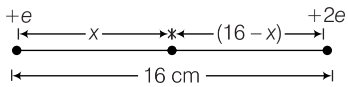
MET 2016 Answer Key PDF

Physics

1. 

For the system to be in equilibrium, we have force on charge q due to + e = force on charge q due to + 2e

i.e. $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e.q}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2e\cdot q}{(16-x)^2}$$

$\Rightarrow \quad 2x^2 = (16-x)^2$

$\Rightarrow \quad 2x^2 = 256 + x^2 - 32x$

$[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow x^2 + 32x - 256 = 0 \ldots$(i)

On solving Eq. (i), we get

$x = 6.63$ cm

2. We know that, $E_n \propto \frac{1}{n^2}$.

As n increases $(E_n - E_{n+1})$ decreases.

3. Ionisation energy of nth state $= -E_n$

For the first excited state, $n = 2$

and energy in that state is, $\frac{13.6}{2^2}\text{eV} = \frac{13.6}{4}\text{eV}$

$= 3.4$ eV

4. Here, $I_g = \frac{I}{10}$, $G = 99\ \Omega$

$\because$ The shunt, $\quad S = \frac{I_g}{I - I_g}\cdot G$

$\therefore \quad S = \frac{I/10}{I - \frac{I}{10}} \times 99$

$\Rightarrow \quad S = 11\ \Omega$

5. From kinetic theory of gases

$$p = \frac{1}{3}\frac{M}{V}\cdot e^2$$

Since, p and V are constant (same)

$\therefore \quad M_1 e_1^2 = M_2 e_2^2$

$\Rightarrow \quad \frac{e_1^2}{e_2^2} = \frac{M_2}{M_1}$

$\Rightarrow \quad \frac{e_1}{e_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{4/1} = 2 \quad \left[\text{given}, \frac{M_1}{M_2} = \frac{1}{4}\right]$

6. Radius of the double loop, $r = \frac{R}{2}$

where, R is the radius of single loop B (centre)

$= \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{R}$.

So, magnetic field due to each coil is double of that due to single coil.

and total magnetic field

$= 2B + 2B = 4B$

7. For minima to occur,

$$2\mu t = 2n\frac{\lambda_1}{2} \quad \ldots\text{(i)}$$

and for maxima

$$2\mu t = (2n-1)\frac{\lambda_2}{2} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$(2n-1)\lambda_2 = 2n\lambda_1$

or $\quad (2n-1)7500 = (2n)6500$

[given, $\lambda_2 = 7500$ Å, $\lambda_1 = 6500$ Å]

or $\quad 1500n - 7500 = 13000n$

or $\quad 2000n = 7500$

or $\quad n = 3.75$

Substitution the value of (n) in Eq. (i), we get

$2 \times \frac{5}{3} \times t = 2 \times 3.75 \times \frac{6500}{2} \times 10^{-10} \quad \left[\because \mu = \frac{5}{3}\right]$

$\Rightarrow \quad t = 7.40 \times 10^{-7}$ cm

8. Since we know that current, $l = 2\pi R = 2\pi n \cdot r$

$\left[\because r = \frac{R}{n}\right]$

The magnetic field at the centre of the coil,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{R} \quad \ldots\text{(i)}$$

and $$B' = \left(\frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{r}\right)\cdot n \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$B' = n^2 B$

9. The frequency of oscillation of a vibrating spring-block, system is given by

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{M}} \quad \ldots\text{(i)}$$

Since, the springs are attached in series, each having spring constant k, so we have

$$\frac{1}{k_{eff}} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k}$$

$\Rightarrow \quad k_{eff} = \frac{k}{2}$

From Eq. (i), we get $\nu = \frac{1}{2\pi}\sqrt{\frac{k_{eff}}{M}} = \frac{1}{2\pi}\sqrt{\frac{k}{2M}}$.

10. It is a case of elastic collision, whole of energy of A is transferred to D.

11. Given, $d\phi = $ Wb, $M = ?$

$$dI = 3A$$

As, $\phi = MI$

$\therefore$ $d\phi = MdI$

$\therefore$ $M = \frac{d\phi}{dI}$

$$= \frac{1.2}{3} = 0.4 \text{ H}$$

12. Since, range is maximum, therefore $\theta = 45°$.

Hence, using $R = u^2 \sin 2\theta / g$.

We find $500 = \frac{u^2}{g}$, i.e. $u = [500g]^{\frac{1}{2}}$.

Distance covered along the inclined plane can be obtained using relation

$\Rightarrow$ $v^2 - v_0^2 = 2ax$

$$0 - u^2 = 2 \times (-g\sin 30°) \times x$$

$[\because a = -g \sin 30°]$

$\therefore$ $x = \frac{u^2}{g} = 500 \text{ m}$

13. Number of atoms in 2 kg of uranium

$$= \frac{6.02 \times 10^{23}}{235} \times 2000$$

$$= 5.12 \times 10^{24}$$

Therefore, energy obtained from these atoms

$$= 5.12 \times 10^{24} \times 185 \text{ MeV}$$

$$= 5.12 \times 10^{24} \times 185 \times 10^6 \text{eV}$$

Energy obtained per second

$$= \frac{5.12 \times 10^{24} \times 185 \times 10^6 \times 1.6 \times 10^{-19}}{30 \times 24 \times 60 \times 60}$$

Solving the above expression

$$= 58.47 \times 10^6 \text{ W}$$

$$\cong 58.5 \text{ MW}$$

14. Given, $C_1 = 3\mu F, C_2 = 6\mu F, V_1 = V_2 = 12$ V

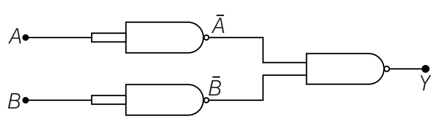
$$V = ?$$

The common potential across capacitors is given by

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$V = \frac{3 \times 10^{-6} \times 12 + 6 \times 10^{-6} \times 12}{(3+6) \times 10^{-6}} = 12 V$$

15.



Here, the output of these gate

$$Y = \overline{\bar{A} \cdot \bar{B}}$$

But according to Boolean algebra,

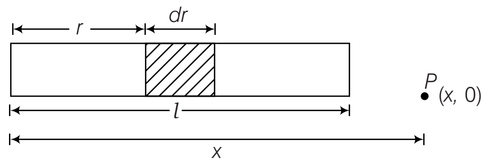
$$\bar{A} \cdot \bar{B} = \overline{A + B}$$

$\therefore$ $Y = \overline{\overline{A + B}} = A + B$

or $Y = A + B$

It means the whole circuit of gates behaves as OR gate.

16.



The potential due to a small element of length dr having charge dQ of the dielectric rod is given by

$$dV = \frac{dQ}{4\pi\varepsilon_0 r}$$

or $dV = \frac{\lambda dr}{4\pi\varepsilon_0 r}$ $\left[\because \lambda = \frac{dQ}{dr}, \lambda dr = dQ\right]$

or $V = \frac{\lambda}{4\pi\varepsilon_0}\int_x^{x-l}\frac{dr}{r}$

$$= \frac{\lambda}{4\pi\varepsilon_0}[\log_e r]_x^{x-l}$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \cdot \log\left(\frac{x-l}{x}\right)$$

$$V = \frac{Q}{4\pi\varepsilon_0 l} \cdot \log e\left(\frac{x-l}{x}\right)$$

When $\lambda = \frac{Q}{l}$ (for whole length of rod)

17. Given, $\mu = \frac{1}{\sqrt{2}}$ w.r.t. air $i = 45°$, $r = ?$

Also, $\mu = \frac{\sin i}{\sin r}$

$\Rightarrow$ $\frac{1}{\sqrt{2}} = \frac{\sin 45°}{\sin r} \Rightarrow \frac{1}{\sqrt{2}} = \frac{1/\sqrt{2}}{\sin r}$

$\Rightarrow$ $\sin r = 1$

$\Rightarrow$ $r = 90°$

Note that the ray is incident from denser medium and is refracted into a rarer medium air.

18. General equation for a plane progressive wave travelling along positive x-direction is given by

$$y = A\sin(\omega t - kx)$$

$$= A\sin\left[2\pi\left(vt - \frac{x}{\lambda}\right)\right] \quad \ldots(i)$$

Here, it is given

$$v = 360 \text{ m/s}, \lambda = 60 \text{ m}$$

We know that, frequency $\nu = \frac{v}{\lambda} = \frac{360}{60} = 6$ Hz

Substituting $A = 0.2$ m/s, $\nu = 6$ Hz

$\lambda = 60$ m in Eq. (i), we get

$$y = 0.2 \sin\left[2\pi\left(6t - \frac{x}{60}\right)\right]$$

19. The resistance in series with galvanometer does not affect the null point.
So, it will obtained at 240 cm.

20. In parallel combination, the resultant inductance L_P,

$$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$= \frac{1}{L} + \frac{1}{L} = \frac{2}{L}$$

$$\Rightarrow \quad L_P = \frac{L}{2}$$

21. When similar poles are on same side time period of oscillation T_1 is given by

$$T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 + m_2) B_H}} \quad \text{[given]}$$

$$m_1 = 2m, \; m_2 = m$$

$$T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(3m)B_H}} \quad \ldots\text{(i)}$$

When the polarity of magnet is reversed, time period of oscillation T_2 is given by

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(m_1 - m_2)B_H}}$$

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{mB_H}} \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$T_2 > T_1$$

or $$T_1 < T_2$$

22. The magnetic induction along the axis of solenoid

$$B = \mu_0 nI$$

where, n = number of turns per unit length

and I = current

$$\because \quad \frac{\mu_0}{4\pi} = 10^{-7} \text{ T / A}$$

$$\Rightarrow \quad \mu_0 = 4\pi \times 10^{-7} \text{ T/A}$$

Given, $$n = 10 \text{ cm}^{-1} = 1000 \text{ m}^{-1}$$

$$I = 0.5 \text{ A}$$

$$\therefore \quad B = 4\pi \times 10^{-7} \times 1000 \times 0.5$$

$$= 2\pi \times 10^{-4} \text{ T}$$

23. In *L-C-R* circuit, inductive reactance is given by

$$X_L = \omega L$$

$$= 2\pi fL \quad [\because \omega = 2\pi f]$$

$$\therefore \quad X_L \propto f$$

Thus, inductive reactance increases with increase frequency.

24. The de-Broglie wavelength associated with electron

$$\lambda_e = \frac{1.227}{\sqrt{V}} \text{ nm}$$

$$V = \frac{(12.27)^2}{\lambda_e^2} \times 10^{-2}$$

$$= \frac{150}{\lambda_e^2} \times 10^{-2} \Rightarrow V = \frac{150}{(0.2)^2}$$

$$= \frac{150 \times 10^{-2}}{4 \times 10^{-2}}$$

$$\therefore \quad E = V \text{ eV} = 37.5 \text{ eV}$$

25. Given, $R = 120\,\Omega$,

$$X_L = 180\ \Omega, \; X_C = 130\ \Omega$$

The impedance of circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{(120)^2 + (180 - 130)^2}$$

$$= 130\ \Omega$$

26. Neutrons, as they are neutral particles.

27. Given, $W_1 = 2W$,

Fringe width, $$W = \frac{\lambda D}{d}$$

$$\Rightarrow \quad 2W = \frac{\lambda D}{d}$$

When, $$D' = 2D$$

$$W_2 = \frac{\lambda D'}{d} \Rightarrow W_2 = \frac{\lambda 2D}{d}$$

$$= 2 \times 2W = 4W$$

28. The given force varies with distance.
Therefore, work done,

$$W = \int_0^{x=5} F \cdot dx$$

$$= \int_0^5 (7 - 2x + 3x^2)dx$$

$$= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5$$

$$= [7 \times 5 - (5)^2 + (5)^3 - 0]$$

$$= 135 \text{ J}$$

29. Conservation of momentum (gives)

$$mu + 0 = mv_1 + mv_2$$

$\Rightarrow \quad v_1 + v_2 = u \quad \ldots(i)$

Newton's experimental formula,

$$v_1 - v_2 = -e[u_1 - u_2] \text{ gives}$$

$$v_1 - v_2 = -eu \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$v_1 = \frac{(1-e)u}{2}$$

$$v_2 = \frac{(1+e)u}{2}$$

$$\therefore \quad \frac{v_1}{v_2} = \frac{1-e}{1+e}$$

30. Given, $C_p = 7.03$ cal/mol °C, $R = 8.315$/mol°C,

$n = 5$ moles, $T_2 = 20°C$, $T_1 = 10°C$,

$$\Delta Q = ?$$

$\therefore$ Heat, $\Delta Q = nC_v \, \Delta T$

$$= n(C_p - R)\Delta T \qquad [\because C_p - C_v = R]$$

$$= 5\left[7.03 - \frac{8.31}{4.2}\right] \times (20 - 10)$$

$$\simeq 253 \text{ cal}$$

31. Activity, $\left(\frac{R}{R_0}\right) = \left(\frac{1}{2}\right)^n$

$$\Rightarrow \quad \frac{1}{16} = \left(\frac{1}{2}\right)^n$$

$$\text{or} \quad \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad n = 4$$

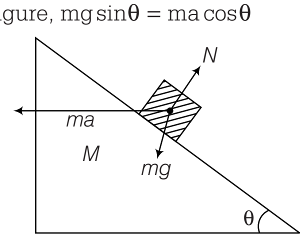
$$\Rightarrow \quad \frac{t}{T} = 4 \qquad \left[\because n = \frac{t}{T}\right]$$

$$\Rightarrow \quad t = 4T$$

$$= 4 \times 100 \ \mu s$$

$$= 400 \ \mu s$$

32. From figure, $mg\sin\theta = ma\cos\theta$



$$\Rightarrow \quad a = g\tan\theta$$

$\therefore$ Force, $\quad F = (M + m)a$

$$= (M + m)g\tan\theta$$

33. Given, $m = 1$ kg, $v = 2$ m/s, $R = 1$ m

$$W = ?$$

From energy conservation,

$$mgh + W = \frac{1}{2}mv^2$$

$$W = \frac{1}{2}mv^2 - mgh$$

$$= \frac{1}{2} \times 1(2)^2 - 1 \times 10 = -8J$$

34. Limiting friction along the plane upward

$$= \mu_s \, mg\cos\theta$$

$$= 0.8 \times 1 \times 10 \times \cos 37°$$

$$= 0.8 \times 10 \times \frac{4}{5}$$

$$= 6.4 \text{ N}$$

Force responsible for downward motion

$$= mg\sin\theta$$

$$= 1 \times 10 \times \sin 37°$$

$$= 1 \times 10 \times \frac{3}{5}$$

$$= 6N$$

As, downward force < limiting friction.

So, frictional force = downward force.

Hence, tension in string is zero.

35. Areal velocity, $\frac{dA}{dt} = \frac{L}{2m}$

where, L = angular momentum

$$\Rightarrow \quad \frac{dA}{dt} = \frac{mr^2\omega}{2m} \qquad [\because L = mr^2\omega]$$

$$\propto r^2\omega$$

36. Given, $F = 49$ N, $M_A = 7$ kg, $M_B = 3$ kg, $\mu = ?$

Equation of motion of two blocks-system

$$F_{max} = \mu(M_A + M_B)\cdot g$$

$$\mu = \frac{F}{(M_A + M_B)\cdot g}$$

$$= \frac{49}{(7+3)\cdot 9\cdot 8}$$

$$= \frac{49}{10 \times 9.8}$$

$$= 0.5$$

37. As $M_1 > M_2$, so tension in thread connecting the block B and support C is $(M_1 - M_2)g$. When this thread is burnt, this tension disappears. In this case tension in spring is $M_1 g$ and it is elongated. So, tension in string connecting A and B is M_1 g.

Hence, resultant force on A remains zero because tension in string is balanced by tension in spring. The block B has a net upward force $(M_1 - M_2)g$, so initial acceleration of block B is

$$\frac{(M_1 - M_2)g}{M_2}$$

Thus, initial acceleration of M_1 is zero and that of M_2 is $\frac{(M_1 - M_2)g}{M_2}$ upward.

38. For refraction at first surface,

$$u = -7.5 \text{ cm}, R_1 = 2.5 \text{ cm}$$

$$\mu_1 = 1, \mu_2 = \frac{4}{3}$$

$$\therefore \quad \frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

$$\frac{4/3}{v'} - \frac{1}{(-7.5)} = \frac{4/3 - 1}{2.5}$$

$$\Rightarrow \quad \frac{4}{3v'} = \frac{1}{7.5} - \frac{1}{7.5} = 0$$

$$\Rightarrow \quad v' = \infty$$

It means the ray is parallel within the sphere.
For refraction at second surface,

$$u = -\infty, \mu_1 = \frac{4}{3}$$

$$\mu_2 = 1, R_2 = -2.5 \text{ cm}$$

$$\therefore \quad \frac{\mu_2}{v'} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_2} \text{ gives}$$

$$\Rightarrow \quad \frac{1}{v'} - \frac{4/3}{(-\infty)} = \frac{1 - (4/3)}{(-2.5)}$$

$$\Rightarrow \quad v = +7.5 \text{ cm}$$

i.e. Image is formed at distance 7.5 cm to the right of P_2. Hence, distance of final image P from centre C = 7.5 + 2.5 = 10 cm.

39. Total energy, $E_0 = -\frac{GMm}{2r}$ ...(i)

and potential energy, $V = -\frac{GMm}{r}$ ...(ii)

From Eqs. (i) and (ii), we get

$$= 2E_0$$

40. Ge conducts at 0.3 V and Si conducts at 0.7 V. Both Ge and Si are connected in parallel. When current begins to flow, then the potential difference remains at 0.3 V, so no current flows through Si-diode.

$\therefore$ Potential difference across resistive load = 12 − 0.3

$$= 11.7 \text{ V}$$

$\therefore$ Potential of Y = 11.7 V

41. From question,

$$n_1 + 55 \times 6 = n_2$$

$$\Rightarrow \quad n + 55 \times 6 = 2n$$

$$\Rightarrow \quad n = 330 \text{ Hz}$$

$$\therefore \quad n_2 = 2n = 2 \times 330 = 660 \text{ Hz}$$

42. Given, $\lambda_1 = 2000 \text{ Å} = 2000 \times 10^{-10}$ m $= 2 \times 10^{-7}$ m

$$\lambda_2 = 2100 \text{ Å} = 2.1 \times 10^{-7} \text{ m}$$

$$\therefore \quad \frac{hc}{\lambda_1} = W + eV_0 \quad \text{...(i)}$$

$$\text{and} \quad \frac{hc}{\lambda_2} = W + eV_0' \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = e(V_0 - V_0')$$

Change in stopping potential

$$\Delta V = V_0 - V_0'$$

$$= \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}}\left(\frac{1}{2 \times 10^{-7}} - \frac{1}{2.1 \times 10^{-7}}\right)$$

$$= \frac{6.6 \times 3}{1.6}\left(\frac{1}{2} - \frac{1}{2.1}\right)$$

$$= \frac{6.6 \times 3 \times 0.1}{1.6 \times 2 \times 2.1} = 0.3 \text{ V}$$

43. Given, $y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$$y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$$

$$= a_2 \sin\left(\frac{\pi}{2} + \omega t - \frac{2\pi x}{\lambda} + \phi\right)$$

$\therefore$ Phase of first wave, $\phi_1 = \left(\omega t - \frac{2\pi x}{\lambda}\right)$

and phase of second wave, $\phi_2 = \frac{\pi}{2} + \omega t - \frac{2\pi x}{\lambda} + \phi$

$\therefore$ Phase difference, $\Delta\phi = \phi_2 - \phi_1 = \frac{\pi}{2} + \phi$

$\therefore$ Path difference $= \frac{\lambda}{2\pi} \times$ phase difference

$$= \frac{\lambda}{2\pi}\left(\frac{\pi}{2} + \phi\right)$$

44. When intensity of incident photon increases number of electrons emitted from the surface increases. Thus current increases. It is independent of frequency of photon.

45. From Stokes' law, $F = 6\pi\eta rv$

wehre, η = coefficient of viscosity

and $F' = 6\pi\eta(2r)(2v) = 6\pi\eta rv \times 4$

$$\because \quad \frac{F'}{F} = 4$$

$$F' = 4F$$

46. From Einstein's photoelectric equation,

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W \quad \ldots(i)$$

and $$\frac{1}{2}mv'^2 = \frac{hc}{3\lambda/4} - W \quad \ldots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\left(\frac{v'}{v}\right)^2 = \frac{\frac{4hc}{3\lambda} - W}{\frac{hc}{\lambda} - W}$$

$$\because \quad W > 0$$

$$\therefore \quad v' > v\left(\frac{4}{3}\right)^{\frac{1}{2}}$$

47. Weight of wooden block = Weight of water displaced

$$Mg = V_{in}\sigma_w g$$
$$= (40 \times 40 \times 5) \times 10^{-6} \times 10^3 \text{ g}$$

$$\Rightarrow \quad M = 8 \text{ kg}$$

48. Time, $T = \dfrac{2u\sin\theta}{g}$

$$T = \frac{2 \times 50 \times \sin 30°}{10}$$

$$= \frac{2 \times 50 \times \frac{1}{2}}{10} = 5s$$

Time spent after crossing the wall, $t_2 = 5 - 3 = 2s$

$\therefore$ Distance, $x_2 = u\cos\theta \times 1_2$

$$= 50 \times \cos 30° \times 2$$
$$= 50 \times \frac{\sqrt{3}}{2} \times 2$$
$$= 50\sqrt{3}$$
$$= 50 \times 1.73 = 86.5 \text{ m}$$

49. Lower (conceptual question).

When temperature is increased, then the density will decrease, so cube will float lower.

50. Given, $D = 40 \text{ cm} = 40 \times 10^{-2}$ m

$$\lambda = 600 \text{ Å} = 6000 \times 10^{-10} \text{ m}$$
$$W = 0.012 \text{ cm} = 0.012 \times 10^{-2} \text{ m}$$

$\therefore$ Fringe width,

$$W = \frac{D\lambda}{d}$$

$$\Rightarrow \quad d = \frac{D\lambda}{W}$$

where, d = distance between the slits

$$d = \frac{40 \times 10^{-2} \times 6000 \times 10^{-10}}{0.012 \times 10^{-2}}$$
$$= 2 \times 10^{-3} \text{m} = 2 \text{ mm}$$
$$= 0.2 \text{ cm}$$

Chemistry

1. Suppose the chemical formula of the compound be $C_xH_yO_z$ chemical reaction related to combustion of the compound.

$C_xH_yO_z$	$+ O_2$	$\longrightarrow CO_2$	$+ H_2O$	(Steam)
1 vol.	2 vol.	2 vol.	2 vol.	Given
1. mol	2 mol or 4 atoms	2 mol or 2 atom C and 4 atom O	2 mol. or 4 atom H and 2 atom O	(By Avogadro 15 hypothesis)

Since, 2 molecules of CO_2 have 2 atoms of carbon and these carbon must have come from the compound.

Hence, the value of x is 2. Again, 2 molecules of steam contain 4 atoms of hydrogen and they have come from the compound. Hence, the value of y will be 4. Again, total oxygen atoms in the product side = 6 But 4 oxygen atoms must have come from the oxygen molecules. Hence 2 oxygen atoms definitely come from the compound. Hence, the value of z = 2. So, the molecular formula of the compound is $C_2H_4O_2$.

2. Molar mass of silicon = 28 g = 28000 mg

$\because$ 28000 mg of silicon contain 6.022×10^{23} atoms

$\therefore$ 5.68 mg of silicon contain $\dfrac{6.022 \times 10^{23}}{28000} \times 5.68$

$= 0.00122 \times 10^{23}$

$= 1.22 \times 10^{20}$ atoms

3. Molar mass of sugar $= 12 \times 12 + 1 \times 22 + 11 \times 16$

$= 144 + 22 + 176$

$= 342$ g

$\because$ Cost of 1000 g of sugar is 50 rupees

$\therefore$ Cost of 342 g of sugar $= \dfrac{50}{1000} \times 342$

$=$ ₹ 17.1

4. Mass of C-atoms = 152 − 150 = 2 mg

Molar mass of C-atoms = 12 g = 12000 mg

$\because$ 12000 mg of carbon contains 6.022×10^{23} atoms carbon

$\therefore$ 2 mg of carbon contains $= \dfrac{6.022 \times 10^{23}}{12000} \times 2$

$= 0.0010036 \times 10^{23}$

$= 1.0036 \times 10^{20}$ atoms

5. Since, molarity normality and formality are calculated against the volume of the solution and the volume of the solution changes with change in temperature. Therefore, these qualities do not remain constant with temperature.

$$\text{Molality (m)} = \frac{\text{Moles of solute}}{\text{Mass of solvent (in kg)}}$$

The molality of a solution remains independent of temperature, because it involves only mass, which is independent of temperature.

6. Energy of photons,

$$E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{(6.626 \times 10^{-34}\ \text{Js}) \times (3 \times 10^{8}\ \text{ms}^{-1})}{5 \times 10^{-7}}$$

$$= 3.975 \times 10^{-19}\ \text{J}$$

7. According to Pauli exclusion principle, no two electron in an atom can have the same set of four quantum numbers.

Hund's rule states that the pairing of electrons in the orbital of a particular sub-shell (p, d or f) does not take place until all the orbitals of the subshell are singly filled.

Option I and II do not follow Hund's rule and Paulis exclusion principle.

8. Chromium and copper have five and ten electrons in 3d-orbitals rather than four and nine because fully-filled and half-filled orbitals have lower energy and thus have extra stability. Therefore, chromium and copper have d^5 and d^{10}-configurations.

9. Since, He^+ and H have one electron each, while H^+ has no electron, hence Pauli exclusion principle does not apply to them. However, H^- has two electrons, hence this principle applies on it.

10. d-block metals, e.g., Ni, Pt, Fe, etc., are used as catalysts.

11. In general, electron affinity increases in a period from left to right. This is due to decrease in size and increase in nuclear charge as the atomic number increases in a period. Potassium is a metallic element which is electron donor. O^- and F^- ions gain stability by taking one electron.
Only O (oxygen atoms) has tendency to gain electron to attain stability.

12. According to Fajan's rule, as the size of cation decreases, it polarising power increases. Hence, K^+ and Cs^+ having bigger cations contain less polarising power. Cu^{2+} polarise Cl^- ions more than Cu^+ due to its smaller size. Therefore, $CuCl_2$ has more covalent character and hence, its boiling point is less.

13. CH_4 has regular tetrahedral shape, so its dipole moment is zero.

14. RCH_2NHCH_3 shows the hydrogen bonding, because H-atom is directly attached to N-atom.

15. van der Waals' equation for 1 mole of a real gas is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT$$

where, b is volume correction. It arises due to effective size of molecules.

16. Viscosity of a liquid increases with decrease in temperature as kinetic energy of the molecules decreases and hence, intermolecular attraction increases which in turn increases the viscosity of a liquid.

17. Thermodynamically, graphite is the most stable form of carbon.

18. HF is more stable than HCl because more energy is produced in the formation of HF. It means HF has less energy than HCl and hence is more stable. So, we need more energy to break H—F bond. Hence, HF is more stable.

19. $\Delta H < 0, \Delta S < 0$

$$\Delta G = \Delta H - T\Delta S$$

When, $\Delta H < 0$ and $\Delta S < 0$, then ΔG will be negative only at low temperature.

20. K_C is a function of temperature

21. The given reaction is exothermic and according to Le-Chatelier principle, high temperature shifts the equilibrium in backward direction, i.e. high temperature favours the decomposition of N_2O_4.

22. Solubility of alkaline earth metal hydroxide in water increases with increase in atomic number.

23. The phenomenon of emission of electrons from the surface of metal by visible light is called photoelectric effect.

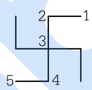
All alkali metals (except lithium) exhibits photoelectric effect. This ability to exhibit photoelectric effect is due to low value of their ionisation energies. Among alkali metals, Cs having least ionisation energy, exhibits maximum photoelectric effect.

24. Generally, as the electronegativity increases, acidic character increases, but in case of boron halides. BI_3 is the most acidic, i.e. strongest Lewis acid. This is because there is back donation of electrons from F-atom to boron in case of BF_3. This makes BF_3 less electron deficient. The tendency of back donation decreases as the size of halogen atom increases due to large energy difference. Hence, BI_3 is the most electron deficient and thus, the strongest Lewis acid.

25. Thiourea contains both S and N and hence gives blood red colouration when its Lassaigne's extract is treated with alkali and $FeCl_3$.

$$3NaCNS + FeCl_3 \longrightarrow \underset{\substack{\text{Ferric} \\ \text{sulphocyanide} \\ \text{(Blood red)}}}{Fe(CNS)_3} + 3NaCl$$

26.



3, 3-diethylpentane

27. $C_6H_5COC_6H_5$ does not exhibit tautomerism as it contains no α-H atom.

28. Inductive effect is a permanent effect operates on σ-electrons, while resonance involves delocalisation of π-electrons.

29. Cl^- is the weakest base, so good leaving group.

30. $C_6H_5\overset{+}{C}HC_6H_5$ is the most stable, since the positive charges can be delocalised over both of phenyl rings.

31. Halogens are the most reactive elements. High reactivity is due to their low dissociation energy. Further, among halogen, reactivity decreases as we move down the group. Hence, the correct order of nucleophilicity would be

$$I^- < Br^- < Cl^-$$

32. Primary alkyl halides undergo bimolecular nucleophilic substitution most readily.

33. To be aromatic the following conditions must be ful-filled.

(i) Molecules should have $(4\pi + 2)\pi$-electrons in conjugation.

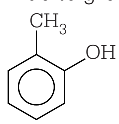
(ii) Molecule should be planar.

Cycloheptatriene has $(4n + 2)\pi$-electrons, but not in conjugation.

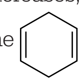
34. Compounds containing two or more carbon atoms are prepared by Wurtz reaction. Thus, methane (CH_4) cannot be prepared by Wurtz reaction.

35. Anti-Markownikoff's rule addition reaction is not observed in case of but-2-ene. The reason is that symmetrical alkenes do not exhibit anti-Markownikoff's addition.

36. Due to greater electron releasing effect,



is the most reactive towards electrophilic reagent.

37. As, the conjugation increases, heat hydrogenation decreases. Thus, alkene  with two isolated double bonds has the highest heat of hydrogenation.

38. The enrichment of water by plant nutrients is known as eutrophication. It is due to the reduction in concentration of dissolved oxygen in water due to phosphate pollution.

39. Peroxy acetyl nitrate is used in photochemical smog. Polycyclic aromatic hydrocarbons are used as carcinogens. Dioxins are used to burn the waste into ashes. IR active molecules are used in global warming. Indigo is called as vat dye because the dying with indigotin was carried out in wooden vats in the form of water soluble indigotin-white.

40. Flame colours of metal ions are due to metal excess defect.

41. Helium-oxygen mixture is used by deep sea divers because of its very low solubility in blood.

42. When benzoic acid dissolve in benzene, it will be dimerise due to the intermolecular hydrogen bonding.

$$2C_6H_5COOH \xrightarrow{\text{Benzene}} (C_6H_5COOH)_2$$

van't Hoff factor =

$$= \frac{\text{Number of particles after association or dissociation}}{\text{Number of particles before associations or dissociations}}$$

$$= \frac{1}{2} = 0.5$$

43. From Faraday's first law,

Mass deposited, $W = Z.Q$

when $Q = 1\,C$, then $W = Z =$ electrochemical equivalent.

44. Nature and concentration of the reactants its and temperature of the reaction influence the rate of reaction. But molecularity does not affect the rate of reaction as it includes the number of atoms, ions or molecules that must collide with one another to result into a chemical reaction.

45. Adsorption is an exothermic process thus, ΔH is negative (i.e. $\Delta H < 0$). Moreover, adsorption results in more ordered arrangements of molecules, thus entropy decreases (i.e. $\Delta S < 0$).

Hence, low temperature favours the reaction.

46. Ligases catalyse reactions in which the pyrophosphate bond of ATP is broken down and linkage takes place between two molecules.

47. It is a method of using dihydrogen in an efficient manner and carried or stored in the form of liquid or gas. dihydrogen releases more energy than petrol and in more eco-friendly and also can be used in fuel cells.

48. In the metallurgy of alumina, purified Al_2O_3 is mixed with Na_3AlF_6 or CaF_2 which lowers the melting point of the mix and brings conductivity.

$$2Al_2O_3 + 3C \longrightarrow 4Al + 3CO_2$$

49. In $(Si_2O_3^{2-})_n$ two oxygen atoms per SiO_4 tetrahedron are shared giving polymeric anions chain. e.g. spodumene, $LiAl(SiO_3)_2$ and diposide, $CaMg(SiO_3)_2$.

50. Myosins are fibrous proteins present in muscles.

Mathematics

1. Consider the function

$$f(x) = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + d$$

We have, $f(0) = d$

and $f(1) = \frac{a}{3} + \frac{b}{2} + c + d$

$$= \frac{2a + 3b + 6c}{6} + d$$

$$= \frac{0}{6} + d \quad [\because 2a + 3b + 6c = 0]$$

Therefore, 0 and 1 are roots of the polynomial f(x). Hence, according to Rolle's Theorem, there exists atleast one root of the polynomial $f'(x) = ax^2 + bx + c$ lying between 0 and 1.

2. We have,

$$a \cdot x = 0$$

Also, $\mathbf{x} = \mathbf{b} + (\mathbf{x} \times \mathbf{a})$

$\Rightarrow \quad x \times a = b \times a + (x \times a) \times a$

$\Rightarrow \quad x \times a = b \times a + (x \cdot a)a - (a \cdot a)x$

$\Rightarrow \quad x - b = b \times a - x$

$|2x| = |b + b \times a|$

$\Rightarrow \quad |x| = \frac{1}{\sqrt{2}}$

3. We have, $b\mathbb{N} = \{bn : n \in \mathbb{N}\}$, $c\mathbb{N} = \{cn : n \in \mathbb{N}\}$

and $d\mathbb{N} = \{dn : n \in \mathbb{N}\}$

Also, $bc \in b\mathbb{N} \cap c\mathbb{N} = d\mathbb{N}$ [given]

$\therefore \quad bc \in b\mathbb{N} \cap c\mathbb{N}$ or $bc \in d\mathbb{N}$

$\therefore \quad bc = d$

[as b and c are coprime]

4. Given, $f(x) = \int_0^x e^{|t-x|}\, dt + \int_x^1 e^{|t-x|}\, dt$

$$= e^x + e^{1-x} - 2$$

$\therefore \quad f'(x) = e^x - e^{1-x}$

Clearly, $x = \frac{1}{2}$ is the point of minima of f(x).

Also, $f'(0) = f(1) = e - 1$

5. Meen $(\bar{x}) = \frac{-1 + 0 + 4}{3}$

$\therefore$ Mean deviation $= \frac{1}{3}[|-1-1| + |0-1| + |4-1|]$

$= 2$

6. $f'(x) - pf(x) \le 0$

$\Rightarrow \quad \frac{d}{dx} e^{-px} f(x) \le 0$

Let $g(x) = e^{-px} f(x)$

$g'(x) \le 0, \forall\ x \ge 1$

$\Rightarrow \quad g(x) \le g(1), \forall\ x \ge 1$

$\Rightarrow \quad g(x) \le 0$

$\Rightarrow \quad f(x) \le 0$

But given, $f(x) \ge 0$

$\therefore \quad f(x) = 0, \ \forall x \ge 1$

7. $S_n = \frac{0}{({}^nC_0)^k} + \frac{1}{({}^nC_1)^k} + \frac{2}{({}^nC_2)^k} + \dots + \frac{n}{({}^nC_n)^k}$

$\Rightarrow \quad S_n = \frac{n}{({}^nC_0)^k} + \frac{n-1}{({}^nC_1)^k} + \frac{n-2}{({}^nC_2)^k} + \dots + \frac{0}{({}^nC_0)^k}$

$\Rightarrow \quad 2S_n = nt_n$

$\Rightarrow \quad \frac{S_n}{nt_n} = \frac{1}{2}$

Now, $\cos^{-1}\left(\frac{S_n}{nt_n}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

8. We have,

$P(|x-4| \le 2) = P(-2 \le x - 4 \le 2) = P(2 \le x \le 6)$

$= 1 - [P(x=0) + P(x=1) + P(x=7) + P(x=8)]$

$$= 1 - \left[{}^8C_0\left(\frac{1}{2}\right)^8 + {}^8C_1\left(\frac{1}{2}\right)^8 + {}^8C_7\left(\frac{1}{2}\right)^8 + {}^8C_8\left(\frac{1}{2}\right)^8\right]$$

$$= 1 - \left(\frac{1}{2}\right)^8 (1 + 8 + 8 + 1) = 1 - \frac{18}{2^8} = \frac{119}{128}$$

9. Since, total number of terms $= 59 + 1 = 60$

$\therefore$ Required sum $= \frac{2^{59}}{2} = 2^{58}$

10. $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

$\Rightarrow \quad \theta = \cot^{-1}x$

Since, $1 \le x < \infty$, therefore $0 \le \theta \le \frac{\pi}{4}$.

11. Given, $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\therefore \quad \tan^{-1}\left(\frac{2\cos x}{1 - \cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \quad \frac{2\cos x}{1 - \cos^2 x} = 2\operatorname{cosec} x$

$\Rightarrow \quad \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

12. $\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = 3\pi$

We know that, if $y = \cos^{-1}x$, then $-1 \le x \le 1$ and $0 \le y \le \pi$.

Hence, the given equation will hold only when each is π.

$\therefore \quad p = q = r = \cos\pi = -1$

$$\therefore \quad p^2 + q^2 + r^2 + 2pqr$$
$$= (-1)^2 + (-1)^2 + (-1)^2 + 2(-1)(-1)(-1)$$
$$= 1 + 1 + 1 - 2$$
$$= 3 - 2 = 1$$

13. Let $P(x, y)$ be such a point, then

$$OP^2 = x^2 + y^2$$

Let $\quad s = OP^2 = x^2 + \frac{1}{x}$

$$\Rightarrow \quad \frac{ds}{dx} = 2x - \frac{1}{x^2}$$

For maximum or minimum, put $\frac{ds}{dx} = 0$

$$\Rightarrow \quad 2x - \frac{1}{x^2} = 0$$

$$\Rightarrow \quad x^3 = \frac{1}{2}$$

$$\therefore \quad x = \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

Also, $\frac{d^2s}{dx^2} = 2 + \frac{2}{x^3}$

Now, $\left(\frac{d^2s}{dx^2}\right)_{x = \left(\frac{1}{2}\right)^{1/3}} = 2 + \frac{2}{1/2} > 0$

$\therefore$ s is minimum at $x = \left(\frac{1}{2}\right)^{1/3}$.

Thus, for nearest point $x = \left(\frac{1}{2}\right)^{1/3}$,

$$y = \pm\left(\frac{1}{2}\right)^{-1/6}$$

14.

q	p	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

$\therefore$ Statement $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

15. $(x^2 + 1) = (x - 2)(x + 2)$

Let $f(x) = x^3 + ax^2 + bx + c$; $a, b, c \in \{1, 2, \ldots, 10\}$

$f(x)$ is divisible by $x^2 + 1$ iff $f(i) = 0$, $f(-i) = 0$

$$\therefore \quad i^3 + ai^2 + bi + c = 0$$
$$\Rightarrow \quad -i - a + bi + c = 0$$
$$\Rightarrow \quad a = c, b = 1$$
and $\quad -i^3 + ai^2 - bi + c = 0$
$$\Rightarrow \quad i - a - bi + c = 0$$
$$\Rightarrow \quad a = c, b = 1$$

$\therefore$ Total number of selection of triplets

$(a, b, c) = (1, 1, 1), (2, 1, 2), \ldots, (10, 1, 10)$

Hence, number of such polynomials $= 10$.

16. Let number of regions for n lines be $R(n)$.

Clearly, $R(1) = 2$, $R(2) = 4, \ldots$

$R(n) = R(n-1) + n$ i.e. $R(n) - R(n-1) = n$

Putting $n = 2, 3, \ldots, n$ and then adding, we get

$$R(n) - R(1) = 2 + 3 + \ldots + n$$
$$= 2 + 2 + 3 + 4 + \ldots + n = 1 + \frac{n(n+1)}{2}$$
$$= \frac{n^2 + n + 2}{2}$$

17. $\lim\limits_{n \to \infty}\left[\sqrt[3]{n^2 - n^3} + n\right] = \lim\limits_{n \to \infty} n\left[\left(-1 + \frac{1}{n}\right)^{1/3} + 1^{1/3}\right]$

$$= \lim_{n \to \infty} n \cdot \frac{\left(\frac{1}{n} - 1\right) + 1}{\left(\frac{1}{n} - 1\right)^{2/3} + 1 - \left(\frac{1}{n} - 1\right)^{1/3}}$$

$$\left[\because a + b = \frac{a^3 + b^3}{a^2 - ab + b^2}\right]$$

$$= \lim_{n \to \infty} \frac{1}{\left(\frac{1}{n} - 1\right)^{2/3} + 1 - \left(\frac{1}{n} - 1\right)^{1/3}}$$

$$= \frac{1}{1 + 1 + 1} = \frac{1}{3}$$

18. $\frac{x + 1}{x^{2/3} - x^{1/3} + 1} - \frac{x - 1}{x - x^{1/2}}$

$$= \frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} - \frac{x - 1}{x^{1/2}(x^{1/2} - 1)}$$

$$= \frac{(x^{1/3} + 1)(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} - \frac{x^{1/2} + 1}{x^{1/2}}$$

$$= x^{1/3} + 1 - 1 - x^{-1/2} = x^{1/3} - x^{-1/2}$$

$\therefore \left(\dfrac{x+1}{x^{2/3}-x^{1/3}+1}-\dfrac{x-1}{x-x^{1/2}}\right)^{10}=(x^{1/3}-x^{-1/2})^{10}$

T_{r+1} for $(x^{1/3}-x^{-1/2})$ is

${}^{10}C_r\,(x^{1/3})^{10-r}\,(-1)^r\,(x^{-1/2})^r.$

For term independent of x,

$$\frac{10-r}{3}-\frac{r}{2}=0$$

$\Rightarrow \quad 20-2r-3r=0$

$\Rightarrow \quad r=4$

Hence, required coefficient $={}^{10}C_4\,(-1)^4=210.$

19. Since, $\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ is a square root of I_2 i.e. two rowed unit matrix.

$\therefore \begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\alpha^2+\beta\gamma & \alpha\beta-\alpha\beta\\ \gamma\alpha-\gamma\alpha & \gamma\beta+\alpha^2\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \alpha^2+\beta\gamma\end{bmatrix}$

$\therefore \quad \alpha^2+\beta\gamma=1$

$\Rightarrow 1-\alpha^2-\beta\gamma=0$

20. Given, $y=Ae^{3x}+Be^{5x}$ ...(i)

$\Rightarrow \quad \dfrac{dy}{dx}=3Ae^{3x}+5Be^{5x}$ ...(ii)

and $\quad \dfrac{d^2y}{dx^2}=9Ae^{3x}+25Be^{5x}$ ...(iii)

From (i), (ii) and (iii), we have

$$\frac{d^2y}{dx^2}-8\frac{dy}{dx}+15y=0$$

which is the required differential equation.

21. Let $y=\sin^{-1}(2x\sqrt{1-x^2})$

and $\quad z=\sin^{-1}(3x-4x^3)$

$\Rightarrow \quad y=2\sin^{-1}x$ and

$\quad z=3\sin^{-1}x$

$\Rightarrow \quad y=\dfrac{2}{3}z$

$\therefore \quad \dfrac{dy}{dz}=\dfrac{2}{3}$

22. Let $I=\displaystyle\int\frac{2x^{12}+5x^9}{(1+x^3+x^5)^3}\,dx=\int\frac{5x^{-6}+2x^{-3}}{(1+x^{-2}+x^{-5})^3}\,dx$

Put $1+x^{-2}+x^{-5}=t$, $(-2x^{-3}-5x^{-6})\,dx=dt$

$\therefore \quad I=-\displaystyle\int\frac{dt}{t^3}=\frac{1}{2t^2}=\frac{1}{2(1+x^{-2}+x^{-5})^2}$

$=\dfrac{x^{10}}{2(x^5+x^3+1)^2}+C$

23. Let $\Delta=\begin{vmatrix}\sin\theta & \cos\theta & \sin 2\theta\\ \sin\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta+\frac{2\pi}{3}\right) & \sin\left(2\theta+\frac{4\pi}{3}\right)\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$

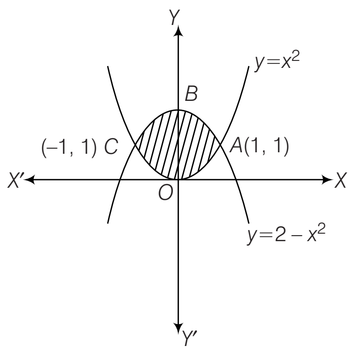
$=\begin{vmatrix}\sin\theta & \cos\theta & \sin 2\theta\\ 2\sin\theta\cos\frac{2\pi}{3} & 2\cos\theta\cos\frac{2\pi}{3} & 2\sin 2\theta\cos\frac{4\pi}{3}\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}$

Applying $R_2\to R_2+R_3$,

$=-\begin{vmatrix}\sin\theta & \cos\theta & \sin 2\theta\\ \sin\theta & \cos\theta & \sin 2\theta\\ \sin\left(\theta-\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \sin\left(2\theta-\frac{4\pi}{3}\right)\end{vmatrix}=0$

as $\quad R_1=R_2$

24. Required area = 2 Area of curve OABO



$=2\displaystyle\int_0^1[(2-x^2)-(x^2)]\,dx$

$=2\displaystyle\int_0^1(2-2x^2)\,dx$

$=4\left[x-\dfrac{x^3}{3}\right]_0^1=\dfrac{8}{3}$ sq units

25. $\lim_{n \to \infty} \frac{1}{1-n^3} \sum_{r=1}^{n} r^2$

$$= \lim_{n \to \infty} \frac{1}{n^3\left(\frac{1}{n^3}-1\right)} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n \to \infty} \frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{n^3\left(\frac{1}{n^3}-1\right)6} = -\frac{1}{3}$$

26. Let $I = \int \frac{\sin x + \cos x}{3 + \sin 2x} dx$

$$= \int \frac{\sin x + \cos x}{4 + \sin 2x - 1} dx$$

$$\Rightarrow \quad I = \int \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2} dx$$

Put $\sin x - \cos x = t$

$\Rightarrow \quad (\cos x + \sin x) dx = dt$

$$\therefore \quad I = \int \frac{dt}{4 - t^2} = \frac{1}{4} \log \frac{2-t}{2+t} + C$$

$$= \frac{1}{4} \log \left(\frac{2 - \sin x + \cos x}{2 + \sin x - \cos x}\right) + C$$

27. Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0.$$

Since, it passes through the point $(2a, 0)$, so

$$4a^2 + 4ag + c = 0 \quad \ldots(i)$$

Also, its radical axis with $x^2 + y^2 = a^2$ is

$$2gx + 2fy + c + a^2 = 0$$

But the radical axis is $x = \frac{a}{2}$, so we get

$$\frac{2g}{1} = \frac{c+a}{-a/2} \text{ and } f = 0$$

or $\quad ag + c + a = 0 \quad \ldots(ii)$

From Eqs. (i) and (ii), we get g and c

Hence, the equation is $x^2 + y^2 - 2ax = 0$

28. Any point on the parabola is $(x, x^2 + 7x + 2)$.

Its distance from the line $y = 3x - 3$ is given by

$$p = \left|\frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{9+1}}\right|$$

$$= \left|\frac{x^2 + 4x + 5}{\sqrt{10}}\right|$$

$$= \frac{x^2 + 4x + 5}{\sqrt{10}}$$

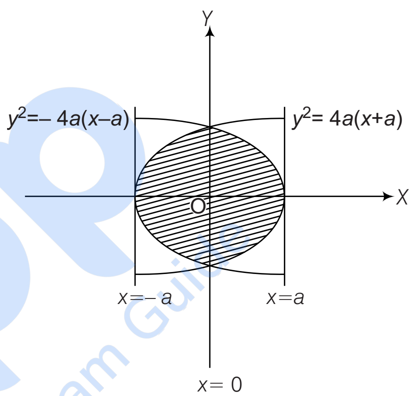
[as $x^2 + 4x + 5 > 0, \forall x \in R$]

$$\therefore \quad \frac{dp}{dx} = 0$$

$\Rightarrow \quad x = -2$

So, the required point is $(-2, -8)$.

29. Required area $= 4\int_0^a \sqrt{4a(a-x)}\, dx$



$$= 4 \cdot 2\sqrt{a} \cdot \left[\frac{-2(a-x)^{3/2}}{3}\right]_0^a$$

$$= \frac{16}{3} a^2$$

30. We have,

$$\frac{dy}{dx} = \sin(x+y)\tan(x+y) - 1 \quad \ldots(i)$$

Put $\quad x + y = v$, we get

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{dv}{dx} - 1$$

Eq. (i) reducing to

$$\frac{dv}{dx} = \sin v \tan v$$

$$\Rightarrow \quad \frac{dv}{\sin v \tan v} = dx$$

$$\Rightarrow \quad \int \operatorname{cosec} v \cot v \, dv = \int dx$$

$$\Rightarrow \quad -\operatorname{cosec} v \cot v = x + C$$

$$\Rightarrow \quad -\operatorname{cosec}(x+y) = x + C$$

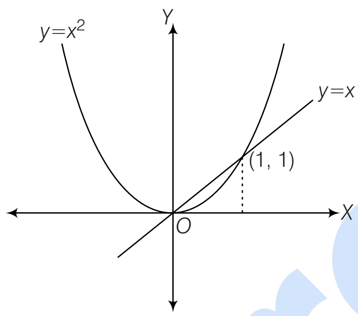
$$\Rightarrow \quad x + \operatorname{cosec}(x+y) = C$$

31. Every time the selected coupon must be from 1 to 9. The probability of such a selection at any time is $\frac{9}{15}=\frac{3}{5}$ and this can happen 7 times. i.e. $\left(\frac{9}{15}\right)^7$.

Put of these $\left(\frac{9}{15}\right)^7, \left(\frac{8}{15}\right)^7$ cases do not contain the number 9.

Thus, the required probability $=\left(\frac{9}{15}\right)^7-\left(\frac{8}{15}\right)^7$

32. $h(x)=\min(x, x^2)=\begin{cases} x, & x<0 \\ x^2, & 0 \le x < 1 \\ x, & x \ge 1 \end{cases}$



It is evident from the above graph that the given function is continuous for all x.

Since, there are sharp edges at $x=0$ and $x=1$, the function is not differentiable at these points.

Also, at $x \ge 1$, the function represents a straight line having slope 1, therefore $h'(x)=1, \forall x \ge 1$.

33. For $f(x)$ to be continuous at $x=0$, we must have

$$\lim_{x \to 0} f(x) = f(0)$$

$$\Rightarrow \lim_{x \to 0} (\cos x)^{1/x} = k$$

$$\Rightarrow \lim_{x \to 0} \{1+(\cos x - 1)\}^{1/x} = k$$

$$\Rightarrow e^{\lim_{x \to 0} \frac{\cos x - 1}{x}} = k$$

$$\Rightarrow e^{-\lim_{x \to 0} \frac{2\sin^2 x/2}{x}} = k$$

$$\Rightarrow e^0 = k$$

$$\therefore k = 1$$

34. $\because f'(0^-) = \lim_{h \to 0} f(0-h)$

$$\Rightarrow f'(0^-) = \lim_{h \to 0} \frac{e^{-1/h}-1}{e^{1/-h}+1}$$

$$\Rightarrow f'(0^-) = \frac{0-1}{0+1}$$

$$\therefore f'(0^-) = -1$$

and $f'(0^+) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{e^{1/h}-1}{e^{1/h}+1}$

$$\Rightarrow f'(0^+) = \frac{1-0}{1+0} = 1$$

35. Let $E = -2i \sum_{p=1}^{6} \left(\cos \frac{2p\pi}{7} + i \sin \frac{2p\pi}{7}\right)$

$$\Rightarrow E = -2i \sum_{p=1}^{6} e^{i\left(\frac{2p\pi}{7}\right)}$$

Let $\alpha = e^{i\left(\frac{2\pi}{7}\right)}$ ...(i)

$$\therefore E = -2i(\alpha + \alpha^2 + \dots + \alpha^6)$$

$$\Rightarrow E = -2i[(1+\alpha+\alpha^2+\dots+\alpha^6)-1]$$

$$\Rightarrow E = -2i\left[\left(\frac{1-\alpha^7}{1-\alpha}\right)-1\right]$$

Since, $\alpha^7 = \left(e^{i\left(\frac{2\pi}{7}\right)}\right)^7 = e^{i2\pi} = 1$

$$\Rightarrow E = -2i\left\{\frac{1-\left(e^{i\frac{2\pi}{7}}\right)^7}{1-e^{i\left(\frac{2\pi}{7}\right)}} - 1\right\}$$

$$\Rightarrow E = -2i\left\{\frac{1-1}{1-e^{i\left(\frac{2\pi}{7}\right)}} - 1\right\}$$

$$\Rightarrow E = -2i(0-1)$$

$$\therefore E = 2i$$

36. Number of options on 1st ring = 4

Number of options on 2nd ring = 4

Number of options on 3rd ring = 4

Number of options on 4rd ring = 4

Number of options on 5rd ring = 4

$\therefore$ Total number of ways of all the options $= 4 \times 4 \times 4 \times 4 \times 4 = 1024$

$\therefore$ Number of all possible unsuccesful attempts to open the lock is $= 1024 - 1 = 1023$

37. Domain of f is defined, if

$$|[|x|-1]|-5>0$$

$$\Rightarrow |[|x|-1]|>5$$

$\Rightarrow$ either $[|x| - 1] < -5$ or $[|x| - 1] > 5$

I

$$[|x| - 1] < -5$$
$$\Rightarrow \quad |x| - 1 < -5$$
$$\Rightarrow \quad |x| < -4, \text{ which is not possible.}$$

II

$$[|x| - 1] > 5$$
$$\Rightarrow \quad |x| - 1 \geq 6$$
$$\Rightarrow \quad |x| \geq 7$$
$$\therefore \quad x \in (-\infty, -7] \cup [7, \infty)$$

38. We have, $y^{1/m} + y^{-1/m} = 2x$

or $\quad y^{1/m} + \dfrac{1}{y^{1/m}} = 2x$

or $\quad (y^{1/m})^2 - 2xy^{1/m} + 1 = 0$

$\therefore\ y^{1/m} = x \pm \sqrt{(x^2 - 1)}$

or $\quad y = \left[x \pm \sqrt{x^2 - 1}\right]^m$

$$\therefore \quad y_1 = m[x \pm \sqrt{(x^2 - 1)}]^{m-1}\left[1 \pm \frac{2x}{2\sqrt{x^2 - 1}}\right]$$

$$\Rightarrow \quad y_1 = \frac{\pm\, m\,[x \pm \sqrt{(x^2 - 1)}]^m}{\sqrt{(x^2 - 1)}} = \frac{\pm\, my}{\sqrt{(x^2 - 1)}}$$

$$\Rightarrow \quad (x^2 - 1)y_1^2 = m^2y^2$$
$$\Rightarrow \quad (x^2 - 1)\,2y_1y_2 + 2xy_1^2 = 2m^2yy_1$$
$$\Rightarrow \quad (x^2 - 1)\,y_2 + xy_1 = m^2y$$

39. We have, $cx - by + b^2 = 0$ is a tangent of circle $x^2 + y^2 - ax - by = 0$.

$\therefore$ Centre $= \left(\dfrac{a}{2}, \dfrac{b}{2}\right)$, Radius$(r) = \sqrt{\dfrac{a^2 + b^2}{4}}$

Distance from centre to the line $cx - by + b^2 = 0$,

$$\Rightarrow \quad r = \left|\frac{\frac{ac}{2} - \frac{b^2}{2} + b^2}{\sqrt{b^2 + c^2}}\right|$$

$$\Rightarrow \quad \sqrt{\frac{a^2 + b^2}{4}} = \frac{\frac{ac + b^2}{2}}{\sqrt{b^2 + c^2}}$$

$$\Rightarrow \quad (b^2 + c^2)(a^2 + b^2) = (ac + b^2)^2$$
$$\Rightarrow \quad b^4 + b^2(a^2 + c^2) + a^2c^2 = b^4 + 2b^2ac + a^2c^2$$
$$\Rightarrow \quad a^2 + c^2 = 2ac$$
$$\Rightarrow \quad (a - c)^2 = 0$$
$$\Rightarrow \quad a = c$$

40. Given, $\alpha + \beta + \gamma = \dfrac{\pi}{2}$

$\Rightarrow \cot\alpha + \cot\beta + \cot\gamma = \cot\alpha\cot\beta\cot\gamma$

$\therefore\ \cot\alpha, \cot\beta, \cot\gamma$ are in AP.

$$\Rightarrow \quad 3\cot\beta = \cot\alpha\cot\beta\cot\gamma$$
$$\Rightarrow \quad \cot\alpha\ \cot\gamma = 3$$

41. Since, $R = \dfrac{b}{2\sin B} = \dfrac{2}{2\sin 30°} = \dfrac{2}{1}$

$\therefore$ Area of circumcircle $= \pi R^2 = \pi\,(2)^2$

$= 4\pi$ sq units

42. $\dfrac{3}{4\cdot 8} - \dfrac{3\cdot 5}{4\cdot 8\cdot 12} + \dfrac{3\cdot 5\cdot 7}{4\cdot 8\cdot 12\cdot 16} - \ldots + \dfrac{3}{4} - \dfrac{3}{4}$

$$= 1 - \frac{1}{4} + \frac{1\cdot 3}{2\cdot 4\cdot 4} - \frac{1\cdot 3\cdot 5}{4\cdot 4\cdot 2\cdot 4\cdot 3} + \ldots - \frac{3}{4}$$

$$= \left[1 + \frac{1}{1!}\left(-\frac{1}{4}\right) + \frac{1(1+2)}{2!}\left(-\frac{1}{4}\right)^2 + \frac{1\,(1+2)\,(1+4)}{3!}\left(-\frac{1}{4}\right)^3 + \ldots\right] - \frac{3}{4}$$

$$= \left(1 - \frac{1}{4}\right)^{-1/2} - \frac{3}{4} = \sqrt{\frac{2}{3}} - \frac{3}{4}$$

43. Let $I = \displaystyle\int_0^{\pi} \sin^3\theta\, d\theta$

[$\because \sin\theta > 0$ for $0 < \theta < \pi$]

$$= \int_0^{\pi} \sin\theta\,(1 - \cos^2\theta)\, d\theta$$

Put $\quad \cos\theta = t$

$$\Rightarrow \quad -\sin\theta\, d\theta = dt$$

$$\therefore \quad I = \int_{-1}^{1} (1 - t^2)\,dt = \left[t - \frac{t^3}{3}\right]_{-1}^{1}$$

$$= 2 - \frac{2}{3} = \frac{4}{3}$$

44. $\displaystyle\int \frac{\cos x + x\sin x}{x^2 + x\cos x}\, dx$

$$= \int \frac{x + \cos x - x + x\sin x}{x^2 + x\cos x}\, dx$$

$$= \int \frac{dx}{x} - \int \frac{x(1 - \sin x)}{x(x + \cos x)}\, dx$$

$$= \log x - \log(x + \cos x) + C$$

$$= \log\left|\frac{x}{x + \cos x}\right| + C$$

45. Equation of tangent with the slope $\dfrac{-3}{4}$ is

$y = \dfrac{-3}{4}x + C.$

According to condition of tangency,

$$C = \sqrt{32 \times \left(\frac{-3}{4}\right)^2 + 18} = \sqrt{18 + 18} = 6$$

$\therefore \qquad y = \frac{-3}{4}x + 6$

$\Rightarrow 4y + 3x = 24$

Since, it meet the coordinate axes in A and B.

$\therefore\;$ A $\equiv (8, 0)$ and B $\equiv (0, 6)$

$\therefore$ Required area $= \frac{1}{2} \times 8 \times 6 = 24$ sq units

46. $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$

$$= \lim_{x \to \frac{\pi}{6}} \frac{4\sin x \cos x + \cos x}{4\sin x \cos x - 3\cos x}$$

[by L′ Hospital's rule]

$$= \lim_{x \to \frac{\pi}{6}} \frac{\cos x(4\sin x + 1)}{\cos x(4\sin x - 3)}$$

$$= \frac{4\sin\frac{\pi}{6} + 1}{4\sin\frac{\pi}{6} - 3} = -3$$

47. On squaring and adding the given equations, we get

$$\sin^2 A + \cos^2 B + 2\sin A\cos B + \sin^2 B + \cos^2 A + 2\sin B\cos A = a^2 + b^2$$

$\Rightarrow\; 2\sin(A + B) + 2 = a^2 + b^2$

$\Rightarrow \qquad \sin(A + B) = \dfrac{a^2 + b^2 - 2}{2}$

48. Given, $\sin A \sin B = \dfrac{ab}{c^2}$

$\Rightarrow\; c^2 = \dfrac{ab}{\sin A \sin B} = \left(\dfrac{a}{\sin A}\right)\left(\dfrac{b}{\sin B}\right)$

$\Rightarrow\; c^2 = \left(\dfrac{c}{\sin C}\right)^2 \qquad \left[\because \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}\right]$

$\Rightarrow \qquad \sin^2 C = 1$

$\Rightarrow \qquad C = 90°$

Hence, ΔABC is a right angled triangle.

49. Given, $l + m + n = 0 \qquad \ldots(i)$

and $\qquad lm = 0$

Either $\qquad m = 0$ or $l = 0$

If $l = 0$, then put in Eq. (i), we get $\quad m = -n$

$\therefore$ Direction ratios are $0, -n, n$ i.e. $0, -1, 1$

If $m = 0$, then put in Eq. (i), we get $l = -n$

$\therefore$ Direction ratios are $-n, 0, n$ i.e. $-1, 0, 1$.

$$\therefore \cos\theta = \frac{0 \times (-1) + (-1) \times 0 + 1 \times 1}{\sqrt{(0)^2 + (-1)^2 + (1)^2}\,\sqrt{(-1)^2 + (0)^2 + (1)^2}} = \frac{1}{2}$$

$\Rightarrow \qquad \theta = \dfrac{\pi}{3}$

50. Given equation can be written as

$$\frac{(x - 3)^2}{16} + \frac{y^2}{25} = 1$$

Here, $a^2 = 16$ and $\;b^2 = 25$

$\therefore \qquad e = \sqrt{1 - \dfrac{16}{25}} = \dfrac{3}{5}$

Hence, the foci of conic section are $(0, \pm\, be)$ i.e. $(3, \pm 3)$.

51. Given, $x - 2y = 4$

Let $\qquad A = xy$

$\Rightarrow \qquad A = 2y^2 + 4y$

$\Rightarrow \qquad \dfrac{dA}{dy} = 4 + 4y$

For extremum value, $\dfrac{dA}{dy} = 0 \;\Rightarrow\; y = -1$

Now, $\quad \dfrac{d^2A}{dy^2} = 4 > 0, \qquad$ (minimum)

At $\quad y = -1, x = 4 + 2(-1) = 2$

$\therefore\quad A = xy = 2(-1) = -2$

$\therefore$ Hence, the minimum value of xy is -2.

52. Let $b = \hat{j}$ and $c = \hat{j}$

$\therefore \qquad |b \times c| = |\hat{k}| = 1$

Again, let $\qquad a = a_1\hat{i} + a_2\hat{j} + a_3\hat{j}$

Now, $\qquad a \cdot b = a \cdot i = a_1,\; a \cdot \bar{c} = a \cdot \hat{j} = a_2$

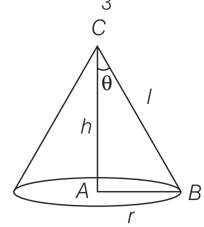
and $\quad a \cdot \dfrac{b \cdot c}{|b \times c|} = a \cdot k = a_3$

$\therefore \quad (a \cdot b)\, b + (a \cdot c)\, c + \dfrac{a \cdot (b \times c)}{|b \times c|} \cdot (b \times c)$

$= a_1 b + a_2 c + a_3(b \times c) = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = a$

53. Volume of cone, $V = \dfrac{\pi}{3} r^2 h$

$\Rightarrow \qquad V = \dfrac{\pi}{3} r^2 \sqrt{l^2 - r^2}$



On differentiating w.r.t. r, we get

$$\frac{dV}{dr} = \frac{\pi}{3}\left[2r\sqrt{l^2 - r^2} + \frac{r^2}{2\sqrt{l^2 - r^2}}(-2r)\right]$$

Put $\frac{dV}{dr} = 0$, we get

$$r = \pm l\sqrt{\frac{2}{3}}$$

At $r = l\sqrt{\frac{2}{3}}, \frac{d^2V}{dr^2} < 0,$ (maximum)

$\therefore$ $h = \sqrt{l^2 - \frac{2}{3}l^2} = \frac{l}{\sqrt{3}}$

In Δ ABC, $\tan\theta = \frac{r}{h} = \frac{l\sqrt{\frac{2}{3}}}{l/\sqrt{3}} = \sqrt{2}$

54. $f(x) = \frac{2\sin 8x\cos x - 2\sin 6x\cos 3x}{2\cos 2x\cos x - 2\sin 3x\sin 4x}$

$$= \frac{(\sin 9x + \sin 7x) - (\sin 9x + \sin 3x)}{(\cos 3x + \cos x) + (\cos 7x - \cos x)}$$

$$= \frac{\sin 7x - \sin 3x}{\cos 7x + \cos 3x} = \frac{2\cos 5x\sin 2x}{2\cos 2x\cos 5x}$$

$$= \tan 2x$$

$\therefore$ Period of $f(x) = \frac{\pi}{2}$

55. We know that the equation of a plane parallel to X-axis is

$$by + cz + d = 0.$$

Since, it passes through the points $(2, 3, 1)$ and $(4, -5, 3)$.

$\therefore$ $3b + c + d = 0$

and $-5b + 3c + d = 0$

$\Rightarrow$ $\frac{b}{1-3} = \frac{c}{-8} = \frac{d}{14}$

$\Rightarrow$ $\frac{b}{-2} = \frac{c}{-8} = \frac{d}{14}$

$\therefore$ Equation of plane is $-2y - 8z + 14 = 0$

i.e. $y + 4z = 7.$

56. Since, the point Q is image of P, therefore PQ is perpendicular to the plane $x - 2y + 5z = 6$

$\therefore$ Required equation of the line is

$$\frac{x-2}{1} = \frac{y-3}{-2} = \frac{z-4}{5}.$$

57. The intersection of two planes is

$$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-6 + 5\lambda) = 0$ ...(i)

Since, this plane is perpendicular to the plane $4x + 5y - 3z - 8 = 0$.

$\therefore$ $(1 + 2\lambda)4 + (1 + 3\lambda)5 + (1 + 4\lambda)(-3) = 0$

$\Rightarrow$ $\lambda = \frac{-6}{11}$

On putting the value of λ in Eq. (i), we get

$$\left(-\frac{1}{11}\right)x + \left(-\frac{7}{11}\right)y + \left(-\frac{13}{11}\right)z + \left(-\frac{96}{11}\right) = 0$$

$\Rightarrow$ $x + 7y + 13z + 96 = 0$

58. Here, $T_n = \frac{n^3}{n!} = \frac{n^2 - 1}{(n-1)!} + \frac{1}{(n-1)!}$

$$= \frac{n+1}{(n-2)!} + \frac{1}{(n-1)!}$$

$$= \frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!}$$

$\therefore \Sigma T_n = \Sigma\left(\frac{1}{(n-3)!} + \frac{3}{(n-2)!} + \frac{1}{(n-1)!}\right)$

$$= e + 3e + e = 5e$$

59. Given, $f(x) = \cos x + \cos\sqrt{2}x$

$\Rightarrow$ $f'(x) = -\sin x - \sqrt{2}\sin\sqrt{2}x$

$\Rightarrow$ $f''(x) = -\cos x - 2\cos\sqrt{2}x$

For extremum value, put $f'(x) = 0$

$\Rightarrow$ $-\sin x - \sqrt{2}\sin\sqrt{2}x = 0$

$\Rightarrow$ $x = 0$

At $x = 0$, $f''(x) < 0$, (maximum)

Hence, $f(x)$ is maximum only once.

60. We know that a vector perpendicular to the plane containing the point A, B and C is given by $A \times B + B \times C + C \times A$.

Given, $\mathbf{A} = \hat{i} - \hat{j} + 2\hat{k}$, $\mathbf{B} = 2\hat{i} + 0\hat{j} - \hat{k}$

and $\mathbf{C} = 0\hat{i} + 2\hat{j} + \hat{k}$

Now, $\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \hat{i} + 5\hat{j} + 2\hat{k}$

$\mathbf{B} \times \mathbf{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

$\mathbf{C} \times \mathbf{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 5\hat{i} + \hat{j} - 2\hat{k}$

Thus, $\mathbf{A} \times \mathbf{B} + \mathbf{B} \times \mathbf{C} + \mathbf{C} \times \mathbf{A}$

$= (\hat{i} + 5\hat{j} + 2\hat{k}) + (2\hat{i} - 2\hat{j} + 4\hat{k}) + (5\hat{i} + \hat{j} - 2\hat{k})$

$= 8\hat{i} + 4\hat{j} + 4\hat{k}$

61. We have,

$$P'P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}\begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad P'P = I$ or $P' = P^{-1}$

As, $\quad Q = PAP'$

$\therefore P'Q^{2005}P = P'[(PAP')(PAP') \dots 2005 \text{ times}]\ P$

$$= \underbrace{(P'P)A\,(P'P)A\,(P'P)\dots(P'P)A\,(P'P)}_{2005 \text{ times}}$$

$$= IA^{2005} = A^{2005}$$

Now, $\quad A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \dots A^{2005}$$

$$= \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad P'Q^{2005}P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

62. $m + n = -\dfrac{3}{7} + \left(\dfrac{-11}{7}\right) = -2$ (–ve integer)

Let $\quad I = \int \cos^{-3/7} x\,(\sin^{(-2+\frac{3}{7})} x)\,dx$

$$= \int \cos^{-3/7} x \sin^{-2} x \sin^{3/7} x\,dx$$

$$= \int \frac{\operatorname{cosec}^2 x}{\left(\dfrac{\cos^{3/7} x}{\sin^{3/7} x}\right)}\,dx = \int \frac{\operatorname{cosec}^2 x\,dx}{\cot^{3/7} x}$$

Put $\cos x = t \Rightarrow -\operatorname{cosec}^2 x\,dx = dt$

$$\therefore \quad I = -\int \frac{dt}{t^{3/7}} = -\frac{t^{-\frac{3}{7}+1}}{-\frac{3}{7}+1} + C$$

$$= -\frac{7}{4} t^{4/7} + c = -\frac{7}{4}\cot^{4/7} x + C$$

$$= -\frac{7}{4}\tan^{-4/7} x + C$$

63. Let $ax + by = 1$

be the chord $\quad \dots$(i)

Making the equation of hyperbola homogeneous using Eq. (i), we get

$$3x^2 - y^2 + (-2x + 4y)(ax + by) = 0$$

or $\quad (3 - 2a)x^2 + (-1 + 4b)y^2 + (-2b + 4a)xy = 0$

Since, the angle subtended at the origin is a right angle.

$\therefore$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$\Rightarrow \quad (3 - 2a) + (-1 + 4b) = 0$

$\Rightarrow \quad a = 2b + 1$

$\therefore$ Chords are $(2b + 1)x + by - 1 = 0$

or $\quad b(2 + y) + (x - 1) = 0,$

which clearly pass through the fixed point $(1, -2)$.

64. The line of intersection of the planes

$\mathbf{r}\cdot(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 1$ and $\mathbf{r}\cdot(\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) = 2$

is common to both the planes.

Therefore, it is perpendicular to normals to the two planes i.e. $\mathbf{n_1} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{n_2} = \hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$. Hence, it is parallel to the vector $\mathbf{n_1} \times \mathbf{n_2} = -2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 13\hat{\mathbf{k}}$.

Thus, we have to find the equation of the plane passing through $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and normal to the vector $\mathbf{n} = \mathbf{n_1} \times \mathbf{n_2}$.

The equation of the required plane is

$$(\mathbf{r} - \mathbf{a})\cdot\mathbf{n} \text{ or } \mathbf{r}\cdot\mathbf{n} = \mathbf{a}\cdot\mathbf{n}$$

or $\quad \mathbf{r}\cdot(-2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 13\hat{\mathbf{k}})$

$$= (\hat{\mathbf{i}} + 2\hat{\mathbf{i}} + \hat{\mathbf{k}})\cdot(-2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 13\hat{\mathbf{k}})$$

or $\quad \mathbf{r}\cdot(2\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - 13\hat{\mathbf{k}}) = 1$

65. Given, $x = 1 + \dfrac{dy}{dx} + \dfrac{1}{2!}\left(\dfrac{dy}{dx}\right)^2 + \dfrac{1}{3!}\left(\dfrac{dy}{dx}\right)^3 + \dots$

or $\quad x = e^{\frac{dy}{dx}}$

$\Rightarrow \quad \dfrac{dy}{dx} = \log_e x$

Hence, the degree of the above differential equation is 1.

66. Let $P(x_1, y_1)$ be the mid-point of a chord of the circle.

The equation of the chord is $T = S_1$

i.e. $\quad xx_1 + yy_1 - 16 = x_1^2 + y_1^2 - 16$

or $\quad y = -\dfrac{x_1}{y_1}x + \dfrac{x_1^2 + y_1^2}{y_1}$

Since, it touches the hyperbola $\dfrac{x^2}{4^2} - \dfrac{y^2}{3^2} = 1$.

$\therefore \left(\dfrac{x_1^2 + y_1^2}{y_1}\right) = 4^2 \cdot \dfrac{x_1^2}{y_1^2} - 3^2 \quad [\because c^2 = a^2m^2 - b^2]$

$\therefore$ Locus of (x_1, y_1) is

$$\frac{(x^2 + y^2)^2}{y^2} = \frac{16x^2 - 9y^2}{y^2}$$

or $\quad (x^2 + y^2)^2 = 16x^2 - 9y^2$

67. Given, ${}^{n+1}C_{n-2} - {}^{n+1}C_{n-1} \le 50$

$$\Rightarrow \quad \frac{(n+1)!}{3!(n-2)!} - \frac{(n+1)!}{2!(n-2)!} \le 50$$

$$\Rightarrow \quad \frac{(n+1)!}{3!}\left[\frac{1}{(n-2)!} - \frac{3}{(n-1)!}\right] \le 50$$

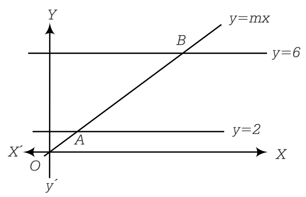
$$\Rightarrow \quad (n+1)!\left(\frac{n-1-3}{(n-1)!}\right) \le 300$$

$$\Rightarrow \quad (n+1)n\,(n-4) \le 300$$

For $n = 8$, it satisfy to the above inequality.

But $n = 1$ it does not satisfy the above inequality.

68. Given lines are $y = mx, y = 2$ and $y = 6$.



Here, coordinates of points A and B are $\left(\frac{2}{m}, 2\right)$, $\left(\frac{6}{m}, 6\right)$, respectively.

$$\therefore \quad AB = \sqrt{\left(\frac{2}{m} - \frac{6}{m}\right)^2 + (2-6)^2} < 5 \qquad \text{[given]}$$

$$\Rightarrow \quad \left(\frac{2}{m} - \frac{6}{m}\right)^2 + (4)^2 < 25$$

$$\Rightarrow \quad \left(\frac{2}{m} - \frac{6}{m}\right)^2 < 9$$

$$\Rightarrow \quad -3 < \frac{2}{m} - \frac{6}{m} < 3$$

$$\Rightarrow \quad -\frac{4}{3} > m > \frac{4}{3}$$

$$\therefore \quad m \in \left]-\infty, -\frac{4}{3}\right[\cup \left]\frac{4}{3}, \infty\right[$$

69. Equation of line perpendicular to $2x + y + 6 = 0$ and passes through origin is $x - 2y = 0$

Now, the point of intersection of $2x + y + 6 = 0$ and $x - 2y = 0$ is $\left(-\frac{12}{5}, -\frac{6}{5}\right)$.

Similarly, the point of intersection of $x - 2y = 0$ and $4x + 2y - 9 = 0$ is $\left(\frac{9}{5}, \frac{9}{10}\right)$.

Let the origin divide the line $x - 2y = 0$ in the ratio $\lambda : 1$

$$\therefore \quad x = \frac{\frac{9}{5}\lambda - \frac{12}{5}}{\lambda + 1} = 0$$

$$\Rightarrow \quad \frac{9}{5}\lambda = \frac{12}{5}$$

$$\Rightarrow \quad \lambda = \frac{12}{9} = \frac{4}{3}$$

70. Given, $P(\overline{A \cup B}) = \frac{1}{6}$

$$\Rightarrow \quad 1 - P(A \cup B) = \frac{1}{6}$$

$$\Rightarrow \quad 1 - P(A) - P(B) + P(A \cap B) = \frac{1}{6}$$

$$\Rightarrow \quad P(\overline{A}) - P(B) + P(A \cap B) = \frac{1}{6}$$

$$\Rightarrow \quad \frac{1}{4} - P(B) + \frac{1}{4} = \frac{1}{6}$$

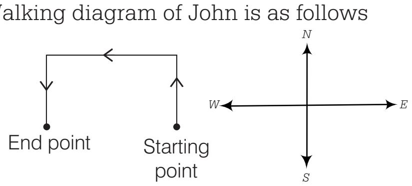
$$\Rightarrow \quad P(B) = \frac{1}{3}$$

and $\quad P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{4} = \frac{3}{4}$

Since, $P(A \cap B) = P(A) \cdot P(B)$, so the events A and B are independent events but not equally likely.

General Aptitude

16. Walking diagram of John is as follows



It is clear from the above diagram that John is in facing South direction now.

17. As, $64 + 7 = 460$

$$\Rightarrow \quad 64 \times 7 + 12 = 460$$

and $\quad 25 + 8 = 212$

$$\Rightarrow \quad 25 \times 8 + 12 = 212$$

Same as, $\quad 43 + 8 = ?$

$$\Rightarrow \quad 43 \times 8 + 12 = \boxed{356}$$

$$\therefore \quad ? = 356$$

19. Given equation

$$6 + 64 - 8 \div 45 \times 8$$

Now, changing the sign as per the question,

$$6 \times 64 \div 8 + 45 - 8$$

$$= \frac{6 \times 64}{8} + 45 - 8$$

$$= 6 \times 8 + 45 - 8$$

$$= 48 + 45 - 8$$

$$= 93 - 8 = 85$$

21.

$$1 \quad 4 \quad 2 \quad 3 \quad 2 \quad \boxed{4}$$

+2

−2

22. As,

$$25 + 17 = 42 \Rightarrow \frac{42}{7} = 6$$

and $\quad 38 + 18 = 56 \Rightarrow \frac{56}{7} = 8$

Similarly, $\quad 89 + 16 = 105$

$$\Rightarrow \quad \frac{105}{7} = \boxed{15}$$

23. Relative speed of both the persons

$$= 12 + 10 = 22 \text{ km/h}$$

Now, distance between both of them

$$= 55 - 11 = 44 \text{ km}$$

$\therefore$ Required time, when distance is 11 km between both of them $= \dfrac{\text{Total distance}}{\text{Relative speed}}$

$$= \frac{44}{22} = 2 \text{ h}$$

Hence, they will be apart 11 km after 2 h.

24. Given, $x - \dfrac{1}{x} = 2$

On, squaring both the sides, we get

$$x^2 + \frac{1}{x^2} - 2 = 4$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 4 + 2 \Rightarrow x^2 + \frac{1}{x^2} = 6$$

25. Population after 2 yr

$$= 20000\left(1 + \frac{20}{100}\right)\left(1 + \frac{30}{100}\right)$$

$$= 20000 \times \frac{120}{100} \times \frac{130}{100}$$

$$= 2 \times 120 \times 130 = 31200$$

26. Let initial price of rice = ₹ x per kg

Now, a reduction of 20% in price,

Reduced price of rice

$$= x \times \frac{80}{100} = ₹ \frac{4x}{5} \text{ per kg}$$

Now, according to the question,

$$\frac{1200}{\frac{4x}{5}} - \frac{1200}{x} = 5$$

$$\Rightarrow \quad \frac{6000}{4x} - \frac{1200}{x} = 5$$

$$\Rightarrow \frac{1500}{x} - \frac{1200}{x} = 5 \Rightarrow \frac{300}{x} = 5$$

$$\Rightarrow \quad x = \frac{300}{5} = 60$$

$\therefore$ Reduced price of rice

$$= \frac{4x}{5} = \frac{4 \times 60}{5} = ₹\ 48 \text{ per kg}$$

27. Let total income of a person = ₹ 100

$\therefore$ Then, spend on different expenses = ₹ 60

$\therefore$ His savings = 100 − 60 = ₹ 40

Now, income after increment of 20%

$$= \frac{120}{100} \times 100 = ₹\ 120$$

and expences after increment of 10%

$$= 120 \times (60 + 10)\%$$

$$= 120 \times \frac{70}{100} = ₹\ 84$$

$\therefore$ New savings = 120 − 84 = ₹ 36

Now, percentage decrease

$$= \left(\frac{40 - 36}{40}\right) \times 100\%$$

$$= \frac{4}{40} \times 100\% = \frac{100}{10}\% = 10\%$$

28. Here, $= ₹\ 4096, \quad = 12\frac{1}{2}\% = \frac{25}{2}\%,$

and $\quad = 18 \text{ months} = \frac{18}{12} \text{ yr} = \frac{3}{2} \text{ yr}$

Now, money received to Shyam

$$= 4096\left(1 + \frac{25}{2 \times 100} \times \frac{1}{2}\right)^{\frac{3}{2} \times 2}$$

$$= 4096\left(1 + \frac{1}{16}\right)^3 = 4096\left(\frac{17}{16}\right)^3$$

$$= 4096 \times \frac{17}{16} \times \frac{17}{16} \times \frac{17}{16}$$

$$= 17 \times 17 \times 17 = ₹\ 4913$$

29. Time taken to complete the work by = 12 days

Since, work twice as fast as .

$\therefore$ Time taken to complete the work by = 6 days

Now, one day work of $= \frac{1}{12}$

and one day work of $= \frac{1}{6}$

$\therefore$ One day work of $(A + B) = \frac{1}{6} + \frac{1}{12}$

$$= \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4}$$

Hence, time taken by and together to finish the work = 4 days.

30. Let the height of embankment = h m

Given, radius of well $= \frac{3}{2} = 1.5$ m

$$\therefore \quad \frac{4}{3} \times \frac{22}{7} \times [(4)^2 - (1.5)^2] \times h = \frac{22}{7} \times (1.5)^2 \times 14$$

$$\Rightarrow \quad \frac{4}{3} \times (16 - 2.25) \times h = 2.25 \times 14$$

$$\Rightarrow \quad \frac{4}{3} \times 13.75 \times h = 2.25 \times 14$$

$$\Rightarrow \quad h = \frac{2.25 \times 14 \times 3}{4 \times 13.75} = \frac{225 \times 7 \times 3}{2 \times 1375} = \frac{189}{110}$$

$$= 1.718 \approx 1.750 \text{ m}$$

Hence, the height of embankment is 1.750 m.