MET 2011 Answer Key PDF

Physics

1. Ammeter is always connected in series with circuit.

3. Time period of proton $T_p = \frac{25}{5} = 5\,\mu s$

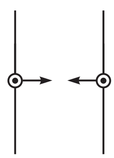
By using $T = \frac{2\pi m}{qB}$

$$\Rightarrow \quad \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha} = \frac{4m_p}{m_p} \times \frac{q_p}{2q_p} = 2$$

$$\Rightarrow \quad T_\alpha = 2T_p = 10\,\mu s$$

4. The magnitude of force acting on length l of each conductor due to other is

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$$



Given $\quad i_1 = i, i_2 = 10\,i$

$$\therefore \quad \frac{F}{l} = \frac{\mu_0}{2\pi}\frac{10 i^2}{r}$$

From Fleming's left hand rule, this is force of attraction.

5. Fundamental frequency of open pipe

$$n_1 = \frac{v}{2l}$$

Frequency of third harmonic of closed pipe

$$n_1 = \frac{3v}{4l}$$

$$\therefore \quad \frac{3v}{4l} - \frac{v}{2l} = 100$$

$$\frac{v}{4l} = 100$$

$$\frac{1}{2}\left(\frac{v}{2l}\right) = 100$$

$$\therefore \quad n_1 = 200 \text{ Hz}$$

6. Separation of slits $d = 4\text{ mm} = 4 \times 10^{-3}$ m

Wavelength $\quad \lambda = 6000\,\text{Å} = 6 \times 10^{-7}$ m

Distance of screen $D = 2$ m

Fringe width $\beta = \frac{\lambda D}{d}$

$$= \frac{6 \times 10^{-7} \times 2}{4 \times 10^{-3}}$$

$$= 3 \times 10^{-4} \text{ m}$$

$$= 0.3 \text{ mm}$$

7. Angle of deviation $\delta = (n-1)A$.

Dispersion produced by both the prism's will be equal.

$$(n_1 - 1)A_1 = (n_2 - 1)A_2$$

$$\Rightarrow \quad A_2 = \frac{(n_1 - 1)A_1}{(n_2 - 1)} = \frac{(1.54 - 1)}{(1.72 - 1)} = 3^\circ$$

Hence, the angle of prism P_2 is 3°.

8. If housefly sits on objective lens, then the intensity of image will decrease, because some of light rays will be stopped by housefly but housefly will not be seen.

9. We know that capacitor offers infinite resistance for DC source.

Current taken from the cell

$$i = \frac{E}{R + r}$$

$$\therefore \quad i = \frac{2.5}{1 + 1 + 0.5} = 1\text{ A}$$

Potential drop across capacitor

$$V = E - ir$$

$$= 2.5 - 1 \times 0.5 = 2\text{ V}$$

The current in 2 Ω resistor is zero.

Charge on capacitor

$$Q = CV$$

$$= 5 \times 2 = 10\,\mu C$$

10. Volume of bigger drop = volume of 64 small drops

i.e., $\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$

or $R = 4r$

Potential on small drop

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Potential on bigger drop

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{64q}{R}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{64q}{4r}$$

$$= \frac{9\times10^9 \times 64\times10^{-9}}{4\times2\times10^{-2}}$$

$$= 7.2\times10^3 \text{ V}$$

11. $p_x = 2\cos t,\ p_y = 2\sin t,$

$\therefore \quad \mathbf{p} = p_x\ \mathbf{i} + p_y\ \mathbf{j}$

$= 2\cos t\ \mathbf{i} + 2\sin t\ \mathbf{j}$

Force = rate of change of momentum

$$= \frac{d\mathbf{p}}{dt}$$

$\Rightarrow \quad \mathbf{F} = -2\sin t\ \mathbf{i} + 2\cos t\ \mathbf{j}$

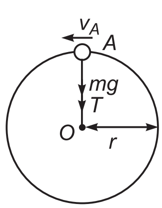
$\therefore \cos\theta = \frac{\mathbf{F}\cdot\mathbf{p}}{|\mathbf{F}|\cdot|\mathbf{p}|}$

$$= \frac{(-2\sin t\ \mathbf{i} + 2\cos t\ \mathbf{j})(2\cos t\ \mathbf{i} + 2\sin t\ \mathbf{j})}{\sqrt{4\sin^2 t + 4\cos^2 t}\sqrt{4\cos^2 t + 4\sin^2 t}}$$

$\Rightarrow \quad \cos\theta = 0$

$\Rightarrow \quad \theta = 90°$

12. When a body tied to the end of a string is revolved in a vertical circle, the speed of the body is different at different points of the circular paths.



The resultant of tension (T) and weight (mg) provides the necessary centripetal force

i.e., $T + mg = \frac{mv_A^2}{r}$

Tension (T) becomes zero at the points of minimum velocity, therefore

$\therefore \quad 0 + mg = \frac{mv_A^2}{r}$

$\Rightarrow \quad v_A = \sqrt{gr}$

13. Drop is stationary, *i.e.,*

weight of drop = electric force

i.e., $mg = qE$

or $mg = q\frac{V}{d}$

or $mg = ne\frac{V}{d}$

$\therefore \quad n = \frac{mgd}{eV}$

$$= \frac{1.8\times10^{-13}\times10\times0.9\times10^{-2}}{1.6\times10^{-19}\times2000}$$

$n = 50$

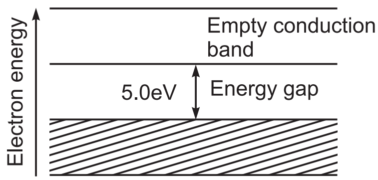
14. Many nuclei are radioactive. This means that they are unstable like radium and will eventually decay by emitting as particle transforming the nucleus into another nucleus. A chain of decays take place until a stable nucleus is reached. In the compound radium bromide, radium is radioactive while bromide is not, hence decay constant of compound is same as that of radium that is λ.

15. Remaing amount $N = N_0\left(\frac{1}{2}\right)^n$

$$N = 6.4\times10^{10}\left(\frac{1}{2}\right)^{12/4}$$

$$= 0.8\times10^{10} \text{ atoms}$$

16. An insulator has its valence band filled with electrons and its conduction band is empty. The energy gap is large approximately 5.0 eV, so it is not possible for ordinary process to provide enough energy for valence electrons to break out of their valence band and jump into the conduction band where they might carry electric current.



17. According to Fleming's right hand rule, magnetic field will be perpendicular to the plane of paper and act downward.

18. The force exerted on electron moving with velocity v is

$$F = evB \sin\theta$$

where $\theta = 90°$

$$\therefore \quad F_B = evB \qquad \ldots(i)$$

Centripetal force is given by

$$F_C = \frac{mv^2}{r} \qquad \ldots(ii)$$

Equating Eqs. (i) and (ii), we get

$$r = \frac{mv^2}{eB}$$

Diameter $d = 2r = \dfrac{2v}{(e/m)B}$

$$\Rightarrow \quad d = \frac{2 \times 2 \times 10^7}{1.76 \times 10^{11} \times 2 \times 10^{-2}}$$

$$= 1.1 \text{ cm}$$

19. Given, $V = 100 \sin 100t$

$$i = 100 \sin\left(100t + \frac{\pi}{3}\right)$$

Comparing with standard forms

$$V = V_0 \sin \omega t$$

$$I = I_0 \sin(\omega t + \phi)$$

$$\therefore \quad V_0 = 100 \text{ V}$$

$$I_0 = 100 \text{ mA} = 100 \times 10^{-3} \text{ A}$$

$$\phi = \frac{\pi}{3}$$

Power $P = \dfrac{V_0 I_0}{2} \cos\phi$

$$\therefore \quad P = \frac{100 \times 100 \times 10^{-3}}{2} \cos\frac{\pi}{3} = 2.5 \text{ W}$$

20. Escape velocity $v_e = \sqrt{\dfrac{2GM}{R}}$

$$= R\sqrt{\frac{8}{3}G\rho}$$

$$\therefore \quad v \propto R$$

We can say, if radius of a planet is double than that of earth then escape velocity from this planet will be 22 km/s.

21. $L = mvr = m\sqrt{\dfrac{GM}{r}}\, r = m\sqrt{GMr}$

$$\therefore \quad L \propto \sqrt{r}$$

$$\Rightarrow \quad \frac{L_2}{L_1} = \sqrt{\frac{r_2}{r_1}}$$

$$= \sqrt{\frac{16r}{r}} = 4$$

$$\Rightarrow \quad L_2 = 4L$$

22. Acceleration $a = \omega^2 y$

$$a = \left(\frac{2\pi}{T}\right)^2 y$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 y}{a}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{a}}$$

$$= 2\pi\sqrt{\frac{3}{12}}$$

$$T = 2\pi \times \frac{1}{2} = 3.14 \text{ s}$$

23. $r = \sqrt{r_1^2 + r_2^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ cm.

24. $\dfrac{\text{Volume of cube submerged}}{\text{Total volume}}$

$$= \frac{\text{density of cube material}}{\text{density of water}}$$

$$\Rightarrow \quad \frac{10 - 4}{10} = \frac{d}{1}$$

$$\therefore \quad d = \frac{6}{10} = 0.6 \text{ gcm}^{-3}$$

25. Temperature gradient of the rod

$$\frac{\Delta\theta}{\Delta x} = \frac{\theta_2 - \theta_1}{l}$$

$$\therefore \quad \frac{\Delta\theta}{\Delta x} = \frac{125 - 25}{50}$$

$$= \frac{100}{50} = 2^\circ \text{C/cm}$$

26. $Q = nC_p \, \Delta T$

For hydrogen

$$C_p = \frac{7}{2} R$$

So, $\quad Q = n \frac{7}{2} R \, \Delta T$

$$= 5 \times \frac{7}{2} \times 2 \times 30$$

$$= 1050 \text{ cal}$$

27. Apparent rise $= d \left(1 - \frac{1}{{}_a\mu_w}\right)$

$$= 12 \times \left(1 - \frac{3}{4}\right)$$

$$= 12 \times \frac{1}{4} = 3 \text{ cm}$$

28. According to Einstein's equation relation between rest energy and moving energy is

$$E = E_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

Given, $\quad v = 0.5\, c$

$$\therefore \quad E = E_0 \left(1 - \frac{(0.5\, c)^2}{c^2}\right)^{-1/2}$$

$$E = E_0 (1 - 0.25)^{-1/2}$$

$$= \frac{E_0}{\sqrt{0.75}}$$

Change in energy $\Delta E = E - E_0$

$$= \frac{E_0}{\sqrt{0.75}} - E_0$$

$$= E_0 \left(\frac{1}{\sqrt{0.75}} - 1\right)$$

$$= 0.511 \left(\frac{1}{\sqrt{0.75}} - 1\right)$$

$$= 0.079 \text{ MeV}$$

29. Force on charge, $F = QE$

or $\quad F = \frac{QV}{d} \quad (\because V = Ed)$

For drop to be stationary,

weight of drop = force due to charge

i.e., $\quad mg = \frac{QV}{d}$

For two drops, $\frac{Q_1}{Q_2} \cdot \frac{V_1}{V_2} = \frac{m_1}{m_2}$

or $$\frac{Q_1}{Q_2} \cdot \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3 \rho}{\frac{4}{3}\pi r_2^3 \rho}$$

or $$\frac{Q_2}{Q_1} = \frac{r_2^3}{r_1^3} \times \frac{V_1}{V_2}$$

$$\therefore \quad \frac{Q_2}{Q} = \frac{(r/2)^3}{r^3} \times \frac{2400}{600}$$

or $$Q_2 = \frac{Q}{2}$$

30. Band spectrum is defined as a molecular spectrum consisting of numerous closely spaced, often unresolved emission lines or absorption lines which occur across a limited range of frequencies. Each line represents an increment of energy due to a change in the rotational state of the molecule. Bands caused by titanium oxide, zirconium and carbon compounds are characteristic of low temperature stars.

31. Decrease in mass number $= 238 - 234 = 4$

No. of α-particles emitted $= \frac{4}{4} = 1$

Decrease in atomic number due to emission of 1 α-particle is 2, but now atomic number is 91, hence number of β-particles $= 91 + 2 - 92 = 1$.

32. $E = 57 \text{ meV} = 57 \times 10^{-3} \text{ eV}$

$$\therefore \quad \lambda = \frac{12375}{E} = \frac{12375}{57 \times 10^{-3}}$$

$$= 217.105 \times 10^3 \text{ Å}$$

$$\lambda = 217105 \text{ Å}$$

33. From law of conservation of linear momentum *i.e.,*

final momentum = initial momentum

$$m_1 v_1 + m_2 v_2 = 0 \quad \text{...(i)}$$

where m_1 and m_2 are masses of pieces.

(initial momentum is zero because body is stationary)

From Eq. (i), $m_1 v_1 = -m_2 v_2$

So, numerical value of momentum of both pieces will be equal.

34. $dp = F \times dt$

$$= 10 \times 10 = 100 \text{ kg-m/s}$$

36. The tension at the highest point is zero, but tension at the lowest point is 6 times the weight of the body. Hence, wire is most likely to break at the lowest point.

37. The time taken by AC in reaching from zero to maximum value is

$$t = \frac{1}{4} = \frac{1}{4f}$$

$$= \frac{1}{50 \times 4} = 5 \times 10^{-3} \text{ s}$$

38. $dQ = nCdT$

$$dQ = nk \frac{T^3}{\theta^3} dT$$

$$Q = \frac{nk}{\theta^3} \int_{T_1}^{T_2} T^3 dT$$

$$= \frac{nK}{\theta^3} \left(\frac{T_2^4 - T_1^4}{4} \right)$$

$$= \frac{2 \times 1940 \times (50^4 - 10^4)}{(281)^3 \times 4}$$

$$= 272.79 \text{ J} = 273 \text{ J}$$

39. Velocity of electron in nth orbit

$$v_n = \frac{Ze^2}{2\varepsilon_0 nh}$$

$$\Rightarrow \quad v \propto \frac{1}{n}$$

$$v_1 = v, n_1 = 2, n_2 = 5$$

$$\therefore \quad \frac{v}{v_2} = \frac{n_2}{n_1}$$

$$\therefore \quad \frac{v}{v_2} = \frac{5}{2}$$

$$\Rightarrow \quad v_2 = \frac{2v}{5}$$

40. If charged particle enters in magnetic field making an angle less than 90°, then its path will be a helix.

41. $C_1 = 10\,\mu\text{F}, V = 50 \text{ V}$

Charge on first capacitor

$$q = C_1 V$$

$$= 10 \times 50 = 500\,\mu\text{C}$$

$$\text{Common potential } V = \frac{\text{total charge}}{\text{total capacitance}}$$

$$\therefore \quad 20 = \frac{500}{C_1 + C_2}$$

$$\text{or} \quad 20 = \frac{500}{10 + C_2}$$

$$\text{or} \quad C_2 = 15\,\mu\text{F}$$

42. Given $E = 200\sqrt{2} \sin(100t)$ ...(i)

Comparing Eq. (i) with

$$E = E_0 \sin \omega t$$

Peak voltage $E_0 = 200\sqrt{2}, \omega = 100$

$$C = 1\mu\text{F} = 1 \times 10^{-6} \text{ F}$$

$$\text{Reading of ammeter, } i = \frac{E_{rms}}{X_C}$$

$$= \frac{E_0 \omega_C}{\sqrt{2}} = \frac{E_{rms}}{1/\omega C} = \frac{E_0 \omega C}{\sqrt{2}}$$

$$= \frac{200\sqrt{2} \times 100 \times 1 \times 10^{-6}}{\sqrt{2}}$$

$$= 20 \times 10^{-3} \text{ A}$$

$$= 20 \text{ mA}$$

43. Transformer works only in alternating current. Primary coil is connected to Lechlanche cell *i. e.,* connected by DC source, so voltage across secondary will be zero.

44. Resistance of galvanometer $G = 99\ \Omega$

$$\text{Resistance of shunt } S = \frac{i_g G}{i - i_g}$$

$$= \frac{10 \times 99}{100 - 10} = 11\ \Omega$$

45. Given $m = 500$ kg, $g = 9.8\ \text{m/s}^2$

AND $R_e = 6.4 \times 10^6$ m

Energy required to scape

$$E = mg\,R_e$$
$$= 500 \times 9.8 \times 6.4 \times 10^6$$
$$= 3.1 \times 10^{10}\ \text{J}$$

46. Direction of current is opposite to the direction of flow of electron and in the same direction of positive ion.

Total current $I = I_{\text{Ne}} + I_{\text{electron}}$

$$= (2.9 \times 10^{18} + 1.2 \times 10^{18})\,e$$
$$= 4.1 \times 10^{18} \times 1.6 \times 10^{-19}$$
$$= 0.66\ \text{A}$$

Direction of current will be the direction of flow of Ne^+ ion *i. e.* towards right.

47. Power $= \dfrac{\text{work}}{\text{time}}$

$$= \frac{mgh}{t} = \left(\frac{m}{t}\right) gh$$

Given, $\dfrac{m}{t} = 100$ kg/s, $h = 100$ m, $g = 10\ \text{m/s}^2$

$\therefore$ Power $= 100 \times 10 \times 100$

$= 100$ kW

48. Velocity of free end

$$v = \sqrt{3gh}$$
$$= \sqrt{3 \times 10 \times 1}$$
$$= 5.47\ \text{m/s}$$

49. Forbidden energy gap for germanium crystal

$$= 0.76\ \text{eV}$$
$$= 0.76 \times 1.6 \times 10^{-19}\ \text{J}$$
$$= 1.216 \times 10^{-19}\ \text{J}$$

50. $V = 200\ \text{V}, \dfrac{e}{m} = 1.6 \times 10^{11}\ \text{C/kg}$

Kinetic energy of electron $= eV$

i.e, $\dfrac{1}{2} mv^2 = eV$

or $v^2 = \dfrac{2e}{m} V$

or $v = \sqrt{2\dfrac{e}{m} V}$

$$= \sqrt{2 \times 1.6 \times 10^{11} \times 200}$$
$$= 8 \times 10^6\ \text{m/s}$$

51. From definition of temperature resistance coefficient

$$R_t = R_0\,(1 + \alpha t)$$
$$3R_0 = R_0\,(1 + 4 \times 10^{-3}\,t)$$
$$3 = 1 + 4 \times 10^{-3}\,t$$
$$4 \times 10^{-3}\,t = 2$$
$$t = \frac{2}{4 \times 10^{-3}}$$
$$= 500^\circ\text{C}$$

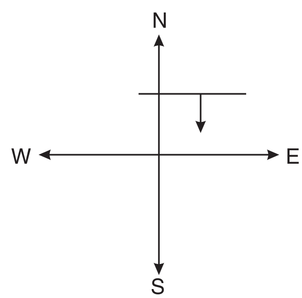
52. Induced emf $e = nA\dfrac{dB}{dt}$

A = area of coil $= 2m^2$

and $n = 1$

$$e = \frac{1 \times 2 \times (4-1)}{2} = 3\ \text{V}$$

53. Given, $l = 10$ m, $v = 5$ m/s



and $H = 0.3 \times 10^{-4}\ \text{Wb/m}^2$

Induced emf $e = Hvl$

$$= 0.3 \times 10^{-4} \times 5 \times 10$$
$$= 1.5 \times 10^{-3}\ \text{V}$$
$$= 1.5\ \text{mV}$$

54. $i = 1.6$ A, $t = 1$ minute $= 60$ s

Charge on one $Cu^{++} = 2e$

Let $n\,Cu^{++}$ are liberated from

$$Q = ne$$

$$it = n\,(2e)$$

$$1.6 \times 60 = n\,(2 \times 1.6 \times 10^{-19})$$

$$n = 3 \times 10^{20} \text{ ions}$$

55. Given, $L = 60$ H

$$R = 30\ \Omega$$

$$V = 100\ \text{V}$$

In L-R circuit current to reach 63.2% of its final value in time constant.

Equation of current in L-R circuit

$$I = I_0\,(1 - e^{-Rt/L})$$

Required time = time constant

$$= \frac{L}{R} = \frac{60}{30} = 2\ \text{s}$$

56. $T_2 = \dfrac{(m_1 + m_2)\,T_3}{m_1 + m_2 + m_3}$

$$= \frac{(10 + 6) \times 40}{10 + 6 + 4}$$

$$= \frac{16 \times 40}{20}$$

$$= 32\ \text{N}$$

57. Maximum KE of the system

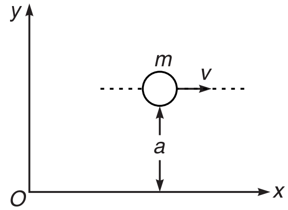
= Maximum PE of the system

$$= \frac{1}{2} kx^2$$

$$= \frac{1}{2} \times 16 \times (5 \times 10^{-2})^2$$

$$= 2 \times 10^{-2}\ \text{J}$$

58. Angular momentum of particle w.r.t. origin



= linear momentum × perpendicular distance of line of action of linear momentum from origin

$= mv \times a = mva =$ constant

59. Isobars are the elements which have same mass number. So, ${}_{15}P^{30}$ and ${}_{14}Si^{30}$ are isobars.

60. Let initial amount be N_0.

Remaining amount $N = N_0 - \dfrac{7N_0}{8} = \dfrac{N_0}{8}$

$$\therefore \qquad N = N_0 \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \qquad \frac{N_0}{8} = N_0 \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \qquad \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \qquad n = 3$$

$$\therefore \qquad \frac{t}{T_{1/2}} = 3$$

$$\Rightarrow \qquad \frac{t}{15} = 3 \quad \text{or} \quad t = 15 \times 3$$

$$t = 45\ \text{h}$$

Chemistry

1. 36 g of water $= \dfrac{36}{18} = 2$ mol

28 g of $CO_2 = \dfrac{28}{44} = 0.64$ mol

46 g of $CH_3OH = \dfrac{46}{32} = 1.44$ mol

58 g of $N_2O_5 = \dfrac{58}{108} = 0.54$ mol

2. $SO_2 + H_2O \longrightarrow 3S + 2H_2O$

Eq. wt. of SO_2 in the reaction

$$= \frac{\text{molar mass of } SO_2}{\text{change in oxidation state}} = \frac{64}{4} = 16$$

Twice of 16 will be 32.

3. An atomic orbital is a three dimensional region of definite shape around the nucleus where the probability of finding the electron is maximum. Therefore, an atom has both characteristic energy and a characteristic shape.

4. For n, $l = 0$ to $(n - 1)$

Therefore, when $n = 3$, $l \neq 3$

5. Configuration of an element $_{9}M = 1s^2, 2s^2, 2p^5$. Hence, its ion will be M^- because it can easily gain one electron to have octet.

6. In S_8 molecules, each sulphur is sp^3 hybridised with two lone pairs of electrons occupying two of the hybrid orbitals and reducing the S—S—S bond angle to 105°.

7. In oxyacids of chlorine, $HClO$, $HClO_2$, $HClO_3$ and $HClO_4$, the central Cl atom is sp^3 hybridised. The half filled unhybridised d-orbitals of Cl and p-orbitals of oxygen overlap to form π-bonds.

8. Anhydrous $AlCl_3$ is highly hygroscopic and hence gives fumes of hydrogen chloride by the hydrolysis of moist air.

$$AlCl_3 + \underset{\text{moist air}}{3H_2O} \longrightarrow Al(OH)_3 + 3HCl$$

9. Fe_2O_3 is amphoteric in nature as it reacts with both acid and alkali.

$$Fe_2O_3 + \underset{\text{acid}}{6HCl} \longrightarrow 2FeCl_3 + 3H_2O$$

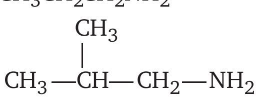
$$Fe_2O_3 + \underset{\text{alkali}}{Na_2CO_3} \longrightarrow 2NaFeO_2 + CO_2$$

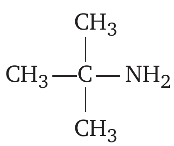
10. $P_4 + 20HNO_3 \longrightarrow 4H_3PO_4 + 20NO_2 + 4H_2O$

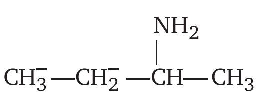
11. CH_3^- is the best nucleophile because carbon has the least electronegativity among key atoms of the given options.

12. Four isomeric primary amines are obtained from $C_4H_{11}N$ are possible.

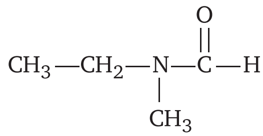
(i) $CH_3CH_2CH_2NH_2$

(ii) 

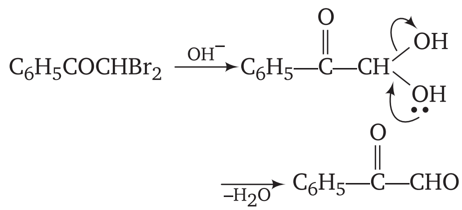
(iii) 

(iv) 

13. Reaction of alkyne with HBr supports it addition nature.

14. 

N-ethyl-N-methyl methanamide.

15. 

16. During catalytic hydrogenation, the hydrogens are transferred from the catalyst to the same side of the double bond. Evidently, smaller the number of R substituents, lesser is the steric hinderance and hence faster is the rate of hydrogenation. Thus, option (a) with two R groups on the same side of the molecule is correct.

17. Pentacetyl derivative can undergo basic hydrolysis.

18. Cellulose is a polysaccharide, composed of D-glucose units which are joined by β-glycosidic linkages. On complete hydrolysis cellulose produces D-glucose.

$$(C_6H_{10}O_5)_n + nH_2O \xrightarrow{H^+} \underset{\text{D-glucose}}{nC_6H_{12}O_6}$$

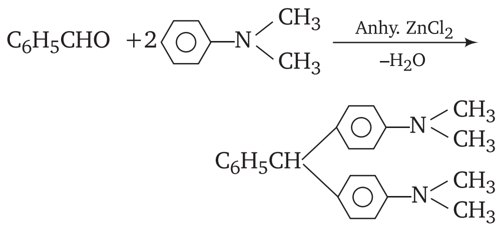
19. Saponification value is the number of mg of KOH required to neutralise the fatty acid resulting from the complete hydrolysis of 1 g of oil or fat.

20. $CH_3CH_2NH_2 \xrightarrow{NaNO_2 + HCl} CH_3CH_2OH + N_2 + H_2O + NaCl$

$NH_2CONH_2 \xrightarrow{NaNO_2 + HCl} 2N_2 + H_2O + CO_2 + NaCl$

$CH_3CONH_2 \xrightarrow{NaNO_2 + HCl} CH_3COOH + N_2 + H_2O + NaCl$

$C_6H_5NH_2 \xrightarrow{NaNO_2 + HCl} C_6H_5N_2^+Cl^- + H_2O + NaCl$

21. 

malachite green

22. When a primary amine reacts with chloroform in presence of alco. KOH, it gives *iso*-cyanide which has abnoxious odour. This reaction is known as carbylamine reaction and it is given only by primary amines.

$$\underset{\text{primary amine}}{RNH_2} + CHCl_3 + \underset{\text{alco.}}{3KOH} \xrightarrow{\Delta} RNC + 3KCl + 3H_2O$$

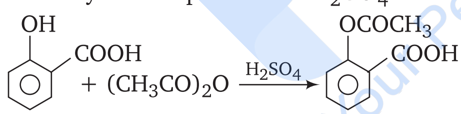
23. $$\underset{\text{acetic acid}}{CH_3COOH} \xrightarrow{NH_3} CH_3COONH_4 \xrightarrow[-H_2O]{\Delta} \underset{\text{acetamide}}{CH_3CO\ NH_2}$$

The isomers of CH_3CONH_2 are

$$\underset{(I)}{NH_2CH_2CHO},\ \underset{(II)}{CH_3-CH=NOH},$$

$$\underset{(III)}{H-CONH-CH_3}$$

24. Salicylic acid gives aspirin on reaction with acetic anhydride in presence of H_2SO_4.



salicylic acid — aspirin

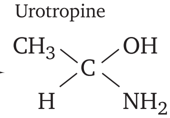
25. Cl^- is the best leaving group being the weakest nucleophile out of NH_2^-, Cl^-, $O^-C_2H_5$ and CH_3COO^-.

26. Acetaldehyde does not show cannizzaro's reaction due to the presence of α-atom.

27. HCHO and CH_3CHO give different reactions with NH_3.

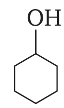
$$\underset{\text{formaldehyde}}{6HCHO} + 4NH_3 \longrightarrow \underset{\text{Urotropine}}{(CH_2)_6N_4} + 6H_2O$$

$$\underset{\text{acetaldehyde}}{CH_3CHO} + NH_3 \longrightarrow$$



acetaldehyde ammonia

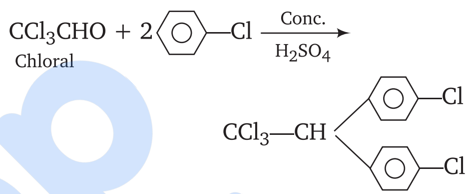
28. Solubility of alcohols in water decreases with increase in molecular masses.

29. 

cyclohexanol
2° alcohol

30. $$C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Invertase}} \underset{\text{glucose}}{C_6H_{12}O_6} + \underset{\text{fructose}}{C_6H_{12}O_6}$$

$$C_6H_{12}O_6 \xrightarrow{\text{Zymase}} 2C_2H_5OH + 2CO_2$$

31. 

Chloral — DDT

32. Vinyl chloride is least reactive towards hydrolysis because of non-reactivity of chlorine atom due to resonance stabilization.

$$CH_2=CH-\ddot{\underset{\cdot\cdot}{Cl}}: \longleftrightarrow \overset{-}{CH_2}-CH=\overset{+}{\underset{\cdot\cdot}{Cl}}:$$

33. Wurtz reaction

$$CH_3I + 2Na + IC_2H_5 \xrightarrow[-2NaI]{\text{Dry ether}} C_2H_6 + C_3H_8 + C_4H_{10}$$

34. Cyanide process is used in the extraction of silver and gold.

35. SnO_2—Cassiterite, Fe_3O_4—Magnetite

Fe_2O_3—Haematite, $Fe_2O_3 \cdot 3H_2O$—Limonite

36. $$Ti + 2I_2 \xrightarrow{500\ K} \underset{\text{volatile stable compound}}{TiI_4} \xrightarrow{1700\ K} \underset{\text{pure metal}}{Ti} + 2I_2$$

37. Cerium is the first element of rare earth metals.

38. $M^{2+} \longrightarrow M^{3+}$, after the removal of $2e^-$, the nuclear charge per electron increases, due to which high energy is required to remove $3e^-$.

39. Transition metal which have low oxidation number show the oxidising nature because of great tendency to lose the electron.

40. In $CoCl_3$, Co is in + 3 state

$Co^{3+} = 1s^2, 2s^2\ 2p^6, 3s^2\ 3p^6\ 3d^6, 4s^\circ$

4 unpaired electrons, so it will be coloured.

41. Spiegeleisn is an alloy of Fe, C and Mn.

42. The magnitude of stability constants for some divalent metal ions of the first transition series with oxygen or nitrogen donor ligands increases in the order

$Mn^{2+} < Fe^{2+} < Co^{2+} < Ni^{2+} < Cu^{2+} < Zn^{2+}$

43. Coordination number is equal to total number of ligands in a complex.

44. In $[Ni(CO)_4]$, Ni has zero oxidation number.

45. Rusting of iron is catalysed by H^+ ions.

46. At 298 K, standard electrode potential of NHE is 0.00 V.

47. $\Delta G^\circ = -nFE^\circ$

$$\Delta G^\circ = -\,2.303\,RT\,\log K_c$$

$$nFE^\circ = 2.303\,RT\,\log K_c$$

$$\log K_c = \frac{nFE^\circ}{2.303RT} = \frac{2 \times 96500 \times 0.295}{2.303 \times 8.314 \times 298}$$

$$\log K_c = 9.97$$

$$K_c = 1 \times 10^{10}$$

48. Equivalence conductance = conducting power of all the ion produced by one gram equivalent of an electrolyte.

Since, Eq. wt. of $BaCl_2 = \dfrac{\text{mol. wt.}}{2}$ then

$$\lambda^\infty_{BaCl_2} = \frac{1}{2}\lambda^\infty\ Ba^{2+} + \lambda^\infty Cl^-$$

$$= \frac{127}{2} + 76$$

$$= 139.5\ \text{ohm}^{-1}\text{cm}^2$$

49. In multimolecular colloids, colloidal size particles or molecules are built by the aggregation of ions or smaller particles. *eg*. gold sol.

50. Physical adsorption involves physical forces *i.e*., van der Waals' forces.

51. $C = \dfrac{5}{342} \times \dfrac{1}{100} \times 1000 = \dfrac{50}{342}\ \text{mol L}^{-1}$

$$\pi = \frac{50}{342} \times 0.082 \times 423$$

$$= 5.07\ \text{atm}$$

52. Mole fraction of $H_2O = \dfrac{80/18}{\dfrac{80}{18} + \dfrac{20}{34}} = \dfrac{34}{77}$

53. When pH = 3 then $[H^+] = 10^{-3}$ M. After that we increased the pH from 3 to 6 then $[H^+] = 10^{-6}$ M means reduced 1000 times.

54. $BaCl_2 \rightleftharpoons \underset{s}{\overset{2+}{Ba}} + \underset{2s}{2Cl^-}\ K_{sp} = 4s^3$,

$$s^3 = \frac{4 \times 10^{-9}}{4} = 10^{-3}$$

$$s = 10^{-3}\ \text{M}$$

55. $AlCl_3$ in a lewis acid because it is a electron pair acceptor, its central atom have a vacant *d*-orbital.

56. Radioactive decay is a first order reaction.

57. $t = \dfrac{2.303}{k} \log \dfrac{[A]_0}{[A]}$

$$= \frac{2.303}{60} = \log \frac{a}{\frac{a}{16}} = \frac{2.303}{60} \log 16$$

$$= \frac{2.303}{60} \times 1.204$$

$$= 0.0462\ \text{s} = 4.6 \times 10^{-2}\ \text{s}$$

58. $C_{(s)} + \dfrac{1}{2} O_{2(g)} \longrightarrow CO_{(s)}\ (\Delta H - \Delta U) = -\,\Delta nRT$

$$= -\frac{1}{2} \times 8.314 \times 298 = -\,1238.78$$

59. At constant temperature and pressure, internal energy of ideal gas remains unaffected.

60. Let RMS speed of nitrogen at T K be u and is equal to that of CO_2 at STP

$$u = \sqrt{\frac{3RT}{28}} = \sqrt{\frac{3R \times 273}{44}}$$

$$T = \frac{273 \times 28}{44} = 173.73\ \text{K} = -\,99.27^\circ\text{C}$$

Mathematics

1. Let $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$

Applying $R_2 \to R_2 - R_3$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$$

Applying $R_3 \to R_3 - (R_1 - 2R_2)$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

2. Given system has infinite solution.

3. As $\det(A) = \pm 1$, A^{-1} exists and

$$A^{-1} = \frac{1}{\det(A)}(\text{adj } A) = \pm \text{ adj } A$$

All entries in adj (A) are integers.

$\therefore A^{-1}$ has integer entries.

4. All statements are correct.

5. Let $I = \int \frac{dx}{x\sqrt{1-(\log x)^2}}$

Put $\quad \log x = t$

$\Rightarrow \quad \frac{1}{x}dx = dt$

$\therefore I = \int \frac{dt}{\sqrt{1-t^2}} = \sin^{-1} t + C$

$= \sin^{-1}(\log x) + C$

6. $\int \frac{x^2}{(x^2+2)(x^2+3)}dx = \int \left(\frac{3}{x^2+3} - \frac{2}{x^2+2}\right)dx$

$= \frac{3}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

$= \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

7. Let the equation of parabola whose axis is x- axis, is

$$y^2 = \pm 4a(x-h)$$

On differentiating w.r.t. x, we get

$2yy_1 = \pm 4a$

$\Rightarrow \quad yy_1 = \pm 2a$

Again differentiating, we get

$(y_1)^2 + yy_2 = 0$

Hence, order and degree are respectively 1 and 2.

8. Given that, $y = ax\cos\left(\frac{1}{x} + b\right)$

On differentiating w.r.t. x, we get

$y_1 = a\left[\cos\left(\frac{1}{x}+b\right) - x\sin\left(\frac{1}{x}+b\right)\left(-\frac{1}{x^2}\right)\right]$

$\Rightarrow y_1 = a\left[\cos\left(\frac{1}{x}+b\right) + \frac{1}{x}\sin\left(\frac{1}{x}+b\right)\right]$

Again differentiating, we get

$y_2 = a\left[-\sin\left(\frac{1}{x}+b\right)\left(-\frac{1}{x^2}\right) - \frac{1}{x^2}\sin\left(\frac{1}{x}+b\right) - \frac{1}{x^3}\cos\left(\frac{1}{x}+b\right)\right]$

$\Rightarrow \quad y_2 = -\frac{ax}{x^4}\cos\left(\frac{1}{x}+b\right)$

$\Rightarrow \quad x^4 y_2 + y = 0$

9. Given differential equation can be rewritten as

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{x^2-1}{x^2+1}$$

Here, $\quad P = \frac{2x}{1+x^2}$ and $Q = \frac{x^2-1}{x^2+1}$

$\therefore \quad \text{IF} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)}$

$= 1 + x^2$

10. $\int_0^{\pi/2} |\sin x - \cos x|\, dx$

$= \int_0^{\pi/4} -(\sin x - \cos x)\,dx$

$+ \int_{\pi/4}^{\pi/2} -(\sin x - \cos x)\,dx$

$$= -[-\cos x - \sin x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= -\left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1\right] + \left[-0 - 1 + \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \sqrt{2} - 1 - 1 + \sqrt{2}$$

$$= 2(\sqrt{2} - 1)$$

11. Let the centre of circle be (h, k). Then

$$\sqrt{(h-2)^2 + (k-3)^2} = \sqrt{(h-4)^2 + (k-5)^2} \quad \text{...(i)}$$

and $\quad k - 4h + 3 = 0 \quad$ ...(ii)

From Eq. (i)

$$-4h - 6k + 8h + 10k = 16 + 25 - 4 - 9$$

$\Rightarrow \quad h + k - 7 = 0 \quad$ ...(iii)

On solving Eqs. (ii) and (iii), we get the centre of circle (2, 5).

Now, radius $= \sqrt{(2-2)^2 + (5-3)^2} = 2$

$\therefore$ Required equation of circle is

$$(x-2)^2 + (y-5)^2 = 2^2$$

$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 25 = 0$

12. Given that, $\log_4 (x-1) = \log_2 (x-3)$

$\therefore \quad x - 1 = (x-3)^2$

$\Rightarrow \quad x^2 - 7x + 10 = 0$

$\Rightarrow \quad (x-5)(x-2) = 0$

$\Rightarrow \quad x = 2, 5$

$\Rightarrow$ $x = 5$, but $x = 2\log_2(x-3)$ is not satisfied.

13. Since, α and β are the roots of $x^2 - 3x + 1 = 0$, then

$$\alpha + \beta = 3 \text{ and } \alpha\beta = 1$$

Now, $\quad S = \frac{1}{\alpha - 2} + \frac{1}{\beta - 2}$

$$= \frac{\alpha + \beta - 4}{\alpha\beta - 2(\alpha + \beta) + 4}$$

$$= \frac{3 - 4}{1 - 2 \cdot 3 + 4} = 1$$

and $\quad P = \frac{1}{\alpha - 2} \times \frac{1}{\beta - 2}$

$$= \frac{1}{\alpha\beta - 2(\alpha + \beta - 2)}$$

$$= 1$$

$\therefore$ Required equation is

$$x^2 - Sx + P = 0$$

$\Rightarrow \quad x^2 - x - 1 = 0$

14. Let $y = \frac{x^2 + 34x - 71}{x^2 + 2x - 7}$

$\Rightarrow \quad x^2(y-1) + 2(y-17)x + (71 - 7y) = 0$

For real values of x, its discriminant, $D \geq 0$

$\therefore \quad 4(y-17)^2 - 4(y-1)(71-7y) \geq 0$

$\Rightarrow \quad (y^2 - 34y + 289) - (71y - 7y^2 - 71 + 7y) \geq 0$

$\Rightarrow \quad y^2 - 14y + 45 \geq 0$

$\Rightarrow \quad (y-5)(y-9) \geq 0$

$\Rightarrow \quad y \leq 5$ and $y \geq 9$

Hence, y does not lie between 5 and 9

15. Now, $\sqrt{50} + \sqrt{48} = 5\sqrt{2} + 4\sqrt{3}$

$$= \sqrt{2}\,[5 + 2 \cdot \sqrt{2} \cdot \sqrt{3}]$$

$$= \sqrt{2}\,(\sqrt{3} + \sqrt{2})^2$$

$\therefore \quad \sqrt{\sqrt{50} + \sqrt{48}} = 2^{1/4}(\sqrt{3} + \sqrt{2})$

16. Since, $I_1 = \int_a^{\pi - a} x\, f(\sin x)\, dx$

$$= \int_a^{\pi - a} (\pi - x)\, f(\sin(\pi - x))\, dx$$

$$= \int_a^{\pi - a} (\pi - x)\, f(\sin x)\, dx$$

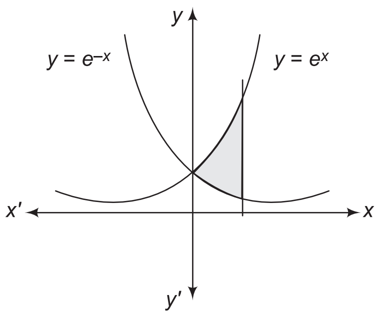
$$= \int_a^{\pi - a} \pi f(\sin x)\, dx - I_1$$

$\Rightarrow \quad 2I_1 = \pi I_2$

$\Rightarrow \quad I_2 = \frac{2}{\pi} I_1$

17. Required area $= \int_0^1 (e^x - e^{-x})\, dx$

$$= [e^x + e^{-x}]_0^1$$



$$= \left(e + \frac{1}{e} - 2\right) \text{ sq unit}$$

18. $f(x) = \sin^{-1}[x]$ is defined, if $-1 \le [x] \le 1$

$\Rightarrow \quad [x] = \{-1, 0, 1\}$

$\therefore \quad x \in [-1, 2)$

$\therefore$ Range is $\{\sin^{-1}(-1), \sin^{-1} 0, \sin^{-1} 1\}$

$$= \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$$

19. Since, $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = 0$

Also, **a**, **b** and **c** are non-collinear vectors, then $x = y = z = 0$

20. Let angle between **b** and **c** is α, then

$$|\mathbf{b} \times \mathbf{c}| = \sqrt{15} \text{ (given)}$$

$\Rightarrow \quad |\mathbf{b}||\mathbf{c}| \sin\alpha = \sqrt{15}$

$\Rightarrow \quad \sin\alpha = \dfrac{\sqrt{15}}{4}$

$\therefore \quad \cos\alpha = \dfrac{1}{4}$

Also, $\quad \mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$

$\Rightarrow \quad (\mathbf{b} - 2\mathbf{c})^2 = \lambda^2 (\mathbf{a})^2$

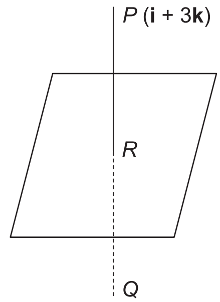
$\Rightarrow \quad |\mathbf{b}|^2 + 4|\mathbf{c}|^2 - 4\mathbf{b}\cdot\mathbf{c} = \lambda^2|\mathbf{a}|^2$

$\Rightarrow \quad 16 + 4 - 4\{|\mathbf{b}||\mathbf{c}|\cos\alpha\} = \lambda^2$

$\Rightarrow \quad \lambda^2 = 16$

$\Rightarrow \quad \lambda = \pm 4$

21. Let Q be the image of the point $P(\mathbf{i} + 3\mathbf{k})$ in the plane $\mathbf{r}\cdot(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 1$. Then, PQ is normal to the plane. Since, PQ passes through P and in normal to the given plane, therefore equation of PQ is $\mathbf{r} = (\mathbf{i} + 3\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$



Since, Q lies on the line PQ, so let the position vector of Q be $(\mathbf{i} + 3\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$ *i.e.*, $(1+\lambda)\mathbf{i} + \lambda\mathbf{j} + (3+\lambda)\mathbf{k}$.

Since, R is the mid-point of PQ, therefore position vector of R is

$$\frac{(1+\lambda)\mathbf{i} + \lambda\mathbf{j} + (3+\lambda)\mathbf{k} + \mathbf{i} + 3\mathbf{k}}{2}$$

$$= \left(\frac{\lambda+2}{2}\right)\mathbf{i} + \frac{\lambda}{2}\mathbf{j} + \left(\frac{6+\lambda}{2}\right)\mathbf{k}$$

$$= \left(\frac{\lambda}{2} + 1\right)\mathbf{i} + \frac{\lambda}{2}\mathbf{j} + \left(3 + \frac{\lambda}{2}\right)\mathbf{k}$$

Since, R lies on the plane

$$\mathbf{r}\cdot(\mathbf{i} + \mathbf{j} + \mathbf{k}) = 1$$

$\therefore \quad \left[\left(\frac{\lambda}{2} + 1\right)\mathbf{i} + \frac{\lambda}{2}\mathbf{j} + \left(3 + \frac{\lambda}{2}\right)\mathbf{k}\right] \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 1$

$\Rightarrow \quad \left[\frac{\lambda}{2} + 1 + \frac{\lambda}{2} + 3 + \frac{\lambda}{2}\right] = 1$

$\Rightarrow \quad \lambda = -2$

Hence, the position vector of Q is $-\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

22. Since, **a**, **b**, **c** form a right handed system

$\therefore \quad \mathbf{c} = \mathbf{b} \times \mathbf{a}$

$= \mathbf{j} \times (x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$

$= x(\mathbf{j} \times \mathbf{i}) + z(\mathbf{j} \times \mathbf{k})$

$= -x\mathbf{k} + z\mathbf{i} = z\mathbf{i} - x\mathbf{k}$

23. Since, plane is parallel to a given vector, it implies that normal of plane must perpendicular to the given vectors. Given point to which plane passes through is $(2, -1, 3)$. Let A, B and C are direction ratios of its normal.

$\therefore$ Equation of plane is

$$A(x-2)+B(y+1)+C(z-3)=0$$

Now, normal to plane $A\,\mathbf{i}+B\,\mathbf{j}+C\,\mathbf{k}$ is perpendicular to the given vectors

$$\mathbf{a}=3\,\mathbf{i}+0\,\mathbf{j}-\mathbf{k}$$

and $\mathbf{b}=-3\,\mathbf{i}+2\,\mathbf{j}+2\mathbf{k}$

$\therefore$ $3A+0B-C=0$ ...(i)

and $-3A+2B+2C=0$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{A}{2}=\frac{B}{-3}=\frac{C}{6}$$

$\therefore$ Equation of plane be

$$2(x-2)-3(y+1)+6(z-3)=0$$

i.e., $2x-3y+6z-25=0$

24. $\lim\limits_{x\to1}\dfrac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$

$$=\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}$$

$$=\lim_{x\to1}\frac{(2x-3)}{(2x+3)(\sqrt{x}+1)}$$

$$=\frac{2-3}{(2+3)(\sqrt{1}+1)}=-\frac{1}{10}$$

25. Since, 10, 3, e, 2 are in decreasing order.

$\therefore$ $\log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$ are in increasing order.

26. Given that, $y=\dfrac{\sqrt5-\sqrt2}{\sqrt5+\sqrt2}$

and $x=\dfrac{\sqrt5+\sqrt2}{\sqrt5-\sqrt2}$

$\Rightarrow$ $y=\dfrac{1}{x}$

$\Rightarrow$ $xy=1$

$\therefore$ $3x^2+4xy-3y^2=3(x-y)(x+y)+4xy$

$$=3\left(\frac{\sqrt5+\sqrt2}{\sqrt5-\sqrt2}-\frac{\sqrt5-\sqrt2}{\sqrt5+\sqrt2}\right)\times\left(\frac{\sqrt5+\sqrt2}{\sqrt5-\sqrt2}+\frac{\sqrt5-\sqrt2}{\sqrt5+\sqrt2}\right)+4$$

$$=3\left[\frac{(\sqrt5+\sqrt2)^2-(\sqrt5-\sqrt2)^2}{(5-2)(5-2)}\right][(\sqrt5+\sqrt2)^2+(\sqrt5-\sqrt2)^2]+4$$

$$=\frac{1}{3}\cdot4\sqrt{10}\cdot2(5+2)+4$$

$$=\frac{56}{3}\sqrt{10}+4$$

$$=\frac{1}{3}(56\sqrt{10}+12)$$

27. $\sum\limits_{r=0}^{n} r^2\ {}^nC_r\ x^r y^{n-r}$

$$=\sum_{r=0}^{n}[r(r-1)+r]\ {}^nC_r\ x^r y^{n-r}$$

$$=\sum_{r=0}^{n} r(r-1)\ {}^nC_r\ x^r y^{n-r}+\sum_{r=0}^{n} r\,{}^nC_r\ x^r y^{n-r}$$

$$=\sum_{r=2}^{n-2} r(r-1)\frac{n}{r}\cdot\frac{n-1}{r-1}\ {}^{n-2}C_{r-2}\ x^2\cdot x^{r-2}y^{n-r}+\sum_{r=1}^{n} r\cdot\frac{n}{r}\ {}^{n-1}C_{r-1}\ x\cdot x^{r-1}y^{n-r}$$

$$=n(n-1)x^2\sum_{r=2}^{n-2}{}^{n-2}C_{r-2}\ x^{r-2}y^{(n-2)-(r-2)}+nx\sum_{r=1}^{n}{}^{n-1}C_{r-1}\ x^{r-1}y^{(n-1)-(r-1)}$$

$$=n(n-1)x^2(x+y)^{n-2}+nx(x+y)^{n-1}$$

$$=n(n-1)x^2+nx \qquad (\because x+y=1)$$

$$=nx(nx-x+1)$$

$$=nx(nx+y) \qquad (\because x+y=1)$$

28. Given that,

$$Z=f(x+ay)+\phi(x-ay)$$

$$Z_x=f'(x+ay)+\phi(x-ay)$$

$\Rightarrow$ $Z_{xx}=f''(x+ay)+\phi''(x-ay)$

and $Z_y=af'(x+ay)-a\phi'(x-ay)$

$\Rightarrow$ $Z_{yy}=a^2f''(x+ay)-a^2\phi'(x-ay)$

Hence, $a^2Z_{xx}=Z_{yy}$

29. For unique solution of the given system, $\Delta \neq 0$

i.e., $\begin{vmatrix} 1 & 1 & 1 \\ 5 & -1 & \mu \\ 2 & 3 & -1 \end{vmatrix} \neq 0$

Hence, it depends only on μ.

30. Given, $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = 10A^{-1}$

$$\therefore \quad \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 10 & 0 & 0 \\ -5+\alpha & 5+\alpha & -5+\alpha \\ 0 & 0 & 10 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow \quad -5+\alpha = 0$$

$$\Rightarrow \quad \alpha = 5$$

31. Given equation of curves are

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ and } x^2 + y^2 = ab$$

$$\therefore \quad \frac{ab - y^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\Rightarrow \quad y^2\left(\frac{a^2 - b^2}{a^2b^2}\right) = \frac{a-b}{a}$$

$$\Rightarrow \quad y^2 = \frac{ab^2}{a+b}$$

$$\text{and} \quad x^2 = \frac{a^2b}{a+b}$$

$$\Rightarrow \quad x = a\sqrt{\frac{b}{a+b}}$$

$$\text{and} \quad y = b\sqrt{\frac{a}{a+b}}$$

Slope of tangent at ellipse $= \dfrac{-b^2x}{a^2y}$

$$= -\frac{b^2}{a^2}\sqrt{\frac{a}{b}}$$

Slope of tangent at circle $= -\dfrac{x}{y} = -\sqrt{\dfrac{a}{b}}$

$$\therefore \quad \theta = \tan^{-1}\left[\frac{\sqrt{\frac{a}{b}} - \frac{b^2}{a^2}\sqrt{\frac{a}{b}}}{1 + \frac{b^2}{a^2}\cdot\frac{a}{b}}\right]$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$$

32. Given that,

$$S = \sum_{n=0}^{\infty} \frac{(\log x)^{2n}}{(2n)!}$$

$$= \left(\frac{e^{\log x} + e^{-\log x}}{2}\right) = \frac{x + x^{-1}}{2}$$

33. $\log 2 + 2\left(\frac{1}{5} + \frac{1}{3}\cdot\frac{1}{5^3} + \frac{1}{5}\cdot\frac{1}{5^5} + \ldots \infty\right)$

$$= \log 2 + \log\left(\frac{1 + \frac{1}{5}}{1 - \frac{1}{5}}\right)$$

$$= \log 2 + \log\left(\frac{3}{2}\right) = \log 3$$

34. Since, vertex is (0, 4) and focus is (0, 2).

$$\therefore \quad a = 2$$

$\therefore$ Equation of parabola is

$$(x-0)^2 = -4\cdot 2(y-4)$$

$$\Rightarrow \quad x^2 + 8y = 32$$

35. The equation of a line parallel to $x + 2y = 4$ is $x + 2y = k$. Since, the distance between these two lines is 3, therefore

$$\frac{k}{\sqrt{1+4}} - \frac{4}{\sqrt{1+4}} = 3$$

$$\Rightarrow \quad k = 4 + 3\sqrt{5}$$

This shifted line cuts x-axis at $(k, 0)$. After rotation the slope of the line is $\tan(\theta - 30°)$, where $\tan\theta =$ (slope of

$$(x + 2y = 4) = -\frac{1}{2}$$

$\therefore$The equation of the line in the new position is

$$y - 0 = \tan(\theta - 30^\circ)(x - k),$$

$$\Rightarrow \quad y = \tan(\theta - 30^\circ)(x - k)$$

where $k = 4 + 3\sqrt{5}$

36. According to given condition, $2ae = 2 \cdot 2a$

$$\Rightarrow \quad e = 2$$

and $\quad 2b = 6 \Rightarrow b = 3$

Hence, $a = \frac{3}{\sqrt{3}} = \sqrt{3}$

$\therefore$Required equation is

$$\frac{x^2}{3} - \frac{y^2}{9} = 1$$

$$\Rightarrow \quad 3x^2 - y^2 = 9$$

37. The second degree equation will represent a pair of perpendicular straight lines, if

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0 \text{ and } a + b = 0$$

Given pair of line is

$$12x^2 + 7xy - py^2 - 18x + qy + 6 = 0$$

$$\therefore \quad \begin{vmatrix} 12 & 7/2 & -9 \\ 7/2 & -p & q/2 \\ -9 & q/2 & 6 \end{vmatrix} = 0$$

and $\quad 12 - p = 0$

$$\Rightarrow \quad p = 12$$

$$\therefore \quad \begin{vmatrix} 12 & 7/2 & -9 \\ 7/2 & -12 & q/2 \\ -9 & q/2 & 6 \end{vmatrix} = 0$$

$$\Rightarrow \quad 12\left(-72 + \frac{q^2}{4}\right) - \frac{7}{2}\left(-21 + \frac{9q}{2}\right) - 9\left(\frac{7q}{4} - 108\right) = 0$$

$$\Rightarrow \quad -864 - 3q^2 - \frac{147}{2} - \frac{63q}{4} - \frac{63q}{4} + 972 = 0 \Rightarrow q = 1$$

38. Given equation can be rewritten as

$$x^2 - 2(\alpha + \beta)x + 3\alpha\beta = 0$$

Let roots be α' and $-\alpha'$.

$$\therefore \quad \alpha' + (-\alpha') = 2(\alpha + \beta)$$

$$\Rightarrow \quad \alpha + \beta = 0$$

39. Let $f(x) = ax^2 + bx + c$

Again, let $f(x) = \int f(x)\,dx$

$$= \frac{a}{3}x^3 + \frac{bx^2}{2} + cx$$

At $\quad x = 0,$

$$f(0) = 0$$

Agin, at $x = 1$

$$f(1) = \frac{a}{3} + \frac{b}{2} + c$$

$$= \frac{2a + 3b + 6c}{6} = 0$$

Hence, one root lies between (0, 1).

40. Since, the lines $3x + 4y + 2 = 0$ and $3x + 4y + 5 = 0$ are on the same side of the origin. The distance between these lines is $5 - 2 = 3$.

And the line $3x + 4y + 2 = 0$ and $3x + 4y - 5 = 0$ are opposite side of the origin, The distance between these lies is 7.

Hence, required ratio is 3 : 7.

41. Given equation of line $y = x$ is written in polar form is

$$\frac{x}{\cos\theta} = \frac{y}{\sin\theta} = r, \text{ where } \theta = \frac{\pi}{4}$$

For point P, $r = 6\sqrt{2}$. Therefore, coordinates of P are given by

$$\frac{x}{\cos\frac{\pi}{4}} = \frac{y}{\sin\frac{\pi}{4}} = 6\sqrt{2}$$

$$\Rightarrow \quad x = 6, y = 6$$

Since, point (6, 6) lies on

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$\therefore \quad 72 + 12(g + f) + c = 0 \quad \text{...(i)}$$

Since, $y = x$ touches the circle, therefore the equation $2x^2 + 2x(g + f) + c = 0$ has equal roots.

$$\Rightarrow \quad 4(g + f)^2 = 8c$$

$$\Rightarrow \quad (g + f)^2 = 2c \qquad \ldots\text{(ii)}$$

From Eq. (i)

$$[(12(g + f)]^2 = [-(c + 72)]^2$$

$$\Rightarrow \quad 144(g + f)^2 = (c + 72)^2$$

$$\Rightarrow \quad 144(2c) = (c + 72)^2 \quad \text{[from Eq. (ii)]}$$

$$\Rightarrow \quad (c - 72)^2 = 0$$

$$\Rightarrow \quad c = 72$$

42. $(1 + x + x^2 + x^3)^n = (1 + x)^n (1 + x^2)^n$

$$= (1 + {}^nC_1\, x + {}^nC_2 x^2 + \ldots\ldots + {}^nC_n\, x^n)$$
$$\times (1 + {}^nC_1\, x^2 + {}^nC_2 x^4 + \ldots\ldots + {}^nC_n x^{2n})$$

$\therefore$ The coefficient of x^4

$$= {}^nC_2 + {}^nC_2 \cdot {}^nC_1 + {}^nC_4$$

$$= {}^nC_4 + {}^nC_2 + {}^nC_1 \cdot {}^nC_2$$

43. $T_n = \dfrac{1 + x + x^2 + \ldots + x^{n-1}}{n!}$

$$= \frac{x^n - 1}{x - 1} \cdot \frac{1}{n!}$$

$$= \frac{1}{1 - x}\left(\frac{x^n}{n!} - \frac{1}{n!}\right)$$

$$\Rightarrow \quad \sum_{n=1}^{\infty} \frac{1}{1 - x}\left(\frac{x^n}{n!} - \frac{1}{n!}\right)$$

$$= \frac{1}{(x - 1)}\left[\sum_{n=1}^{\infty} \frac{x^n}{n!} - \sum_{n=1}^{\infty} \frac{1}{n!}\right]$$

$$= \frac{1}{(x - 1)}(e^x - e)$$

44. Given inequation is

$$n! > 2^{n-1}$$

For $\quad n = 1, 2$

$$1! \not> 1$$

$$\Rightarrow \quad 1 \not> 1$$

and $\quad 2! \not> 2^1 \Rightarrow 2 \not> 2$

For $n = 3$

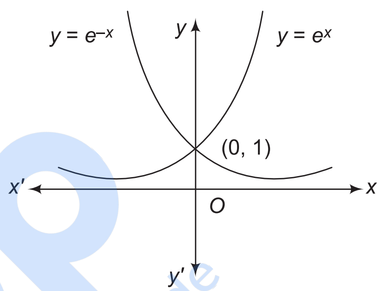
$$3! > 2^2$$

$$\Rightarrow \quad 6 > 4$$

Hence, it is true for $n > 2$.

45. Given sets are

$A = \{(x, y) : y = e^x,\ x \in R\}$ and
$B = \{(x, y) : y = e^{-x},\ x \in R\}$



It is clear from the graph that two curves intersect at one point.

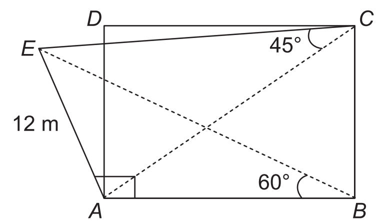
$$\therefore \quad A \cap B \neq \phi$$

46. Now, $\dfrac{b - c}{r_1} + \dfrac{c - a}{r_2} + \dfrac{a - b}{r_3}$

$$= \frac{[(b - c)(s - a) + (c - a)(s - b) + (a - b)(s - c)]}{\Delta}$$

$= 0$

47. Let AE is a vertical lamp post



In ΔAEC,

$$\tan 45° = \frac{AE}{AC}$$

$$\Rightarrow \quad AC = AE = 12\text{ m}$$

and in ΔABE,

$$\tan 60° = \frac{AE}{AB}$$

$$\Rightarrow \quad AB = \frac{AE}{\sqrt{3}} = 4\sqrt{3}$$

Now, $BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{144 - 48} = \sqrt{96}$

$= 4\sqrt{6}$

$\therefore$ Area of rectangle $ABCD$

$= AB \times BC$

$= 4\sqrt{3} \times 4\sqrt{6}$

$= 48\sqrt{2}\ m^2$

48. Given that,

$$f(x) = \max\{(1-x), 2, (1+x)\}$$

For $x \le -1$, we find that $1 - x \ge 2$

and $1 - x \ge 1 + x$

$\therefore \max\{(1-x), 2, (1+x)\} = 1 - x$

For $-1 < x < 1$, we find that

$0 < 1 - x < 2$ and $0 < 1 + x < 2$

$\therefore \max\{(1-x), 2, (1+x)\} = 2$

For $x \ge 1$, we find that

$1 + x \ge 2, 1 + x > 1 - x$

$\therefore \max\{(1-x), 2, (1+x)\} = 1 + x$

Hence, $f(x) = \begin{cases} 1-x & , \quad x \le -1 \\ 2 & , \quad -1 < x < 1 \\ 1+x & , \quad x \ge 1 \end{cases}$

49. Given, $3\sin^2 x - 7\sin x + 2 = 0$

$\Rightarrow \quad (2\sin x - 1)(\sin x - 2) = 0$

$\Rightarrow \quad \sin x = \frac{1}{3} \qquad (\because \sin x \ne 2)$

Let $\sin^{-1}\frac{1}{3} = \alpha, 0 < \alpha < \frac{\pi}{2}$ are the solutions in $[0, 5\pi]$. Then,

$\alpha, \pi - \alpha, 2\pi + \alpha, 3\pi - \alpha, 4\pi + \alpha, 5\pi - \alpha$ are the solutions.

50. $\left(1 + \cos\frac{\pi}{8}\right)\left(1 + \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{3\pi}{8}\right)\left(1 - \cos\frac{\pi}{8}\right)$

$= \left(1 - \cos^2\frac{\pi}{8}\right)\left(1 - \cos^2\frac{3\pi}{8}\right)$

$= \left(\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right)^2$

$= \frac{1}{4}\left(2\sin\frac{3\pi}{8}\sin\frac{\pi}{8}\right)^2$

$= \frac{1}{4}\left(\cos\frac{\pi}{4} - \cos\frac{\pi}{2}\right)^2 = \frac{1}{8}$

51. Since, $\sin^{-1}\sqrt{x}, \cos^{-1}\sqrt{x}$ are defined for $x \le 1$ and $x \ge 0$.

$\therefore \quad 0 \le \sqrt{x^2 - x + 1} \le 1$

and $\quad 0 \le \sqrt{x^2 - x} \le 1$

$\Rightarrow \quad -1 \le x^2 - x \le 0$

and $\quad 0 \le x^2 - x \le 0$

$\Rightarrow \quad x^2 - x = 0$

$\Rightarrow \quad x = 0, 1$

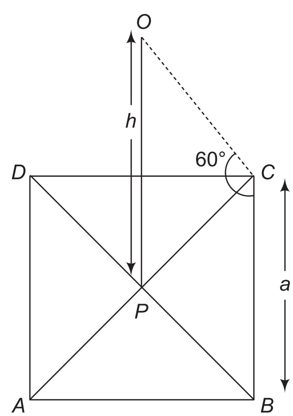
Hence, number of solutions are two.

52. Let $ABCD$ be a square of each side of length a. It is given that $\angle OCP = 60°$.

Length of diagonal

$AC = \sqrt{a^2 + a^2} = a\sqrt{2}$

$\therefore \quad PC = \frac{AC}{2} = \frac{a}{\sqrt{2}}$



In $\quad \Delta OCP, \tan 60° = \frac{OP}{PC}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{a/\sqrt{2}}$

$\Rightarrow \quad \sqrt{3}a = \sqrt{2}h$

$\Rightarrow \quad 3a^2 = 2h^2$

53. $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right)=\cos^{-1}\left(\cos\left(2\pi-\dfrac{5\pi}{6}\right)\right)$

$$=\cos^{-1}\left(\cos\left(\frac{5\pi}{6}\right)\right)=\frac{5\pi}{6}$$

54. Let $T=|\sin nx|$

Put $n=2$, then

$$T=|\sin 2x|$$
$$=|2\sin x\cos x|\le 2|\sin x|$$

55. Given sides of a triangle are

$$a=30,\ b=24,\ c=18$$

$$\therefore\quad s=\frac{a+b+c}{2}$$
$$=\frac{30+24+18}{2}=36$$

Now, $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

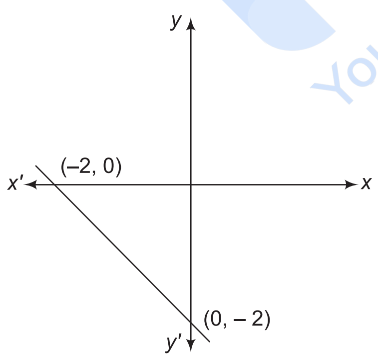
$$=\sqrt{36\,(36-30)\,(36-24)\,(36-18)}$$
$$=\sqrt{36\times 6\times 12\times 18}$$
$$=\sqrt{36\times 36\times 36}=216$$

$$\therefore\quad r_3=\frac{\Delta}{s-c}=\frac{216}{18}=12$$

56. Given lines are

$$xy+2x+2y+4=0 \qquad \text{...(i)}$$

and $\quad x+y+2=0 \qquad$ ...(ii)



From Eqs. (i) and (ii), we get

$$xy=0$$

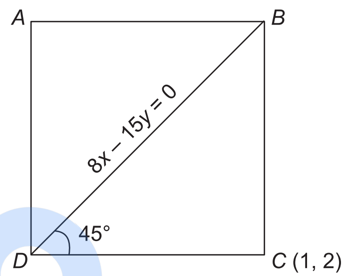
$\Rightarrow \quad x=y=0$

$\therefore$ Vertices of triangle are $(-2,0),(0,0),(0,-2)$.

In a right angled triangle circumcentre is the mid-point of hypotenuse.

$\therefore$ Point $(-1,-1)$ is the required coordinate of the circumcentre.

57. Slop of BD is $\dfrac{8}{15}$ and angle made by BD with AD and DC is 45°.



So, let slope of DC be m, then

$$\tan 45^\circ=\pm\frac{m-\frac{8}{15}}{1+\frac{8}{15}m}$$

$\Rightarrow \quad (15+8m)=\pm(15m-8)$

$\Rightarrow \quad m=\dfrac{23}{7}$ and $-\dfrac{7}{23}$

Hence, the equation of DC and AD are

$$y-2=\frac{23}{7}(x-1)$$

$\Rightarrow \quad 23x-7y-9=0$

and $\quad y-2=-\dfrac{7}{23}(x-1)$

$\Rightarrow \quad 7x+23y-53=0$

58. Given curves are

$$x^3-3xy^2+2=0 \qquad \text{...(i)}$$

and $\quad 3x^2y-y^3-2=0 \qquad$ ...(ii)

On differentiating Eqs. (i) and (ii), with respect to x, we get

$$\left(\frac{dy}{dx}\right)_{C_1}=\frac{x^2-y^2}{2xy}$$

and $\quad \left(\dfrac{dy}{dx}\right)_{C_2}=\dfrac{2xy}{x^2-y^2}$

$$\left(\frac{dy}{dx}\right)_{C_1}\times\left(\frac{dy}{dx}\right)_{C_2}=-1$$

59. Given that,

$$\frac{2x+3}{(x+1)(x-3)} = \frac{a}{(x+1)} + \frac{b}{(x-3)}$$

$$\Rightarrow \quad (2x+3) = a(x-3) + b(x+1)$$

On comparing the coefficient of x and constant, we get

$$2 = a + b \text{ and } 3 = -3a + b$$

$$\Rightarrow \quad a = -\frac{1}{4} \text{ and } b = \frac{9}{4}$$

$$\therefore \quad a + b = -\frac{1}{4} + \frac{9}{4} = 2$$

60. $$\lim_{n \to \infty} \frac{1}{n}\left[\sec^2 \frac{\pi}{4n} + \sec^2 \frac{2\pi}{4n} + \ldots + \sec^2 \frac{n\pi}{4n}\right]$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} \sec^2 \frac{r\pi}{4n}$$

$$= \int_0^1 \sec^2 \frac{\pi x}{4} dx$$

$$= \frac{4}{\pi}\left[\tan \frac{\pi x}{4}\right]_0^1 = \frac{4}{\pi}$$

61. Given that, $f(x) = |\cos x|$ and $g(x) = [x]$,

$$\therefore \quad gof(x) = g(|\cos x|)$$

$$= [|\cos x|]$$

62. Since, $\lim_{x \to \infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$

$$\Rightarrow \quad \lim_{x \to \infty}\left(1 + \frac{ax+b}{x^2}\right)^{2x} - e^2 \quad \ldots\text{(i)}$$

$\Rightarrow \quad \lim_{x \to \infty} \frac{ax+b}{x^2}$ must be equal to zero.

$$\Rightarrow \quad \lim_{x \to \infty} \frac{ax+b}{x^2} = 0$$

$$\Rightarrow \quad \lim_{x \to \infty}\left(\frac{a}{x} + \frac{b}{x^2}\right) = 0$$

$\Rightarrow \quad a$ and $b \neq 0$

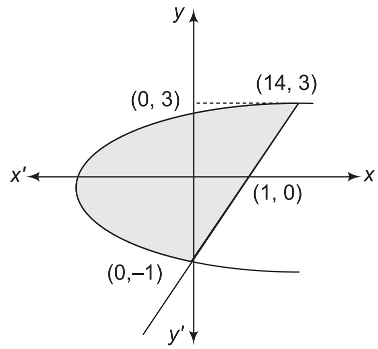
$\therefore$ From Eq. (i)

$$e^{\lim_{x \to \infty} \frac{2(ax+b)}{x}} = e^2$$

$$\Rightarrow \quad e^{2a} = e^2$$

$\Rightarrow \quad a = 1$ and $b \in R$

63. The point of intersection of curve $y^2 = 2x + 1$ and line $x - y - 1 = 0$ is (14, 3) and (0, – 1)



$$\therefore \text{Required area} = \int_{-1}^{3} (x_2 - x_1)\, dy$$

$$= \int_{-1}^{3}\left\{(y+1) - \frac{(y^2-1)}{2}\right\} dy$$

$$= \frac{1}{2}\int_{-1}^{3} (2y + 3 - y^2)\, dy$$

$$= \frac{1}{2}\left[y^2 + 3y - \frac{y^3}{3}\right]_{-1}^{3}$$

$$= \frac{1}{2}\left[(9 + 9 - 9) - \left(1 - 3 + \frac{1}{3}\right)\right]$$

$$= \frac{1}{2}\left(9 + \frac{5}{3}\right)$$

$$= \frac{16}{3} \text{ sq units}$$

64. For $f(x)$ to be continuous everywhere, we must have

$$f(0) = \lim_{x \to 0} f(x)$$

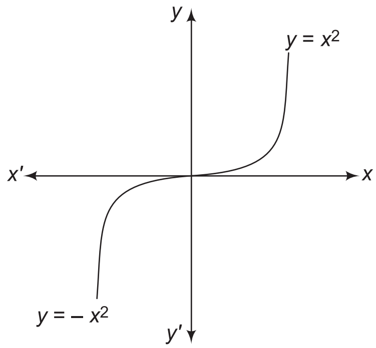
$$= \lim_{x \to 0} \frac{2 - (256 - 7x)^{1/8}}{(5x + 32)^{1/5} - 2}$$

$$= \lim_{x \to 0} \frac{\frac{7}{8}(256 - 7x)^{-7/8}}{(5x + 32)^{-4/5}}$$

$$= \frac{7}{8} \times \frac{2^{-7}}{2^{-4}} = \frac{7}{64}$$

65. Given that, $f(x) = x|x|$

$$\therefore \quad f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$



It is clear from the graph that $f(x)$ is differentiable everywhere.

66. Since, $f(x) = |x|^{\sin x}$

when $x < 0$

$$f(x) = (-x)^{(-\sin x)}$$
$$= e^{-\sin x \log(-x)}$$
$$\Rightarrow \quad f'(x) = e^{-\sin x \log(-x)} \left(-\cos x \log(-x) - \frac{\sin x}{x}\right)$$
$$\Rightarrow \quad f'(x) = (-x)^{-\sin x} \left(-\cos x \cdot \log(-x) - \frac{\sin x}{x}\right)$$
$$\Rightarrow \quad f'\left(-\frac{\pi}{4}\right) = \left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(-\frac{1}{\sqrt{2}} \log\frac{\pi}{4} + \frac{4}{\pi}\left(-\frac{1}{\sqrt{2}}\right)\right)$$
$$= \left(\frac{\pi}{4}\right)^{1/\sqrt{2}} \left(\frac{\sqrt{2}}{2} \log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$$

67. Let the centre of a circle be $C_1(h, k)$

Since, $\quad C_1C_2 = r_1 + r_2 \quad$ (given)

$$\Rightarrow \quad \sqrt{(h-0)^2 + (k-3)^2} = |k+2|$$
$$\Rightarrow \quad h^2 = 5(2k-1)$$

Hence, locus of a point is $x^2 = 5(2y - 1)$ which represents a equation of parabola.

68. Let $S(a, 0)$ be the focus of the parabola $y^2 = 4ax$ and $P(at^2, 2at)$ be a point on it. Then, the equation of a circle on SP as diameter is $(x - a)(x - at^2) + (y - 0)(y - 2at) = 0$

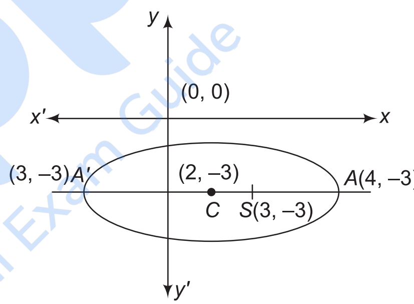
It meets y-axis at $x = 0$

$$\therefore \quad y^2 - 2aty + a^2t^2 = 0$$
$$\Rightarrow \quad (y - at)^2 = 0$$

This shows that y-axis meets the circle in two coincident points. Hence, the circle touches the tangent at the vertex.

69. Let $2a$ and $2b$ be the major and minor axes of the ellipse. Then, its equation is

$$\frac{(x-2)^2}{a^2} + \frac{(y+3)^2}{b^2} = 1 \quad \text{...(i)}$$



Here, semi-major axis $CA = a$

$$\Rightarrow \quad \sqrt{(4-2)^2 + (-3+3)^2} = a$$
$$\Rightarrow \quad a = 2 \quad \text{...(ii)}$$

Here, $CS = ae$

$$\Rightarrow \quad \sqrt{(2-3)^2 + (-3+3)^2} = ae$$
$$\Rightarrow \quad ae = 1 \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$e = \frac{1}{2}$$

Now, $\quad b^2 = a^2(1 - e^2)$

$$\Rightarrow \quad b^2 = 4\left(1 - \frac{1}{4}\right) = 3$$

On substituting the values of a and b in Eq. (i) we get

$$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{3} = 1$$

70. The diameters $y = m_1 x$ and $y = m_2 x$ are conjugate diameters of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ if } m_1 m_2 = \frac{b^2}{a^2}$$

Given equation is

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Here, $a^2 = 9, b^2 = 16$

Also given, $x = 2y$

Then, $m_1 = \frac{1}{2}$

$$\because \quad \frac{1}{2} \times m_2 = \frac{16}{9}$$

$$\Rightarrow \quad m_2 = \frac{32}{9}$$

Hence, the required equation of diameter is

$$y = \frac{32x}{9}$$

71. $\therefore (2x + 3) = a(x - 3) + b(x + 1)$

Put $x = -1$ and $x = 3$ respectively, we get

$$1 = -4a \text{ and } 9 = 4b$$

$$\Rightarrow \quad a = -\frac{1}{4} \text{ and } b = \frac{9}{4}$$

$$\therefore \quad a + b = -\frac{1}{4} + \frac{9}{4} = 2$$

72. Here, $e^x + 2 = -3(2e^x - 3) + B(e^x - 1)$

On equating the coefficient of e^x and constant, we get

$$1 = -6 + B,$$

$$2 = 9 - B \Rightarrow B = 7$$

73. $$\frac{3\sqrt{2}}{\sqrt{6} + \sqrt{3}} - \frac{4\sqrt{3}}{\sqrt{6} + \sqrt{2}} + \frac{\sqrt{6}}{\sqrt{3} + \sqrt{2}}$$

$$= \frac{3\sqrt{2}(\sqrt{6} - \sqrt{3})}{6 - 3} - \frac{4\sqrt{3}(\sqrt{6} - \sqrt{2})}{6 - 3} + \frac{\sqrt{6}(\sqrt{3} - \sqrt{2})}{3 - 2}$$

$$= \sqrt{2}(\sqrt{6} - \sqrt{3}) - \sqrt{3}(\sqrt{6} - \sqrt{2}) + \sqrt{6}(\sqrt{3} - \sqrt{2})$$

$$= 0$$

74. Since, it is an AP.

$$\therefore \quad \log_3 (2^x - 5) = \frac{\log_3 2 + \log_3 \left(2^x - \frac{7}{2}\right)}{2}$$

$$\Rightarrow \quad 2\log_3 (2^x - 5) = \log_3 \left(2 \cdot \left(2^x - \frac{7}{2}\right)\right)$$

$$\Rightarrow \quad (2^x - 5)^2 = 2^{x+1} - 7$$

$$\Rightarrow \quad 2^{2x} - 12 \cdot 2^x - 32 = 0$$

$$\Rightarrow \quad x = 2, 3$$

But $x = 2$, $\log_3 (2^x - 5)$ is not defined.

75. Let $\frac{A}{R}, A, AR$ be the roots of the equation $ax^3 + bx^2 + cx + d = 0$, then

Product of roots

$$A^3 = -\frac{d}{a}$$

$$\Rightarrow \quad = A = -\left(\frac{d}{a}\right)^{1/3}$$

Since, A is a root of the equation.

$$\therefore \quad aA^3 + bA^2 + cA + d = 0$$

$$\Rightarrow \quad a\left(-\frac{d}{a}\right) + b\left(-\frac{d}{a}\right)^{2/3} + c\left(-\frac{d}{a}\right)^{1/3} + d = 0$$

$$\Rightarrow \quad b\left(\frac{d}{a}\right)^{2/3} = c\left(\frac{d}{a}\right)^{1/3}$$

$$\Rightarrow \quad b^3 \cdot \frac{d^2}{a^2} = c^3 \cdot \frac{d}{a}$$

$$\Rightarrow \quad b^3 d = c^3 a$$

76. The composition table is as given below

$\times_5$	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Clearly, identity element = 1

$$\therefore \quad 2^{-1} \times_5 (3 \times_5 x) = 4$$

$$\Rightarrow \quad 3 \times_5 (3 \times_5 x) = 4$$

$$\Rightarrow \quad (3 \times_5 3) \times_5 x = 4$$

$\Rightarrow \quad 4 \times_5 x = 4$

$\Rightarrow \quad x = 4^{-1} \times_5 4 = 1$

77. Let p : Ram is in class x, q : Rashmi is in class XII

Given proposition is $p \vee q$.

Its negation is $\sim (p \vee q) = \sim p \wedge \sim q$

Ram is not in class X and Rashmi is not in class XII.

78. Inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$

$\therefore$ Inverse of $(p \wedge \sim q) \Rightarrow r$ is

$\sim (p \wedge \sim q) \Rightarrow \sim r$

i. e., $\quad (\sim p \vee q) \Rightarrow \sim r$

79. $(17)^2 = 289 \equiv (-1) \pmod 5$

$\Rightarrow \quad (17^2)^{15} \equiv (-1)^{15} \pmod 5$

$\Rightarrow \quad 17^{30} \equiv - \pmod 5$

i. e., $\quad 17^{30} \equiv 4 \pmod 5$

80. Given equation can be rewritten as

$$\operatorname{cosec} y \, dy = \sin x \, dx$$

On integrating both sides, we get

$$\log \tan \frac{y}{2} = -\cos x + \log C$$

$$\Rightarrow \quad \log \frac{\tan \frac{y}{2}}{C} = -\cos x$$

$$\Rightarrow \quad \frac{\tan \frac{y}{2}}{C} = e^{-\cos x}$$

$$\Rightarrow \quad e^{\cos x} \cdot \tan \frac{y}{2} = C$$

General English and Aptitude

21. Knot, Watt and Fathom, all are units of measurement.

22. Majlis, Diet and Knesset, all are parliament name of countries.

23. Kanha, Periyar and Dachigam, all are animal sanctuaries.

24. Stirrup, Anvil and Drum, all are parts of ear.

25. Kennedy, Indira and Palme, all of them were assassinated.

26. Chair, Door and Stick all are made of the same raw material.

27. As, a performer plays music on a guitar. Similarly, an acrobat performs tricks on a rope.

28. First is used to make the second and the third.

29. Second and third are bigger and more sophisticated forms than the first and second respectively.

30. Second is more intense form of the first, while third is opposite of second.