MET 2013 Answer Key PDF

Physics

1. Rate of change of displacement is known as velocity. The equation for displacment (y) of a body in SHM with angular velocity ω is given by

$$y = a\sin \omega t$$

where a is amplitude

velocity v = rate of change of displacement $= \frac{dy}{dt}$

$$v = \frac{dy}{dt} = \frac{d}{dt}(a\sin \omega t) = a\omega \cos \omega t$$

Given, $v_1 = v_2 = v_3$ and $a_1 = A_1$, $a_2 = A_2$.

$$a_3 = A_3$$

$$A_1\omega_1 = A_2\omega_2 = A_3\omega_3$$

2. The unit dyne/cm is CGS system of unit.

The CGS unit of force is dyne and SI unit is Newton. Also there are 10^5 dynes in one newton and 100 cm in 1 cm.

$$\therefore \frac{70 \text{ dynes}}{\text{cm}} = \frac{70 \times 10^{-5}}{10^{-2}} = 7 \times 10^{-2} \text{ N/m}$$

3. The general gas equation is $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$ = gas constant

where p_1, V_1, p_2, V_2 are pressure and volume at temperatures T_1 and T_2 respectively.

Given $V_1 = 1500 \text{ m}^3$, $T_1 = 27 + 273 = 300$ K,

$$p_1 = 4 \text{ atm.}$$

$$T_2 = -3°\text{C} = 273 - 3 = 270 \text{ K}, \; p_2 = 2 \text{ atm.}$$

$$\therefore V_2 = \frac{p_1 V_1}{T_1} \times \frac{T_2}{p_2}$$

$$V_2 = \frac{4 \times 1500}{300} \times \frac{270}{2}$$

$$V_2 = 2700 \text{ m}^3$$

4. Work done against frictional force equals the kinetic energy of the body.

when a body of mass m, moves with velocity v, it has kinetic energy $k = \frac{1}{2}mv^2$, this energy is utilized in doing work against the frictional force between the tyres of the car and road.

$\therefore$ kinetic energy = work done against friction force

$$\frac{1}{2}m^2 = \mu\, mgs$$

where s is the distance in which the car is stopped and μ is coefficient of kinetic friction.

Given $\quad v = v_0$

$$s = \frac{v_0^2}{2\mu\, g}.$$

5. Average velocity is equal to ratio of total displacement to total time. Velocity is a vector quantity and average velocity is defined as the total displacement undergone by the body per unit time.

Displacement in first instance $= v_1 t$

Displacement in second instance $= v_2 t$

$$\therefore \text{Average velocity} = \frac{v_1 t + v_2 t}{t + t} = \frac{v_1 + v_2}{2}$$

6. If the maximum kinetic energy of photoelectron emitted from the surface of a metal is E_K and W is the work function of the metal, the from Einstein's photoelectric equation we have

$$E_K = h\nu - W$$

where, $h\nu$ is the energy of the photon absorbed by the electron in the metal.

Also $\quad \nu = \frac{c}{\lambda}$

$$= \frac{\text{velocity}}{\text{wavelength}}$$

$$= E_K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Given, $\lambda = 3 \times 10^{-7}$ m,

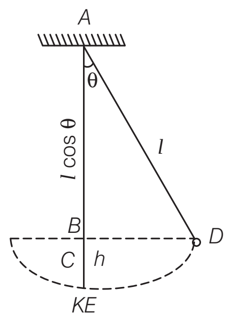
$$\lambda_0 = 4 \times 10^{-7} \text{ m.}$$

$$\therefore E_K = 6.6 \times 10^{-34} \times 3 \times 10^8 \times \left(\frac{1}{3 \times 10^{-7}} - \frac{1}{4 \times 10^{-7}}\right) \text{J}$$

$$= \frac{19.8 \times 10^{-19}}{12 \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.03 \approx 1 \text{eV}$$

7. In the equilibrium position energy is in the form of kinetic energy.



Height of bob at maximum angular displacement is

$$h = l - l\cos\theta$$

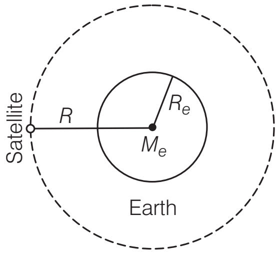
$$= l(1 - \cos\theta)$$

Also PE = KE

$$mgh = mgl(1 - \cos\theta)$$

8. Gravitational force provides the required centripetal force.

The gravitational force provides the required centripetal force in orbit of earth.



$$\therefore \quad \frac{GM_e M}{R^2} = \frac{mv_0^2}{R}$$

$$v_0 = \sqrt{\frac{GM_e}{R}}$$

kinetic energy $= \frac{1}{2} mv_0^2$

$$\therefore \quad KE = \frac{1}{2} m\left(\frac{GM_e}{R}\right)^{2/2}$$

$$= \frac{1}{2}\frac{mGM_e}{R}$$

$$KE \propto \frac{1}{R}$$

9. The surface tension (T) of a liquid is equal to the work (W) required to increase the surface area of the liquid film by unity at constant temperature.

As per key idea, tension

$$= \frac{\text{work done}}{\text{surface area}}$$

or $\quad T = \frac{W}{\Delta A}$

Since soap bubble has two surface and surface area of soap bubble is $4\pi R^2$

where R is radius of bubble.

then $\quad W = T \times 2 \times 4\pi R^2$

Given $\quad R = 2R$

therefore $\quad W = T \times 2 \times 4\pi \times (2R)^2$

$$W = 32\ \pi R^2 T$$

10. Form Einstein's mass energy relation

$$E = \Delta mc^2 \qquad \ldots\text{(i)}$$

where Δm is mass lost, c is speed of light. Also heat given by body is

$$E = mC\Delta\theta \qquad \ldots\text{(ii)}$$

where, C is specific heat.

Equating Eqs. (i) and (ii) we get

$$\Delta m = \frac{MC\Delta\theta}{C^2} = \frac{M \times 0.2 \times 100 \times 4.2 \times 10^3}{(3 \times 10^8)^2} \text{ J}$$

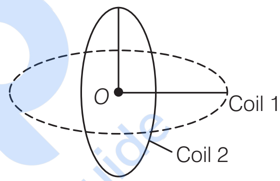
$$\frac{\Delta m}{M} = \frac{20 \times 4.2 \times 10^3}{(3 \times 10^8)^2}$$

% increase in mass

$$= \frac{\Delta m}{M} \times 100 = \frac{20 \times 4.2 \times 10^3}{(3 \times 10^8)^2} \times 100$$

$$= 9.3 \times 10^{-11}\%$$

11. Where the two coils are kept perpendicular to each other then resultant field in one coil is directed along the axis of coil and the other downwards, hence resultant field is

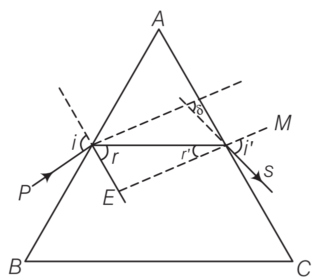


$$B' = \sqrt{B^2 + B^2} = \sqrt{2}\ B$$

Ratio of magnetic field due to one coil and resultant is $\quad \frac{B}{B\sqrt{2}} = \frac{1}{\sqrt{2}}$

12. In the position of minimum deviation angle of incidence is equal to angle of emergence.

Let a ray of monochromatic light PQ be incident on face AB. $PQRS$ is path of light ray,



where i is angle of incidence r angle of refraction, r angle of incidence and i angle of emergence.

In position of minimum deviation

$$i' = i,\ r' = r,\ \delta = \delta m$$

$$2r = A \text{ or } r = \frac{A}{2}$$

Given $A = 60°$,

$$r = \frac{60°}{2} = 30°$$

Also from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin 30^\circ}$$

$$\sqrt{2} = \frac{\sin i}{\sin 30^\circ}$$

$$\Rightarrow \quad \sin i = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad i = 45^\circ$$

13. Net acceleration on the elevator car is more than acceleration due to gravity.

Since elevator car is ascending upwards, from Newton's second law, the net force is acting upwards hence resultant acceleration is

$$a_r = (g + a)$$
$$= (9.8 + 1.2) = 11\,\text{m/s}^2$$

Relative velocity of observer to elevator is

$$u_r = 0$$

from the equation

$$s = u_r t + \frac{1}{2} a_r t^2$$

$$2.7 = 0 + \frac{1}{2} \times 11 \times t^2$$

$$t^2 = \frac{5.4}{11}$$

$$t = \sqrt{\frac{5.4}{11}}\,\text{s}$$

14. From equation of motion

$$h = ut + \frac{1}{2} gt^2$$

where u is initial velocity and t is time.

Since, $u = 0$

$$\therefore \quad t = \sqrt{\frac{2h}{g}} = \frac{\sqrt{2 \times 4.9}}{9.8} = 1\,\text{s}$$

The horizontal range of the drop $= x$, then

$$x = \left(\frac{v_t}{o}\right) t$$

$$\text{Also,} \quad \omega = \frac{\Delta\theta}{\Delta t} = \frac{21 \times 2\pi}{44} = 3\,\text{rad/s}$$

Tangential speed $v_t = r\omega = 0.5 \times 3 \times 1.5$ m/s

$$x = 1.5 \times 1 = 1.5\,\text{m}$$

$$\text{Locus of drop} = \sqrt{x^2 + r^2} = \sqrt{(1.5)^2 + (0.5)^2}$$
$$= \sqrt{2.5}\,\text{m}$$

15. Kinetic energy of rotation is half the product of the moment of inertia (I) of the body and the square of the angular velocity (ω) of the body.

Kinetic energy of rotation $= \frac{1}{2} \times$ moment of inertia $\times$ angular velocity

i.e.,
$$K = \frac{1}{2} I\omega^2$$

$$\omega^2 = \frac{2K}{I}$$

Given $I = 1.2$ kg m^2, $K = 1500$ J

$$\omega^2 = \frac{2 \times 1500}{1.2}$$

$$\Rightarrow \quad \omega = 50\,\text{rad/s}$$

From the equation of angular motion, we have

$$\omega = \omega_0 + \alpha t$$

where ω_0 is initial angular velocity, α is angular acceleration and t is time

given $\omega_0 = 0$,

$$\omega = 50\,\text{rad/s},$$
$$\alpha = 25\,\text{rad/s}^2$$
$$t = \frac{\omega}{\alpha} = \frac{50}{25} = 2s$$

16. The kinetic energy of moving body is equal to half the product of the mass (m) of the body and the square of its speed (v^2).

$$\text{kinetic energy} = \frac{1}{2} \times \text{mass} \times (\text{speed})^2$$

$$\text{i.e.,} \quad = \frac{1}{2} mv^2$$

Let mass of man is M kg and speed is v and speed of body is v_1, then

$$\frac{1}{2} Mv^2 = \frac{1}{2}\left\{\frac{1}{2}\frac{M}{2} v_1^2\right\} \quad \ldots\text{(i)}$$

when man speed up by 1 m/s, than

$$v = v + 1$$

$$\therefore \quad \frac{1}{2} M(v+1)^2 = \frac{1}{2}\left\{\frac{M}{2}\right\} \cdot v_1^2 \quad \ldots\text{(ii)}$$

Dividing Eqs. (i) by (ii), we get

$$\frac{v^2}{(v+1)^2} = \frac{1}{2}$$

$$\sqrt{2}v = v + 1$$

$$v = \frac{1}{\sqrt{2} - 1}\,\text{m/s}$$

17. Larger the distance of planet from the sun, larger will be its period of revolution around the sun.

Kepler's third low planetary motion the square of the period of revolution of any planet around the sun is directly proportional to the cube of the semi-major axis of its elliptical orbit

$$\therefore \quad T^2 \propto R^3$$

$$\frac{T_s}{T_c} = \left(\frac{R_s}{R_c}\right)^{3/2}$$

given $\quad R_s = 4\,R_c$

$$\frac{T_s}{T_c} = \left(\frac{4\,R_c}{R_c}\right)^{3/2} = 8$$

for $T_c = 1$ day

$T_s = 8$ days.

18. Force experienced by the surface is equal to change in momentum in one second.

From Newton's second law of motion, the rate of change of momentum of a body is equal to the external force applied on the body and the change in momentum always takes place in the direction of the force.

Momentum of ball before impact $= mu$

Momentum after impact $= -\,mu$

Change in momentum $= mu - (-\,mu) = 2mu$

Change in momentum due to n balls in one second is $= 2\,mnu$.

Change of momentum = force experienced per second $= 2\,mnu$.

19. When no external force acts on a body its angular moment remains conserved.

From law of conservation of angular momentum, we have

$$J = I\omega = \text{constant}$$

where I is moment of inertia of the body and ω is angular velocity.

Also $I = \frac{1}{2}MR^2$ (for disc) where M is mass and R is radius of disc.

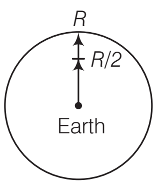
$$J = \frac{1}{2}MR^2\,\omega = \text{constant}$$

Since, man moves towards the centre of the disc, distance of mass distribution R decreases hence, ω increases.

20. Use parallel axis theorem, from the theorem of parallel axis, the moment of inertia (I) of a body about given axis is equal to its moment of inertia I about its diameter, plus the product of the mass M of the body and square of perpendicular distance between the two axes. That is

$$I = I_d + MR^2 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

21. Value of acceleration due to gravity decreases on going below the surface of the earth.



weight (w) of a body is defined as product of mass (m) and acceleration due to gravity (g)

$w = mg$

The value of g below the surface of earth at a distance h is given by

$$g' = g\left(1 - \frac{h}{R}\right)$$

$$\Rightarrow \quad W' = W\left(1 - \frac{h}{R}\right)$$

Given, $h = \frac{R}{2}$, $W = 500$ N

$$W' = 500\left(1 - \frac{1}{2}\right)$$

$$= \frac{500}{2} = 250\text{ N}$$

22. At a crtain velocity of projection the body will go out of the gravitational field of the earth and will never return to the earth, this velocity is known as escape velocity.

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

Given, $\quad M_e = M_p,\ R_p = \frac{R_e}{4}$

$$\therefore \quad \frac{v_p}{v_e} = \sqrt{\frac{M_e}{M_e} \times \frac{R_e}{R_e/4}} = \sqrt{4} = 2$$

$$\Rightarrow \quad v_p = 2v_e = 2 \times 11.2$$

$$= 22.4\text{ km/s}$$

23. Since liquid is incompressible there is no change in volume.

When strain is small the ratio of the normal stress to the volume strain is called the bulk modulus of material of the body.

$$B = \frac{\text{normal stress}}{\text{volume strain}}$$

$= $ normal stress

change in volume/original volume

since, the liquid is incompressible change in volume is zero.

$$B = \frac{\text{normal stress}}{0} = \infty$$

Hence, bulk modulus is infinite.

24. The narrower the tube, the higher is the rise in water.

when a glass capillary

$$h = \frac{2T\cos\theta}{R\rho g}$$

$$h \propto \frac{1}{R}$$

$$\frac{h_1}{h_2} = \frac{R_2}{R_1}$$

$$\frac{3}{h_2} = \frac{1}{3}$$

$$h_2 = 9\text{ mm}$$

25. $u = -20$ cm, $f = 20$ cm

From mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$
$$\frac{1}{20} = \frac{1}{v} + \frac{1}{-20}$$
$$\frac{1}{v} = \frac{1}{20} + \frac{1}{20}$$
$$\frac{1}{v} = \frac{2}{20} = v = 10 \text{ cm}$$

26. According to law of conservation of angular momentum if there is no torque on the system, then the angular momentum remains constant.

27. Equivalent focal length

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$
$$= \frac{1}{20} + \frac{1}{30}$$
$$F = \frac{20 \times 30}{20 + 30} = \frac{600}{50} = 12 \text{ cm}$$
$$F = 12 \text{ cm}$$

28. Work done $W = F \times d$

$$= (6i + 2j) \times (3i - j)$$
$$= 6 \times 3 + 2 \times -1$$
$$= 18 - 2 = 16 \text{ J}$$
$$W = 16 \text{ J}$$

29. From Einstein's theorem

Relativistic momentum $= \dfrac{m_0 v}{\sqrt{1 - v^2/c^2}}$

Putting $v = 2v$

we get $p = \dfrac{m_0 (2v)}{\sqrt{1 - (2v)^2/c^2}}$

$$= 2\left(\frac{m_0 v}{\sqrt{1 - (2v)^2/c^2}}\right)$$

So the momentum becomes more than the double.

30. Position $x = Ka^m t^n$

Writing the dimension on the both sides.

$$[M^0 L T^0] = [L T^{-2}]^m [T]^n$$
$$[M^0 L^m T^{-2m+n}]$$

On comparing both sides.

$$m = 1$$
$$-2m + n = 0$$
$$n = 2m = 2 \times 1 = 2$$

31. We know that

$$\alpha = \left(\frac{\Delta i_C}{\Delta i_E}\right) \quad \ldots\text{(i)}$$

and $$\beta = \left(\frac{\Delta i_C}{\Delta i_B}\right) = \frac{\Delta i_C}{\Delta i_E} \times \frac{\Delta i_E}{\Delta i_B}$$
$$= \alpha \frac{\Delta i_E}{\Delta i_B} \quad \ldots\text{(ii)}$$

But $\Delta i_B = \Delta i_E - \Delta i_C$

from (ii)

$$\beta = \alpha \frac{\Delta i_E}{\Delta i_E - \Delta i_C} = \frac{\alpha}{1 - \frac{\Delta i_C}{\Delta i_E}}$$

$$\therefore \quad \beta = \frac{\alpha}{1 - \alpha}$$

32. $v_r^2 = v_m^2 - v^2$

$$v = \frac{320}{4} \text{ m/min} = 80 \text{ m/min}$$
$$v_m = \frac{5}{3} v_r$$
$$v_r^2 = t\,\frac{5}{3}(v_r)^2 - (80)^2$$
$$\frac{16}{9} v_r^2 = (80)^2$$
$$v_r = 60 \text{ m/min}$$

u^2, u_0 form canours linear

33. Let the radii of the thin spherical and the solid sphere are R_1 and R_2 respectively.

Then the moment of inertia of the spherical shell about their diameter

$$I = \frac{2}{3} MR_1^2 \quad \ldots\text{(i)}$$

and the moment of inertia of the solid sphere is given by

$$I = \frac{2}{5} MR_2^2 \quad \ldots\text{(ii)}$$

Given that the masses and moment of inertia for both the bodies are equal, then from Eqs. (i)and (ii)

$$\frac{2}{3} MR_1^2 = \frac{2}{5} MR_2^2 \Rightarrow \frac{R_1^2}{R_2^2} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{R}{R_2} = \sqrt{\frac{3}{5}} \Rightarrow R_1 : R_2 = \sqrt{3} : \sqrt{5}$$

34. The apparent weight of person on the equator (latitude $\lambda = 0$) is given by

$$\omega' = w - mR\omega^2$$

Here, $\therefore$ $\omega' = (3/5)\,w = (3/5)\,mg \quad [\because \omega = mg]$

or $\quad mR\omega^2 = mg - (3/5)\,mg = \left(\frac{2}{5}\right) mg$

or $\quad \omega^2 = \frac{2g}{5R}$

$$\omega = \sqrt{\frac{2g}{5R}}$$

Here, $g = 9.8\,\text{ms}^{-2}$

and $\quad R = 6400\,\text{km} = 6400 \times 10^3\ \text{m}$

$$\therefore \quad \omega = \sqrt{\left(\frac{2}{5} \times \frac{9.8}{6400 \times 10^3}\right)}\ \text{rads}^{-1}$$

$$= 7.82 \times 10^{-4}\ \text{rads}^{-1}$$

35. From Bernoulli's theorem

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{\alpha}\rho v_2^2$$

$$\therefore \quad p_1 - p_2 = \frac{1}{2}\rho\,(v_2^2 - v_1^2)$$

$$\therefore \quad 10 = \frac{1}{2} \times 1.25 \times 10^3\ (v_2^2 - v_1^2)$$

$$\therefore \quad v_2^2 - v_1^2 = \frac{10 \times 2}{1.25 \times 10^3} = 16 \times 10^{-3} \qquad \ldots\text{(i)}$$

Also from equation of continuity

$$= A_1 v_1 = A_2 v_2$$

$$\pi r_1^2 v_1 = \pi r_2^2\ v_2$$

$$\therefore \quad \frac{v_1}{v_2} = \left[\frac{r_2}{r_1}\right]^2 = \frac{0.04}{0.1} = 0.4$$

$$v_1 = 0.4\,v_2 \qquad \ldots\text{(ii)}$$

Substituting this value in Eq. (i)

$$v_2^2 - (0.4\,v_2)^2 = 16 \times 10^{-3}$$

$$v_2 = 1.38 \times 10^{-1} = 0.138\ \text{ms}^{-1}$$

Rate of flow of glycrine $v = A_2 v_2$

$$= \pi r_2^2 v_2$$

$$= 6.93 \times 10^{-4}\ \text{m}^3\text{s}^{-1}$$

36. $E = \sigma T^4$

$T_1 = T, \qquad T_2 = 2T$

$$\frac{E_1}{E_2} = \frac{T_1^{\,4}}{T_2^{\,4}} = \left(\frac{T}{2T}\right)^4 = \frac{1}{16}$$

$E_2 = 16\,E$

37. Since, police jeep and thief are moving in the same direction relative velocity is

$$v_\sigma = v_1 - v_2 = 153 - 45$$

$$= 108\ \text{km/h}$$

$$= 108 \times \frac{5}{18} = 30\ \text{min}$$

38. $E = \frac{-mz^2e^4}{8\,\varepsilon_0\,h^2} \cdot \frac{1}{n^2}$

$$Z_H = 1,$$

$$Z_{He} = 2$$

$$\frac{E_H}{E_{He}} = \frac{Z_H^2}{Z_{He}^2} = \frac{1}{4}$$

$$E_H = \frac{Z_H^2}{Z_{He}^2} = \frac{2}{4}$$

$$E_{He} = 4E_H$$

$$E_H = E_n$$

$$E_{He} = 4E_n$$

39. $W = \frac{hc}{\lambda}$

where h is planck's constant, c is speed of light, λ is wavelength

$$\lambda = \frac{hc}{W} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{6.825 \times 1.6 \times 10^{-19}}$$

$$\lambda = 1800\ \text{Å}$$

40. A field effert transistor is a unipolar device since the conduction of current is due to majority charge carriers only.

41. The periodic-time of a simple pendulum is given by

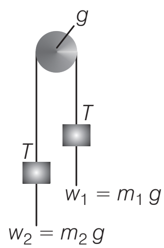
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T_1 = T\ ,\ l_1 = l,\ l_2 = 9l$$

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

$$T_2 = 3T_1 = 3T$$

42. The free body diagram of the given situation is shown.



Taking the upward and downward motions respectively, we get

$$T_1 - mg = m_1\,(g + a)$$

$$T - 2mg = m_1 a$$

for second weight

$$m_2 g - T = m_2\,(g - a)$$

$$2m_2 g - T = m_2 a$$

$$T = \frac{4m_1 m_2 g}{(m_1 + m_2)}$$

$w_1 = m_1 g,\ m_2 = m_2 g$ hence switch

$$T = \frac{4\,w_1\,w_2}{(w_1 + w_2)}$$

43. The angle subtended at the eye becomes 10 times larger, this happens only when the tree appear 10 times nearer.

44. When the switch is closed, the current grows in the circuit but due to self-inductance of the switch inductor the growth is slow. On disclosing the switch the current suddenly decrease to zero on account of the infinite resistance of the circuit. Thus, rate cf growth of the current is lower as compared to the rate of decays

45. We know that $v = \sqrt{\dfrac{T}{m}}$

$$v \propto \sqrt{T}$$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\right)^2$$

$$\frac{T_2 - T_1}{T_1} = \frac{v_2^2 - v_1^2}{v_1^2}$$

$$v_2 = v_1 + \frac{20}{100} v_1 = \frac{120}{100} v_1 = \frac{6}{5} v_1$$

$$= \frac{6}{5} \times 150$$

$$\frac{T_2 - T_1}{T_1} = \frac{(180)^2 - (150)^2}{(150)^2}$$

$$= \frac{30 \times 330}{150 \times 180} = 0.47$$

$$= \frac{T_2 - T_1}{T_1} \times 100 = 0.44 \times 100 = 44\%$$

46. $E - 40 \text{ GWh} = 40 \times 10^9 \text{ J/s}$

$$(60 \times 60) = 1.44 \times 10^{14} \text{ J}$$

$$E = mc^2$$

$$m = \frac{E}{c^2} = \frac{1.44 \times 10^{14}}{(3 \times 10^8)^2} = 1 \times 10^3 \text{ kg}$$

$$m = 1.6 \text{ g}$$

47. $y = k t^2$

$$\frac{d^2 y}{dt^2} = 2k$$

$$a = 2 \text{ m/s}^2$$

$$T_1 = 2\pi \sqrt{\frac{l}{g}}$$

$$T_2 = 2\pi \sqrt{\frac{l}{g + a_y}}$$

$$\frac{T_1^2}{T_2^2} = \frac{g + a_y}{g}$$

$$= \frac{10 + 2}{10} = \frac{6}{5}$$

48. $F = -\dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^2} = \text{r}$

$$F = -k \frac{q_1 q_2}{r^2} = \text{r}$$

$$F = -k \frac{e.e}{r^2} \hat{r} = -k \frac{e^2}{r^2} \hat{r}$$

$$\hat{r} = \frac{\text{r}}{|\text{r}|} = \frac{\text{r}}{r}$$

$$F = -k \frac{e^2}{r^2} \cdot \frac{\text{r}}{r} = -k \frac{e^2}{r^3} \text{r}$$

49. Gauss's law

$$\phi = \frac{q}{\varepsilon_0}$$

$$\phi' = \frac{\phi}{6} = \frac{q}{6\varepsilon_0} = \frac{4\pi q}{6(4\pi\varepsilon_0)}$$

50. $\overline{A} \cdot \overline{B} = \overline{A + B}$

51. $F = mr\,\omega^2 = mr \times (2\pi mrn)^2 = 4\pi^2 mrn^2$

$$F \propto 2n^2$$

$$F = F_1 + F_2$$

$$n^2 = n_1^2 + n_2^2$$

$$n = \sqrt{n_1^2 + n_2^2}$$

$$n = \sqrt{(6)^2 + (8)^2}$$

$$n = \sqrt{100} = 10 \text{ Hz}$$

52. $K = K_0$ = total energy

As total energy remains conserved itself hence when U is maximum in SHM, $K = 0$

i.e., E is also equal to U_{max} *i.e.*,

$$U_{max} = E = K_0$$

53. Relative velocity of the parrot w.r.t. the train

$$[10 - (-5)] = (10 - (-5)) \text{ m/s} = 15 \text{ m/s}$$

Time taken by the parrot to cross the train

$$= \frac{150}{15} = 10 \text{ s}$$

54. Time taken by ice to grow a thickness of

$$t = \frac{eL}{2K\theta} y^2$$

Hence time interval to change thickness from 0 to y, from y to $2y$ and so on will be in the ratio

$$\Delta t_1 : \Delta t_2 : \Delta t_3$$

$$\therefore \quad = (1 - 0^2) : (2^2 - 1^2) : (3^2 - 2^2)$$

$$\Delta t_1 : \Delta t_2 : \Delta t_3 = 1 : 3 : 5$$

According to question = Δt_1 < terims

$$\Delta t_2 = 3\,\Delta H$$

$$= 3 \times 12 = 36 \text{ min}$$

55. As,

$$\sqrt{V} = a(Z-b)$$
$$\sqrt{V} = a(Z-1)$$
$$V = a^2 (Z-1)^2$$
$$\frac{C}{\lambda} = a^2(Z-1)^2$$
$$\lambda = \frac{c}{a^2 (Z-1)^2}$$

$z = 43$, wavelength $= \lambda$

$$\lambda = \frac{c}{a^2 (43-1)^2}$$
$$\lambda = \frac{c}{a^2 \times 42}$$

for $z = 29$, wavelength $= \lambda'$

$$\lambda' = \frac{C}{a^2(29-1)^2}$$
$$\lambda' = \frac{c}{28a^2}$$
$$\frac{\lambda'}{\lambda} = -\left(\frac{42}{28}\right)^2 = \left(\frac{3}{2}\right)^2$$
$$\lambda' = \left(\frac{9}{4}\right)\lambda$$

56. $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

$$-\frac{1}{u^2}\frac{du}{dt} - \frac{1}{v^2}\frac{dv}{dt} = 0$$
$$= -\frac{1}{u^2}u_0 - \frac{1}{v^2}v_0 = 0$$
$$\frac{u_0}{v_0} = -\frac{u^2}{v^2} = -\frac{1}{m^2}$$

57. As, $I^2R = hA$

$$I^2\left(\frac{\rho L}{\pi r^2}\right) = h \times 2\pi rl, \quad I \propto r^{3/2}$$

58. $f = \frac{\Delta V}{(\Delta Q_P)} = \frac{\Delta Q_V}{(\Delta Q_P)_p} = \frac{\mu\, Cv\, \Delta T}{\mu\, Cp\, \Delta T} = \frac{1}{T}$

$$f = \frac{7}{5}$$
$$f = \frac{5}{7}$$

59. In interference of light energy is neither created nor destroyed.

60. From colour triangle for pigment and dyes we observe that the given combination give black colour.

Chemistry

1. i for KCl = 2, i for for $CaCl_2$ = 3

$$\Delta T_f \propto i$$
$$\frac{\Delta T_f(KCl)}{\Delta T_f(CaCl_2)} = \frac{2}{3}$$
$$\Delta T_f(CaCl_2) = \frac{3}{2} \times 2 = 3°C$$

Freezing point of $CaCl_2 = -3°C$

2. $\frac{r_1}{r_2} = \frac{p_1}{p_2}\sqrt{\frac{M_2}{M_1}}$

$$P_{He} = x_{He} \cdot p_{total}$$
$$= \frac{1}{4} \times 16 = 4 \text{ atom}$$
$$P_{N_2} = x_{N_2} \cdot p_{total}$$
$$= \frac{3}{4} \times 16 = 12 \text{ atom} = (p_{total}^{-}\ p_{He})$$
$$\frac{r_{He}}{r_{N_2}} = \frac{p_{He}}{p_{N_2}}\sqrt{\frac{M(N_2)}{M(He)}}$$
$$= \frac{4}{12}\sqrt{\frac{28}{4}} = 0.88$$

Hence, moles of He and N_2 effusing out initially are in the ratio 0.88 : 1.

3. Sc (21) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^1\ s^2$

19th electron goes into 4s

Hence, $n = 4$, $\quad l = 0$,

$m = 0$, $\quad m_s = +\frac{1}{2}$

4. (A) + tap water $\rightarrow$ white turbidity soluble in aq. NH_3.
Tap water has Cl^- and turbidity is soluble in $aq\,NH_3$.
Turbidity is of AgCl and thus (A) has Ag^+
(A) also gives ring test of NO_3^-
(A) has nitrate and thus (A) is $AgNO_3$

5. Greater the number of ions, greater the conduction hence, order of increasing conductivity is

$$c<d<b<a$$

(a) $[Pt(NH_3)_6]Cl_4 \rightleftharpoons [Pt(NH_3)_6]^{4+} + 4Cl^- = 5$ ions

(b) $[Cr(NH_3)_6]Cl_3 \rightleftharpoons [Cr(NH_3)_6] + 3Cl^- = 4$ ions

(c) $[Co(NH_3)_4Cl_2]Cl \rightleftharpoons [Co(NH_3)_4Cl_2] + Cl^- = 2$ ions

(d) $K_2[PtCl_6] \rightleftharpoons 2K + [PtCl_6]^{2-} = 3$ ions

6. EAN = Z – (O.N.) + $2x$ or $36 = 26 - 0 + 2x$ or $x = 5$

7. **For reaction *A***

$$\Delta G^\circ = 2\Delta G^\circ_f\,(ZnO) + 2\Delta\, G^\circ_f\,(SO_2) \;\; - 2G^\circ_f\,(ZnS)$$
$$= 2\,[-318.2 - 300.4 + 205.4]$$
$$= -826.4\text{ kJ} \qquad (\Delta G^\circ_f\text{ (element)} = 0)$$

For reaction *B*

$$\Delta G^\circ = \Delta G^\circ_f\,(CO) - \Delta G^\circ_f\,(ZnO)$$
$$= -137.3 + 328.2$$
$$= +180.9\text{ kJ}$$

8. Ore- copper pyrites , $CuFeS_2$

Concentrated pyrites ore (froth- floatation process) is roasted below 800°C. FeS is converted to FeO while copper remains as sulphide.

Roasting $2FeS + 3O_2 \xrightarrow{\Delta} 2FeO + 2\,SO_2\uparrow$

Smelting $FeO + SiO_2 \longrightarrow \underset{\text{slag}}{FeSIO_3}$

(SiO_2 either is present ore or is added)

Bessemerisation

$$2Cu_2S + 3O_2 \longrightarrow 2Cu_2O + 2SO_2$$
$$2Cu_2O + Cu_2S \longrightarrow 6Cu + SO_2 \qquad \text{(final step)}$$

9. $CO_2 + H_2O \rightleftharpoons H_2CO_3 \rightleftharpoons H^+ + HCO_3^-$

$$\underset{\text{yellow}}{2CrO_4^{2-}} + 2H^+ \longrightarrow \underset{\text{orange red}}{Cr_2O_7^{2-}} + H_2O$$

Aqueous CO_2 is acidic and changes yellow CrO_4^{2-} into orange $Cr_2O_7^{2-}$.

10. Transition elements form complexers due to their smaller size, higher nuclear charge and presence of low energy vacant orbitals to accept lone pair of electrons donated by ligands.

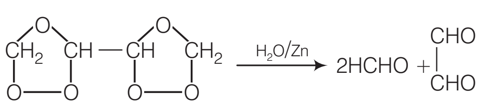
11. ClO_2 reacts with KOH forming KCl_3 and $KClO_2$

$$2KOH + 2ClO_2 \longrightarrow KClO_2 + KClO_3 + H_2O$$

This reaction indicates that ClO_2 is a mixed anhydride of $HClO_2$ and $HClO_3$.

12. Alkene *A* is $CH_2{=}CH{-}CH{=}CH_2$

$$CH_2{=}CH{-}CH{=}CH_2 + O_3 \longrightarrow$$



13. It can be prepared in the laboratory by reacting sodium, nitrite ($NaNO_2$) with a reducing agent such as Fe^{2+} in an acidic medium. NH_4NO_3 on heating at 250°C gives N_2O.

14. It absorbs CO_2 and releases O_2.

$$4KO_2 + 2CO_2 \longrightarrow 2K_2CO_3 + 3O_2$$

15. Stepwise formation of MgO involves

Step	Reaction	Δ *H* in kJ mol^{-1}
(i)	$Mg(s) \to Mg(g)$	147.7
(ii)	$Mg(g) \to Mg^{2+} + 2e^-$	2189.0
(iii)	$\frac{1}{2}O_2(g) \to O(g)$	498.4/2
(iv)	$O(g) + 1e^- \to O^-(g)$	−141.0
(v)	$O^-(g)te^- \to O^{2-}(g)$	*q*
(vi)	$Mg^{2+}(g) + O^{2-}(g) \to MgO(s)$	−3791.0

$$Mg(s) + \frac{1}{2}O_2(g) \longrightarrow MgO(s)\;\Delta H = -1346.1 + q$$

$$Mg(s) + \frac{1}{2}O_2(g) \longrightarrow MgO(s)\;\Delta H_2 = -601.7$$

By Born Herber cycle (based on Hes's law)

$$\Delta H_1 = \Delta H_2$$
$$-1346.1 + q = -601.7$$
$$q = 744.4\text{ Jmol}^{-1}$$

16. $H_2S_2O_7 + H_2O \longrightarrow 2H_2SO_4$

SO_3 of $H_2S_2O_7$ is converted into H_2SO_4, hence SO_3 acts also as a dibasic acid.

Eq. wt. $\quad (SO_3) = \frac{M}{2} = 40$

Let H_2SO_4 in fuming $H_2SO_4 = x$ g

$$SO_3 = (1 - x)\text{ g}$$

Equivalent of $H_2SO_4 = \frac{x}{49}$

$$SO_3 = \frac{1 - x}{40}$$

Equivalent of NaOH used $= \frac{26.7 \times 0.8}{1000}$

$$= 0.02136$$

$$\frac{x}{49} + \frac{1 - x}{40} = 0.02136$$

$x = 0.7927$ g H_2SO_4 in 1 g oleum.

Percentage of $H_2SO_4 = 79.27\%$

$$SO_3 = 20.73\%$$

17. ${}^{14}_{7}N + {}^{1}_{0}n \longrightarrow {}^{14}_{6}C + {}^{1}_{1}H$

This reaction can be represented as

$${}^{14}_{7}N(n, p){}^{14}_{6}C$$

18. Ag^+ is reduced hence, it is oxidant Hydroquinone is oxidised hence, it is reductant.

19. N(7) and O^+ $(8-1=7) = 1s^2\ 2s^2\ \underbrace{2p_x^1 2p_y^1 2p_z^1}_{\text{Maximum multiplicity most stable}}$

$$O = 1s^2, 2s^2\ 2p_x^2, 2p_y^1, 2p_z^1$$

Ionisation potential is dependent on the value of Z.

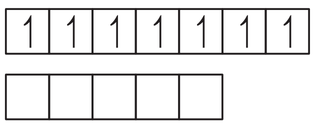
$$Z \text{ of } O^+ (8) > Z \text{ of } N(7)$$

Thus, ionisation potential of $O^+ > N$

Na with one valence electron has least ionisation potential.

20. Gd. (64) = $[Xe]4f^7 5d^1$

$4f^4$



$5d^1$

All electrons are unpaired, hence greater stability

21. He^+, Li^{2+} and Be^{3+} each has one electron, hence no screening.

22. Symmetrical molecules will have net dipole zero while unsymmetrical will have dipoles. I is symmetrical and have no dipole while II and III have dipoles.

23. In PCl_5 No. of e^- pairs around central atom P = 5

No of lone pairs = 0 and hybridisation-sp^3d Shape = trigonal bipyramidal. In PCl_4^+– number. of e^- pairs = 4, lone pairs = 0 , hybridisation = sp^3 and shape – tetrahedral

In PCl_6^-– number of e^- pairs = 6, lone pairs = 0, hybridisation = sp^3d^2 and shape is octahedral

24. Repulsive interactions in decreasing order

$$= lp-lp > lp-bp > bp-bp$$

25. Based on Pauling's scale

$$(EN)_F - (EN)_{Cl} = k\sqrt{\Delta} = 0.208\sqrt{\Delta}$$

where, Δ = resonance energy

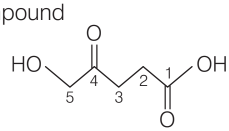
and k = conversion factor which is 0.208 for converting k cal into eV.

$$\Delta = (BE)_{Cl-F} - \sqrt{(BE)_{Cl-Cl}(BE)_{F-F}}$$
$$= 61 - \sqrt{58 \times 38}$$
$$= 61 - 46.95 = 14.05 \text{ kcal}$$
$$(EN)_{Cl} = (EN)_F - 0.208\sqrt{\Delta}$$
$$= 4.0 - 0.208\sqrt{14.05}$$
$$= 4.0 - 0.78 = 3.22 \text{ eV.}$$

26. Number of moles of O atoms $= \frac{1}{3}$ moles of H atoms

$$= \frac{1}{3} \times 3.18 = 1.06 \text{ mol}$$

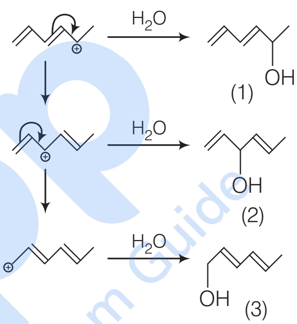
27. In the compound



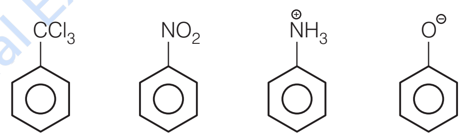
keto group is at 4C – atom

28. S_N1 reaction proceed through formation of carbocation which can form a pair of enantiomers. Thus, recemisation takes place (statement I). S_N2 reaction takes place forming pantavalent intermediate which loses Cl^- giving inverted alcohol. (Statement II) Hence, statements I and II both are correct.

29.

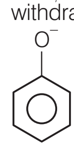


30. Electrophile $\overset{\oplus}{N}O_2$ attacks the following



I II III IV

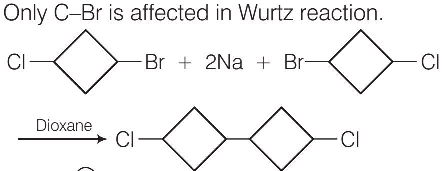
CCl_3, NO_2 and NH_3 groups are electron withdrawing groups. All deactivating and *m*-directing.



In negative charge on O is delocalised increasing electron density at *o*-and *p*-positions.

31. BE of C– Br bond < BE of C–Cl

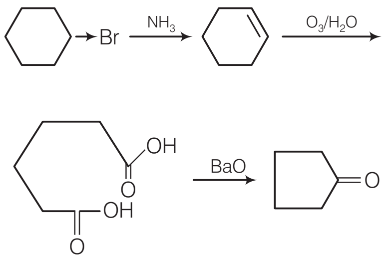
Only C–Br is affected in Wurtz reaction.



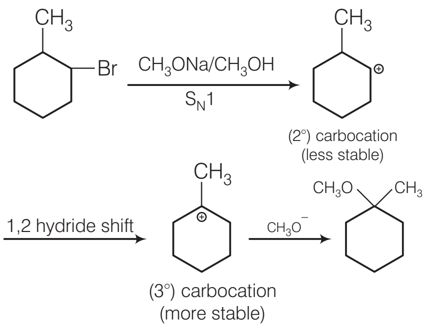
32. $CH_3CH{=}CH_2 \xrightarrow{D^-(NaBD_4)} CH_3\underset{D}{C}H\overset{\oplus}{C}H_2$

$$\xrightarrow{H_2O_2/OH^-} CH_3\underset{\mid \atop D}{C}HCH_2OH$$

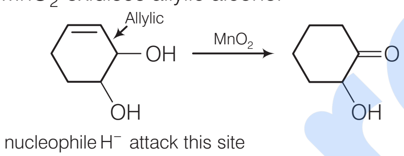
33.



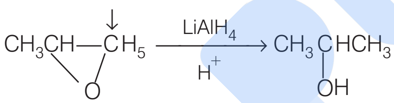
34.



35. MnO_2 oxidises allylic alcohol



36.



37. $CH_3\overset{O}{\overset{\|}{C}}OC_2H_5 + HCH_2\overset{O}{\overset{\|}{C}}OC_2H_5 \xrightarrow{C_2H_5ONa} CH_3\overset{O}{\overset{\|}{C}}CH_2\overset{O}{\overset{\|}{C}}OC_2H_5$

(A) Keto ester

(b) *A* forms oxime due to keto group

(c) *A* shows tautomerism

(d) *A* shows iodoform test due to $CH_3\overset{O}{\overset{\|}{C}}—$

38. When a nucleophile encounters a ketone a site of attack is the carbon atom of the carbonyl group.

39. —COOH attached to ring is eliminated due to presence of keto group at β – position.

40. $CH_3—\overset{O}{\overset{\|}{C}}—NHCH_2CH_2CH_3$

N- propylacetamide.

41. (a) $CH_3CH_2NO_2 \xrightarrow{HNO_2} CH_3\underset{\underset{N—OH}{\|}}{C}NO_2$

blue (nutroic acid)

$\xrightarrow{NaOH} CH_3—\underset{\underset{NONa}{\|}}{C}—NO_2$

red

(b) $CH_3\underset{CH_3}{\overset{|}{C}}HNO_2 \xrightarrow{HNO_2} CH_3—\overset{CH_3}{\underset{N=O}{\overset{|}{\underset{|}{C}}}}—NO_2$

preudonitrol (blue)

$\xrightarrow{NaOH}$ No reaction blue

(c) $(CH_3)_3CNO_2 \xrightarrow{HNO_2}$ No reaction

42. $CH_3\underset{NH_2}{\underset{|}{C}}HCOOH \; CH_3\underset{\oplus NH_3}{\underset{|}{C}}HCOO^-$

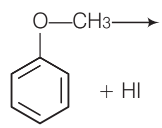
$\xrightarrow[\text{acidic}]{H^+} CH_3\underset{\oplus NH_3}{\underset{|}{C}}HCOOH$

(pH = 4, 1 or 2)

43. Sucrose has glucose and fructose, maltose has two units of glucose, raffinose has glucose, fructose and galactose. Galactose is a monosaccharide.

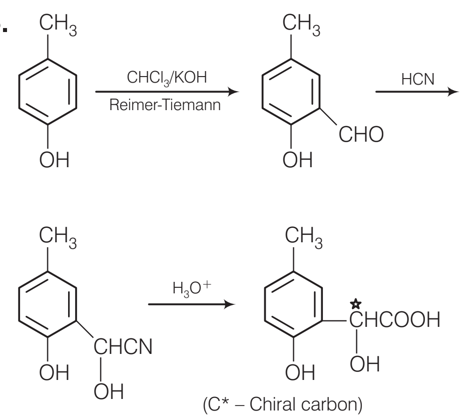
44. Amylose is a straight chain polymer have D- glucose units joined to gather by α – glycosidic linkage.

45.

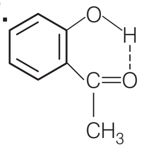


This bond is cleaved due to greater stability of phenxide ion. Thus, C_6H_5OH and CH_3I is formed.

46.



(C* – Chiral carbon)

47. 

Boiling point of of *A* is lower than that of *B*. (In *A*, there is interamolecular H-bonding and in *B*, there is intermolecular H- bonding.) Thus, *A* is more volatile than *B*.

48. Sodium extract of thiourea will give red colour in Lassaigne's test due to the presence of both the elements nitrogen and sulphur.

49. The given half reactions are

$$BrO_3^- + 4e^- + 4H^+ \longrightarrow BrO^- + 2H_2O \qquad E^\circ(\text{in V})\ 1.50 \qquad \Delta G^\circ = -nFE^\circ\ \ -6\,F$$

$$BrO^- + e^- + 2H^+ \longrightarrow \frac{1}{2}Br_2 + H_2O \qquad 1.60 \qquad -1.6\,F$$

Adding $BrO_3^- + 5e^- + 6H^+ \longrightarrow \frac{1}{2}Br_2 + 3H_2O$

$$\Delta G^\circ = -7.6\,F = -5\,E^\circ F$$

$$E^\circ = \frac{7.6}{5} = 1.52\text{ V}$$

50. Zn is at lower potential, it will be oxidised and the cell reaction will be

$$Zn(s) + Ni^{2+}(aq) \longrightarrow Zn^{2+}(aq) + Ni(s)$$

51. $0.65 = i \times 1.86 \times 0.15$

$$i = 2.33 = 1 + 4\alpha$$

$$\alpha = -0.33$$

52. $p = p^\circ x_1$

$$99.50 = 100\frac{n_1}{n_1 + n_2}$$

$$\frac{n_2}{n_1} = \frac{1}{191}$$

$$\Delta T_b = k_b m = K_b \frac{n_2}{n_1} \times \frac{1000}{m_1}$$

$$= 0.52 \times \frac{1}{191} \times \frac{1000}{18} = 0.15$$

$$T_b = 100.15°C$$

53. In reverse direction, product becomes reactant and *vice versa*. Hence, (b) is most endothermic in reverse direction

54. $\log \frac{k_2}{k_1} = \log 1000$

$$= \frac{E_1 - E_2}{RT}$$

$$= \frac{98000 - E_2}{8.314 \times 300}$$

E_2 (catalyst) = 80.77 kJ

55. From experiment no –3 and 4, doubling $[OCl^-]$ doubles the rate; *i.e.*, order with respect to OCl^- is one. Also, from experiment no 1 and 2, doubes the concentration of I^- double the rate, *i.e*. order with respect to I^- is also one.
rate $= k[I^-][OCl^-]$

56. $HO\overset{+}{N}H_3 + H_2O \rightleftharpoons HONH_3OH + H^+$

$$K_h = \frac{K_w}{k_b} = \frac{10^{-14}}{10^{-8}} = 10^{-6}$$

$$[H^+] = \sqrt{K_h C}$$

$$= \sqrt{10^{-6} \times 0.024}$$

$$= 1.5 \times 10^{-4}$$

and pH = 3.8

57. $AgBr + 2CN^- \rightleftharpoons Ag(CN)_2^- + Br^-$

$$0.1 - 2s \qquad\qquad s \qquad s$$

$$K_{sp} = k_{sp} \cdot k = 4.3 \times 10^{-4}$$

$$4.3 \times 10^{-4} = \left(\frac{s}{0.1 - 2s}\right)$$

$$s = 2 \times 10^{-3}$$

58. Cetyl trimethyl ammonium bromide forms cationic micelles above a certain minimum concentration.

59. $\frac{d(Cs)}{d(Li)} = \frac{1.87}{0.53} = \frac{133}{7}\left(\frac{r(Li)}{r(Cs)}\right)^3$

$$\frac{r(Cs)}{r(Li)} = 1.753$$

or $\qquad r_{Cs} = 1.753\,r(Li)$

60. Nylon threads are made up of polyamide. Some common examples are nylon 6 and nylon 66.

Nylon 6 $H_2N-(CH_2)_5-COOH$
Monomer

$$-NH-(CH_2)_5-\overset{O}{\overset{\|}{C}}-NH-(CH_2)_5-\overset{O}{\overset{\|}{C}}-NH-$$

Polymer

Nylon 66 $\quad HO-\overset{O}{\overset{\|}{C}}-(CH_2)_4COOH$

and $\quad H_2N-(CH_2)_6-NH_2$ are monomers

$$HN-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-NH-(CH_2)_6-NH$$

nylon-66 (polymer)

$$-\overset{O}{\overset{\|}{C}}-(CH_2)_4-\overset{O}{\overset{\|}{C}}-$$

Mathematics

1. For $n = 1$, $\quad 1! < (1)^1$

that is, $1 < 1$ is not true.

For $n = 2$, $\quad 2! < \left(\frac{3}{2}\right)^1$

that is, $2 < \frac{9}{4}$ is true,

By induction, $n! < \left(\frac{n+1}{2}\right)^n \ \forall\, n \in N$ for $n > 2$.

$\therefore$ least $n = 2$.

2. Let T_r be the r^{th} term of the given series.

Then, $\quad T_r = \frac{r}{1 + r^2 + r^4}, \qquad r = 1, 2, 3, \ldots, n$

$$= \frac{r}{(r^2 + r + 1)(r^2 - r + 1)}$$

$$= \frac{1}{2}\left[\frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1}\right]$$

$\therefore$ Sum of the series $= \sum_{r=1}^{n} T_r$

$$= \frac{1}{2}\left\{\sum_{r=1}^{n}\left(\frac{1}{r^2 - r + 1} - \frac{1}{r^2 + r + 1}\right)\right\}$$

$$= \frac{1}{2}\left\{\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{13}\right) + \ldots + \left(\frac{1}{n^2 - n + 1} - \frac{1}{n^2 + n + 1}\right)\right\}$$

$$= \frac{1}{2}\left\{1 - \frac{1}{n^2 + n + 1}\right\} = \frac{n^2 + n}{2(n^2 + n + 1)}$$

3. Clearly, the given series is an arithmetico-geometric series whose corresponding AP and GP are respectively, 1, 4, 7, 10, … and $1, \frac{1}{5}, \frac{1}{5^2}, \frac{1}{5^3}, \ldots$

The n^{th} term of AP $= [1 + (n - 1) \times 3] = 3n - 2$

The n^{th} term of GP $= \left[1 \times \left(\frac{1}{5}\right)^{n-1}\right] = \left(\frac{1}{5}\right)^{n-1}$

So, the n^{th} term of the given series is

$$(3n - 2) \times \frac{1}{5^{n-1}} = \frac{3n - 2}{5^{n-1}}$$

Let, $\quad S_n = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots + \frac{3n - 5}{5^{n-2}} + \frac{3n - 2}{5^{n-1}} \quad \ldots\text{(i)}$

$$\frac{1}{5}S_n = \frac{1}{5} + \frac{4}{5^2} + \frac{7}{5^3} + \ldots + \frac{(3n - 5)}{5^{n-1}} + \frac{3n - 2}{5^n} \quad \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$S_n - \frac{1}{5}S_n = 1 + \left\{\frac{3}{5} + \frac{3}{5^2} + \frac{3}{5^3} + \ldots + \frac{3}{5^{n-1}}\right\} - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow \frac{4}{5}S_n = 1 + \frac{3}{5}\frac{\left\{1 - \left(\frac{1}{5}\right)^{n-1}\right\}}{\left(1 - \frac{1}{5}\right)} - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow \frac{4}{5}S_n = 1 + \frac{3}{5}\frac{\left\{1 - \frac{1}{5^{n-1}}\right\}}{\left(\frac{4}{5}\right)} - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow \frac{4}{5}S_n = 1 + \frac{3}{4}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{(3n - 2)}{5^n}$$

$$\Rightarrow S_n = \frac{5}{4} + \frac{15}{16}\left(1 - \frac{1}{5^{n-1}}\right) - \frac{(3n - 2)}{4 \cdot 5^{n-1}}$$

4. The sequence of differences between successive terms is 2, 6, 18, 54, … .

Clearly, it is a GP. Let T_n be the n^{th} term of the given series and S_n be the sum of its n terms.

Then,

$$S_n = 5 + 7 + 13 + 31 + 85 + \ldots + T_{n-1} + T_n \quad \ldots\text{(i)}$$

$$S_n = 5 + 7 + 13 + 31 + \ldots + T_{n-1} + T_n \quad \ldots\text{(ii)}$$

Subtracting (ii) from (i), we get

$$O = 5 + [2 + 6 + 18 + 54 + \ldots + (T_n - T_{n-1})] - T_n$$

$$\Rightarrow O = 5 + 2 \cdot \frac{3^{n-1} - 1}{3 - 1} - T_n$$

$$\Rightarrow T_n = 5 + (3^{n-1} - 1) = 4 + 3^{n-1}$$

$$\therefore S_n = \sum_{k=1}^{n} T_k = \sum_{k=1}^{n}(4 + 3^{K-1})$$

$$= \sum_{k=1}^{n} 4 + \sum_{k=1}^{n} 3^{k-1}$$

$$= 4n + (1 + 3 + 3^2 + \ldots + 3^{n-1})$$

$$= 4n + 1 \times \left(\frac{3^n - 1}{3 - 1}\right)$$

$$= 4n + \left(\frac{3^n - 1}{2}\right)$$

$$= \frac{1}{2}(3^n + 8n - 1)$$

5. The discriminant of the quadratic equation $2x^2 + 6x + b = 0$ is given by $D = 36 - 8b > 0$.

Therefore, the given equation has real roots.

we have,

$$\frac{x_1}{x_2} + \frac{x_2}{x_1} = \frac{x_1^2 + x_2^2}{x_1 \cdot x_2} = \frac{(x_1 + x_2)^2 - 2x_1x_2}{x_1 \cdot x_2}$$

$$= \frac{(-3)^2 - 2(b/2)}{(b/2)} = \frac{18}{b} - 2 < -2 \qquad [\because\ b < 0]$$

6. $x^4 + x^2 + 1 = (x^2 + 1)^2 - x^2$

$$= (x^2 + x + 1)(x^2 - x + 1)$$

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \neq 0 \qquad \forall\ x.$$

Now, $(a - 1)(x^2 + x + 1)^2 = (a + 1)(x^4 + x^2 + 1)$

$$(a - 1)(x^2 + x + 1) = (a + 1)(x^2 - x + 1).$$

$\Rightarrow$ $x^2 - ax + 1 = 0$ which has real and distinct roots

$\therefore \quad \Delta = a^2 - 4 > 0$

$\Rightarrow \quad a^2 > 4$

$\Rightarrow \quad a \in (-\infty, -2) \cup (2, \infty)$

7. Given that, $\alpha + \beta = 0$

$$\alpha + \beta + \gamma = -p$$

$\Rightarrow \quad \gamma = -p$

Substituting $\gamma = -p$ in the given equation,

$\Rightarrow \quad -p^3 + p^3 - pq + r = 0$

$\Rightarrow \quad pq = r.$

8. Selection of 2 parallel lines from $m = {}^mC_2$

Selection of 2 parallel lines from $n = {}^nC_2$

Hence, number of parallelograms $= {}^mC_2 \cdot {}^nC_2$

$$= \frac{1}{4}(m - 1)(n - 1)\, mn$$

9. We have

$$ {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 255 \quad \ldots \text{(i)}$$

Also, the sum of binomial coefficients

$$ {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n + {}^{2n+1}C_{n+1} + + {}^{2n+1}C_{n+2} + \ldots + {}^{2n+1}C_{2n+1}$$

$$= (1 + 1)^{2n+1} = 2^{2n+1}$$

$$\Rightarrow {}^{2n+1}C_0 + 2\{{}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n\} + {}^{2n+1}C_{2n+1} = 2^{2n+1}$$

$\Rightarrow \quad 1 + 2(255) + 1 = 2^{2n+1}$

$\Rightarrow \quad 1 + 255 = 2^{2n}$

$\Rightarrow \quad 2^{2n} = 2^8$

$\Rightarrow \quad n = 4$

10. $\log x$ is defined only when $x > 0$

Now, the 3rd term in the expansion

$T_{2+1} = {}^5C_2 \cdot x^{5-2} \cdot (x^{\log_{10} x})^2 = 1{,}000{,}000 \quad$ (given)

$\Rightarrow \quad x^{3 + 2\log_{10} x} = 10^5.$

Taking logarithm of both sides, we get

$$(3 + 2\log_{10} x) \cdot \log_{10} x = 5$$

$\Rightarrow \quad 2y^2 + 3y - 5 = 0,$

where $\log_{10} x = y$

$\Rightarrow \quad (y - 1)(2y + 5) = 0$

$\Rightarrow \quad y = 1 \text{ or } -5/2$

$\Rightarrow \quad \log_{10} x = 1 \text{ or } -5/2$

$\Rightarrow \quad x = 10^1 = 10 \text{ or } 10^{-5/2}$

11. $\displaystyle\sum_{k=0}^{10} {}^{20}C_k = {}^{20}C_0 + {}^{20}C_1 + \ldots + {}^{20}C_{10}$

$$= \frac{1}{2}[2 \cdot {}^{20}C_0 + 2 \cdot {}^{20}C_1 + \ldots + 2 \cdot {}^{20}C_{10}]$$

$$= \frac{1}{2}[({}^{20}C_0 + {}^{20}C_{20}) + ({}^{20}C_1 + {}^{20}C_{19}) + \ldots + 2\,{}^{20}C_{10}]$$

$$= \frac{1}{2}[({}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + \ldots + {}^{20}C_{19} + {}^{20}C_{20}) + {}^{20}C_{10}]$$

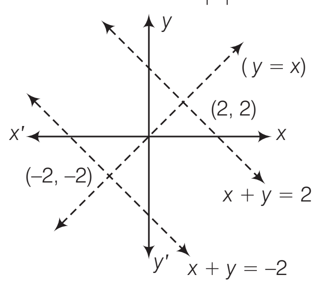
$$= \frac{1}{2}[2^{29} + {}^{20}C_{10}]$$

$$= 2^{19} + \frac{1}{2}\,{}^{20}C_{10}.$$

12. Lines $x + y = 4$ and $x + y = -4$ are parallel and points $(2, 2)$ and $(-2, -2)$ lie on these lines.

If point (a, a) lies between the lines then $a > -2$ and $a < 2$ i.e.,

$$-2 < a < 2 \Rightarrow |a| < 2$$



13. From the given equation,

$$3(my + 2) + 2y = 0$$

$\Rightarrow \quad y(3m + 2) = 4$

$\Rightarrow \quad y = \dfrac{4}{3m + 2},$

since, $y \in I \Rightarrow 3m + 2$

$\Rightarrow \quad \pm 1, \pm 2, \pm 4$

$\Rightarrow \quad m = -1, -\frac{1}{3}, -\frac{4}{3}, 0, \frac{2}{3}, -2$

14. Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ ...(i)

Its intercepts on y and x axes are b and a respectively.

According to the question,

$$\frac{1}{a} + \frac{1}{b} = \text{constant} = K \quad \text{(say)}$$

$$\therefore \quad \frac{1}{aK} + \frac{1}{bK} = 1$$

$$\Rightarrow \quad \frac{1/K}{a} + \frac{1/K}{b} = 1 \quad \text{...(ii)}$$

From (ii) it follows that line (i) passes through the fixed point $\left(\frac{1}{K}, \frac{1}{K}\right)$.

15. Variable line is $\frac{x}{a} + \frac{y}{b} = 1$ ...(i)

Any line perpendicular to (i) and passing through the origin will be

$$\frac{x}{b} - \frac{y}{a} = 0 \quad \text{...(ii)}$$

Now foot of the perpendicular from the origin to line (i) is the point of intersection (i) and (ii)

Let it be $P(\alpha, \beta)$, then $\frac{\alpha}{a} + \frac{\beta}{b} = 1$ ...(iii)

and $\frac{\alpha}{b} - \frac{\beta}{a} = 1$ ...(iv)

Squarring and adding Eqs. (iii) and (iv), we get

$$\alpha^2\left(\frac{1}{a^2} + \frac{1}{b^2}\right) + \beta^2\left(\frac{1}{b^2} + \frac{1}{a^2}\right) = 1$$

$$\therefore \quad (\alpha^2 + \beta^2)\frac{1}{c^2} = 1$$

Hence, the locus of $P(\alpha, \beta)$ is $x^2 + y^2 = c^2$

which is a circle.

16. The given equation of pair of straight lines can be rewritten as $(\sqrt{3}x - y)(x - \sqrt{3}y) = 0$. Their separate equations are,

$$y = \sqrt{3}\, x \text{ and } y = \frac{1}{\sqrt{3}}x$$

$$\Rightarrow \quad y = \tan 60^\circ x \quad \text{and} \quad y = \tan 30^\circ x$$

After rotation, the separate equations are

$$y = \tan 90^\circ x$$

and $\quad y = \tan 60^\circ x$

$\Rightarrow \quad x = 0$ and $y = \sqrt{3} \cdot x$

$\therefore$ Combined equation in the new position is

$x(\sqrt{3}x - y) = 0$ or $\sqrt{3}x^2 - xy = 0$

17. $\lim\limits_{x \to 1} \frac{\left[\sum\limits_{K=1}^{100} x^K\right] - 100}{(x-1)}$

$$= \lim_{x \to 1} \frac{(x + x^2 + x^3 + \ldots + x^{100}) - 100}{(x-1)}$$

$$= \lim_{x \to 1} \frac{(x-1) + (x^2 - 1) + (x^3 - 1) + \ldots + (x^{100} - 1)}{(x-1)}$$

$$= \lim_{x \to 1} \left\{\left(\frac{x-1}{x-1}\right) + \left(\frac{x^2-1}{x-1}\right) + \left(\frac{x^3-1}{x-1}\right) + \ldots + \left(\frac{x^{100}-1}{x-1}\right)\right\}$$

$$= \lim_{x \to 1}\left(\frac{x-1}{x-1}\right) + \lim_{x \to 1}\left(\frac{x^2-1}{x-1}\right) + \lim_{x \to 1}\left(\frac{x^3-1}{x-1}\right) + \ldots + \lim_{x \to 1}\left(\frac{x^{100}-1}{x-1}\right)$$

$$= 1 + 2 + 3 + \ldots + 100$$

$$= \Sigma 100 = \frac{100 \times (100+1)}{2}$$

$$= 50 \times 101 = 5050 \qquad \left\{\because \Sigma n = \frac{n(n+1)}{2}\right\}$$

18. Let $y = \cfrac{x}{x + \cfrac{\sqrt[3]{x}}{x + \sqrt[3]{x} \ldots \text{infinity}}}$

$$= \frac{x}{x + \frac{1}{x^{2/3}} \cdot \frac{x}{x + \sqrt[3]{x} \ldots \infty}} = \frac{x}{x + \frac{y}{x^{2/3}}}$$

$$\Rightarrow \quad y = \frac{x^{5/3}}{x^{5/3} + y}$$

$$\Rightarrow \quad y^2 + (x^{5/3}) \cdot y - x^{5/3} = 0$$

$$\therefore \quad y = \frac{-x^{5/3} \pm \sqrt{x^{10/3} + 4x^{5/3}}}{2}$$

$$= \frac{-x^{5/3} + \sqrt{x^{10/3} + 4x^{5/3}}}{2} \qquad (\because y > 0)$$

$$= \frac{4x^{5/3}}{2\left(\sqrt{x^{10/3} + 4x^{5/3}} + x^{5/3}\right)} = \frac{2}{\sqrt{\left(1 + \frac{4}{x^{5/3}}\right)} + 1}$$

$$\therefore \quad \lim_{x \to \infty} y = \frac{2}{\sqrt{1 + 0} + 1}$$

$$= \frac{2}{2} = 1.$$

19. We have; $f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - x} = 1, & \text{if } x < 0 \text{ or } x > 1 \\ -\dfrac{(x^2 - x)}{(x^2 - x)} = -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } x = 0 \\ -1, & \text{if } x = 1 \end{cases}$

$= \begin{cases} 1, & \text{if } \; x \le 0 \text{ or } x > 1 \\ -1, & \text{if } \; 0 < x \le 1 \end{cases}$.

Now, $\lim_{x \to 0^-} 1 = 1$ and $\lim_{x \to 0^+} f(x) = \lim_{x \to 0} -1 = -1$

Clearly, $\lim_{x \to 0^-} f(x) \ne \lim_{x \to 0^+} f(x)$.

So, $f(x)$ is not continuous at $x = 0$. It can be easily seen that it is not continuous at $x = 1$ also.

20. Given; $f(x)$ is continuous at $x = 1$

$\therefore \; f(1) = \text{RHL}$

$\Rightarrow \; f(1) = \lim_{x \to 1^+} f(x) \Rightarrow f(x) = \lim_{h \to 0} f(1 + h)$

$\Rightarrow \; a - b = \lim_{h \to 0} 3(1 + h) \Rightarrow a - b = 3 \qquad \ldots(\text{i})$

Again, given $f(x)$ is discontinuous at $x = 2$.

$\therefore \;$ LHL $\ne f(x)$

$\Rightarrow \; \lim_{x \to 2^-} f(x) \ne f(2) \Rightarrow \lim_{h \to 0} f(2 - h) \ne f(2)$

$\Rightarrow \; \lim_{h \to 0} 3(2 - h) \ne 4b - a \Rightarrow 6 \ne 4b - a \qquad \ldots(\text{ii})$

Assume, $6 = 4b - a$

then from (i) and (ii), we get $b = 3$.

$\therefore \;$ locus $\quad y = 3$

Which is impossible $\qquad (\because \; 6 \ne 4b - a)$

Hence, locus of (a, b) is $x - y = 3$ excluding the point when it cuts the line $y = 3$.

21. We are given that,

$(\cot\alpha_1) \cdot (\cot\alpha_2) \ldots (\cot\alpha_n) = 1$

$\Rightarrow \; (\cos\alpha_1 \cdot (\cos\alpha_2) \ldots (\cos\alpha_n) = (\sin\alpha_1) \cdot (\sin\alpha_2) \ldots (\sin\alpha_n) \qquad \ldots(\text{i})$

Let $y = (\cos\alpha_1) \cdot (\cos\alpha_2) \ldots (\cos\alpha_n)$ (to be max.)

Squarring both sides, we get

$y^2 = (\cos^2\alpha_1) \cdot (\cos^2\alpha_2) \ldots (\cos^2\alpha_n)$

$= \cos\alpha_1 \cdot \sin\alpha_1 \cdot \cos\alpha_2 \cdot \sin\alpha_2 \ldots \cos\alpha_n \cdot \sin\alpha_n$ [using (i)]

$= \dfrac{1}{2^n} [\sin 2\alpha_1 \cdot \sin 2\alpha_2 \ldots \sin 2\alpha_n]$

As $\quad 0 \le \alpha_1, \alpha_2, \ldots, \alpha_n \le \dfrac{\pi}{2}$

$\therefore \quad 0 \le 2\alpha_1, 2\alpha_2, \ldots, 2\alpha_n \le \pi$

$\Rightarrow \; 0 \le \sin 2\alpha_1, \sin 2\alpha_2, \ldots, \sin 2\alpha_n \le 1$

$\therefore \; y^2 \le \dfrac{1}{2^n} \cdot 1 \qquad \Rightarrow y \le \dfrac{1}{2^{n/2}}$

$\therefore \;$ Maximum value of y is $\dfrac{1}{2^{n/2}}$.

22. For $\alpha = -\dfrac{\pi}{2}, \beta = -\dfrac{\pi}{2}$ and $r = 2\pi$

$\sin\alpha + \sin\beta + \sin\gamma = -2$

$\Rightarrow$ minimum value of the expression is negative

23. $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = |\mathbf{a}^2| + |\mathbf{b}^2| + 2\mathbf{a} \cdot \mathbf{b}$

$= 1 + 1 + 2\,(1)\,(1) \cos\theta = 2 + 2\cos\theta$

$\Rightarrow \quad |\mathbf{a} + \mathbf{b}|^2 = 2 \cdot 2\cos^2 \dfrac{\theta}{2}$

$\Rightarrow \quad \cos\dfrac{\theta}{2} = \dfrac{1}{2} |\mathbf{a} + \mathbf{b}|$

24. Let $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$

then, $\mathbf{a} \cdot \mathbf{i} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot \mathbf{i} = x$

and $\; \mathbf{a} \cdot (\mathbf{i} + \mathbf{j}) = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j}) = x + y$

and $\; \mathbf{a} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k})$

$= x + y + z$

Given that $\quad \mathbf{a} \cdot \mathbf{i} = \mathbf{a} \cdot (\mathbf{j} + \mathbf{i}) = \mathbf{a} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k})$

$\because \quad x = x + y = x + y + z$

Now, $\quad x = x + y \qquad \Rightarrow y = 0$

and $x + y = x + y + z \qquad \Rightarrow z = 0$

Hence; $\quad x = 1$

$\therefore \quad \mathbf{a} = \mathbf{i}$

25. Projection of $x\mathbf{i} - \mathbf{j} + \mathbf{k}$ on $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$

$= \dfrac{(x\mathbf{i} - \mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})}{\sqrt{4 + 1 + 25}} = \dfrac{2x + 1 + 5}{\sqrt{30}}$

But, given, $\dfrac{2x + 6}{\sqrt{30}} = \dfrac{1}{\sqrt{30}}$

$\Rightarrow \quad 2x + 6 = 1$

$\Rightarrow \quad x = \dfrac{-5}{2}$

26. Since, $\mathbf{a} \times \mathbf{b} = \mathbf{c}$

$\therefore \quad (\mathbf{b} \times \mathbf{c}) \times \mathbf{b} = \mathbf{c}$

$\Rightarrow \quad (\mathbf{b} \cdot \mathbf{b})\,\mathbf{c} - (\mathbf{b} \cdot \mathbf{c})\,\mathbf{b} = \mathbf{c}$

$\Rightarrow \quad (\mathbf{b} \cdot \mathbf{b}) = 1, \quad \mathbf{c} \cdot \mathbf{b} = 0$

$\Rightarrow \quad b^2 = 1 \;$ *i.e.*, $\; b = 1$

$\therefore \quad \mathbf{b}$ is a unit vector

$\therefore |\mathbf{c}| = |\mathbf{a}| \qquad \Rightarrow \; c = a.$

27. Let $\mathbf{a} = t\,(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + s(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$

$= (t + 3s)\,\mathbf{i} + (t + 2s)\,\mathbf{j} + (2t + s)\,\mathbf{k}$

given, $\mathbf{a} \cdot (\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0$

$\therefore \quad t + s + t + 2s + 2t + s = 0$

$\Rightarrow \quad 4\,(t + s) = 0$

$\Rightarrow \quad t + s = 0$

$\therefore \quad \mathbf{a} = t\,(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) - t\,(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$

$= t\,(-\mathbf{j} + \mathbf{k})$

But $|\mathbf{a}| = 1$

$\therefore \quad 1 = 2t^2$

$\Rightarrow \qquad t = \pm \dfrac{1}{\sqrt{2}}$

$\therefore \qquad a = \pm \left(\dfrac{k - j}{\sqrt{2}}\right)$

28. As, $A^2 = O$, $A^K = O \quad \forall \ K \geq 2$

Thus, $\qquad (A + I)^{50} = I + 50\,A$

$\Rightarrow \quad (A + I)^{50} - 50\,A = I$

$\therefore \qquad I = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow \ a = 1, b = 0, c = 0, d = 1$

$\therefore \ a + b + c + d = 1 + 0 + 0 + 1 = 2.$

29. $\begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix}^{-1} \begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix}^{-1}$

$= \left\{ \begin{bmatrix} z_1 & z_2 \\ -\bar{z}_2 & \bar{z}_1 \end{bmatrix} \begin{bmatrix} \bar{z}_1 & -z_2 \\ \bar{z}_2 & z_1 \end{bmatrix} \right\}^{-1}$

$= \begin{bmatrix} z_1\bar{z}_1 + z_2\bar{z}_2 & 0 \\ 0 & z_2\bar{z}_2 + z_1\bar{z}_1 \end{bmatrix}^{-1}$

$= \begin{bmatrix} |z_1|^2 + |z_2|^2 & 0 \\ 0 & |z_1|^2 + |z_2|^2 \end{bmatrix}^{-1}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$

30. We know that, in a square matrix of order n

$$|\text{adj } A| = |A|^{n-1}$$

$\Rightarrow \ |\text{adj (adj } A)| = |\text{adj } A|^{n-1} = |A|^{(n-1)^2}$

$\Rightarrow \ n^2 - 2n - 8 = 0$

$\Rightarrow \quad n = 4$ as $n = -2$ is not possible.

31. Coefficient x in $f(x)$ is equal to coefficient of x in

$$\begin{vmatrix} x & \left(1 + x - \dfrac{x^3}{3!} \ldots\right)^3 & 1 - \dfrac{x^2}{2!} \\ 1 & x - \dfrac{x^2}{2} & 2 \\ x^2 & 1 + x^2 & 0 \end{vmatrix}$$

$= \text{coefficient of } x \text{ in} \begin{vmatrix} x & 1 & 1 \\ 1 & x & 2 \\ x^2 & 1 & 0 \end{vmatrix}$

$= \text{Coefficient of } x \text{ in } [x\,(0 - 2) - 1\,(0 - 2x^2) + 1\,(1 - x^3)]$

$= -2$

32. Given system has non-trivial solution then

$$\begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0$$

$\Rightarrow \qquad \alpha_1\beta_2 = \alpha_2\beta_1$

$\Rightarrow \qquad \dfrac{\alpha_1}{\beta_1} = \dfrac{\alpha_2}{\beta_2} = \dfrac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \sqrt{\dfrac{\alpha_1\,\alpha_2}{\beta_1\,\beta_2}}$

$\Rightarrow \qquad \dfrac{-b/a}{-q/p} = \sqrt{\dfrac{c/a}{r/p}}$

$\Rightarrow \qquad \dfrac{bp}{aq} = \sqrt{\dfrac{cp}{ar}}$

$\Rightarrow \qquad b^2p^2ar = a^2q^2cp$

$\Rightarrow \qquad b^2pr = q^2ac$

33. The given circle

$$S(x, y) \equiv x^2 + y^2 - x - y - 6 = 0 \qquad \ldots\text{(i)}$$

has centre at $C \equiv \left(\dfrac{1}{2}, \dfrac{1}{2}\right)$.

According to the given conditions, the given point $P\,(\alpha - 1, \alpha + 1)$ must lie inside the given circle.

i.e., $\qquad S\,(\alpha - 1, \alpha + 1) < 0$

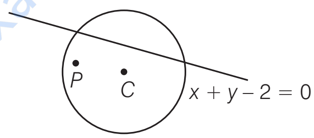
$\Rightarrow \ (\alpha - 1)^2 + (\alpha + 1)^2 - (\alpha - 1) - (\alpha + 1) - 6 < 0$

$\Rightarrow \ \alpha^2 - \alpha - 2 < 0 \ $ *i.e.,* $\ (\alpha - 2)\,(\alpha + 1) < 0$

$\Rightarrow \ -1 < \alpha < 2$

[using sign – scheme from algebra] $\qquad \ldots$(ii)

and also P and C must lie on the same side of the line.



$$L\,(x, y) = x + y - 2 = 0$$

i.e., $L\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ and $L\,(\alpha - 1, \alpha + 1)$ must have the same sign.

Now, Isince $L\left(\dfrac{1}{2}, \dfrac{1}{2}\right) = \dfrac{1}{2} + \dfrac{1}{2} - 2 < 0$

therefore, we have

$$L\,(\alpha - 1, \alpha + 1) = (\alpha - 1) + (\alpha + 1) - 2 < 0$$

$\Rightarrow \qquad \alpha < 1 \qquad \ldots$(iv)

Ineqalities (ii) and (iv) together give the permissible values of α as $-1 < \alpha < 1$.

34. The two circles are

$$x^2 + y^2 - 2ax + c^2 = 0$$

and $\qquad x^2 + y^2 - 2by + c^2 = 0$

centres : $C_1\,(a, 0) \qquad C_2\,(0, b)$

radii : $\ r_1 = \sqrt{a^2 - c^2}, \quad r_2 = \sqrt{b^2 - c^2}$

Since, the two circles touch each other externally, therefore

$$C_1C_2 = r_1 + r_2$$

$$\Rightarrow \quad \sqrt{a^2+b^2} = \sqrt{a^2-c^2} + \sqrt{b^2-c^2}$$

$$\Rightarrow \quad a^2 + b^2 = a^2 - c^2 + b^2 - c^2 + 2\sqrt{a^2-c^2}\sqrt{b^2-c^2}$$

$$\Rightarrow \quad c^4 = a^2b^2 - c^2(a^2+b^2) + c^4$$

$$\Rightarrow \quad a^2b^2 = c^2(a^2+b^2)$$

$$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$$

35. Let the coordinates of P be (h, k). Let the equation of a tangent from $P(h, K)$ to the circle $x^2 + y^2 = a^2$ be $y = mx + a\sqrt{1+m^2}$

Since, $P(h, K)$ lies on $y = mx + a\sqrt{1+m^2}$ therefore, $k = mh + a\sqrt{1+m^2} = (k-mh)^2 = a^2(1+m^2)$ this is a quadratic equation in m. Let the two roots be m_1 and m_2, then

$$m_1 + m_2 = \frac{2hK}{K^2-a^2}$$

But $\tan\alpha = m_1$, $\tan\beta = m_2$ and it is given that

$$\cot\alpha + \cot\beta = 0$$

$$\therefore \quad \frac{1}{m_1} + \frac{1}{m_2} = 0$$

$$\Rightarrow \quad m_1 + m_2 = 0$$

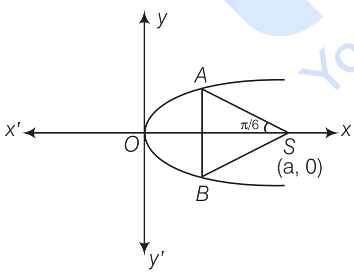
$$\Rightarrow \quad \frac{2hK}{K^2-a^2} = 0$$

$$\Rightarrow \quad hK = 0$$

Hence, the locus of (h, K) is $xy = 0$.

36. Let $A(at_1^2, 2at_2)$, $B(at_2^2, -2at_2)$.

We have,



$$m_{AS} = \tan\left(\frac{5\pi}{6}\right)$$

$$\Rightarrow \quad \frac{2at_1}{at_1^2 - a} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad t_1^2 + 2\sqrt{3}\,t_1 - 1 = 0 \quad \Rightarrow t_1 = -\sqrt{3} \pm 2$$

Clearly, $t_1 = -\sqrt{3} - 2$ is rejected.

thus, $t_1 = (2 - \sqrt{3})$,

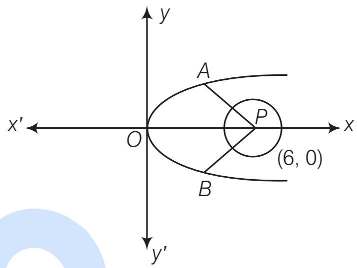
Hence, $AB = 4at_1 = 4a(2-\sqrt{3})$.

37. Centre and radius of the given circle is $P(6, 0)$ and $\sqrt{5}$, respectively.

Now minimum distance between two curves always occurs along a line which normal to both the curves.

Equation of normal to $y^2 = 4x$ at $(t^2, 2t)$ is

$$y = -tx + 2t + t^3.$$



If it is normal to circle also, then it must pass though $(6, 0)$.

$\therefore \quad 0 = t^3 - 4t \quad \Rightarrow \quad t = 0 \quad$ or $\quad t = \pm 2$.

$\Rightarrow \quad A(4, 4)$ and $C(4, -4)$.

$\Rightarrow \quad PA = PC = \sqrt{20} = 2\sqrt{5}$.

$\Rightarrow$ Required minimum distance $= 2\sqrt{5} - \sqrt{5} = \sqrt{5}$.

38. Let the points of intersection of the line and the ellipse be $(a\cos\theta, b\sin\theta)$ and $\left\{a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right\}$, since they lie on the given line $\quad lx + my + n = 0$.

$$la\cos\theta + mb\sin\theta + n = 0$$

$$\Rightarrow \quad la\cos\theta + mb\sin\theta = -n$$

and $$la\sin\theta + mb\cos\theta + n = 0$$

$$la\sin\theta - mb\cos\theta = n$$

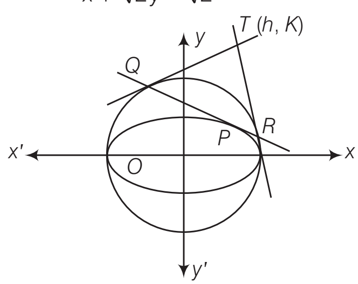
squarring and adding, we get,

$$a^2l^2 + b^2m^2 = 2n^2 \quad \Rightarrow \frac{a^2l^2 + b^2m^2}{n^2} = 2.$$

39. Equation of tangent to ellipse at given point is

$$x\left(\frac{1}{\sqrt{2}}\right) + 2y\left(\frac{1}{2}\right) = 1.$$

$$\Rightarrow \quad x + \sqrt{2}y = \sqrt{2} \qquad \ldots\text{(i)}$$



Now, QR is chord of contact of circle $x^2 + y^2 = 1$ with respect to point $T\ (h, K)$.

then, $\quad QR \equiv hx + Ky = 1 \quad \ldots$(ii)

Equations (i) and (ii) represent same straight line, then comparing ratio of coefficients we have

$$\frac{h}{1} = \frac{K}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Hence; $(h, K) \equiv \left(\frac{1}{\sqrt{2}}, 1\right)$

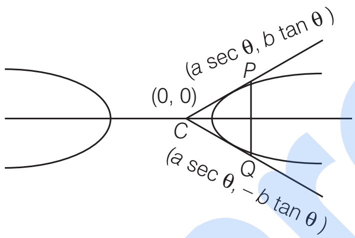
40. $e^2 = 1 + \frac{b^2}{a^2} = 1 + \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$

and $\quad a^2 = \cos^2\alpha$

$\therefore \quad a^2e^2 = 1$

$\therefore$ *foci* $(\pm\, ae, 0) = (\pm\, 1, 0)$, which is independent of α.

41. Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and any double ordinate PQ be $(a \sec\theta, b\tan\theta)$, $(a\sec\theta, -\,b\tan\theta)$ and O is centre $(0, 0)$.



ΔOPQ being equilateral.

$\therefore \quad \tan 30° = \frac{b\tan\theta}{a\sec\theta}$

$\Rightarrow \quad 3 \cdot \frac{b^2}{a^2} = \text{cosec}^2\,\theta$

$\Rightarrow \quad 3\,(e^2 - 1) = \text{cosec}^2\theta$

Now, $\quad \text{cosec}^2\theta \geq 1$

$\therefore \quad 3\,(e^2 - 1) \geq 1 \Rightarrow e^2 \geq \frac{4}{3}$

$\Rightarrow \quad e > 2/\sqrt{3}$

42. $iz^4 = -\,1$

$\Rightarrow \quad z^4 = -\frac{1}{i}$

$\Rightarrow \quad z^4 = i$

$\Rightarrow \quad z = (i)^{1/4} = (0 + i)^{1/4}$

$\Rightarrow \quad z = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/4}$

$\Rightarrow \quad z = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$

(using De Moirrels theorem)

43. Let $a = \cos\alpha + i\sin\alpha$,

$b = \cos\beta + i\sin\beta$,

$c = \cos\gamma + i\sin\gamma$

then; $a + b + c = (\cos\alpha + \cos\beta + \cos\gamma) + i\,(\sin\alpha + \sin\beta + \sin\gamma)$

$= 0 + i0 = 0$

$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \quad (\cos 3\alpha + i\sin 3\alpha) + (\cos 3\beta + i\sin 3\beta) + (\cos 3\gamma + i\sin 3\gamma)$

$= 3\,[\cos(\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)]$

$\Rightarrow \quad \cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos\,(\alpha + \beta + \gamma)$

44. $z_1^2 + z_2^2 + 2z_1z_2\cos\theta = 0$

$\Rightarrow \quad \left(\frac{z_1}{z_2}\right)^2 + 2\left(\frac{z_1}{z_2}\right)\cos\theta + 1 = 0$

$\Rightarrow \quad \left(\frac{z_1}{z_2} + \cos\theta\right)^2 = -\,(1 - \cos^2\theta) = -\,\sin^2\theta$

$\Rightarrow \quad \frac{z_1}{z_2} = -\cos\theta \pm i\sin\theta$

$\Rightarrow \quad \left|\frac{z_1}{z_2}\right| = \sqrt{(-\cos\theta)^2 + (\sin\theta)^2} = 1$

$\Rightarrow \quad |z_1| = |z_2|$

$\Rightarrow \quad |z_1 - 0| = |z_2 - 0|$

Thus, triangle with vertices 0, z_1, z_2 is iscosceles.

45. We have $f(xy) = f(x)\,f(y)$ for all $x,\ y \in R$.

Putting $\ x = y = 1$, we get

$f(1) = f(1)\,f(1)$

$\Rightarrow \quad f(1)\,[1 - f(1)] = 0$

$\Rightarrow \quad f(1) = 1 \qquad [\because\ f(1) \neq 0] \quad \ldots$(i)

Now, $\ f'(1) = 2$

$\Rightarrow \quad \lim_{h \to 0} \frac{f(1 + h) - f(1)}{h} = 2$

$\Rightarrow \quad f(1)\lim_{h \to 0}\frac{f(h) - 1}{h} = 2$

$\Rightarrow \quad \lim_{h \to 0}\frac{f(1)\,f(h) - f(1)}{h} = 2 \qquad$ [using $f(1) = 1$] $\quad \ldots$(ii)

$\Rightarrow \quad \lim_{h \to 0}\frac{f(h) - 1}{h} = 2$

Now $\ f'(4) = \lim_{h \to 0}\frac{f(4 + h) - f(4)}{h}$

$= \lim_{h \to 0}\frac{f(4) \cdot f\,(h) - f(4)}{h}$

$= \left\{\lim_{h \to 0}\frac{f(h) - 1}{h}\right\} \cdot f(4) = 2f(4) \qquad$ {from (i)}

$= 2 \times 4 = 8$

46. Let $x = a\cos^2\theta + b\sin^2\theta$

$\therefore \quad a - x = a - a\cos^2\theta - b\sin^2\theta = (a-b)\sin^2\theta$

and $x - b = a\cos^2\theta - b\sin^2\theta - b = (a-b)\cos^2\theta$

$\therefore \quad y = (a-b)\sin\theta\cdot\cos\theta - (a-b)\tan^{-1}\tan\theta$

$$= \frac{a-b}{2}\sin 2\theta - (a-b)\theta$$

$$\therefore \quad \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(a-b)\cos 2\theta - (a-b)}{(b-a)\sin 2\theta}$$

$$= \frac{1-\cos 2\theta}{\sin\theta} = \tan\theta = \sqrt{\frac{a-x}{x-b}}$$

47. We have $\sin^{-1}\left(\dfrac{x^2-y^2}{x^2+y^2}\right) = \log a$

$$\Rightarrow \quad \frac{x^2-y^2}{x^2+y^2} = \sin(\log a)$$

$$\Rightarrow \quad \frac{1-\tan^2\theta}{1+\tan^2\theta} = \sin(\log a)$$

(on putting $y = x\tan\theta$.)

$\Rightarrow \quad \cos 2\theta = \sin(\log a)$

$\Rightarrow \quad 2\theta = \cos^{-1}(\sin\log a)$

$$\Rightarrow \quad \theta = \frac{1}{2}\cos^{-1}\{\sin(\log a)\}$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2}\cos^{-1}\{\sin\log a\}$$

$$\Rightarrow \quad \frac{1}{1+\dfrac{y^2}{x^2}}\cdot\frac{x\dfrac{dy}{dx} - y}{x^2} = 0$$

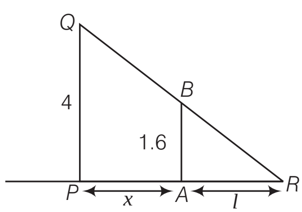
$$\Rightarrow \quad x\frac{dy}{dx} - y = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} \qquad \ldots\text{(i)}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-y}{x^2} + \frac{1}{x}\cdot\frac{dy}{dx} = \frac{-y}{x^2} + \frac{1}{x}\left(\frac{y}{x}\right); \qquad \{\text{from (i)}\}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{-y}{x^2} + \frac{y}{x^2} = 0.$$

48. Let $PQ = 4m$ be the height of pole and, $AB = 1.6m$ be height of man. Let the end of shadow is R and it is at a distance of l from A when the man is at a distance x from PQ at some instant.



Since, ΔPQR and ΔABR are similar,

we have, $\dfrac{PQ}{AB} = \dfrac{PR}{AR}$

$$\Rightarrow \quad \frac{4}{1.6} = \frac{x+l}{l}$$

$\Rightarrow \quad 2x = 3l$

$$\Rightarrow \quad \frac{2\,dx}{dt} = 3\cdot\frac{dl}{dt} \qquad \left\{\text{given } \frac{dx}{dt} = 30 \text{ m/min}\right\}$$

$$\Rightarrow \quad \frac{dl}{dt} = \frac{2}{3}\times 30 \text{ m/min} = 20 \text{ m/min.}$$

49. $\dfrac{dy}{dx} = Pe^{px} + P$

$$\left(\frac{dy}{dx}\right)_{(0,\,1)} = 2P$$

Subtangent $= \left|y\dfrac{dx}{dy}\right|$, subnormal $= \left|y\dfrac{dy}{dx}\right|$

Given; Subtangent = Subnormal

$$\Rightarrow \quad \frac{dy}{dx} = \pm 1 \;\Rightarrow 2p = \pm 1 \quad \Rightarrow \quad p = \pm\frac{1}{2}.$$

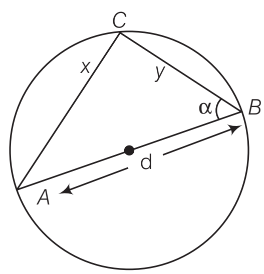
50. Area of ΔABC, $A = \dfrac{1}{2}x\sqrt{d^2-x^2}$

for max/min $\dfrac{dA}{dx} = 0$

$$\Rightarrow \quad \frac{1}{2}\sqrt{d^2-x^2} + \frac{1}{2}\times\left\{\frac{-2x}{2\sqrt{d^2-x^2}}\right\} = 0$$

$$\Rightarrow \quad \frac{d^2-x^2-x^2}{2\sqrt{d^2-x^2}} = 0$$

$$\Rightarrow \quad x = \frac{d}{\sqrt{2}}$$



Also for $\quad x < \dfrac{d}{\sqrt{2}}, \dfrac{dA}{dx} > 0$

and for $\quad x > \dfrac{d}{\sqrt{2}}, \dfrac{dA}{dx} < 0.$

Hence; $x = \dfrac{d}{\sqrt{2}}$ is the point of maxima.

$\therefore$ A is max at $x = \dfrac{d}{\sqrt{2}}$, the area is maximum $y = \dfrac{d}{\sqrt{2}}$

$\therefore$ when Δ is isosceles,

51. Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right) = \alpha$. then $\cos\alpha = \frac{\sqrt{5}}{3}$, where $0 < \alpha < \frac{\pi}{2}$.

Now, $\tan\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = \sqrt{\frac{1-\sqrt{5}/3}{1+\sqrt{5}/3}}$

$= \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}} = \frac{1}{2}(3-\sqrt{5})$

52. We know that;

if $|x| \le 1$, then $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

and if $|x| > 1$, then $\pi - 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

thus, if $|x| > 1$.

$2\tan^{-1}x + \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x + \pi$

$-2\tan^{-1}x = \pi$

which is independent of x.

53. $1 + \sin x \cdot \sin^2\frac{x}{2} = 0$

$\Rightarrow \quad 2 + 2\sin x \cdot \sin^2\frac{x}{2} = 0$

$\Rightarrow \quad 2 + \sin x(1-\cos x) = 0$

$\Rightarrow \quad 4 + 2\sin x(1-\cos x) = 0$

$\Rightarrow \quad 4 + 2\sin x - \sin 2x = 0$

$\Rightarrow \quad \sin 2x = 2\sin x + 4$

Above is not possible for any value of x as LHS has maximum value 1 and RHS has minimum value 2.

Hence, there is no solution.

54. $\sin x \cdot \cos y = 1$

$\Rightarrow \quad \sin x = 1, \cos y = 1$

or $\quad \sin x = -1, \cos y = -1$.

if $\quad \sin x = 1, \cos y = 1 \quad \Rightarrow x = \frac{\pi}{2}, y = 0, 2\pi$

if $\quad \sin x = -1, \cos y = -1 \quad \Rightarrow x = \frac{3\pi}{2}, y = \pi$

Thus possible ordered pairs are

$\left(\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, 2\pi\right), \left(\frac{3\pi}{2}, \pi\right)$.

55. Let $I = \int x^x(\log ex)\,dx$

$= \int x^x(1+\log x)\,dx$

let $t = x^x = e^{x\log x}$

$\Rightarrow \quad \frac{dt}{dx} = x^x\left\{x \cdot \frac{1}{x} + \log x\right\}$

$\Rightarrow \quad dt = x^x(1+\log x)\,dx$

$\therefore \quad I = \int t + c = x^x + c.$

56. Let $I = \int\sqrt{1+\operatorname{cosec} x}\,dx = \int\frac{\sqrt{1+\sin x}}{\sqrt{\sin x}}\,dx$

$= \int \pm \frac{\sin\frac{x}{2} + \cos\frac{x}{2}}{\sqrt{2\sin\frac{x}{2}\cdot\cos\frac{x}{2}}}\cdot dx$

$= \pm\int\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sqrt{1-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}\cdot dx$

Put, $\quad \sin\frac{x}{2} - \cos\frac{x}{2} = t$

$\Rightarrow \quad \left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)dx = 2dt$

$\therefore \quad I = \pm\int\frac{2\,dt}{\sqrt{1-t^2}} = \pm 2\sin^{-1}t + c$

$= \pm 2\sin^{-1}\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + c$

57. We have;

$\int\frac{dx}{x^2(x^n+1)^{\frac{(n-1)}{n}}} = \int\frac{dx}{x^2\cdot x^{n-1}\left(1+\frac{1}{x^n}\right)^{\frac{(n-1)}{n}}}$

$= \int\frac{dx}{x^{n+1}(1+x^{-n})^{\frac{(n-1)}{n}}}$

Put, $\quad 1 + x^{-n} = t$

$\therefore \quad -nx^{-n-1}dx = dt$

$\Rightarrow \quad \frac{dx}{x^{n+1}} = \frac{-dt}{n}$

$\Rightarrow \quad \int\frac{dx}{x^2(x^n+1)^{\frac{(n-1)}{n}}} = -\frac{1}{n}\int\frac{dt}{t^{\frac{(n-1)}{n}}}$

$= -\frac{1}{n}\int t^{(1/n)-1}\,dt = -\frac{1}{n}\cdot\frac{t^{\left(\frac{1}{n}-1+1\right)}}{\left(\frac{1}{n}-1+1\right)} + c$

$= -t^{1/n} + c = -(1+x^{-n})^{\frac{1}{n}} + c$

58. The given limit

$L = \lim_{n\to\infty}\left\{\frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + \ldots + \frac{1}{na+n(b-a)}\right\}$

$$= \lim_{n \to \infty} \sum_{r=0}^{n(b-a)} \frac{1}{na + r}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{r=0}^{(b-a)n} \frac{1}{a + r/n}$$

$$= \int_0^{(b-a)} \frac{dx}{a + x} = [\log (a + x)]_0^{b-a} \qquad \left(\frac{r}{n} = x\right)$$

$$= \log b - \log a$$

$$= \log \left(\frac{b}{a}\right).$$

59. Case I. If $0 \le a < b$, then $\frac{|x|}{x} = 1$

$\therefore \qquad I = \int_a^b 1 \cdot dx = b - a = |b| - |a|$

Case II : If $a < b \le 0$, then $|x| = -x$

$\therefore \qquad I = \int_a^b \frac{-x}{x} dx = \int_a^b (-1)\, dx = [-x]_a^b$

$$= -b - (-a) = |b| - |a|$$

Case III If $a < 0 < b$.

then $|x| = -x$ when $a < x < 0$

and $|x| = x$ when $0 < x < b$

$$I = \int_a^b \frac{|x|}{x} dx = \int_a^0 \frac{|x|}{x} dx + \int_0^b \frac{|x|}{x} dx$$

$$= \int_a^0 \frac{-x}{x} dx + \int_0^b \frac{x}{x} dx$$

$$= \int_a^0 (-1)\, dx + \int_0^b (1)\, dx$$

$$= [-x]_a^0 + [x]_0^b$$

$$= a + b = b - (-a)$$

$$= |b| - |a|$$

Hence, in all cases,

$$I = \int_a^b \frac{|x|}{x} dx = |b| - |a|$$

60. Here, we have to prove that,

y = constant or derivative of y with respect to x is zero.

$$y = \int_{1/8}^{\sin^2 x} \sin^{-1} \sqrt{t}\, dt + \int_{1/8}^{\cos^2 x} \cos^{-1} \sqrt{t}\, dt \quad \ldots \text{(i)}$$

$$\frac{dy}{dx} = \sin^{-1} \sqrt{\sin^2 x} \cdot 2 \sin x \cdot \cos x + \cos^{-1} \sqrt{\cos^2 x}$$

$$(-2 \sin x \cdot \cos x) = 2x \sin x \cdot \cos x - 2x \cdot \sin x \cdot \cos x$$

$= 0$ for all x.

61. $x^2 + y^2 = 1$, $x + y = 1$ meet when

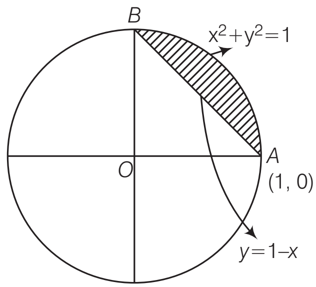
$$x^2 + (1 - x)^2 = 1$$

$\Rightarrow \qquad x^2 + 1 + x^2 - 2x = 1$

$\Rightarrow \qquad 2x^2 - 2x = 0$

$\Rightarrow \qquad 2x(x - 1) = 0$

$\Rightarrow \quad x = 0, x = 1$



$\Rightarrow$ Meet at points A (1, 0), B (0, 1)

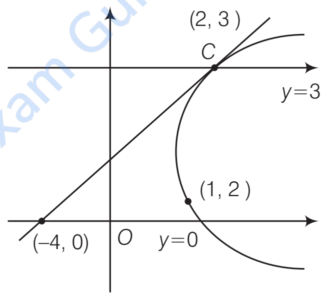
$\therefore$ Required area $= \frac{\pi}{4} - \frac{1}{2} (1) \cdot (1)$

$= \frac{\pi}{4} - \frac{1}{2}$ i.e., [quad. $OAB - \Delta OAB$].

62. Given parabola, is

$$(y - 2)^2 = x - 1$$

$\Rightarrow \qquad \frac{dy}{dx} = \frac{1}{2(y - 2)}$



when, $y = 3$, $x = 2$

$\therefore \qquad \frac{dy}{dx} = \frac{1}{2(3 - 2)} = \frac{1}{2}$

Tangent at $(x, 3)$ is

$$y - 3 = \frac{1}{2}(x - 2)$$

$\Rightarrow \qquad x - 2y + 4 = 0.$

$\therefore$ Required area

$$\int_0^3 \{(y - 2)^2 + 1\}\, dy - \int_0^3 (2y - 4)\, dy$$

$$= \left[\frac{(y - 2)^3}{3} + y\right]_0^3 - [y^2 - 4y]_0^3$$

$$= \frac{1}{3} + 3 + \frac{8}{3} - (9 - 12) = 9.$$

63. Clearly, the given differential equation is not a polynomial indifferential coefficients. So, its degree is not defined.

64. We have, $\frac{dy}{dx} = \frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$

Putting $y = vx$, so that

$$\frac{dy}{dx} = v + x\frac{dv}{dx},$$

$$\therefore \quad v + x\frac{dv}{dx} = v - \cos^2 v$$

$$\Rightarrow \quad \frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

$$\Rightarrow \quad \sec^2 v\, dv = -\frac{1}{x} dx$$

on integration, we get

$$\tan v = -\log x + \log c$$

$$\Rightarrow \tan\left(\frac{y}{x}\right) = -\log x + \log c$$

this passes through $(1, \pi/4)$. therefore,

$$1 = \log c$$

So; $\quad \tan\left(\frac{y}{x}\right) = -\log x + 1$

$$\Rightarrow \quad \tan\left(\frac{y}{x}\right) = -\log x + \log c$$

$$\Rightarrow \quad y = x\tan^{-1}\left(\log\frac{c}{x}\right).$$

65. $(y\cos y + \sin y)\,dy = (2x\log x + x)\,dx$

$$y\sin y - \int \sin y\, dy + \int \sin y\, dx$$

$$= x^2\log x - \int x^2 \cdot \frac{1}{x}dx + \int x\,dx + c$$

$$\therefore \quad y\sin y = x^2 \log x + c$$

66. We have length of the

normal = radius vector

$$\Rightarrow \quad y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad y^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\} = x^2 + y^2$$

$$\Rightarrow \quad y^2\left(\frac{dy}{dx}\right)^2 = x^2$$

$$\Rightarrow \quad x = \pm\, y\frac{dy}{dx}$$

$$\Rightarrow \quad x = y\frac{dy}{dx} \qquad \text{or} \quad x = -\,y\frac{dy}{dx}$$

$$\Rightarrow \quad x\,dx - y\,dy = 0$$

or $\quad x dx + y dy = 0$

$\Rightarrow x^2 - y^2 = c_1 \quad$ or $\quad x^2 + y^2 = c_2$

Clearly, $x^2 - y^2 = c_1$ represents a rectangular hyperbola and $x^2 + y^2 = c_2$ represents circle.

67. We have,

$$\frac{1}{x(x+1)(x+2)\ldots(x+n)} = \frac{A_0}{x} + \frac{A_1}{x+1} + \ldots + \frac{A_r}{x+r} + \ldots + \frac{A_n}{x+n}$$

$\Rightarrow 1 = A_0\,(x+1)(x+2)\ldots(x+n)$
$+ \ldots + A_r\; x(x+1)(x+2)\ldots(x+r-1)(x+r+1)$
$\ldots(x+n) + \ldots + A_n x(x+1)\ldots(x+n-1)$

Putting $x = -r$, we get;

$1 = A_r\,(-r)(-r+1)(-r+2)\ldots(-3)(-2)(-1)(1)(2)$
$\ldots(n-r)$

$\Rightarrow 1 = A_r\,(-1)^r\,\{r(r-1)(r-2)\ldots(3\cdot2\cdot1\}$
$(1\cdot2\cdot3\ldots(n-r))$

$$\Rightarrow \quad A_r = (-1)^r / r!\,(n-r)!$$

68. $\log_2(9-2^x) = 10^{\log_{10}(3-x)}$

$\Rightarrow \quad \log_2(9-2^x) = (3-x) \qquad [\because\; a^{\log_a b} = b]$

$$\Rightarrow \quad 2^{3-x} = 9 - 2^x$$

$$\Rightarrow \quad \frac{2^3}{2^x} = 9 - 2^x$$

$$\Rightarrow \quad 8 = 2^x \times (9 - 2^x)$$

$\Rightarrow 2^{2x} - 2^x \times 9 + 8 = 0$

Let $2^x = y$, then;

$$y^2 - 9y + 8 = 0$$

$$\Rightarrow \quad (y-8)(y-1) = 0$$

$\Rightarrow \quad y = 8 \quad$ or $\quad y = 1.$

$\Rightarrow 2^x = 2^3 \quad$ or $2^x = 2^0$

$\Rightarrow \quad x = 3 \quad$ or $\quad x = 0.$

But $x = 3$ does not satisfy the given equation, since log 0 is not defined.

69. a, b, c are in GP $\Rightarrow b^2 = ac$

taking log on both the sides, we get

$$2\log b = \log a + \log c$$

$$\Rightarrow \quad \log b = \frac{\log a + \log c}{2}$$

$\Rightarrow \log a, \log b, \log c$ are in AP.

70. We have; $\Delta = \frac{\sqrt{3}}{4}a^2,\; s = \frac{3a}{2}$

$$\therefore \quad r = \frac{\Delta}{S} = \frac{a}{2\sqrt{3}},\; R = \frac{abc}{4\Delta}$$

$$= \frac{a^3}{\sqrt{3}\cdot a^2} = \frac{a}{\sqrt{3}}$$

and $\quad r_1 = \frac{\Delta}{S-a} = \frac{\sqrt{3}/4 \cdot a^2}{a/2} = \frac{\sqrt{3}}{2}\cdot a$

Hence; $\; r : R : r_1 = \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}}{2}\cdot a = 1 : 2 : 3$

71. Since the angles are in AP, therefore

$2B = A + C \quad \Rightarrow \quad 3B = A + B + C$

$\Rightarrow \quad 3B = 180° \quad \Rightarrow \quad B = 60°$

using the formula,

$$\frac{\sin A}{a} = \frac{\sin B}{b}, \text{ we get}$$

$$\sin A = \frac{a}{b}\sin B = \frac{24}{22}\cdot \sin 60° = \frac{6\sqrt{3}}{11}$$

Now; $\sin C = \sin(180° - (A+B)) = \sin(A+B)$

$$= \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$= \left(\frac{6\sqrt{3}}{11}\right)\left(\frac{1}{2}\right) + \sqrt{\left\{1 - \left(\frac{6\sqrt{3}}{11}\right)^2\right\}} \cdot \left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{22}\left(\frac{\sqrt{3}}{2}\right)(12 + 2\sqrt{3})$$

Now, $c = \dfrac{b}{\sin B}\sin C$

$\Rightarrow \quad c = 12 + 2\sqrt{3}.$

72. $2^{-1} * x * 3^{-1} = 5$

$\Rightarrow \quad x = 2 * (5 * 3) = 2 * (5 + 3 + 15)$

$= 2 * (23) = 2 + 23 + 46 = 71$

73. $b^{-1} * a^{-1} * b * a = (b^{-1} * a^{-1}) * (b * a) = (a * b)^{-1}$

$* (b * a) = (a * b)^{-1} * (a * b) = e.$

74. $(a * b)^2 = (a * a) * (b * b)$ for all $a, b \in G$

$\Rightarrow \quad (a * b) * (a * b) = (a * a) * (b * b)$ for all $a, b \in G$

$a * (b * a) * b = a * ((a * b) * b$

for all $a, b \in G$

$\Rightarrow \quad b * a = a * b$ for all $a, b \in G$

{by cancellation laws}

$\Rightarrow \quad G$ is abelian.

77. By hit and trial method taking $x = y = z = 2$,

we get $xy + yz + zx = 12$

and $xyz = 8.$

78. Since, c is a prime number so it can have only two factors 1 and c. $ab = c$ gives two more factors of c as a and b, which is not possible.

79. $(p \wedge \sim q)(\sim p \wedge q) = (p \wedge \sim p) \wedge (\sim q \wedge q)$

$= f \wedge f = f$

(By using associative laws and commutative laws.)

$\therefore \quad (p \wedge \sim q) \wedge (\sim p \wedge q)$ is a contradiction.

80. $\sim p \wedge q = \sim (q \rightarrow p)$

English

6. **Radiant** means giving a warm bright light.

7. **Prune** means to make something smaller by removing parts.

8. **Dilettante** means a person, who does something, but is not serious about it.

9. **Foster** means to encourage something to develop.

10. **Enigma** means a person or thing that is mysterious or difficult to understand.

11. **Conform** means to agree with or match something.

12. **Aboriginal** means relating to the original people, animals etc.

13. **Amenable** means easy to control.

14. **Acquit** means to state officially that somebody is not guilty of a crime.

15. **Forbidden** means not allowed.

31. Remove 'do not'.

32. Remove 'about'.

33. Use 'but' in place of 'and'.

34. Use 'that' in place of 'because'.

35. Use 'were' in place of 'was'.

36. The correct spelling is 'Honorary'.

37. The correct spelling is 'Maintenance'.

38. The correct spelling is 'Vehicle'.

39. The correct spelling is 'Daffodil'.

40. The correct spelling is 'Curable'.