} + (1-i)^{n_2}$

$$= 2^{n_1/2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{n_1} + 2^{n_1/2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^{n_1} + 2^{n_2/2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)^{n_2} + 2^{n_2/2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^{n_2}$$

$$= 2^{n_1/2}\left\{\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{n_1} + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{n_1}\right\} + 2^{n_2/2}\left\{\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{n_2} + \left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^{n_2}\right\}$$

$$= n^{n_1/2}\left\{\cos\frac{n_1\pi}{4} + i\sin\frac{n_1\pi}{4} + \cos\frac{n_1\pi}{4} - i\sin\frac{n_1\pi}{4}\right\}$$

$$+ 2^{n_2/2}\left\{\cos\frac{n_2\pi}{4} + i\sin\frac{n_2\pi}{4} + \cos\frac{n_2\pi}{4} - \sin\frac{n_2\pi}{4}\right\}$$

$$= 2^{n_1/2}\cdot 2\cos\frac{n_1\pi}{4} + 2^{n_2/2}\cdot 2\cos\frac{n_2\pi}{4} = \text{real}$$

*ie,* $n_1$, $n_2$ are any two positive integers.

**21.** Let $z = x + iy$, then $|z + i| - |z - i| = k$ becomes

$$\therefore \quad \sqrt{x^2 + (y+1)^2} - \sqrt{x^2 + (y-1)^2} = k \quad \ldots(i)$$

or $\quad x^2 + (y+1)^2 - x^2 - (y-1)^2$

$$= k\{\sqrt{x^2 + (y+1)^2} + \sqrt{x^2 + (y-1)^2}\}$$

$$\therefore \quad \sqrt{x^2 + (y-1)^2} + \sqrt{x^2 + (y+1)^2} = \frac{4y}{k} \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$$2\sqrt{x^2 + (y+1)^2} = k + \frac{4y}{k}$$

$$\Rightarrow \quad 4x^2 + 4y^2 + 8y + 4 = k^2 + \frac{16y^2}{k^2} + 8y$$

$$\Rightarrow \quad 4x^2 + \left(4 - \frac{16}{k^2}\right)y^2 = k^2 - 4$$

For an hyperbola, $\frac{4k^2 - 16}{k^2} < 0 \Rightarrow k^2 - 4 < 0$

$$\Rightarrow \quad |k| < 2 \text{ or } -2 < k < 2$$

**22.** Since, $|\sin x| + |\cos x|$ is a periodic function with period $\pi/2$.

$\therefore \quad f(x) = |\sin 4x| + |\cos 4x|$ is a periodic

function with period $\frac{1}{4}\cdot\frac{\pi}{2}$ *ie,* $\frac{\pi}{8}$.

**23.** $\cos x = \dfrac{2\cos y - 1}{2 - \cos y}$

$$\Rightarrow \quad \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}} = \frac{2\left(\dfrac{1 - \tan^2\frac{y}{2}}{1 + \tan^2\frac{y}{2}}\right) - 1}{2 - \left(\dfrac{1 - \tan^2\frac{y}{2}}{1 + \tan^2\frac{y}{2}}\right)}$$

$$\Rightarrow \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}} = \frac{2 - 2\tan^2\frac{y}{2} - 1 - \tan^2\frac{y}{2}}{2 + 2\tan^2\frac{y}{2} - 1 + \tan^2\frac{y}{2}}$$

$$\Rightarrow \quad \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}} = \frac{1 - 3\tan^2\frac{y}{2}}{1 + 3\tan^2\frac{y}{2}}$$

$$\Rightarrow 1 + 3\tan^2\frac{y}{2} - \tan^2\frac{x}{2} - 3\tan^2\frac{x}{2}\tan^2\frac{y}{2}$$

$$= 1 - 3\tan^2\frac{y}{2} + \tan^2\frac{x}{2} - 3\tan^2\frac{x}{2}\tan^2\frac{y}{2}$$

$$\Rightarrow \quad 6\tan^2\frac{y}{2} = 2\tan^2\frac{x}{2}$$

$$\Rightarrow \quad \tan\frac{x}{2}\cdot\cot\frac{y}{2} = \sqrt{3}$$

**24.** $\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$

$$= \frac{1}{2\sin\frac{2\pi}{15}}\cdot 2\sin\frac{2\pi}{15}\cdot\cos\frac{2\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$\Rightarrow \frac{1}{2\sin\frac{2\pi}{15}}\cdot\sin\frac{4\pi}{15}\cdot\cos\frac{4\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$\Rightarrow \quad \frac{1}{4\sin\frac{2\pi}{15}}\cdot\sin\frac{8\pi}{15}\cdot\cos\frac{8\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$= \frac{1}{8\sin\frac{2\pi}{15}}\cdot\sin\frac{16\pi}{15}\cdot\cos\frac{16\pi}{15}$$

$$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{32\pi}{15}$$

$$= \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\left(2\pi + \frac{2\pi}{15}\right) = \frac{1}{16\sin\frac{2\pi}{15}}\cdot\sin\frac{2\pi}{15}$$

$$= \frac{1}{16}$$

**25.** We have, $\cos x\cdot\cos\left(\frac{\pi}{3} + x\right)\cos\left(\frac{\pi}{3} - x\right) = \frac{1}{4}$

$$\Rightarrow \quad \cos x\left(\frac{1}{4}\cos^2 x - \frac{3}{4}\sin^2 x\right) = \frac{1}{4}$$

or $$\frac{\cos x}{4}(4\cos^2 x - 3) = \frac{1}{4}$$

$$\Rightarrow \quad 4\cos^3 x - 3\cos x = 1$$

$$(\because \cos 3x = 4\cos^3 x - 3\cos x)$$

or $$\cos 3x = 1$$

$$\Rightarrow \quad 3x = 2n\pi$$

$\Rightarrow x = \frac{2n\pi}{3}$, where $n = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

$$(\because x \in [0, 6\pi])$$

$\therefore$ The required sum $= \frac{2\pi}{3}\sum_{n=0}^{9} n = 30\pi$

**26.** The given expression

$$= 2\tan^{-1}[\text{cosec} \tan^{-1} x - \tan \cot^{-1} x]$$

$$= 2\tan^{-1}\left[\text{cosec}\left\{\text{cosec}^{-1} \frac{\sqrt{1+x^2}}{x}\right\} - \tan\left\{\tan^{-1}\left(\frac{1}{x}\right)\right\}\right]$$

$$= 2\tan^{-1}\left[\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right] = 2\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$

$$= 2\tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] \qquad [\text{putting } x = \tan\theta]$$

$$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$$

$$= 2\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right]$$

$$= 2\tan^{-1}\tan\frac{\theta}{2} = 2\cdot\frac{\theta}{2} = \theta = \tan^{-1} x$$

**27.** We have,

$$\sin^{-1}\left[\cos\left(\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)} + \cos^{-1}\frac{\sqrt{12}}{4} + \sec^{-1}\sqrt{2}\right)\right]$$

$$= \sin^{-1}\left[\cos\left(\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \cos^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{1}{\sqrt{2}}\right)\right]$$

$$\left[\because \left(\frac{2-\sqrt{3}}{4}\right)^{1/2} = \frac{\sqrt{3}-1}{2\sqrt{2}}\right]$$

$$= \sin^{-1}[\cos(15° + 30° + 45°)]$$

$$= \sin^{-1}[\cos 90°] \qquad \left(\because \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

$$= \sin^{-1} 0$$

$$= 0$$

**28.** In given right angled triangle, we have

$$\cos A = c/b$$

$$\Rightarrow \frac{1-\tan^2\frac{A}{2}}{1+\tan^2\frac{A}{2}} = c/b$$

$$\Rightarrow \frac{\left(1+\tan^2\frac{A}{2}\right) - \left(1-\tan^2\frac{A}{2}\right)}{\left(1+\tan^2\frac{A}{2}\right) + \left(1-\tan^2\frac{A}{2}\right)} = \frac{(b-c)}{(b+c)}$$

(by componendo and dividendo rule)

Hence, $\tan^2\frac{A}{2} = \frac{b-c}{b+c}$

**29.** We have, $r_1 > r_2 > r_3$

$$\Rightarrow \frac{\Delta}{s-a} > \frac{\Delta}{s-b} > \frac{\Delta}{s-c}$$

$$\Rightarrow \frac{s-a}{\Delta} < \frac{s-b}{\Delta} < \frac{s-c}{\Delta}$$

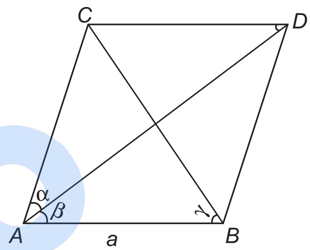
$$\Rightarrow s-a < s-b < s-c$$

$$\Rightarrow -a < -b < -c \Rightarrow a > b > c$$

**30.** Clearly,

$\angle CDA = \beta$, $\angle DCB = \gamma$, $\angle ACB = \pi - (\alpha + \beta + \gamma)$

Using since rule in $\Delta CAB$, we get




$$\frac{AB}{\sin(\pi - (\alpha+\beta+\gamma))} = \frac{CA}{\sin\gamma}$$

$$\Rightarrow CA = \frac{a\cdot\sin\gamma}{\sin(\alpha+\beta+\gamma)}$$

Now, using sine rule in $\Delta CAD$,

we get $\frac{CA}{\sin\beta} = \frac{CD}{\sin\alpha}$

$$\Rightarrow CD = \frac{CA\cdot\sin\alpha}{\sin\beta} = \frac{a\sin\alpha\cdot\sin\gamma}{\sin\beta\cdot\sin(\alpha+\beta+\gamma)}$$

**31.** We have, $\vec{\mathbf{P}} = \vec{\mathbf{AC}} + \vec{\mathbf{BD}} = \vec{\mathbf{AC}} + \vec{\mathbf{BC}} + \vec{\mathbf{CD}}$

$$= \vec{\mathbf{AC}} + \lambda\,\vec{\mathbf{AD}} + \vec{\mathbf{CD}} = \lambda\,\vec{\mathbf{AD}} + (\vec{\mathbf{AC}} + \vec{\mathbf{CD}})$$

$$= \lambda\,\vec{\mathbf{AD}} + \vec{\mathbf{AD}} = (\lambda+1)\,\vec{\mathbf{AD}}$$

But, $\vec{\mathbf{P}} = \mu\,\vec{\mathbf{AD}}$

$\therefore \qquad \mu = \lambda + 1$

**32.** Given, $3\vec{\mathbf{p}} + 8\vec{\mathbf{q}} = 6\vec{\mathbf{r}} + 5\vec{\mathbf{s}}$

$$\Rightarrow \frac{3\vec{\mathbf{p}} + 8\vec{\mathbf{q}}}{8+3} = \frac{6\vec{\mathbf{r}} + 5\vec{\mathbf{s}}}{6+5}$$

$\Rightarrow$ The point which divides PQ in ratio 8 : 3 in the same as the point which divides RS in the ratio 5 : 6. Hence, the line PQ and RS intersect.

**33.** We have, $\vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = |\vec{\mathbf{c}}|$ and $|\vec{\mathbf{c}} - \vec{\mathbf{a}}| = 2\sqrt{2}$

$$\Rightarrow \vec{\mathbf{a}}\cdot\vec{\mathbf{c}} = |\vec{\mathbf{c}}| \text{ and } |\vec{\mathbf{c}}|^2 + |\vec{\mathbf{a}}|^2 - 2(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}}) = 8$$

$$\Rightarrow |\vec{\mathbf{c}}|^2 + 9 - 2|\vec{\mathbf{c}}| = 8$$

$$\Rightarrow \quad (|\vec{c}|-1)^2 = 0$$

$$\Rightarrow \quad |\vec{c}| = 1$$

$$\Rightarrow \quad |(\vec{a}\times\vec{b})\times\vec{c}| = |\vec{a}\times\vec{b}||\vec{c}| \sin 30°$$

$$= \frac{1}{2}|\vec{a}\times\vec{b}| = \frac{3}{2}$$

$$[\because \ \vec{a}\times\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}]$$

**34.** We have,

$$\vec{V}_1 = \frac{1}{2}(\vec{a}\times\vec{b}), \vec{V}_2 = \frac{1}{2}(\vec{b}\times\vec{c}), \vec{V}_3 = \frac{1}{2}(\vec{c}\times\vec{a})$$

and $\vec{V}_4 = \frac{1}{2}\{(\vec{c}-\vec{a})\times(\vec{b}-\vec{a})\}$

$$\therefore \ \vec{V}_1+\vec{V}_2+\vec{V}_3+\vec{V}_4 = \frac{1}{2}(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$$

$$+\vec{c}\times\vec{b}-\vec{c}\times\vec{a}-\vec{a}\times\vec{b}) = \vec{0}$$

$$|\vec{V}_1+\vec{V}_2+\vec{V}_3+\vec{V}_4| = \vec{0}$$

**35.** We have,

$$|[\vec{a}\ \vec{b}\ \vec{c}]| = V$$

Let $V_1$ be the volume of parallelopiped formed by the vectors $\vec{\alpha}$, $\vec{\beta}$ and $\vec{\gamma}$. Then,

$$V_1 = |[\vec{\alpha}\ \vec{\beta}\ \vec{\gamma}]|$$

$$\Rightarrow \quad V_1 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} [\vec{a}\ \vec{b}\ \vec{c}]$$

$$\Rightarrow \quad V_1 = [\vec{a}\ \vec{b}\ \vec{c}]^2[\vec{a}\ \vec{b}\ \vec{c}]$$

$$\Rightarrow \quad V_1 = [\vec{a}\ \vec{b}\ \vec{c}]^3$$

$$\Rightarrow \quad V_1 = V^3$$

**36.** We have, $|a\hat{i} + b\hat{j} + c\hat{k}| = |a| + |b| + |c|$

$\Rightarrow \sqrt{a^2+b^2+c^2} = |a|+|b|+|c|$

$\Rightarrow a^2+b^2+c^2$

$= a^2+b^2+c^2 + 2(|a||b|+|b||c|+|c||a|)$

$\Rightarrow |a||b|+|b||c|+|c||a| = 0$

$\Rightarrow ab = bc = ca = 0$

$\Rightarrow$ Any two of a, b and c are zero.

**37.** We have, $\vec{u}\cdot\hat{n} = 0$ and $\vec{v}\cdot\hat{n} = 0$

$$\Rightarrow \quad \hat{n} \perp \vec{u} \text{ and } \hat{n} \perp \vec{v}$$

$$\Rightarrow \quad \hat{n} = \pm\frac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|}$$

Now, $\vec{u}\times\vec{v} = (\hat{i}+\hat{j})\times(\hat{i}-\hat{j}) = -2\hat{k}$

$$\hat{n} = \pm\hat{k}$$

Hence, $|\vec{w}\cdot\hat{n}| = |(\hat{i}+2\hat{j}+3\hat{k})\cdot(\pm\hat{k})| = 3$

**38.** We have, $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$. This shows that A and B are independent events. So,

$$P(B) = P\left(\frac{B}{A}\right) = \frac{1}{2}$$

Now, $P\left(\frac{A}{B}\right) = \frac{1}{4} \Rightarrow P\left(\frac{\bar{A}}{B}\right) = 1 - \frac{1}{4} = \frac{3}{4}$

**39.** Suppose the coin is tossed n times. Let X be the number of the heads obtained. Then, X follows a binomial distribution with parameters n and $p = \frac{1}{2}$

Now, $P(X \geq 1) \geq 0.8 \Rightarrow 1 - P(X=0) \geq 0.8$

$$\Rightarrow \quad 1 - {}^nC_0 p^0 (1-p)^n \geq 0.8$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^n \leq 0.2 \text{ or } \left(\frac{1}{2}\right)^n \leq \frac{1}{5}$$

$$\Rightarrow \quad 2^n \geq 5$$

$$\Rightarrow \quad n \geq 3$$

Hence, the least value of n is 3.

**40.** Since, $(n+1)p = \frac{101}{3}$ is not an integer.

Therefore, $p(X = r)$ is maximum when $r = \left[\frac{101}{3}\right] = 33$

**41.** Let $n_1$ and $n_2$ be the number of observations in two groups having means $\bar{X}_1$ and $\bar{X}_2$ respectively.

Then $\quad \bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2} - \bar{X}_2$

Now, $\quad \bar{X} - \bar{X}_1 = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2} - \bar{X}_1$

$$= \frac{n_2(\bar{X}_2 - \bar{X}_1)}{n_1+n_2} > 0 \quad [\because \bar{X}_2 > \bar{X}_1]$$

$$\Rightarrow \quad \bar{X} > \bar{X}_1 \quad \ldots(i)$$

And $\quad \bar{X} - \bar{X}_2 = \frac{n(\bar{X}_1 - \bar{X}_2)}{n_1+n_2} < 0 \quad [\because \bar{X}_2 > \bar{X}_1]$

$$\Rightarrow \quad \bar{X} < \bar{X}_2 \quad \ldots(ii)$$

From Eqs. (i) and (ii), $\bar{X}_1 < \bar{X} < \bar{X}_2$

**42.** Here, $\tan\theta = 2$,

So, $\quad \cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

For x and y, we have

$$x = X\cos\theta - Y\sin\theta = \frac{X - 2Y}{\sqrt{5}}$$

and $$y = X\sin\theta + Y\cos\theta = \frac{2X + Y}{\sqrt{5}}$$

The equation $4xy - 3x^2 = a^2$ reduces to

$$\frac{4(X-2Y)}{\sqrt{5}} \cdot \frac{(2X+Y)}{\sqrt{5}} - 3\left(\frac{X-2Y}{\sqrt{5}}\right)^2 = a^2$$

$$\Rightarrow \quad 4(2X^2 - 2Y^2 - 3XY) - 3(X^2 - 4XY + 4Y^2) = 5a^2$$

$$\Rightarrow \quad 5X^2 - 20Y^2 = 5a^2$$

$$\Rightarrow \quad X^2 - 4Y^2 = a^2$$

**43.** The given points are collinear, if

$$\begin{vmatrix} t_1 & 2at_1 + at_1^3 & 1 \\ t_2 & 2at_2 + at_2^3 & 1 \\ t_3 & 2at_3 + at_3^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad a\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 & 2t_2 + t_2^3 & 1 \\ t_3 & 2t_3 + t_3^3 & 1 \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ t_2 - t_1 & 2(t_2 - t_1) + (t_2^3 - t_1^3) & 0 \\ t_3 - t_1 & 2(t_3 - t_1) + (t_3^3 - t_1^3) & 0 \end{vmatrix} = 0$$

$$\Rightarrow (t_2 - t_1)(t_3 - t_1)\begin{vmatrix} t_1 & 2t_1 + t_1^3 & 1 \\ 1 & 2 + t_2^2 + t_1^2 + t_2t_1 & 0 \\ 1 & 2 + t_3^2 + t_1^2 + t_3t_1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow (t_2 - t_1)(t_3 - t_1)(t_3 - t_2)(t_3 + t_2 + t_1) = 0$$

$$\Rightarrow \quad t_1 + t_2 + t_3 = 0 \quad [\because t_1 \neq t_2 \neq t_3]$$

**44.** The slope of the line $x - y + 1 = 0$ is 1. So, it makes an angle of 45° with x-axis.

The equation of a line passing through (2, 3) and making an angle of 45°

$$\frac{x-2}{\cos 45^\circ} = \frac{y-3}{\sin 45^\circ} = r$$

Coordinates of any point on this line are

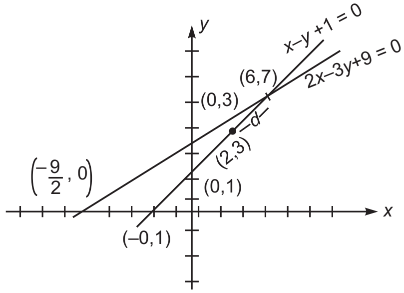
$$(2 + r\cos 45^\circ, 3 + r\sin 45^\circ)$$

or $$\left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$$

If this point lies on the line $2x - 3y + 9 = 0$, then $4 + \sqrt{2}r - 9 - \frac{3r}{\sqrt{2}} + 9 = 0$

$$\Rightarrow \quad r = 4\sqrt{2}$$

**Alternate Method**




Since the point (2, 3) lies on the line $x - y + 1 = 0$. Therefore the distance from (2, 3) to the line $2x - 3y + 9 = 0$ along the line $x - y + 1 = 0$ is equal to the distance between the points (2, 3) and intersection point of $2x - 3y + 9 = 0$ and $x - y + 1 = 0$ *ie*, (6, 7).

Hence required distance

$$d = \sqrt{(6-2)^2 + (7-3)^2} = \sqrt{32}$$

$$d = 4\sqrt{2}$$

**45.** The equation of any line through the point of intersection of the lines $x - y - 1 = 0$ and $2x - 3y + 1 = 0$ is

$$(x - y - 1) + \lambda(2x - 3y + 1) = 0$$

$$\Rightarrow (2\lambda + 1)x - y(3\lambda + 1) + (\lambda - 1) = 0 \quad \ldots(i)$$

The line in Eq. (i) will be parallel to x-axis, if it is of the form y = constant, therefore coefficient of x in Eq. (i) = 0

*ie*, $$2\lambda + 1 = 0 \Rightarrow \lambda = -\frac{1}{2}$$

On putting $\lambda = -\frac{1}{2}$ in Eq. (i), we get $y = 3$

This is the equation of the required line.

**46.** Since, given straight lines are the sides of a rhombus. Therefore, distance between the parallel lines

$a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_2 = 0$ and

$a_2x + b_2y + d_1 = 0$, $a_2x + b_2y + d_2 = 0$ must be equal

$$\therefore \quad \frac{|c_1 - c_2|}{\sqrt{a_1^2 + b_1^2}} = \frac{|d_1 - d_2|}{\sqrt{a_2^2 + b_2^2}}$$

$$\Rightarrow (a_2^2 + b_2^2)(c_1 - c_2)^2 = (a_1^2 + a_1^2)(d_1 - d_2)^2$$

**47.** The given equation is of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, on comparing the given equation with it

We obtain $a = 3$, $h = \frac{7}{2}$, $b = 2$, $g = \frac{5}{2}$, $f = \frac{5}{2}$ and $c = 2$

Now, $abc + 2fgh - af^2 - bg^2 - ch^2$

$$= 12 + 2 \times \frac{5}{2} \times \frac{5}{2} \times \frac{7}{2} - 3\left(\frac{5}{2}\right)^2 - 2\left(\frac{5}{2}\right)^2 - 2\left(\frac{7}{2}\right)^2$$

$= 0$

Hence, the given equation represents a pair of straight lines.

**48.** If the two pairs of straight lines have the same bisectors, then the two pairs are equally inclined.

The equation of the bisectors of the angle between the lines given by $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h} \quad \ldots(i)$$

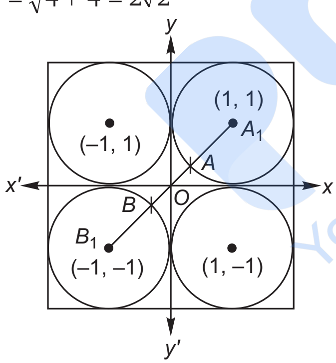
The equation of the bisectors of the angle between the lines given by $a^2x^2 + 2h(a + b)xy + b^2y^2 = 0$ is

$$\frac{x^2 - y^2}{a^2 - b^2} = \frac{xy}{h(a + b)}$$

$$\Rightarrow \quad \frac{x^2 - y^2}{a - b} = \frac{xy}{h} \quad \ldots(ii)$$

Clearly Eqs. (i) and (ii) are the same. The two pairs of straight lines are equally inclined.

**49.** $A_1B_1 = \sqrt{4 + 4} = 2\sqrt{2}$




$$AB = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$$

$\Rightarrow OA = OB = \frac{AB}{2} = \sqrt{2} - 1$ which is the radius of required circle.

Thus, equation of required circle is

$$x^2 + y^2 = (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}$$

**50.** We have, the given circle $x = a\cos\theta$, $y = a\sin\theta$

$$\Rightarrow \quad x^2 + y^2 = a^2 \quad \ldots(i)$$

Tangents to the circle (i) at $\theta$ and $\left(\theta + \frac{\pi}{2}\right)$ are

$$x\cos\theta + y\sin\theta = a$$

and $x\cos\left(\theta + \frac{\pi}{2}\right) + y\sin\left(\theta + \frac{\pi}{2}\right) = a$

or $\quad x\cos\theta + y\sin\theta = a$

and $\quad -x\sin\theta + y\cos\theta = a$

Squaring and adding, we get

$$x^2 + y^2 = 2a^2$$

which is a circle.

**51.** Let the equation of the required circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$$

This passes through $A(1, 0)$ and $B(0, 1)$, therefore,

$$1 + 2g + c = 0 \text{ and } 1 + 2f + c = 0$$

$$\Rightarrow \quad g = -\left(\frac{c+1}{2}\right) \text{ and } f = -\left(\frac{c+1}{2}\right)$$

Let r be the radius of circle (i), then

$$r = \sqrt{g^2 + f^2 - c}$$

$$\Rightarrow \quad r = \sqrt{\left(\frac{c+1}{2}\right)^2 + \left(\frac{c+1}{2}\right)^2 - c}$$

$$\Rightarrow \quad r = \sqrt{\frac{c^2 + 1}{2}}$$

$$\Rightarrow \quad r^2 = \frac{1}{2}(c^2 + 1)$$

Clearly, r is minimum when $c = 0$ and the minimum value of r is $\frac{1}{\sqrt{2}}$.

For $c = 0$, we have

$$g = -\frac{1}{2} \text{ and } f = -\frac{1}{2}$$

On substituting the values of g, f and c in Eq. (i), we get,

$$x^2 + y^2 - x - y = 0$$

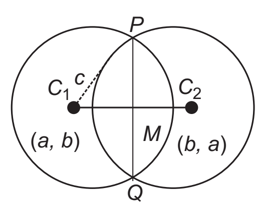
This is equation of the required circle.

**52.** The equations of two circles are

$$S_1 \equiv (x - a)^2 + (y - b)^2 = c^2 \quad \ldots(i)$$

and $\quad S_2 \equiv (x - b)^2 + (y - a)^2 = c^2 \quad \ldots(ii)$

The equation of the common chord of these circle is




$$S_1 - S_2 = 0$$

$\Rightarrow (x-a)^2-(x-b)^2+(y-b)^2-(y-a)^2=0$

$\Rightarrow (2x-a-b)(b-a)+(2y-b-a)(a-b)=0$

$\Rightarrow 2x-a-b-2y+b+a=0$

$\Rightarrow x-y=0$

Let $C_1$ and $C_2$ be the centres of circles (i) and (ii) respectively. Then, the coordinates of $C_1$ and $C_2$ are (a, b) and (b, a) respectively.

$$C_1M=\left|\frac{a-b}{\sqrt{1+1}}\right|=\frac{|a-b|}{\sqrt{2}}$$

In right $\Delta C_1PM$, we have,

$$PM=\sqrt{C_1P^2-C_1M^2}=\sqrt{c^2-\frac{(a-b)^2}{2}}$$

$$\therefore \quad PQ=2PM=2\sqrt{c^2-\frac{(a-b)^2}{2}}$$

$$=\sqrt{4c^2-2(a-b)^2}$$

**53.** Let the points $\left(m_i,\frac{1}{m_i}\right)$, $i=1,2,3,4$ lie on the circle $x^2+y^2+2gx+2fy+c=0$

Then,

$$\Rightarrow m_i^2+\frac{1}{m_i^2}+2gm_i+\frac{2f}{m_i}+c=0,\ i=1,2,3,4$$

$$\Rightarrow \quad m_i^4+2gm_i^3+cm_i^2+2fm_i+1=0,$$
$$i=1,2,3,4$$

$\Rightarrow$ $m_1, m_2, m_3$ and $m_4$ are the roots of the equation

$$m^4+2gm^3+cm^2+2fm+1=0$$

$$\Rightarrow \quad m_1m_2m_3m_4=\frac{1}{1}=1$$

**54.** Equation of the normal of the parabola $y^2=4ax$ at the point $(at^2, 2at)$ is

$$y+tx=2at+at^3 \qquad \dots(i)$$

$\because$ Eq. (i) cuts the parabola again at $(aT^2, 2aT)$.

Then, $\quad 2aT+taT^2=2at+at^3$

$$\Rightarrow \quad 2a(T-t)=-at(T^2-t^2)$$

$$\Rightarrow \quad 2=-t(T+t) \qquad (\because t\neq T)$$

$$\Rightarrow \quad t^2+tT+2=0$$

$\because$ *t* is real, $\therefore T^2-4\cdot1\cdot2\geq 0$

$$\Rightarrow \quad T^2\geq 8$$

**55.** Since, $3(3)^2+5(5)^2-32>0$. So, the given point lies outside the ellipse. Hence, two real tangents can be drawn from the point to the ellipse.

**56.** Let $y=m_1x$ and $y=m_2x$ be the lines represented by $lx^2+2mxy+ny^2=0$, then

$$m_1m_2=\frac{l}{n} \qquad \dots(i)$$

But $y=m_1x$ and $y=m_2x$ are conjugate diameters of the hyperbola.

$$\therefore \quad m_1m_2=\frac{b^2}{a^2} \qquad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{l}{n}=\frac{b^2}{a^2}\Rightarrow la^2=nb^2$$

**57.** The equations of the asymptotes of the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \text{ are } y=\pm\frac{b}{a}\ ie,$$

$$y=\frac{b}{a}x \text{ and } y=\frac{-b}{a}x$$

If $\theta$ is the angle between the two asymptotes, then,

$$\tan\theta=\frac{\frac{b}{a}+\frac{b}{a}}{1-\frac{b}{a}\cdot\frac{b}{a}}=\frac{2b}{a}\times\frac{a^2}{a^2-b^2}=\frac{2ab}{a^2-b^2}$$

From this, we get $\cos\theta=\dfrac{a^2-b^2}{a^2+b^2}$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2}-1=\frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2}=1+\frac{a^2-b^2}{a^2+b^2}$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2}=\frac{2a^2}{a^2+b^2}$$

$$\Rightarrow \quad \cos^2\frac{\theta}{2}=\frac{a^2}{a^2+b^2}$$

But $b^2=a^2(e^2-1)=a^2e^2-a^2\Rightarrow a^2+b^2=a^2e^2$

$$\therefore \quad \cos^2\left(\frac{\theta}{2}\right)=\frac{a^2}{a^2e^2}=\frac{1}{e^2}$$

$$\Rightarrow \quad \cos\left(\frac{\theta}{2}\right)=\frac{1}{e}$$

**58.** Let the point be $P\left(ct,\frac{c}{t}\right)$

The equation of the tangent to the hyperbola $xy=c^2$ at *P* is

$$x\frac{c}{t}+cty=2c^2$$

$$\Rightarrow \quad x+yt^2-2ct=0 \qquad \dots(i)$$

Then, putting $y=0, x=0$ successively in Eq. (i), we get $a_1 = 2ct$ and $b_1 = \frac{2c}{t}$

($\because$ intercepts of tangent on the axes are $a_1$ and $b_1$)

Again the equation of the normal to the hyperbola $xy = c^2$ at $P$ is

$$xt^3 - yt - ct^4 + c = 0$$

As before, $a_2 = ct - \frac{c}{t^3}$ and $b_2 = -ct^3 + \frac{c}{t}$

$$\therefore a_1a_2 + b_1b_2 = 2ct\left(ct - \frac{c}{t^3}\right) + \frac{2c}{t}\left(-ct^3 + \frac{c}{t}\right)$$
$$= 0$$

**59.** The equation of xy-plane is $z = 0$

$\therefore$ Direction cosines of its normal are 0, 0, 1.

**60.** $xy + yz = 0 \Leftrightarrow (x+z)y = 0 \Leftrightarrow y = 0$

or $x + z = 0$

The equations $y = 0$ and $x + z = 0$ represents planes which are at right angles as their altitudes numbers are $< 0, 1, 0 >$ and $< 1, 0, 1 >$ respectively.

**61.** By image formula,

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{z_2 - z_1}{c} = -\frac{2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2}$$

$$\Rightarrow \frac{x_2 - 2}{3} = \frac{y_2 + 1}{-2} = \frac{z_2 - 3}{-1} = -2\frac{(6 + 2 - 3 - 9)}{9 + 4 + 1}$$

$$\Rightarrow \frac{x_2 - 2}{3} = \frac{y_2 + 1}{-2} = \frac{z_2 - 3}{-1} = \frac{-2(-4)}{14}$$

$$\therefore x_2 = \frac{26}{7}, y_2 = \frac{-15}{7}, z_2 = \frac{17}{7}$$

**62.** $$\lim_{x \to 0} \frac{\sqrt{1 - \cos 2x}}{\sqrt{2} \cdot x} = \lim_{x \to 0} \frac{\sqrt{1 - (1 - 2\sin^2 x)}}{\sqrt{2} \cdot x}$$
$$= \lim_{x \to 0} \frac{\sqrt{2\sin^2 x}}{\sqrt{2} \cdot x} = \lim_{x \to 0} \frac{|\sin x|}{x}$$

Let $f(x) = \frac{|\sin x|}{x}$

Then,

$$f(0+0) = \lim_{h \to 0} \frac{|\sin(0+h)|}{h} = \lim_{h \to 0} \frac{\sin h}{h} = 1$$

and

$$f(0-0) = \lim_{h \to 0} \frac{|\sin(0-h)|}{-h} = \lim_{h \to 0} \frac{\sin h}{-h} = -1$$

$\because \quad f(0+0) \neq f(0-0)$

$\therefore$ The limit does not exist.

**63.** We have,

$$\lim_{h \to 0} f(1-h) = \lim_{h \to 0} a(1-h)^2 + b = a + b$$

$$\Rightarrow \lim_{h \to 0} f(1+h) = \lim_{h \to 0} (1+h) + 3 = 4$$

and $f(1) = 4$

$\therefore$ f(x) will not be continuous at $x = 1$, if $a + b \neq 4$

**64.** We have, $f(x) = \log_x(\log x) = \frac{\log(\log x)}{\log x}$

$$\Rightarrow f'(x) = \frac{\log x \cdot \frac{1}{\log x} \cdot \frac{1}{x} - \log(\log x) \cdot \frac{1}{x}}{(\log x)^2}$$
$$= \frac{1 - \log(\log x)}{x(\log x)^2}$$

$$\therefore f'(e) = \frac{1 - \log(\log e)}{e(\log e)^2} = \frac{1}{e}$$

**65.** Given, circle is intersection of sphere.

$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ ...(i)

and plane $x - 2y + 2z + 8 = 0$ ...(ii)

Centre of sphere is $(-1, 1, 2)$.

P = length of the perpendicular from $(-1, 1, 2)$ upon Eq. (ii)

$$= \frac{-1 - 2 + 4 + 8}{\sqrt{1 + 4 + 4}} = \frac{9}{3} = 3$$

R = Radius of the sphere $= \sqrt{1 + 1 + 4 + 19} = 5$

Radius of the circle $= \sqrt{R^2 - P^2} = \sqrt{25 - 9}$
$$= \sqrt{16} = 4$$

**66.** We have, $x^y = e^{x-y}$

$$\Rightarrow y \log x = (x - y)\log e = x - y$$

$$\Rightarrow y = \frac{x}{1 + \log x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \log x) \cdot 1 - x \cdot \frac{1}{x}}{(1 + \log x)^2} = \frac{\log x}{(1 + \log x)^2}$$

**67.** We have, $y = \sin(m \sin^{-1} x)$

$$\frac{dy}{dx} = \cos(m \sin^{-1} x) \cdot \frac{m}{\sqrt{1 - x^2}}$$

$$\Rightarrow (1 - x^2)\left(\frac{dy}{dx}\right)^2 = m^2 \cos^2(m \sin^{-1} x)$$
$$= m^2(1 - y^2)$$

On differentiating w.r.t. $x$, again, we get

$$(1-x^2)2\cdot\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\cdot(-2x)=-2m^2y\frac{dy}{dx}$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-m^2y$$

[$\because$ Dividing by $2dy/dx$]

**68.** We have, $f(x)=(ax+b)\sin x+(cx+d)\cos x$

$f'(x)=a\sin x+(ax+b)\cos x+c\cos x-(cx+d)\sin x$

But $f'(x)=x\cos x$ for all x (given)

$\therefore\ x\cos x=(a-d)\sin x+(b+c)\cos x+ax\cos x-cx\sin x$

Equating the coefficients of $\sin x$, $\cos x$, $x\cos x$ and $x\sin x$, we get

$$a-d=0,\ b+c=0,\ a=1,\ c=0$$

$\therefore b=c=0$ and $a=d=1$

**69.** Let $v$ be the velocity of the particle when the distance covered is $s$. Then,

$$v\propto\sqrt{s} \qquad \text{(given)}$$

$$\Rightarrow \quad v=\lambda\sqrt{s}$$

$$\Rightarrow \quad \frac{dv}{dx}=\frac{\lambda}{2\sqrt{s}}$$

$$\Rightarrow \quad v\frac{dv}{ds}=\frac{\lambda v}{2\sqrt{s}}=\frac{\lambda^2}{2}=\text{constant}$$

$\therefore$ The acceleration is constant

**70.** We have, $f(x)=e^x\sin x$

$$\Rightarrow \quad f'(x)=e^x\cos x+\sin x\cdot e^x$$

and $f''(x)=-\sin x\cdot e^x+\cos x\cdot e^x+\cos x\cdot e^x+\sin x\cdot e^x$

Now, for maximum or minimum slope put $(f'(x))'=0\Rightarrow f''(x)=0$

$$\Rightarrow \quad 2\cos x\cdot e^x=0$$

$$\Rightarrow \quad \cos x=0\Rightarrow x=\frac{\pi}{2}$$

Also, $f'''(x)=-2\sin x\cdot e^x+2\cos x\cdot e^x=$ negative

$\therefore$ Slope is maximum at $x=\pi/2$.

**71.** Clearly, $f(2)=-1$

$$\Rightarrow \quad -1=\frac{2a+b}{(2-1)(2-4)}\Rightarrow 2a+b=2$$

Now, $\quad f'(x)=\dfrac{4a+5b-2bx-ax^2}{(x-1)^2(x-4)^2}$,

$$f'(2)=0$$

$$\Rightarrow \quad b=0\Rightarrow a=1$$

$$\Rightarrow \quad f'(x)=-\frac{(x-2)(x+2)}{(x-1)^2(x-4)^2}$$

Clearly, for $x>2$, $f'(x)<0$ and for $x<2$, $f'(x)>0$.

Thus, $x=2$ is indeed the point of local maxima for $y=f(x)$

**72.** We have, $z=\tan(y+ax)+\sqrt{y-ax}$

$$\Rightarrow z_x=a\sec^2(y+ax)+\frac{(-a)}{2\sqrt{y-ax}}$$

$$\Rightarrow z_{xx}=2a^2\sec^2(y+ax)\tan(y+ax)-\frac{a^2}{4}(y-ax)^{-3/2}$$

and $\quad z_y=\sec^2(y+ax)+\dfrac{1}{2\sqrt{y-ax}}$

$$\Rightarrow z_{yy}=2\sec^2(y+ax)\tan(y+ax)-\frac{1}{4}(y-ax)^{-3/2}$$

$$a^2z_{yy}=2a^2\sec^2(y+ax)\cdot\tan(y+ax)-\frac{a^2}{4}(y-ax)^{-3/2}$$

Now, $\quad z_{xx}-a^2z_{yy}=0$

**73.** We have, $\int f(x)\,dx=F(x)$

$$\therefore\ \int x^3 f(x^2)\,dx=\frac{1}{2}\int x^2f(x^2)\,d(x^2)$$

$$=\frac{1}{2}[x^2F(x^2)-\int F(x^2)d(x^2)]$$

**74.** $\displaystyle\int\frac{x^2+4}{x^4+16}dx$

$$=\int\frac{1+\frac{4}{x^2}}{x^2+\frac{16}{x^2}}dx=\int\frac{d\left(x-\frac{4}{x}\right)}{\left(x-\frac{4}{x}\right)^2+8}$$

$$=\int\frac{dt}{t^2+(2\sqrt{2})^2},\ \text{where } t=x-\frac{4}{x}$$

$$=\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-4}{2\sqrt{2}\,x}\right)+c$$

**75.** Let $I=\displaystyle\int\frac{1}{(x+1)\sqrt{x^2-1}}dx$

Put $\quad x+1=\dfrac{1}{t}$

$$\Rightarrow \quad dx=-\frac{1}{t^2}dt,\text{ then}$$

$$I = \int \frac{1}{\frac{1}{t}\sqrt{\left(\frac{1}{t}-1\right)^2 - 1}}\left(\frac{-1}{t^2}\right)dt$$

$$\Rightarrow \quad I = -\int \frac{dt}{\sqrt{1-2t}} = -\int (1-2t)^{-1/2}dt$$

$$= -\frac{(1-2t)^{1/2}}{(-2)\left(\frac{1}{2}\right)} + c$$

$$\Rightarrow \quad I = \sqrt{1-2t} + c$$

$$\Rightarrow \quad I = \sqrt{1 - \frac{2}{x+1}} + c$$

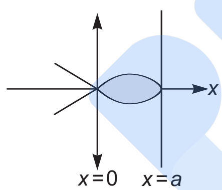
$$\Rightarrow \quad I = \sqrt{\frac{x-1}{x+1}} + c$$

**76.** $\int_{\pi/2}^{3\pi/2} [\sin x]dx = \int_{\pi/2}^{\pi} [\sin x]dx + \int_{\pi}^{3\pi/2} [\sin x]\, dx$

$$= \int_{\pi/2}^{\pi} 0\, dx + \int_{\pi}^{3\pi/2} (-1)dx = -[x]_{\pi}^{3\pi/2}$$

$$= -\left(\frac{3\pi}{2} - \pi\right) = -\frac{\pi}{2}$$

**77.** The area of the loop of the curve $ay^2 = x^2(a-x)$ is given by




$$2\int_0^a y\, dx = 2\int_0^a x\sqrt{\frac{a-x}{a}}dx$$

$$= 4a^2 \int_0^{\pi/2} \sin^3\theta \cdot \cos^2\theta \cdot d\theta$$

[Let $x = a\sin^2\theta$

$\Rightarrow \quad dx = 2a\sin\theta \cdot \cos\theta \cdot d\theta$]

$$= 4a^2 \cdot \frac{2\cdot 1}{5\cdot 3\cdot 1} = \frac{8a^2}{15} \text{ sq unit.}$$

**78.** The given differential equation can be written as

$$x = e^{xy\frac{dy}{dx}}$$

$$\Rightarrow \quad \log x = xy\frac{dy}{dx} \Rightarrow ydy = \frac{\log x}{x}dx$$

$$\Rightarrow \quad ydy = \log x\, d(\log x)$$

On integrating, we get

$$\frac{y^2}{2} = \frac{(\log x)^2}{2} + c$$

$$\Rightarrow \quad y^2 = (\log_e x)^2 + 2c$$

$$\Rightarrow \quad y = \pm\sqrt{(\log_e x)^2 + 2c}$$

**79.** We have, $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$$\Rightarrow \quad (ax+3)\,dx = (2y+f)dy$$

On integrating, we obtain

$$a\frac{x^2}{2} + 3x = y^2 + fy + c$$

$$\Rightarrow \quad -\frac{a}{2}x^2 + y^2 - 3x + fy + c = 0$$

This will represent a circle, if

$$-\frac{a}{2} = 1$$

($\because$ coefficient of $x^2$ = coefficient of $y^2$)

$$\Rightarrow \quad a = -2$$

**80.** $\int_1^5 x^2dx$

$$= \frac{4}{4}\left[\frac{1}{2}f(1) + f(2) + f(3) + f(4) + \frac{1}{2}f(5)\right]$$

$$= \left[\frac{1}{2} + 4 + 9 + 16 + \frac{25}{2}\right] = 42 \text{ [using } f(x) = x^2]$$