MET 2012 Answer Key PDF

Physics

1. Let one mole of each gas has same volume as V. When they are mixed, then density of mixture is

$$\rho_{mixture} = \frac{\text{mass of } O_2 + \text{mass of } H_2}{\text{volume of } O_2 + \text{volume of } H_2}$$

$$= \frac{32+2}{V+V} = \frac{34}{2V} = \frac{17}{V}$$

Also, $\rho_{H_2} = \dfrac{2}{V}$

Now, velocity $v = \left(\dfrac{\gamma P}{\rho}\right)^{1/2}$ or $v \propto \dfrac{1}{\sqrt{\rho}}$

$$\frac{v_{mixture}}{v_{H_2}} = \sqrt{\left(\frac{\rho_{H_2}}{\rho_{mixture}}\right)}$$

$$= \sqrt{\left(\frac{2/V}{17/V}\right)} = \sqrt{\left(\frac{2}{17}\right)}$$

2. Let the distance between the two cliffs be d. Since, the man is standing midway between the two cliffs, then the distance of man from either end is $d/2$.

The distance travelled by sound (in producing an echo)

$$2 \times \frac{d}{2} = v \times t \quad \Rightarrow \quad d = 340 \times 1 = 340 \text{ m}$$

3. The distance between the first dark (minima) fringes on either side of the central maximum is twice the fringe width of either maximum or minimum.

i.e., $\beta = 2\dfrac{D\lambda}{d} = \dfrac{2 \times 2 \times 600 \times 10^{-9}}{1 \times 10^{-3}}$

$$= 2.4 \times 10^{-3} = 2.4 \text{ mm}$$

4. After silvering the plane surface, plano-convex lens behaves as a concave mirror of focal length

$$\frac{1}{F} = \frac{2}{f_{lens}}$$

But $F = 0.2\,\text{m}$

$\therefore$ $f_{lens} = 2F = 2 \times 0.2 = 0.4$ m

Now, from lens maker's formula

$$\frac{1}{f_{lens}} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ $\dfrac{1}{0.4} = (1.5-1) \times \dfrac{1}{R_1}$ $[\because R_2 = \infty]$

$\Rightarrow$ $R_1 = 0.5 \times 0.4 = 0.2$ m

5. For a stone which is thrown downwards from a balloon rising upwards, the equation of motion is

$$h = -ut + \frac{1}{2}gt^2$$

$$= -29 \times 10 + \frac{1}{2} \times 9.8 \times (10)^2$$

$$= -290 + 490$$

$$= 200 \text{ m}$$

6. The distance between two consecutive dark or bright fringes is recognised as β (fringe width) and that between central fringe and first dark fringe on either side is $\dfrac{\beta}{2}$.

Given, spacing between second dark fringe and central fringe $= \beta + \dfrac{\beta}{2}$

or $\dfrac{3\beta}{2} = 1$ mm (Given)

or $\beta = \dfrac{2}{3} \times 1$ mm or $\dfrac{\lambda D}{d} = \dfrac{2}{3}$ mm

$\therefore$ $\lambda = \dfrac{2}{3} \times 10^{-3} \times \dfrac{0.9 \times 10^{-3}}{1}$

$\Rightarrow$ $\lambda = 0.6 \times 10^{-6}$ m

$\therefore$ Wavelength of the monochromatic source of light

$$\lambda = 600 \times 10^{-9} \text{ m} = 600 \text{ nm}$$

7. The number of counts left after time t

$$N = N_0\left(\frac{1}{2}\right)^{t/T_{1/2}}$$

$\therefore$ $30 = 240\left(\dfrac{1}{2}\right)^{60/T_{1/2}}$

$[\because t = 1 \text{ h} = 60 \text{ min}]$

or $\dfrac{30}{240} = \left(\dfrac{1}{2}\right)^{60/T_{1/2}}$

or $\left(\dfrac{1}{2}\right)^3 = \left(\dfrac{1}{2}\right)^{60/T_{1/2}}$

Comparing the powers, we get

$$\therefore \quad \frac{60}{T_{1/2}} = 3$$

$$T_{1/2} = \frac{60}{3}$$

or $$T_{1/2} = 20 \text{ min}$$

8. The resistance of conductor

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi r^2}$$

or $$R \propto \frac{1}{r^2}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{r_2^2}{r_1^2}$$

$$= \frac{2}{1} \times \left(\frac{2}{1}\right)^2 = \frac{8}{1}$$

Thermal potentials between the ends of the rods are same. So, heat conducted per second

$$H = \frac{Q}{t} \propto \frac{1}{R}$$

$$\therefore \quad \frac{H_1}{H_2} = \frac{R_2}{R_1} = \frac{1}{8} = 1:8$$

9. Using the relation

$$\frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

or $$\frac{W}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

or $$\frac{W}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \left(\because \frac{Q_1}{Q_2} = \frac{T_1}{T_2}\right)$$

or $$W = Q_1\left(1 - \frac{T_2}{T_1}\right)$$

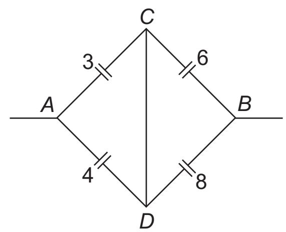
$$\therefore \quad W = 6 \times 10^4 \left[1 - \frac{127 + 273}{227 + 273}\right]$$

or $$W = 6 \times 10^4 \left(1 - \frac{400}{500}\right)$$

$$= 6 \times 10^4 \times \frac{100}{500}$$

$$= 1.2 \times 10^4 \text{ J}$$

10. The points *C* and *D* will be at same potenitals since, $\frac{3}{6} = \frac{4}{8}$. Therefore, capacitance of 2 μF will be unaffected. So, the equivalent circuit can be shown as the effective capacitance in upper arm in series, is given by



$$C_1 = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = 2\mu\text{F}$$

The effective capacitance in lower arm in series, is given by

$$C_2 = \frac{4 \times 8}{4 + 8} = \frac{32}{12} = \frac{8}{3}\mu\text{F}$$

Hence, the resultant capacitance in parallel is given by

$$C = C_1 + C_2 = 2 + \frac{8}{3} = \frac{14}{3}\mu\text{F}$$

11. Tension in the string is along the radius of circular path adopted by the bob; while displacement of the bob is along the circumference of the path. Hence, again **F** and **s** are at 90° and so, W = 0.

12. $K_{max} = K_0$ = total energy

As total energy remains conserved in SHM, hence when *U* is maximum in SHM, K = 0, *i.e.*, *E* is also equal to U_{max}, *i.e.*, $U_{max} = E = K_0$.

13. When the particle completes half revolution, change in velocity

$$\Delta v = [5 - (-5)] \text{ m/s} = 10 \text{ m/s}$$

Time taken to complete the half revolution is

$$t = \frac{\pi r}{v} = \frac{\pi \times 5}{5} = \pi \text{ s}$$

Thus, average acceleration $= \frac{\Delta v}{t} = \frac{10}{\pi} \text{ m/s}^2$

14. Mass of copper liberated

$$m = zit = z\left(\frac{P}{V}\right)t$$

$$= (3.3 \times 10^{-7})\left(\frac{100 \times 36 \times 10^5}{33}\right)$$

$$= 3.6 \text{ kg}$$

15. At resonant frequency

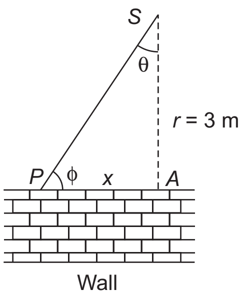
$$X_L = X_C \qquad \left(\omega L = \frac{1}{\omega C}\right)$$

At frequencies higher than resonance frequencies

$$X_L > X_C$$

i.e., behaviour is inductive.

16. The situation is shown in figure.



From figure,

$$x = r \tan \theta$$

∴ Velocity of *P* is

$$v = \frac{dx}{dt} = r \sec^2 \theta \left(\frac{d\theta}{dt}\right)$$

where, $\frac{d\theta}{dt}$ = angular velocity of rotation of spot

$$= \omega$$

$$\therefore \quad v = \omega r \sec^2 \theta$$

At $\phi = 45°$, so $\theta = 45°$

Hence, $v = 0.1 \times 3 \times \sec^2 45°$

$$= 0.1 \times 3 \times 2 = 0.6 \text{ m/s}$$

17. Using standard gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad \text{or} \quad V_2 = \frac{p_1V_1T_2}{p_2T_1}$$

$$\therefore \quad V_2 = \frac{1 \times 500 \times (273 - 3)}{0.5 \times (273 + 27)}$$

$$V_2 = \frac{1 \times 500 \times 270}{0.5 \times 300}$$

$$V_2 = 900 \text{ m}^3$$

18. Amplitude is independent of wavelength, velocity and frequency of oscillation.

19. Breaking stress $= \frac{\text{Breaking force}}{\text{Area}}$ = constant

$$\frac{F}{\left(\frac{\pi D^2}{4}\right)} = \frac{F'}{\pi D^2}$$

$$F' = 4F$$

20. $\frac{pV}{T}$ = constant; V = constant

$$\therefore \quad \frac{p}{T} = \text{constant}$$

$$\therefore \quad \frac{p}{T} = \frac{100.4p}{1000} \times \frac{1}{(T+1)}$$

$$\frac{1}{T} = \frac{100.4}{100} \times \frac{1}{(T+1)}$$

$$\frac{1}{T} = \frac{100.4}{100T + 100}$$

$$T = 250 \text{ K}$$

21. Maximum rate of energy flow, $S = E_0 \times H_0$

Given, $E_0 = 100$ V/m, $H_0 = 0.265$ A/m

$$\therefore \quad S = 100 \times 0.265 = 26.5 \text{ W/m}^2$$

22. Conservation of linear momentum gives,

$$m_1v_1 + m_2v_2 = 0$$

$$m_1v_1 = -m_2v_2$$

$$v_1 = \frac{-m_2v_2}{m_1}$$

Given, $m_1 = 10 \text{ g} = \left(\frac{10}{1000}\right)$ kg, $m_2 = 1$ kg,

$$v_2 = -5 \text{ m/s}$$

$$\therefore \quad v_1 = \frac{+1 \times 5}{10/1000}$$

or $\quad v_1 = 500$ m/s

23. Work done

$$W = p(V_2 - V_1)$$

Given, $p = 2 \times 10^5 \text{ N/m}^2$, $V_1 = 50 \times 10^{-3} \text{ m}^3$,

$$V_2 = 150 \times 10^{-3} \text{ m}^3$$

$$\therefore \quad W = 2 \times 10^5 (150 - 50) 10^{-3}$$

$$W = 2 \times 10^5 \times 100 \times 10^{-3}$$

$$W = 2 \times 10^4 \text{ J}$$

24. In an electron gun, the control grid is given a negative potential relative to cathode in order to repel the electrons so that a converging beam of electrons emerges from gun.

25. From the formula, $\omega = \frac{1}{\sqrt{LC}}$

Given, L = 8 H,

$$C = 0.5 \mu F = 0.5 \times 10^{-6} \text{ F}$$

$$\therefore \quad \omega = \frac{1}{\sqrt{8 \times 0.5 \times 10^{-6}}}$$

$$\Rightarrow \qquad \omega = \frac{1}{2\times 10^{-3}} = 500$$

$$\Rightarrow \qquad \omega = \frac{250}{\pi} \text{ rad/s}$$

26. Total power

$$P = P_1 + P_2$$
$$P = -4D + 2D$$
$$P = -2D$$

27. Molecular spectra due to vibrational motion lies in the microwave region of electromagnetic spectrum. Due to Kirchhoff's law in spectroscopy the same will be absorbed.

28. During inelastic collision of two particles kinetic energy after collision is not equal to kinetic energy before collision. Here, KE appears in other forms. In some cases, $(KE)_{final} < (KE)_{initial}$ such as when initial KE is converted into internal energy of the product (as heat, elastic or excitation energy) while in other cases $(KE)_{final} > (KE)_{initial}$ such as when internal energy stored in the colliding particles is released.

29. From the relation,

$$\overline{KE} = \frac{3}{2}kT$$

$$\Rightarrow \qquad \frac{(\overline{KE})_1}{(\overline{KE})_2} = \frac{T_1}{T_2} \qquad \text{(become } k \text{ is constant)}$$

Given, $\quad T_1 = 300\text{ K}, T_2 = 350\text{ K}$

$$\therefore \qquad \frac{(\overline{KE})_1}{(\overline{KE})_2} = \frac{300}{350} = \frac{6}{7}$$

30. From the formula, $\varepsilon = -L\dfrac{dI}{dt}$

Take magnitude on both sides.

$$|\varepsilon| = \left| -L\frac{dI}{dt} \right| \quad \text{or} \quad \varepsilon = L\frac{dI}{dt}$$

$$\therefore \qquad 80 = L\left(\frac{2}{0.05}\right) \quad \Rightarrow \quad 80 = L \times 40$$

$$\Rightarrow \qquad L = \frac{80}{40} \quad \Rightarrow \quad L = 2\text{ H}$$

32. As same force is applied to each wire having equal area of cross-section, so stress is same but extensions will be different for wires of different materials. So, same stress but different strains are there.

33. $L_1 = \dfrac{g_1 T^2}{4\pi^2} = \dfrac{g_1}{\pi^2}$; $L_2 = \dfrac{g_2 T^2}{4\pi^2} = \dfrac{g_2}{\pi^2}$

Since, length is decreased, g_2 is less than g_1.

$$\therefore \qquad L_1 - L_2 = \frac{g_1 - g_2}{\pi^2}$$

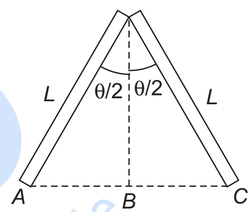
or $\qquad (L_1 - L_2)\pi^2 = g_1 - g_2$

or $\qquad 0.3 \times 10 = g_1 - g_2$

$$\therefore \qquad g_2 = 981 - 3 = 978\text{ cm/s}^2$$

34. Electric field due to outer plate is zero and so the electric field due to inner cylindrical charged conductor varies inversely as the distance from the axis.

35. On bending the magnet, the length of the magnet



$$AC = AB + BC$$
$$= L\sin\left(\frac{\theta}{2}\right) + L\sin\left(\frac{\theta}{2}\right)$$
$$= 2L\sin\left(\frac{\theta}{2}\right) = 2L\sin 30°$$
$$= 2L \times \frac{1}{2} = L$$

36. When a capacitor is fully charged, then no current is drawn by it. When no current flows in the circuit, potential difference across the cell = emf of cell = potential difference across the capacitor.

37. For hydrogen like atom, the radius of nth orbit

$$r_n^Z = \frac{n^2}{Z}a_0$$

Here, $\qquad a_0 = 0.51 \times 10^{-10}$ m

$$\therefore \qquad r_n^Z = \frac{0.51 \times 10^{-10}}{4}\text{ m}$$

In the ground state, $n = 1$

$$\therefore \qquad \frac{0.51 \times 10^{-10}}{4} = \frac{1^2}{Z} \times 0.51 \times 10^{-10}$$

$$\therefore \qquad Z = 4$$

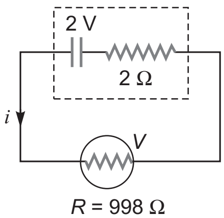
So, the atom is triply ionised beryllium (Be^{3+}).

38. $\lambda = \dfrac{h}{mv} = \dfrac{6.6 \times 10^{-34}}{1 \times 2000}$

$$= 3.3 \times 10^{-37}\text{ m} = 3.3 \times 10^{-27}\text{ Å}$$

39. From Ohm's law

$$i = \frac{E}{R + r} = \frac{2}{998 + 2} = 2 \times 10^{-3} \text{ A}$$

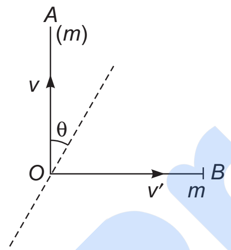


Potential difference across the voltmeter is

$$V = iR = (2 \times 10^{-3}) \times 998$$
$$= 1.996 \text{ V}$$

40. Initially, due to upward acceleration apparent weight of the body increases but then it decreases due to decrease in gravity.

41. From law of conservation of momentum,



$$mv - mv' \cos\theta = 0 \qquad \ldots\text{(i)}$$
$$mv - mv' \sin\theta = 0 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\theta = 45°$$
$$v' = \frac{v}{\cos\theta} = v\sqrt{2}$$

42. The decay equation is

$$_{92}X^{A} \longrightarrow {}_{Z}Y^{228} + \alpha$$

α-particle is nucleus of $_2He^4$

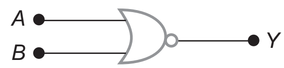
$$_{92}X^{A} \longrightarrow {}_{Z}Y^{228} + {}_{2}He^{4}$$
$$A = 228 + 4 = 232$$
$$Z = 92 - 2 = 90$$

43. Insulators have large energy gap between valence and conduction band (about 6 eV), while semiconductors have a smaller one and conductors, the smallest energy gap.

44. NOR gate is obtained when the output of OR gate is made as the input of NOT gate. Boolean expression for NOR gate is

$$Y = \overline{A + B}$$

Symbol of NOR gate



Truth Table

A	*B*	*C*
0	0	1
1	0	0
0	1	0
1	1	0

45. In common-emitter amplifier, the emitter current is much larger than the base current.

46. $$L = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

$$\therefore \quad L = 10 \log_{10}\left(\frac{2 \times 10^{-8}}{10^{-12}}\right)$$

$$(\because \; I_0 = 10^{-12} \text{ W/m}^2)$$

$$L = 10 \log_{10} (2 \times 10^4)$$
$$L = 10 [\log_{10} 2 + \log_{10} (10)^4]$$
$$L = 10 \, (4.3)$$
$$L = 43 \text{ decibel}$$

47. $D = 360° - 2\theta \quad \Rightarrow 240° = 360° - 2\theta$

$2\theta = 120°, \quad \therefore \quad \theta = 60°$

Then, number of images observable

$$N = \frac{360°}{60°} - 1$$
$$N = 6 - 1$$
$$N = 5$$

48. Number of Cu ions liberated is

$$n = \frac{I \cdot t}{e \text{ (valence electron)}}$$
$$n = \frac{1 \times 10}{1.6 \times 10^{-19} \times 2}$$
$$n = 3.125 \times 10^{19}$$
$$\therefore \quad n \approx 3.1 \times 10^{19}$$

49. For a magnet $B = \frac{\mu_0}{4\pi}\frac{2M}{r^3}$ (nearly)

$$\Rightarrow \qquad \frac{B_2}{B_1} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \qquad \frac{B_2}{B_1} = \left(\frac{x}{2x}\right)^3 \quad \Rightarrow \quad \frac{B_2}{B_1} = \frac{1}{8}$$

Thus, $B_1 : B_2 = 8 : 1$ approximately

50. Electromagnetic waves travel through oscillating electric and magnetic fields whose directions are perpendicular to each other.

51. According to question,

$$\frac{M_1}{d_1^3} = \frac{M_2}{d_2^3}$$

$$\Rightarrow \qquad \frac{M_1}{M_2} = \left(\frac{d_1}{d_2}\right)^3 = \left(\frac{12}{18}\right)^3$$

$$\therefore \qquad \frac{M_1}{M_2} = \frac{8}{27}$$

52. Effective current is the rms value. Here, 220 V is the labelled value of AC which is also the rms value. Hence,

$$I_{rms} = \frac{E_{rms}}{R}$$

$$I_{rms} = \frac{220}{100 \times 10^3}$$

$$I_{rms} = 2.2 \text{ mA}$$

54. The refractive index of glass is the greatest for violet colour and is smaller for red colour and as $\delta \propto (\mu - 1)$ therefore deviation for violet will be maximum.

55. Given, $y_1 = A \sin(kx - \omega t)$

$$y_2 = A \cos(kx - \omega t)$$

or $$y_2 = A \sin\left(kx - \omega t + \frac{\pi}{2}\right)$$

Phase difference of two waves $= \frac{\pi}{2}$

$\because$ Resultant amplitude

$$R = \sqrt{A^2 + A^2 + 2AA\cos\phi}$$

$$= \sqrt{A^2 + A^2 + 2A^2 \cos\frac{\pi}{2}}$$

$$= \sqrt{2A^2} \qquad \left(\because \cos\frac{\pi}{2} = 0\right)$$

$$R = \sqrt{2}A$$

56. Momentum of photon, $p = \frac{h}{\lambda}$

Kinetic energy of photon of mass *M*, $K = \frac{p^2}{2M}$

or $$K = \frac{h^2}{2M\lambda^2}$$

57. $$E = \frac{L^2}{2I}$$

$$\therefore \qquad E \propto L^2$$

$$\frac{E_2}{E_1} = \left(\frac{L_2}{L_1}\right)^2$$

$$\frac{E_2}{E_1} = \left[\frac{L_1 + 200\% \text{ of } L_1}{L_1}\right]$$

$$= \left[\frac{L_1 + 2L_1}{L_1}\right]^2 = (3)^2$$

$$E_2 = 9E_1$$

Increment in kinetic energy

$$\Delta E = E_2 - E_1$$

$$= 9E_1 - E_1$$

$$\Delta E = 8E_1$$

$$\frac{\Delta E}{E_1} = 8$$

or percentage increase = 800%

58. The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second.

Volume of water flowing out per second $= A\sqrt{2gh}$

Volume of water flowing in per second $= 70 \text{ cm}^3/\text{s}$

$$\therefore \qquad A\sqrt{2gh} = 70$$

$$1\sqrt{2 \times 980 \times h} = 70$$

$$h = \frac{4900}{1960} = 2.5 \text{ cm}$$

59. The production of X-rays is an atomic property whereas the production of γ-rays is a nuclear property.

60. Wood is non-crystalline.

Chemistry

1. The reaction is

$$N_2O_4 \rightleftharpoons 2NO_2$$

Molecular weight of $N_2O_4 = 2 \times 14 + 4 \times 16$
$= 92$

$\therefore$ Vapour density (D) of N_2O_4

$$= \frac{\text{molecular weight}}{2}$$

$$= \frac{92}{2} = 46$$

From, $\alpha = \dfrac{D - d}{(n-1)d}$

$$\alpha = \frac{46 - 30}{(2-1)30} = \frac{16}{30}$$

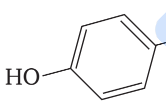
$$= 0.533 = 53.3\%$$

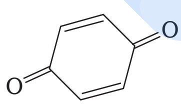
2. Protective power of a lyophilic colloid is measured in terms of gold number. Lower the value of gold number, better is the protective power of a protective colloid. Thus, the order will be

$$(a) > (b) > (d) > (c)$$

3. (a) $\frac{1}{2} H_2(g) \longrightarrow H^+(aq) + e^-$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log [H^+]$$

(b)  $\rightleftharpoons$

 $+ 2H^+ + 2e^-$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log [H^+]^2$$

(c) $Ag(s) \longrightarrow Ag^+(aq)$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log [Ag^+]$$

(d) $Ag(s) + Cl^-(aq) \longrightarrow AgCl(s) + e^-$

$$E_{cell} = E^\circ_{cell} - 0.0591 \log \frac{1}{[Cl^-]}$$

Thus, in case (d), the E_{cell} increases with increase in the concentration of ions.

4. According to Raoult's law, "Relative lowering of vapour pressure = mole fraction of solute"

$$(X_B) = 0.0125$$

$\because$ $X_B = \dfrac{m \cdot M_A}{10000 + m \cdot M_A}$

$\therefore$ $0.0125 = \dfrac{m \times 18}{1000 + m \times 18}$

$$12.5 + 0.225m = 18m$$

$$17.775m = 12.5$$

$$m = \frac{12.5}{17.775} = 0.70$$

5.

Formula	Name
(a) $Na_2S_2O_3 \cdot 5H_2O$	hypo
(b) $Na_2SO_4 \cdot 10H_2O$	Glauber's salt
(c) $Na_2CO_3 \cdot 10H_2O$	Washing soda
(d) $Na(NH_4)HPO_4$	Microcosmic salt

6. $PhCHClBr \xrightarrow[-t\text{-}BuOH]{tBuO^-} PhC^-ClBr \longrightarrow Ph\ddot{C}Cl$

7. 3° alcohol offers resistance to oxidation. Hence, Cr^{3+} ions are not formed, and the solution does not turn green.

8. Molisch's test is a general test for carbohydrates, while ninhydrin test is given by all proteins. Nitroprusside test is given by proteins which contains —SH group. Due to the presence of —CONH linkage, alkaline solution of proteins give biuret test.

9. Rocket propellants are a combination of an oxidiser and fuel. Phenyl hydrazine can't act as propellant.

10. $Cl^- = 2, 8, 8$

$$O^{2-} = 2, 8$$

$$Na^+ = 2, 8$$

$$Mg^{2+} = 2, 8$$

Cl^- contains same number of electrons in outermost and penultimate shells.

11. Mixture of $MgCl_2$ and MgO is called Sorel's cement. It is $MgCl_2 \cdot 5MgO \cdot xH_2O$.

12. Substracting equation (II) from (I), we get

$$S_R \longrightarrow S_M, \quad \Delta H = +2.5 \text{ kJ}$$

Thus, S_R to S_M transition is an endothermic reaction.

13. (a) PSC (polar stratospheric clouds) are special type of clouds formed over Antarctica in winter that play important role in ozone layer depletion.

(b) PCB (poly chlorinated biphenyls) are toxic chemicals with high stability and resistance to oxidation. These are used for fluids in transformers and capacitors.

(c) BOD (biochemical oxygen demand) is a measure of dissolved oxygen that would be needed by the microorganisms to oxidise organic and inorganic compounds present in polluted water.

(d) COD (chemical oxygen demand) is the amount of oxygen, in ppm, that would be required to oxidise the contaminants.

14. For a first order reaction

$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

At half-time,

$$x = \frac{a}{2}$$

$$\therefore \quad k = \frac{2.303}{1.6} \times \log \frac{a}{a - \frac{a}{2}}$$

$$= \frac{2.303}{1.6} \times \log 2$$

$$= \frac{2.303}{1.6} \times 0.3010$$

$$= 0.4332$$

For 90% completion,

$$x = 0.9a$$

$$\therefore \quad k = \frac{2.303}{t} \log \frac{a}{a - 0.9a}$$

$$0.4332 = \frac{2.303}{t} \log \frac{a}{0.1\,a}$$

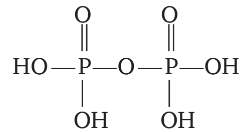
$$= \frac{2.303}{t} \log 10$$

$$= \frac{2.303}{t} \times 1$$

$$t = \frac{2.303}{0.4332}$$

$$= 5.3 \text{ min}$$

15. $H_4P_2O_7$ (pyrophosphoric acid) is tetrabasic because it gives four H^+ ions.



16. Bond angles of ClF_3, PF_3, NF_3 and BF_3 are $80°90'$, $101°$, $106°$ and $120°$ respectively.

17. Isotonic species have the same the number of neutrons.

No. of neutrons in

$${}_6C^{14} = 14 - 6 = 8$$

$${}_7N^{14} = 14 - 7 = 7$$

$${}_9F^{19} = 19 - 9 = 10$$

$${}_7N^{15} = 15 - 7 = 8$$

$${}_9F^{17} = 15 - 7 = 8$$

$${}_6C^{12} = 12 - 6 = 6$$

Hence, ${}_6C^{14}$, ${}_7N^{15}$, ${}_9F^{17}$ are isotonic.

18. $K[Co(CO)_4]$

$$+1 + x + 0 \times 4 = 0$$

$$\therefore \quad x = -1$$

19. The metal placed above in electrochemical series can displace the metals placed below from their salt solution.

The order of given metals in the series is Zn, Fe, Cu, Ag. Thus, Zn metal can displace all other three metals from their salt solutions.

20. Magic numbers are 2, 8, 20, 28, 50 or 82 protons or 2, 8, 20, 28, 50, 82 or 126 neutrons in the nucleus. These numbers impart stability to the nucleus.

(a) ${}_{13}Al^{27}$, no. of neutrons $= 27 - 13 = 14$

(b) ${}_{26}Fe^{56}$, no. of neutrons $= 56 - 26 = 30$

(c) ${}_{83}Bi^{209}$, no. of neutrons $= 209 - 83 = 126$

(d) ${}_{92}U^{238}$, no. of neutrons $= 238 - 92 = 146$

21. $[H_3O^+] = \sqrt{K_a \times C}$

$$= \sqrt{4 \times 10^{-10} \times 2.5 \times 10^{-1}}$$

$$= \sqrt{10^{-10}} = 10^{-5} \text{ M}$$

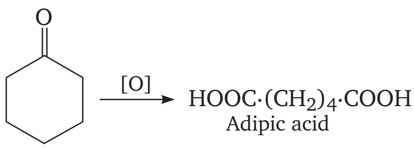
$$\because \quad pH = -\log [H_3O^+]$$

$$\therefore \quad pH = -\log (10^{-5}) = 5$$

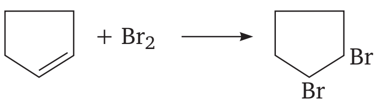
22. $\Delta H = ms\Delta T = (125 + 10.5) \times 1 \times 3.1$

$= 420.05 \text{ cal} = \dfrac{420.05}{10.5} \text{ cal/g}$

$= 40 \text{ cal/g} = 167.7 \text{ J/g}$

23. 

24. The conversion of $—CN$ to $—CH_2NH_2$ by catalytic reduction is called Mendius reaction.

25. 

26. No. of atoms of *A* from corners of unit cell $= \dfrac{7}{8}$

No. of atoms of *B* from faces of unit cell = 3

$\therefore \quad A : B = \dfrac{7}{8} : 3 = 7 : 24$

Thus, formula of the compound is A_7B_{24}.

27. The elements of II period show similar properties as the elements of III period, which are diagonally placed to them. This is known as diagonal relationship.

II Period	:	Li	Be	B	C	N
III Period	:	Na	Mg	Al	Si	P

28. Molecular weight of $X_4O_6 = (4a + 96)$

$\because$ $(4a + 96)$g X_4O_6 contains X = 4ag

$\therefore$ 10 g X_4O_6 contains $X = \dfrac{4a \times 10}{4a + 96}$

$5.72 = \dfrac{4a \times 10}{4a + 96}$

$22.88a + 549.12 = 40a$

$17.12a = 549.12$

$a = 32.07$ amu.

29.

	Formula	Name
(a)	ZnO	Philosopher's wool
(b)	$ZnS + BaSO_4$	Lithopone
(c)	$ZnSO_4 \cdot 7H_2O$	White vitriol
(d)	$ZnO \cdot CoO$	Rinmann's green

30. $_{29}Cu = 2, 8, 18, 1$

$= 1s^2, 2s^2 2p^6, 3s^2 3p^6 3d^{10}, 4s^1$

$\therefore$ Quantum numbers for $4s^1$ electron are

$n = 4, \quad l = 0, \quad m = 0, \quad s = +\dfrac{1}{2}$

31. $_5B$(G.S.) = 2s [↑↓] 2p [↑ | |]

B(E.S.) = 2s [↑] 2p [↑ | ↑ |]

sp^2-hybridisation

Due to sp^2-hybridisation, BF_3 has trigonal planar structure.

32. Normality, $N = \dfrac{w}{E \cdot V}$

$= \dfrac{10 \text{ g}}{60 \times \dfrac{100}{1000} \text{ L}}$

$= 1.66$ N

33.

$$H—O—\overset{\overset{\displaystyle O}{\|}}{\underset{\underset{\displaystyle O}{\|}}{S}}—O—O—\overset{\overset{\displaystyle O}{\|}}{\underset{\underset{\displaystyle O}{\|}}{S}}—O—H$$

$\because$ 2O-atoms are in peroxide linkage

$\therefore$ Their oxidation state = –1

and, oxidation state of remaining 6O-atoms = –2

Oxidation state of H-atoms = +1

Let oxidation state of S atom = x

$S_2 \quad H_2 \quad O_6 \quad O_2$

$\therefore \quad 2x + 2(+1) + 6(-2) + 2(-1) = 0$

$2x + 2 - 12 - 2 = 0$

$x = +6$

34. Reaction (a) and (b) are exothermic and spontaneous so do not need catalyst. On the other hand, conversion of SO_2 into SO_3 need catalyst. Catalyst used for this reaction may be Pt or V_2O_5 (contact process) or NO (lead chamber process)

$$2SO_2 + O_2 \xrightarrow{V_2O_5} 2SO_3$$

35. $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + 22$ kcal

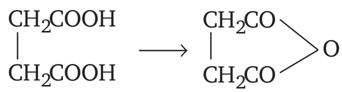
$\because$ Activation energy for the forward reaction $= 50$ kcal

$\therefore$ Activation energy for the backward reaction

$= 50 + 22 = 72$ kcal

36. $CH_2(COOH)_2 \xrightarrow{\Delta} CH_3COOH + CO_2$

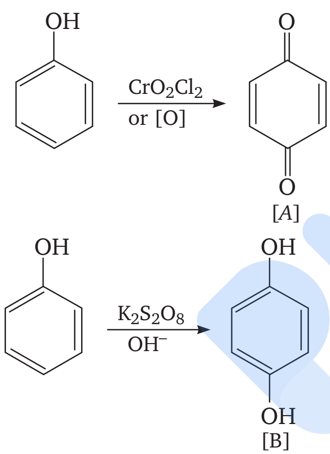
Malonic acid



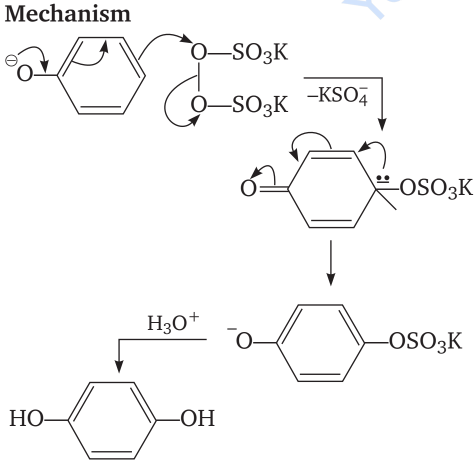
Succinic acid

37. $CHCl_3 + Cl_2 \xrightarrow{h\nu} \underset{\text{Carbon tetra chloride or pyrene}}{CCl_4} + HCl$

38.



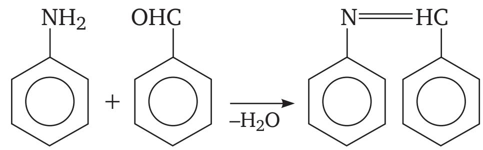
Mechanism



39. In transcription, synthesis of RNA takes place while in translation, synthesis of protein occurs in the cytoplasm of the cell.

40. Penicillin G is a narrow spectrum antibiotics. It is effective against single organism or disease.

41.



42. Only *A* and *C* are anomers.

A and B
B and C } are structural isomers.

43. According to IUPAC nomenclature  is bicyclo (2,1,0) pentane.

44. $CH_3CH_2OH \xrightarrow[K_2Cr_2O_7/H_2SO_4]{[O]} CH_3CHO$

$\xrightarrow[K_2Cr_2O_7/H_2SO_4]{[O]} CH_3COOH \xrightarrow[\Delta]{NaOH + CaO}$

$CH_4 \xrightarrow{Cl_2} CH_3Cl \xrightarrow[-KCl]{aq.\ KOH} CH_3OH$

45. PDI abbreviates as polydispersity index of polymer.

$$PDI = \frac{\overline{M}_w}{\overline{M}_n}$$

For natural polymers, PDI $= 1$
and for synthetic polymers, PDI > 1

46. Diamagnetic species have all electrons paired, whereas paramagnetic species have unpaired electrons.

$H_2 = \sigma 1s^2$

No. of unpaired electron $= 0 \Rightarrow$ diamagnetic

$H_2^- = \sigma 1s, \sigma^* 1s^1$

No. of unpaired electron $= 1 \Rightarrow$ paramagnetic

$H_2^- = \sigma 1s^1$

No. of unpaired electron $= 1 \Rightarrow$ paramagnetic

$He_2^+ = \sigma 1s^2, \sigma^* 1s^1$

No. of unpaired electron $= 1 \Rightarrow$ paramagnetic

47. Non-metals are electronegative in nature.

48. $\because \quad \Delta G = \Delta H - T\Delta S$

At equilibrium, $\Delta G = 0$

$\therefore \quad \Delta H = T\Delta S$

$$\Delta H = 273 \times (60.01 - 38.20)$$
$$= 5954.13 \text{ J mol}^{-1} = 5.954 \text{ kJ mol}^{-1}$$
$$\approx 6 \text{ kJ mol}^{-1}$$

49. $$\underset{\text{White phosphorus}}{P_4} + 3NaOH + 3H_2O \xrightarrow{\Delta} \underset{\text{Phosphene}}{PH_3} + \underset{\text{Sodium hypophosphate}}{3NaH_2PO_2}$$

50. $\because \quad \Delta T_b = \dfrac{1000 \cdot K_b \cdot w}{m \cdot W} \times i$

$\therefore \quad \Delta T_b \propto i \cdot \dfrac{w}{m \cdot \dfrac{W}{1000}} \quad (\because \; K_b = \text{constant})$

or $\quad \Delta T_b \propto i \cdot M$

$\left(\because \text{ Molality} = \dfrac{w}{m \cdot \dfrac{W}{1000}} \text{ and assuming, Molarity, M = molality, } m\right)$

Now, for the given solutions,

(a) For 0.08 M $BaCl_2$, $i \cdot M = 3 \times 0.08 = 0.24$

(b) For 0.15 M KCl, $i \cdot M = 2 \times 0.15 = 0.30$

(c) For 0.10 M glucose, $i \cdot M = 1 \times 0.10 = 0.10$

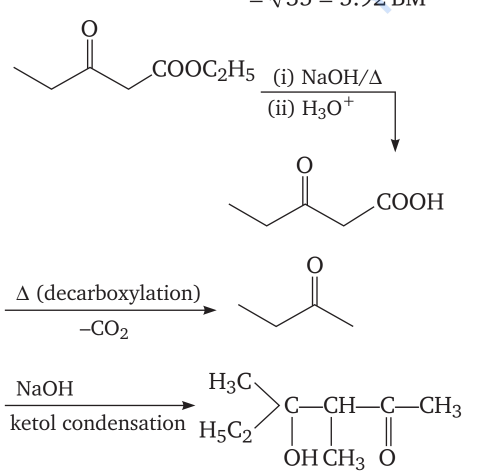
(d) For 0.06 M $Ca(NO_3)_2$, $i \cdot M = 3 \times 0.06 = 0.18$

Hence, ΔT_b is maximum for 0.15 M KCl solution.

51. No. of unpaired electrons in Fe = 5

Magnetic moment, $\mu = \sqrt{n(n+2)} = \sqrt{5(5+2)}$
$$= \sqrt{35} = 5.92 \text{ BM}$$

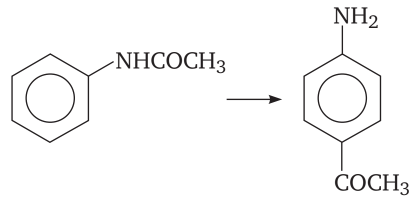
52.



53. $\because \quad \dfrac{r_H}{r_{He}} = \sqrt{\dfrac{M_{He}}{M_H}}$
$$= \sqrt{\frac{4}{2}} = \sqrt{2} = 1.4$$

$\therefore \; r_H : r_{He} = 1.4 : 1$

54.



Above reaction is an example of rearrangement reaction.

55. Temporary hardness of water is due to the presence of bicarbonates of calcium and magnesium while permanent hardness is due to the presence of soluble chlorides and sulphates of calcium and magnesium.

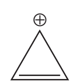
56. Vasopressin, oxytocin, insulin, angiotensin, are peptide hormones whereas thyroid, adrenaline are amine hormones.

57. Indigo is a vat dye. Vat dyes are insoluble in water and cann't be applied directly.

58. Solder is an alloy of tin and lead. Its melting point is quite low, hence, it is very useful in stitching ICs in various electrical instruments.

59. Hg_2^{2+} and Pb^{2+} have been placed in I group of basic radicals as their K_{sp} are less than the II group ions (Hg^{2+}, Cd^{2+}). Hence, only these will precipitate.

60.



$2\pi e^- \Rightarrow 4 \times 0 + 2 = 4n + 2$

$\therefore$ According to Huckel rule, cyclopropenyl carbocation is aromatic in nature.

Mathematics

1. Let $S = 1 + \dfrac{2^2}{2!} + \dfrac{3^2}{3!} + \dfrac{4^2}{4!} + \ldots$

$$\therefore \quad T_n = \frac{n^2}{n!} = \frac{n}{(n-1)!}$$

$$\Rightarrow \quad T_n = \frac{n-1+1}{(n-1)!} = \frac{1}{(n-2)!} + \frac{1}{(n-1)!}$$

$$\therefore \quad S_n = \Sigma T_n = \Sigma \frac{1}{(n-2)!} + \Sigma \frac{1}{(n-1)!}$$

$$= e + e$$

$$= 2e$$

2. We have, $f(x) = (x+b)^2 + 2c^2 - b^2$

$$\Rightarrow \quad \min f(x) = 2c^2 - b^2$$

and $\quad g(x) = b^2 + c^2 - (x+c)^2$

$$\Rightarrow \quad \max g(x) = b^2 + c^2$$

Thus, $\quad \min f(x) > \max g(x)$

$$\Rightarrow \quad 2c^2 - b^2 > b^2 + c^2$$

$$\Rightarrow \quad c^2 > 2b^2 \quad \Rightarrow \quad |c| > \sqrt{2}|b|$$

3. Let $z_1 = \gamma_1 (\cos\theta_1 + i \sin\theta_1)$.

Given that, $\left|\dfrac{z_1}{z_2}\right| = 1$

$$\Rightarrow \quad |z_1| = |z_2| \quad \Rightarrow \quad |z_1| = |z_2| = \gamma_1$$

Now, $\arg(z_1 z_2) = 0 \Rightarrow \arg(z_1) + \arg(z_2) = 0$

$$\Rightarrow \quad \arg(z_2) = -\theta_1$$

Therefore, $z_2 = \gamma_1(\cos(-\theta_1) + i\sin(-\theta_1))$

$$= \gamma_1(\cos\theta_1 - i\sin\theta_1) = \bar{z}_1$$

$$\Rightarrow \quad \bar{z}_2 = (\bar{\bar{z}}_1) = z_1 \quad \Rightarrow \quad z_2\bar{z}_2 = z_1 z_2$$

$$\Rightarrow \quad |z_2|^2 = z_1 z_2$$

4. We have, $\cos A = m \cos B$

$$\Rightarrow \quad \frac{\cos A}{\cos B} = \frac{m}{1}$$

On applying componendo and dividendo rule, we get

$$\Rightarrow \quad \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$$

$$\Rightarrow \quad \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)} = \frac{m+1}{m-1}$$

$$\Rightarrow \quad \cot\left(\frac{A+B}{2}\right) = \left(\frac{m+1}{m-1}\right)\tan\left(\frac{B-A}{2}\right)$$

But, $\quad \cot\dfrac{A+B}{2} = \lambda \tan\left(\dfrac{B-A}{2}\right) \qquad$ (given)

$$\Rightarrow \quad \lambda = \frac{m+1}{m-1}$$

5. Let the angles be $A = x - d$, $B = x$, $C = x + d$,
Then, $x - d + x + x + d = 180°$

$$\Rightarrow \quad 3x = 180° \quad \Rightarrow \quad x = 60°$$

Therefore, two larger angles are B and C.
Hence, $b = 9$ and $c = 10$.
Now,

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$

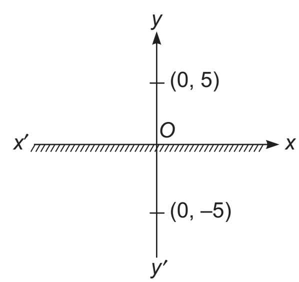
$$\Rightarrow \quad \frac{1}{2} = \frac{100 + a^2 - 81}{20a}$$

$$\Rightarrow \quad a^2 - 10a + 19 = 0$$

$$\Rightarrow \quad a = 5 \pm \sqrt{6}$$

6. $\left|\dfrac{z - 5i}{z + 5i}\right| = 1 \quad \Rightarrow \quad |z - 5i| = |z + 5i|$

$\left(\begin{array}{l}\text{Using definition } |z - z_1| = |z - z_2| \text{ gives} \\ \text{Perpendicular bisector of } z_1 \text{ and } z_2.\end{array}\right)$



$\Rightarrow$ Perpendicular bisector of points $(0, 5)$ and $(0, -5)$.
whch lies on y-axis.

7. Let the common ratio of the GP be r. Then, $y = xr$ and $z = xr^2$.

$$\Rightarrow \quad \ln y = \ln x + \ln r \text{ and } \ln z = \ln x + 2\ln r$$

Putting $A = 1 + \ln x$, $D = \ln r$

Then, $\quad \dfrac{1}{1 + \ln x} = \dfrac{1}{A}$,

$$\frac{1}{1+\ln y} = \frac{1}{1+\ln xr} = \frac{1}{1+\ln x+\ln r} = \frac{1}{A+D}$$

and $$\frac{1}{1+\ln z} = \frac{1}{1+\ln x+2\ln r} = \frac{1}{A+2D}$$

Here we see that A, $A + D$ and $A + 2D$ are in AP.

$\therefore$ $\frac{1}{A}, \frac{1}{A+D}$ and $\frac{1}{A+2D}$ are in HP.

Therefore, $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$, are in HP.

8. We have,

$$\frac{e^x}{1-x} = B_0 + B_1x + B_2x^2 + \ldots + B_nx^n + \ldots$$

$$\Rightarrow e^x(1-x)^{-1} = B_0 + B_1x + B_2x^2 + \ldots + B_nx^n + \ldots$$

$$\Rightarrow \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!} + \ldots\right) \times$$
$$(1 + x + x^2 + \ldots + x^{n-1} + x^n + \ldots \infty)$$
$$= B_0 + B_1x + B_2x^2 + \ldots + B_nx^n + \ldots$$

On comparing the coefficients of x^n and x^{n-1} on both sides, we get

$$\frac{1}{n!} + \frac{1}{(n-1)!} + \ldots + \frac{1}{2!} + \frac{1}{1!} + 1 = B_n$$

and $$\frac{1}{(n-1)!} + \frac{1}{(n-2)!} + \ldots + \frac{1}{2!} + \frac{1}{1!} + 1 = B_{n-1}$$

$$B_n - B_{n-1} = \frac{1}{n!}$$

9. As, $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$

$$\Rightarrow f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & x \in R - [0, 1) \\ 0, & 0 \leq x < 1 \end{cases}$$

$\therefore$ RHL at $x = 0$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} \frac{\sin[0+h]}{[0+h]} = 0$$

LHL at $x = 0$

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} \frac{\sin[0-h]}{[0-h]}$$
$$= \lim_{h \to 0} \frac{\sin(-1)}{-1} = \sin 1$$

Since, RHL $\neq$ LHL

$\therefore$ Limit does not exist.

10. Here, $\frac{dy}{dt} - \left(\frac{t}{1+t}\right)y = \frac{1}{1+t}$ and $y(0) = -1$

which represents linear differential equation of first order.

$$\text{IF} = e^{-\int\left(\frac{t}{1+t}\right)dt} = e^{-\int\frac{(t+1-1)}{1+t}dt}$$
$$= e^{-t+\log(1+t)}$$
$$= e^{-t}(1+t)$$

$\therefore$ Required solution is

$$y(\text{IF}) = [\textstyle\int Q \cdot (\text{IF})\, dt] + C$$

$$ye^{-t}(1+t) = \int \frac{1}{1+t} e^{-t}(1+t)\, dt + C$$
$$= \int e^{-t}\, dt + C = -e^{-t} + C$$

At $t = 0, y = -1$

$\therefore$ $-1(1+0) = -e^0 + C$

$\Rightarrow$ $C = 0$

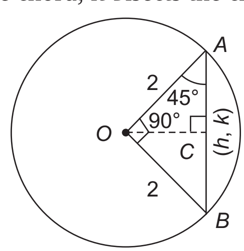
$\therefore$ $ye^{-t}(1+t) = -e^{-t}$

At $t = 1$

$ye^{-1}(1+1) = -e^{-1}$

$\Rightarrow$ $y = -\frac{1}{2}$

11. As, we have to find the locus of mid-point of chord and we know perpendicular drawn from centre to the chord, it bisects the chord.



Given, $\angle OAB = 90°$

Also, OC bisects the $\angle AOB$

$\therefore$ $\angle COA = \angle OAC = 45°$

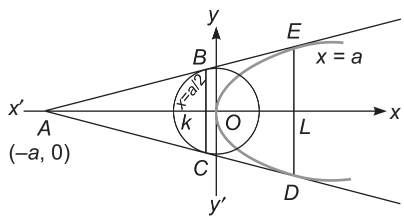
In ΔOAC, $\frac{OC}{OA} = \sin 45° \Rightarrow OC = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\therefore$ $h^2 + k^2 = OC^2$

$\Rightarrow$ $h^2 + k^2 = (\sqrt{2})^2$

or $x^2 + y^2 = 2$ is required equation of locus of mid-point of chord subtending right angle at centre.

12. Equation of any tangent of the parabola, $y^2 = 4ax$ is $y = mx + \frac{a}{m}$. This line will touch the circle $x^2 + y^2 = \frac{a^2}{2}$.



$$\therefore \quad BC = 2\sqrt{OB^2 - OK^2}$$
$$= 2\sqrt{\frac{a^2}{2} - \frac{a^2}{4}}$$
$$= 2\sqrt{\frac{a^2}{4}} = a$$

and we know that *DE* is the latusrectum of the parabola, so its length is 4a.

Thus, area of trapezium *BCDE*

$$= \frac{1}{2}(BC + DE)(KL)$$
$$= \frac{1}{2}(a + 4a)\left[\frac{3a}{2}\right]$$
$$= \frac{15a^2}{4} \text{ sq units}$$

13. Applying $R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_1$, the given determinant becomes

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -2-\omega^2 & \omega^2 - 1 \\ 0 & \omega^2 - 1 & \omega - 1 \end{vmatrix}$$

$$= 1[(\omega - 1)(-2 - \omega^2) - \{(\omega^2 - 1)(\omega^2 - 1)\}] - 1(0) + 1(0)$$
$$= (-2 - \omega^2)(\omega - 1) - (\omega^2 - 1)^2$$
$$= -2\omega + 2 - \omega^3 + \omega^2 - (\omega^4 + 1 - 2\omega^2)$$
$$= -2\omega + 1 + \omega^2 - (\omega + 1 - 2\omega^2)$$
$$= 3\omega^2 - 3\omega$$
$$= 3\omega(\omega - 1)$$

14. Let α, β be the roots of given quadratic equation. Then,

$$\alpha + \beta = \frac{4 + \sqrt{5}}{5 + \sqrt{2}} \quad \text{and} \quad \alpha\beta = \frac{8 + 2\sqrt{5}}{5 + \sqrt{2}}$$

Again, *H* be the harmonic mean between α and β, then $H = \frac{2\alpha\beta}{\alpha + \beta} = \frac{16 + 4\sqrt{5}}{4 + \sqrt{5}} = 4$

15. For $x = -3, 3; |x^2 - 9| = 0$

Therefore, $\log|x^2 - 9|$ does not exist at

$$x = -3, 3$$

Hence, domain of function is $R - \{-3, 3\}$.

16. Let $\quad y = \left[\frac{f(1 + x)}{f(1)}\right]^{1/x}$

$$\Rightarrow \quad \log y = \frac{1}{x}[\log f(1 + x) - \log f(1)]$$
$$\Rightarrow \lim_{x \to 0} \log y = \lim_{x \to 0} \frac{[\log f(1 + x) - \log f(1)]}{x}$$
$$\Rightarrow \lim_{x \to 0} \log y = \lim_{x \to 0}\left[\frac{1}{f(1 + x)} f'(1 + x)\right]$$

(using L' Hospital's rule)

$$= \frac{f'(1)}{f(1)} = \frac{6}{3} = 2$$
$$\Rightarrow \lim_{x \to 0} y = e^2$$

17. Now, $[\mathbf{a} \quad \mathbf{a} + \mathbf{b} \quad \mathbf{a} + \mathbf{b} + \mathbf{c}]$

$$= [\mathbf{a} \times (\mathbf{a} + \mathbf{b})] \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c})$$
$$= (\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$$
$$= [\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$$

18. $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{b} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$, $\mathbf{c} = -4\mathbf{i} - \mathbf{k}$

$$\therefore \quad \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b}$$
$$= (4 + 1)(\mathbf{i} + \mathbf{j} - 3\mathbf{k})$$
$$= 5\mathbf{i} + 5\mathbf{j} - 15\mathbf{k}$$

19. Here, $I(m, n) = \int_0^1 t^m (1 + t)^n \, dt$

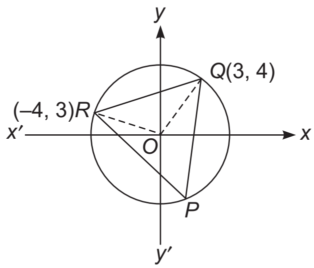
[We apply integration by parts, taking $(1 + t)^n$ as first and t^m as second function]

$$I(m, n) = \left[(1 + t)^n \cdot \frac{t^{m+1}}{m + 1}\right]_0^1 - \int_0^1 n(1 + t)^{n-1} \cdot \frac{t^{m+1}}{m + 1} dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1}\int_0^1 (1+t)^{n-1} \cdot t^{m+1}\, dt$$

$$\therefore \quad I(m, n) = \frac{2^n}{m+1} - \frac{n}{m+1} \cdot I(m+1, n-1)$$

20. Let *O* is the point on centre and *P* is the point on circumference. Therefore, angle *QOR* is double the angle *QPR*. So, it is sufficient to find the angle *QOR*.



Now, slope of OQ, $m_1 = \frac{4-0}{3-0} = \frac{4}{3}$

Slope of OR, $m_2 = \frac{3-0}{-4-0} = -\frac{3}{4}$

Again, $m_1 m_2 = -1$

Therefore, $\angle QOR = 90°$

Which implies that $\angle QPR = \frac{90°}{2} = 45°$.

21. We have, $y^2 + 4y + 4x + 2 = 0$

$$\Rightarrow \quad (y+2)^2 + 4x - 2 = 0$$

$$\Rightarrow \quad (y+2)^2 = -4\left(x - \frac{1}{2}\right)$$

Replace, $y + 2 = Y,\ x - \frac{1}{2} = X$

We have, $Y^2 = -4X$

This is a parabola with directrix at $X = 1$

$$\Rightarrow \quad x - \frac{1}{2} = 1$$

$$\Rightarrow \quad x = \frac{3}{2}$$

22. Given, $y = mx - b\sqrt{1+m^2}$ touches both the circles.

So, distance from centre = Radius of both the circles

$$\therefore \quad \frac{|-b\sqrt{1+m^2}|}{\sqrt{m^2+1}} = b$$

and $$\frac{|ma - 0 - b\sqrt{1+m^2}|}{\sqrt{m^2+1}} = b$$

$$\Rightarrow \quad |ma - b\sqrt{1+m^2}| = |-b\sqrt{1+m^2}|$$

$$\Rightarrow \quad m^2a^2 - 2abm\sqrt{1+m^2} + b^2(1+m^2) = b^2(1+m^2)$$

$$\Rightarrow \quad ma - 2b\sqrt{1+m^2} = 0$$

$$\Rightarrow \quad m^2a^2 = 4b^2(1+m^2)$$

$$\Rightarrow \quad m^2(a^2 - 4b^2) = 4b^2$$

$$\Rightarrow \quad m = \frac{2b}{\sqrt{a^2 - 4b^2}}$$

23. Here, $\alpha \in \left[0, \frac{\pi}{2}\right) \Rightarrow \tan\alpha$ is (+ve)

As, we know, if $a, b > 0 \Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$

i.e., AM ≥ GM

$$\therefore \quad \frac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2} \geq \sqrt{\sqrt{x^2+x} \cdot \frac{\tan^2\alpha}{\sqrt{x^2+x}}}$$

(using AM ≥ GM)

$$\Rightarrow \quad \sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \geq 2\tan\alpha$$

24. Let *x* be the length of an edge of a cube and *V* be the volume of that cube.

$$\therefore \quad V = x^3$$

On differentiating w.r.t. *x*, we get

$$\frac{dV}{dx} = 3x^2$$

Let δV be error in *V* and corresponding error δx in *x*.

$$\therefore \quad \delta V = \frac{dV}{dx}\delta x = 3x^2\,\delta x$$

Given that, $x = 2$ m and $\delta x = 0.5$ cm $= \frac{0.5}{100}$ m

$$\therefore \quad \delta V = 3(2)^2\left(\frac{0.5}{100}\right)$$

$$= \frac{12 \times 0.5}{100} = \frac{6}{100} = 0.06\ m^3$$

25. Let $g(x) = f(x) - x^2$

$\Rightarrow$ $g(x)$ has atleast 3 real roots which are $x = 1, 2, 3$ (by mean value theorem)

$\Rightarrow$ $g'(x)$ has atleast 2 real roots in $x \in (1, 3)$

$\Rightarrow$ $g''(x)$ has atleast 1 real root in $x \in (1, 3)$

$\Rightarrow$ $f''(x) = 2$ for atleast one $x \in (1, 3)$

26. We have,

$$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5} \quad \ldots(i)$$

$$\Rightarrow y = (C_1 + C_2)\cos(x + C_3) - C_4 e^x \cdot e^{C_5}$$

Now, let $C_1 + C_2 = A, C_3 = B, C_4 e^{C_5} = C$

$$\Rightarrow \quad y = A\cos(x + B) - Ce^x \quad \ldots(ii)$$

On differentiating w.r.t. *x*, we get

$$\frac{dy}{dx} = -A\sin(x + B) - Ce^x \quad \ldots(iii)$$

Again, differentiating w.r.t. *x*, we get

$$\frac{d^2y}{dx^2} = -A\cos(x + B) - Ce^x \quad \ldots(iv)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -y - 2Ce^x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + y = -2Ce^x \quad \ldots(v)$$

Again, differentiating w.r.t. *x*, we get

$$\frac{d^3y}{dx^3} + \frac{dy}{dx} = -2Ce^x \quad \ldots(vi)$$

$$\Rightarrow \quad \frac{d^3y}{dx^3} + \frac{dy}{dx} = \frac{d^2y}{dx^2} + y \text{ [from Eq. (v)]}$$

which is a differential equation of order 3.

27. Let $I = \int (\sqrt{\tan x} + \sqrt{\cot x})\,dx$

$$= \int \frac{\tan x + 1}{\sqrt{\tan x}}\,dx$$

Put $\tan x = t^2$

$$\Rightarrow \quad \sec^2 x\,dx = 2t\,dt$$

$$\Rightarrow \quad dx = \frac{2t\,dt}{1 + \tan^2 t} = \frac{2t}{1 + t^4}\,dt$$

$$\therefore \quad I = \int \frac{t^2 + 1}{\sqrt{t^2}} \cdot \frac{2t}{t^4 + 1}\,dt$$

$$= 2\int \frac{t^2 + 1}{t^4 + 1}\,dt$$

$$= 2\int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2} - 2 + 2}\,dt$$

$$= 2\int \frac{1 + \frac{1}{t^2}}{\left(t - \frac{1}{t}\right)^2 + (\sqrt{2})^2}\,dt$$

$$= 2\int \frac{du}{u^2 + (\sqrt{2})^2}$$

where, $\quad u = t - \frac{1}{t} \Rightarrow du = \left(1 + \frac{1}{t^2}\right)dt$

$$\Rightarrow \quad I = \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C$$

$$= \sqrt{2}\tan^{-1}\left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}}\right) + C$$

28. Let number of candidates be *n*. Therefore, $n - 2$ are to be elected and so one can vote up to $(n - 2)$. Hence, number of ways in which one can vote

$$= {}^nC_1 + {}^nC_2 + \ldots + {}^nC_{n-2} = 56 \quad \text{(given)}$$

$$\Rightarrow \quad 2^n - ({}^nC_0 + {}^nC_{n-1} + {}^nC_n) = 56$$

$$\Rightarrow \quad 2^n - n = 58 \Rightarrow 2^n = 58 + n$$

Which is satisfied by $n = 6$ only.

29. We have, $\quad x = t^2 + t + 1 \quad \ldots(i)$

and $\quad y = t^2 - t + 1 \quad \ldots(ii)$

Now, $\quad x + y = 2(1 + t^2) \quad \ldots(iii)$

and $\quad x - y = 2t \quad \ldots(iv)$

Now, from Eqs. (iii) and (iv), we get

$$x + y = 2\left[1 + \left(\frac{x - y}{2}\right)^2\right]$$

$$\Rightarrow \quad x + y = 2\left[\frac{4 + x^2 + y^2 - 2xy}{4}\right]$$

$$\Rightarrow \quad x^2 + y^2 - 2xy - 2x - 2y + 4 = 0 \quad \ldots(v)$$

$\therefore$ On comparing with

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

We get, $a = 1, b = 1, c = 4, h = -1, g = -1, f = -1$

$\because \quad \Delta = abc + 2fgh - af^2 - bg^2 - ch^2$

Now, $\Delta = 1 \cdot 1 \cdot 4 + 2(-1)(-1)(-1) - 1 \times (-1)^2 - 1 \times (-1)^2 - 4(-1)^2$

$= 4 - 2 - 1 - 1 - 4$

$= -4$ therefore, $\Delta \neq 0$

and $ab - h^2 = 1 \cdot 1 - (1)^2 = 1 - 1 = 0$

So, it is equation of a parabola.

30. Equation of tangent is drawn at a $(3\sqrt{3}\cos\theta, \sin\theta)$ to the curve

$$\frac{x^2}{27} + \frac{y^2}{1} = 1$$

is $\frac{x\cos\theta}{3\sqrt{3}} + \frac{y\sin\theta}{1} = 1$

Thus, sum of intercepts

$= (3\sqrt{3}\sec\theta + \operatorname{cosec}\theta) = f(\theta)$ (say)

$\Rightarrow f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$

$$= \frac{3\sqrt{3}\sin^3\theta - \cos^3\theta}{\sin^2\theta\cos^2\theta}$$

Put $f'(\theta) = 0$

$\Rightarrow \sin^3\theta = \frac{1}{3^{3/2}}\cos^3\theta$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = \frac{\pi}{6}$

31. Since, *a*, *b*, *c* are in AP.

$\therefore$ $a = A - D,\ b = A,\ c = A + D$

Where, *A* is the first term and *D* is the common difference of an AP.

Given, $a + b + c = \frac{3}{2}$

$\Rightarrow (A - D) + A + (A + D) = \frac{3}{2}$

$\Rightarrow 3A = \frac{3}{2}$

$\Rightarrow A = \frac{1}{2}$

$\therefore$ The numbers are $\frac{1}{2} - D, \frac{1}{2}, \frac{1}{2} + D$

Also, $\left(\frac{1}{2} - D\right)^2, \frac{1}{4}, \left(\frac{1}{2} + D\right)^2$ are in GP.

$\Rightarrow \left(\frac{1}{4}\right)^2 = \left(\frac{1}{2} - D\right)^2\left(\frac{1}{2} + D\right)^2$

$\Rightarrow \frac{1}{16} = \left(\frac{1}{4} - D^2\right)^2$

$\Rightarrow \frac{1}{4} - D^2 = \pm\frac{1}{4}$

$\Rightarrow D^2 = \frac{1}{2}$ ($\because$ D = 0 is not possible)

$\Rightarrow D = \pm\frac{1}{\sqrt{2}}$

$\therefore a = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$

So, out of the given value $a = \frac{1}{2} - \frac{1}{\sqrt{2}}$ is the right choice.

32. For $f(x)$ to be continuous, we must have

$$f(0) = \lim_{x \to 0} f(x)$$

$\therefore \lim_{x \to 0} \frac{\log(1 + ax) - \log(1 - bx)}{x}$

$= \lim_{x \to 0} \frac{a\log(1 + ax)}{ax} + \frac{b\log(1 - bx)}{-bx}$

$= a \cdot 1 + b \cdot 1$ $\left[\text{using, } \lim_{x \to 0}\frac{\log(1 + x)}{x} = 1\right]$

$= a + b$

$\therefore f(0) = a + b$

33. Here, $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$, to find domain, we must have,

$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$ $\left(\text{but } -\frac{\pi}{2} \leq \sin^{-1}\theta \leq \frac{\pi}{2}\right)$

$\therefore -\frac{\pi}{6} \leq \sin^{-1}(2x) \leq \sin\frac{\pi}{2}$

$\Rightarrow \sin\left(-\frac{\pi}{6}\right) \leq 2x \leq \sin\left(\frac{\pi}{2}\right)$

$\Rightarrow -\frac{1}{2} \leq 2x \leq 1$

$\Rightarrow -\frac{1}{4} \leq x \leq \frac{1}{2}$

$\Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$

34. $x\,dy = y\,(dx + y\,dy),\ y > 0$

$\Rightarrow x\,dy - y\,dx = y^2\,dy$

$\Rightarrow \frac{x\,dy - y\,dx}{y^2} = dy$

$$\Rightarrow \quad -d\left(\frac{x}{y}\right) = dy$$

On integrating both sides, we get

$$\frac{x}{y} = -y + C \quad \ldots(i)$$

As $y(1) = 1 \Rightarrow x = 1, y = 1$

$\therefore \quad C = 2$

$\therefore$ Eq. (i) becomes

$$\frac{x}{y} + y = 2$$

Again, for $x = -3$

$$\Rightarrow \quad -3 + y^2 = 2y$$

$$\Rightarrow \quad y^2 - 2y - 3 = 0$$

$$\Rightarrow \quad (y+1)(y-3) = 0$$

As $y > 0$, we take $y = 3$, (neglecting $y = -1$)

35. Let $I = \int \frac{(x-1)e^x}{(x+1)^3} dx$

$$\Rightarrow \quad I = \int \left\{\frac{x+1-2}{(x+1)^3}\right\} e^x \, dx$$

$$= \int \left\{\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right\} e^x \, dx$$

$$= \int e^x \cdot \frac{1}{(x+1)^2} dx - 2\int e^x \frac{1}{(x+1)^3} dx$$

Applying integrating by parts, we get

$$= \left(\frac{1}{(x+1)^2} e^x - \int e^x \frac{(-2)}{(x+1)^3} dx\right) - 2\int e^x \frac{1}{(x+1)^3} dx$$

$$= \frac{e^x}{(x+1)^2} + C$$

Note We can use the formula

$$\int e^x [f(x) + f'(x)] \, dx = e^x f(x) + C$$

36. Now, $\log_{140} 63 = \log_{2^2 \times 5 \times 7} (3 \times 3 \times 7)$

$$= \frac{\log_2 (3 \times 3 \times 7)}{\log_2 (2^2 \times 5 \times 7)} = \frac{\log_2 3 + \log_2 3 + \log_2 7}{2\log_2 2 + \log_2 5 + \log_2 7}$$

$$= \frac{2a + \frac{1}{c}}{2 + b + \frac{1}{c}} = \frac{2ac + 1}{2c + bc + 1}$$

37. Let $f(x) = \frac{\log_e x}{x}$

On differentiating, w.r.t. x, we get

$$f'(x) = \frac{1}{x^2} - \frac{\log_e x}{x^2}$$

For maximum or minimum value of $f(x)$,

Put $\quad f'(x) = 0$

$$\Rightarrow \quad \frac{1 - \log_e x}{x^2} = 0$$

$$\Rightarrow \quad \log_e x = 1$$

$\Rightarrow \quad x = e$, which lies in $(0, \infty)$.

For $x = e$, $f''(x) = -ve$

Hence, y is maximum at $x = e$ and its maximum value $= \frac{\log_e e}{e} = \frac{1}{e}$

38. LHS

$$= \frac{(\cos x + i \sin x)(\cos y + i \sin y)}{(\cos u + i \sin u)(\cos v + i \sin v)} \sin u \cos v$$

$$= \sin u \cos v \, e^{ix} \cdot e^{iy} \cdot e^{-iu} \cdot e^{-iv}$$

$$= \sin u \cos v [\cos(x + y - u - v) + i \sin(x + y - u - v)]$$

39. Let e be the identity,

$\therefore \quad a * e = a$

$$\Rightarrow \quad \frac{ae}{6} = a$$

$$\Rightarrow \quad e = 6$$

$\because \quad a * a^{-1} = e$

$\therefore \quad a * a^{-1} = 6$

$$\Rightarrow \quad \frac{aa^{-1}}{6} = 6$$

$$\Rightarrow \quad a^{-1} = \frac{6 \times 6}{a}$$

$$\therefore \quad 9^{-1} = \frac{6 \times 6}{9} = 4$$

40. $\therefore \quad \sin^2 \alpha + \sin(\beta - \gamma)\sin(\beta + \gamma)$

$$= \sin^2 \alpha + \sin(\pi - \alpha)\sin(\beta - \gamma)$$

$$= \sin \alpha [\sin \alpha + \sin(\beta - \gamma)]$$

$$= \sin \alpha [\sin(\pi - (\beta + \gamma)) + \sin(\beta - \gamma)]$$

$$= \sin \alpha [\sin(\beta - \gamma) + \sin(\beta + \gamma)]$$

$$= 2 \sin \alpha \sin \beta \cos \gamma$$

41. We have, $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ac}$

On multiplying both sides by *abc*

$\Rightarrow 2bc\cos A+ac\cos B+2ab\cos C=a^2+b^2$

$$\Rightarrow (b^2+c^2-a^2)+\frac{(c^2+a^2-b^2)}{2}+(a^2+b^2-c^2)=a^2+b^2$$

$$\Rightarrow (c^2+a^2-b^2)=2a^2-2b^2$$

$$\Rightarrow b^2+c^2=a^2$$

42. $\frac{\sin^2 A+\sin A+1}{\sin A}=\sin A+\frac{1}{\sin A}+1$

$$=\left[\left(\sqrt{\sin A}-\frac{1}{\sqrt{\sin A}}\right)^2+3\right]\geq 3$$

43. The given system has a non-trivial solution, if

$$\begin{vmatrix}\lambda & \sin\alpha & \cos\alpha\\ 1 & \cos\alpha & \sin\alpha\\ -1 & \sin\alpha & -\cos\alpha\end{vmatrix}=0$$

By expanding the determinant along (C_1), we get

$\lambda(-\cos^2\alpha-\sin^2\alpha)-1(-\sin\alpha\cos\alpha-\sin\alpha\cos\alpha)$

$$-1(\sin^2\alpha-\cos^2\alpha)=0$$

$$\lambda=\sin 2\alpha+\cos 2\alpha$$

For $\lambda=1$, $\sin 2\alpha+\cos 2\alpha=1$

$$\Rightarrow \frac{1}{\sqrt2}\sin 2\alpha+\frac{1}{\sqrt2}\cos 2\alpha=\frac{1}{\sqrt2}$$

$$\Rightarrow \cos\left(2\alpha-\frac{\pi}{4}\right)=\cos\left(2n\pi\pm\frac{\pi}{4}\right)$$

$\Rightarrow 2\alpha=2n\pi\pm\frac{\pi}{4}+\frac{\pi}{4}$, *n* being an integer.

$$\Rightarrow \alpha=n\pi+\frac{\pi}{4},\ n\pi$$

44. Let $f(x)=x^2-2ax+a^2+a-3=0$. Since, $f(x)$ has real roots both less than 3. Therefore, $D>0$ and $f(3)>0$

$\Rightarrow 4a^2-4(a^2+a-3)>0$

and $a^2-5a+6>0$

$\Rightarrow a<3$ and $a<2$ or $a>3$

$\Rightarrow a<2$

45. $\because |z_1|=|z_2|+|z_1-z_2|$

$\Rightarrow |z_1|-|z_2|=|z_1-z_2|$

$\Rightarrow (|z_1|-|z_2|)^2=|z_1-z_2|^2$

$\Rightarrow |z_1|^2+|z_2|^2-2|z_1||z_2|=|z_1|^2+|z_2|^2-2|z_1||z_2|\cos(\theta_1-\theta_2)$

$\Rightarrow \cos(\theta_1-\theta_2)=1$

$\Rightarrow \theta_1-\theta_2=0$

$\Rightarrow \arg(z_1)-\arg(z_2)=0$

$\Rightarrow \frac{z_1}{z_2}$ is purely real.

$$\Rightarrow \operatorname{Im}\left(\frac{z_1}{z_2}\right)=0$$

46. Since, total number of numbers 1 to 5 digits are 99999 and total number of numbers 1 to 4 digits are 9999.

Hence, the total number of numbers of exactly 5 digits = 99999 – 9999 = 90000.

47. $\because \left(x^2+\frac{1}{x^2}+2\right)^n=\left[\left(x+\frac{1}{x}\right)^2\right]^n=\left(x+\frac{1}{x}\right)^{2n}$

The number of terms in the expansion of $\left(x+\frac{1}{x}\right)^{2n}$ is $2n+1$ which is odd. Therefore, $(n+1)$th term will be middle term.

$$\therefore T_{n+1}={}^{2n}C_n\,(x)^{2n-n}\left(\frac{1}{x}\right)^n$$

$$={}^{2n}C_n\,x^n\cdot\frac{1}{x^n}$$

$$={}^{2n}C_n=\frac{(2n)!}{n!\,n!}=\frac{(2n)!}{(n!)^2}$$

48. We know,

$$\sin^2 A=\left(\frac{1-\cos 2A}{2}\right)$$

$$\Rightarrow \sin^2(22°30')=\frac{1-\cos 45°}{2}$$

$$=\frac{\left(1-\frac{1}{\sqrt2}\right)}{2}$$

$$=\frac{\sqrt2-1}{2\sqrt2}$$

$$\Rightarrow \sin(22°30')=\sqrt{\frac{\sqrt2-1}{2\sqrt2}}$$

49. $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \dfrac{1-\sin\theta}{\sqrt{\cos^2\theta}} = \dfrac{1-\sin\theta}{|\cos\theta|}$

$= \dfrac{1-\sin\theta}{-\cos\theta} \qquad \left(\because \dfrac{\pi}{2} < \theta < \dfrac{3\pi}{2}\right)$

$= -\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= -\sec\theta + \tan\theta$

50. Let $\quad I = \displaystyle\int \dfrac{2^x}{\sqrt{1-4^x}}\,dx$

Let $\quad 2^x = t$

$\Rightarrow \quad 2^x \log 2\,dx = dt$

$\therefore \quad I = \dfrac{1}{\log 2}\displaystyle\int \dfrac{1}{\sqrt{1-t^2}}\,dt$

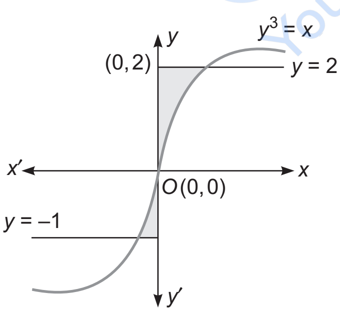
$= \dfrac{1}{\log 2}\sin^{-1}(t) + C$

$= \dfrac{1}{\log 2}\sin^{-1}(2^x) + C$

But $\quad I = k\sin^{-1}(2^x) + C,$

$\Rightarrow \quad k = \dfrac{1}{\log 2}$

51. Required area $= \displaystyle\int_{-1}^{2} |x_1|\,dy$



$= \displaystyle\int_{-1}^{0}(-x_1)\,dy + \int_{0}^{2} x_1\,dy$

$= -\displaystyle\int_{-1}^{0} y^3\,dy + \int_{0}^{2} y^3\,dy$

$= -\left[\dfrac{y^4}{4}\right]_{-1}^{0} + \left[\dfrac{y^4}{4}\right]_{0}^{2} = \dfrac{1}{4} + 4$

$= \dfrac{17}{4}$ sq units.

52. Let (x_1, y_1) be the mid-point of the line joining the common points of the given line and the given parabola. Then, the equation of line is

$$yy_1 - 4(x + x_1) = y_1^2 - 8x_1$$

$\Rightarrow \quad 4x - yy_1 + y_1^2 - 4x_1 = 0$

This line and $2x - 3y + 8 = 0$ represents the same line.

$\therefore \quad \dfrac{4}{2} = \dfrac{-y_1}{-3} = \dfrac{y_1^2 - 4x_1}{8}$

$\Rightarrow \quad y_1 = 6,\ y_1^2 - 4x_1 = 16$

$\Rightarrow \quad y_1 = 6,\ x_1 = \dfrac{36-16}{4}$

$\Rightarrow \quad y_1 = 6,\ x_1 = 5$

$\therefore$ The required point is (5, 6).

53. $\cos^2 48° - \sin^2 12° = \dfrac{1}{2}(2\cos^2 48° - 2\sin^2 12°)$

$= \dfrac{1}{2}[(1+\cos 96°) - (1 - \cos 24°)]$

$= \dfrac{1}{2}(\cos 96° + \cos 24°)$

$= \dfrac{1}{2}\left[2\cos\left(\dfrac{96° + 24°}{2}\right)\cos\left(\dfrac{96° - 24°}{2}\right)\right]$

$= \cos 60° \cos 36°$

$= \dfrac{1}{2} \times \left(\dfrac{\sqrt{5}+1}{4}\right)$

$= \dfrac{\sqrt{5}+1}{8}$

54. $\quad \sin^2\theta + 3\cos\theta = 3$

$\Rightarrow \quad 1 - \cos^2\theta + 3\cos\theta = 3$

$\Rightarrow \quad \cos^2\theta - 3\cos\theta + 2 = 0$

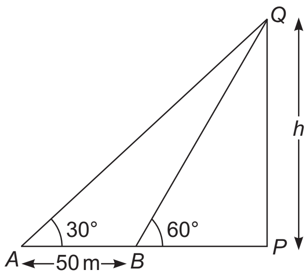
$\Rightarrow \quad (\cos\theta - 1)(\cos\theta - 2) = 0$

$\therefore \quad \cos\theta = 1 \qquad (\because \cos\theta \neq 2)$

$\Rightarrow \quad \theta = 0° \text{ as } \theta \in [-\pi, \pi]$

$\therefore$ The number of solution is 1.

55. Let $PQ = h$ be the height of chimney. A and B are the two points 50 m apart.



In ΔAPQ,

$$\tan 30° = \frac{h}{AP}$$

$$\Rightarrow \quad AP = h \cot 30° \quad \ldots(i)$$

and in ΔQBP,

$$\tan 60° = \frac{h}{BP}$$

$$\Rightarrow \quad BP = h \cot 60° \quad \ldots(ii)$$

$$\because \quad AP - BP = 50$$

$$\therefore \quad h(\cot 30° - \cot 60°) = 50$$

$$\Rightarrow \quad h = \frac{50}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)} = \frac{50\sqrt{3}}{3-1}$$

$$= \frac{50\sqrt{3}}{2} = 25\sqrt{3} \text{ m}$$

56. $$\lim_{x \to \infty} \left(\frac{x^3+1}{x^2+1} - (ax+b)\right) = 2$$

$$\Rightarrow \lim_{x \to \infty} \frac{x^3(1-a) - bx^2 - ax + (1-b)}{x^2+1} = 2$$

$$\Rightarrow \lim_{x \to \infty} \frac{x(1-a) - b - \frac{a}{x} + \frac{1-b}{x^2}}{1 + \frac{1}{x^2}} = 2$$

$$\Rightarrow \quad 1 - a = 0 \quad \text{and} \quad -b = 2$$

$$\Rightarrow \quad a = 1 \quad \text{and} \quad b = -2$$

57. $$f(x) = \begin{cases} \dfrac{|x+2|}{\tan^{-1}(x+2)}, & x \neq -2 \\ 2, & x = -2 \end{cases}$$

$$\therefore \lim_{x \to -2^-} f(x) = \lim_{h \to 0} f(-2-h)$$

$$= \lim_{h \to 0} \frac{|-2-h+2|}{\tan^{-1}(-2-h+2)}$$

$$= \lim_{h \to 0} \frac{-h}{\tan^{-1} h} = -1$$

and $$\lim_{x \to -2^+} f(x) = \lim_{h \to 0} f(-2+h)$$

$$= \lim_{h \to 0} \left[\frac{|-2+h+2|}{\tan^{-1}(-2+h+2)}\right]$$

$$= \lim_{h \to 0} \frac{h}{\tan^{-1} h} = 1$$

$$\because \lim_{x \to -2^-} f(x) \neq \lim_{x \to -2^+} f(x)$$

So, $f(x)$ is not continuous as well as not differentiable at $x = -2$.

58. Let the coordinates of the point P be (x_1, y_1). This point lies on the curve

$$x^{2/3} + y^{2/3} = a^{2/3} \quad \ldots(i)$$

$$\therefore \quad x_1^{2/3} + y_1^{2/3} = a^{2/3} \quad \ldots(ii)$$

On differentiating Eq. (i) w.r.t. x, we get

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$

$$\Rightarrow \quad \left(\frac{dy}{dx}\right)_{(x_1, y_1)} = -\left(\frac{y_1}{x_1}\right)^{1/3}$$

The equation of the tangent at (x_1, y_1) to the given curve is

$$y - y_1 = -\frac{y_1^{1/3}}{x_1^{1/3}}(x - x_1)$$

$$\Rightarrow \quad xx_1^{-1/3} + yy_1^{-1/3} = x_1^{2/3} + y_1^{2/3}$$

$$\Rightarrow \quad xx_1^{-1/3} + yy_1^{-1/3} = a^{2/3} \quad \text{[using Eq. (ii)]}$$

This tangent meets the coordinate axes at $A(a^{2/3}x_1^{1/3}, 0)$ and $B(0, a^{2/3}y_1^{1/3})$.

$$\therefore \quad AB = \sqrt{(0 - a^{2/3}x_1^{1/3})^2 + (a^{2/3}y_1^{1/3} - 0)^2}$$

$$= \sqrt{a^{4/3}(x_1^{2/3} + y_1^{2/3})} \quad \text{[from Eq. (ii)]}$$

$$= \sqrt{a^{4/3} \cdot a^{2/3}}$$

$$= \sqrt{a^2} = a$$

59. Given that, $x = (2t - 3)^2$

On differentiating w.r.t. x, we get

$$\frac{dx}{dt} = 2(2t - 3)(2)$$

At origin, $x = 0$

$\therefore \quad 2t - 3 = 0 \Rightarrow t = \frac{3}{2}$

$\therefore \quad \text{Velocity} = \frac{dx}{dt} = 4\left(2 \times \frac{3}{2} - 3\right) = 0$

60. $\therefore \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}$

$$= \frac{1}{2\sin\frac{\pi}{7}} \times \left[2\sin\frac{\pi}{7}\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{4\pi}{7} + 2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}\right]$$

$$= \frac{1}{2\sin\frac{\pi}{7}} \times \left[\sin\frac{3\pi}{7} - \sin\frac{\pi}{7} + \sin\frac{5\pi}{7} - \sin\frac{3\pi}{7} + \sin\pi - \sin\frac{5\pi}{7}\right]$$

$$= -\frac{1}{2}$$

61. The maximum value of $3\cos\theta + 4\sin\theta$

$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{25} = 5$$

62. 'Where are you going' is not a statement.

63. Since, $123 = -1 \pmod{124}$

$125 = 1 \pmod{124}$

$127 = 3 \pmod{124}$

$\therefore \quad 123 \times 125 \times 127 = (-1)(1)(3) \bmod (124)$

$= -3 \bmod (124)$

$= 121 \bmod (125)$

64. The vertex connectivity of any tree is one.

65. Since, a five digit number is formed by using digits 0, 1, 2, 3, 4 and 5, divisible by 3 *i.e.,* only possible when sum of digits is multiple of three which gives two cases.

Case I Using digits 0, 1, 2, 4, 5 the number of ways $= 4 \times 4 \times 3 \times 2 \times 1 = 96$.

Case II Using digits 1, 2, 3, 4, 5 the number of ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

$\therefore$ Total numbers formed $= 120 + 96 = 216$

66. 3r th term in the expansion of $(1 + x)^{2n}$

$$= {}^{2n}C_{3r-1}\, x^{3r-1}$$

and $(r + 2)$th term in the expansion of $(1 + x)^{2n}$

$$= {}^{2n}C_{r+1}\, x^{r+1}$$

Given that the binomial coefficients of $(3r)$th and $(r + 2)$th terms are equal.

$\therefore \quad {}^{2n}C_{3r-1} = {}^{2n}C_{r+1}$

$\Rightarrow \quad 3r - 1 = r + 1$

or $\quad 2n = (3r - 1) + (r + 1)$

$\Rightarrow \quad 2r = 2 \quad \text{or} \quad 2n = 4r$

$\Rightarrow \quad r = 1 \quad \text{or} \quad n = 2r$

But $\quad r > 1$

$\therefore \quad n = 2r$

67. $\log_4(x - 1) = \log_2(x - 3) = \log_{4^{1/2}}(x - 3)$

$\Rightarrow \quad \log_4(x - 1) = 2\log_4(x - 3)$

$\Rightarrow \quad \log_4(x - 1) = \log_4(x - 3)^2$

$\Rightarrow \quad (x - 1) = (x - 3)^2$

$\Rightarrow \quad x^2 + 9 - 6x = x - 1$

$\Rightarrow \quad x^2 - 7x + 10 = 0$

$\Rightarrow \quad (x - 2)(x - 5) = 0$

$\Rightarrow \quad x = 2 \quad \text{or} \quad x = 5$

Hence, $x = 5$

[$\because$ $x = 2$ makes $\log(x - 3)$ undefined.]

68. $\lim_{x \to 0} \frac{\int_0^{x^2} \cos^2 t\, dt}{x \sin x}$

Using L' Hospital's rule, we get

$$= \lim_{x \to 0} \frac{\cos^2(x^2) \cdot 2x - 0}{x\cos x + \sin x}$$

$$= \lim_{x \to 0} \frac{2\cos^2(x^2)}{\cos x + \frac{\sin x}{x}}$$

$$= \frac{2}{2} = 1$$

69. Let $\quad I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5 - x} + \sqrt{x}}\, dx \quad \ldots(i)$

$\therefore \quad I = \int_2^3 \frac{\sqrt{5 - x}}{\sqrt{x} + \sqrt{5 - x}}\, dx \quad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$$2I = \int_2^3 \frac{\sqrt{x} + \sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}}\, dx$$

$$= \int_2^3 1\, dx = 3 - 2 = 1$$

$$\Rightarrow \quad I = \frac{1}{2}$$

70. The given differential equation is

$$(e^x + 1) \cdot y\, dy = (y + 1)e^x\, dx$$

$$\Rightarrow \quad \frac{y\, dy}{(y+1)} = \frac{e^x}{(e^x + 1)}\, dx$$

$$\Rightarrow \quad \left(\frac{y + 1 - 1}{y + 1}\right) dy = \frac{e^x}{e^x + 1}\, dx$$

$$\Rightarrow \quad dy - \frac{1}{y+1}\, dy = \frac{e^x}{e^x + 1}\, dx$$

On integrating, we get

$$\Rightarrow \quad y - \log|y + 1| = \log(e^x + 1) + \log k$$

$$\Rightarrow \quad y = \log|(y+1)(e^x + 1)|\, k$$

$$\Rightarrow \quad (y + 1)(e^x + 1) = e^y C$$

71. The required equation of circle is

$$(x^2 + y^2 + 13x - 3y) + \lambda\left(11x + \frac{1}{2}y + \frac{25}{2}\right) = 0 \quad \ldots(i)$$

This circle passes through (1, 1).

$$(1^2 + 1^2 + 13 - 3) + \lambda\left(11 + \frac{1}{2} + \frac{25}{2}\right) = 0$$

$$\therefore \quad 12 + \lambda(24) = 0$$

$$\Rightarrow \quad \lambda = -\frac{1}{2}$$

On putting this value of λ in Eq. (i), we get

$$x^2 + y^2 + 13x - 3y - \frac{11}{2}x - \frac{1}{4}y - \frac{25}{4} = 0$$

$$\Rightarrow \quad 4x^2 + 4y^2 + 52x - 12y - 22x - y - 25 = 0$$

$$\Rightarrow \quad 4x^2 + 4y^2 + 30x - 13y - 25 = 0$$

72. Tangent to the curve $y^2 = 8x$ is $y = mx + \frac{2}{m}$.

So, it must satisfy $xy = -1$

$$\Rightarrow x\left(mx + \frac{2}{m}\right) = -1 \quad \Rightarrow \quad mx^2 + \frac{2}{m}x + 1 = 0$$

Since, it has equal roots.

$$\therefore \quad D = 0$$

$$\Rightarrow \quad \frac{4}{m^2} - 4m = 0$$

$$\Rightarrow \quad m^3 = 1 \quad \Rightarrow \quad m = 1$$

So, equation of common tangent is $y = x + 2$.

73. $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$

$$= \frac{b}{2}(1 + \cos C) + \frac{c}{2}(1 + \cos B)$$

$$= \frac{b}{2} + \frac{c}{2} + \frac{1}{2}(b\cos C + c\cos B)$$

$$= \frac{b}{2} + \frac{c}{2} + \frac{a}{2} \qquad (\because \; a = b\cos C + c\cos B)$$

$$= \frac{a + b + c}{2} = \frac{k}{2}$$

74. $I_1 = \int_{1-k}^{k} xf\{x(1-x)\}\, dx$

$$= \int_{1-k}^{k} (1-x)f\{(1-x)(1-(1-x))\}\, dx$$

$$\left(\because \int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx\right)$$

$$= \int_{1-k}^{k} (1-x)f\{(1-x)x\}\, dx$$

$$= \int_{1-k}^{k} f\{x(1-x)\}\, dx - \int_{1-k}^{k} xf\{x(1-x)\}\, dx$$

$$\Rightarrow \quad I_1 = I_2 - I_1 \quad \Rightarrow \quad 2I_1 = I_2$$

$$\Rightarrow \quad \frac{I_1}{I_2} = \frac{1}{2}$$

75. The maximum area corresponds to when *P* is at either end of the minor axis.

$\therefore$ Required area $= \frac{1}{2}(2a)b = ab$ sq unit

76. We have, $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$$\Rightarrow \quad \frac{\cos A}{k\sin A} = \frac{\cos B}{k\sin B} = \frac{\cos C}{k\sin C}$$

$$\Rightarrow \quad \cot A = \cot B = \cot C$$

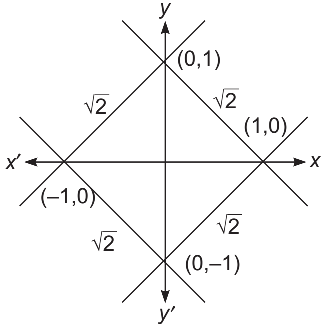
$$\Rightarrow \quad A = B = C = 60°$$

$\therefore$ ΔABC is an equilateral triangle.

$\therefore$ Area of triangle $= \frac{\sqrt{3}}{4}a^2$

$$= \frac{\sqrt{3}}{4} \times 2^2 = \sqrt{3} \text{ sq units}$$

77. The region is clearly square with vertices at the points (1, 0), (0, 1), (–1, 0) and (0, –1). So, its area $= \sqrt{2} \times \sqrt{2} = 2$ sq units.



78. $\sqrt{3}$ cosec 20° – sec 20°

$$= \tan 60° \operatorname{cosec} 20° - \sec 20°$$

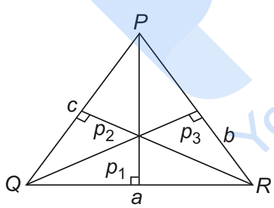
$$= \frac{\sin 60° \cos 20° - \cos 60° \sin 20°}{\cos 60° \sin 20° \cos 20°}$$

$$= \frac{\sin(60° - 20°)}{\cos 60° \sin 20° \cos 20°}$$

$$= \frac{\sin 40°}{\frac{1}{2} \sin 20° \cos 20°}$$

$$= \frac{2 \sin 20° \cos 20°}{\frac{1}{2} \sin 20° \cos 20°} = 4$$

79. By sine rule



$$\frac{a}{\sin P} = \frac{b}{\sin Q} = \frac{c}{\sin R} = k \quad \text{(say)}$$

Also, $\frac{1}{2} ap_1 = \Delta$

$\Rightarrow \quad \frac{2\Delta}{a} = p_1$

$\Rightarrow \quad p_1 = \frac{2\Delta}{k \sin P}$

Similarly, $p_1 = \frac{2\Delta}{k \sin Q}$ and $p_3 = \frac{2\Delta}{k \sin R}$

Since, sin P, sin Q, sin R are in AP.

$\therefore \quad \frac{1}{\sin P} \cdot \frac{1}{\sin Q}, \frac{1}{\sin R}$ are in HP

$\Rightarrow \quad \frac{2\Delta}{k \sin P}, \frac{2\Delta}{k \sin Q}, \frac{2\Delta}{k \sin R}$ are in HP

$\Rightarrow p_1, p_2, p_3$ are in HP.

80. Let E_1 be the events that man will be selected and E_2 the events that woman will be selected.

Then, $\quad P(E_1) = \frac{1}{4}$

So, $\quad P(\bar{E}_1) = 1 - \frac{1}{4} = \frac{3}{4}$

and $\quad P(E_2) = \frac{1}{3}$

$\Rightarrow \quad P(\bar{E}_2) = \frac{2}{3}$

$\therefore$ E_1 and E_2 are independent events.

$\therefore \quad P(\bar{E}_1 \cap \bar{E}_2) = P(\bar{E}_1) \times P(\bar{E}_2)$

$$= \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$$