

MET 2017 Answer Key PDF

Physics

1. (b) Using red light ($\lambda = 6600 \text{ \AA}$) 60 fringes are seen.

Hence, range of field of view is

$$60 \times w = 60 \times \frac{D\lambda}{d}$$

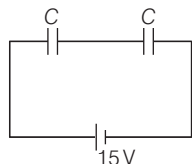
Using light of wavelength λ' , n fringes are seen, then

$$\Rightarrow 60 \times \frac{D\lambda}{d} = n \times \frac{D\lambda'}{d}$$

$$60 \times \lambda = n \times \lambda'$$

$$\Rightarrow n = 60 \times \frac{\lambda}{\lambda'} = 60 \times \frac{6600}{4400} = 90$$

2. (c) Let capacitance of each capacitor is C . Then equivalent capacitance in series is



$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$\Rightarrow C' = \frac{C}{2}$$

Charge $Q = C'V = \frac{C}{2} \times 15$... (i)

When filled with dielectric

$$C_1 = 4C, C_2 = C$$

$$\frac{1}{C'} = \frac{1}{4C} + \frac{1}{C} = \frac{5}{4C}$$

$$\Rightarrow C' = \frac{4C}{5}$$

Since, charge is conserved

$$Q = C'V' = \frac{4C}{5} V' \quad \dots (ii)$$

From Eqs. (i) and (ii), we get

$$\frac{C}{2} \times 15 = \frac{4C}{5} V'$$

$$\Rightarrow V' = \frac{15 \times 5}{4 \times 2} = 9.4 \text{ V} \approx 10 \text{ V}$$

3. (a) The speed of transverse wave

$$v = \sqrt{\frac{T}{m}}$$

Given, $T = 20 \text{ N}$, $\frac{M}{l} = \frac{d \times Al}{l} = d \times A$

$$\therefore v = \sqrt{\frac{20 \times 10^{-3}}{7.5 \times 0.2 \times (10^{-3})^2}}$$

$$v \approx 116 \text{ ms}^{-1}$$

4. (d) Common acceleration

$$a = \frac{F}{m_1 + m_2 + m_3}$$

$$a = \frac{40}{10 + 6 + 4} = 2 \text{ m/s}^2$$

Equation of motion of m_3 is

$$T_3 - T_2 = m_3 a$$

$$40 - T_2 = 4 \times 2 \Rightarrow T_2 = 32 \text{ N}$$

5. (d) Let n cells be in series and m in parallel, then

$$\frac{nE}{R + nr} = \frac{E}{R + \frac{r}{m}}$$

$$\Rightarrow n \left[R + \frac{r}{m} \right] = R + nr$$

$$\Rightarrow nRm + nr = Rm + mn r$$

$$\Rightarrow 6 + 2r = 3 + 4r$$

$$\Rightarrow 2r = 3$$

$$\Rightarrow R = 1.5 \Omega$$

6. (d) From Coulomb's law the force (F) between two charges is

$$F = \frac{1}{4\pi\epsilon_0 k} \frac{q^2}{r^2}$$

First case : $k = 1$

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q^2}{r^2} \quad \dots (i)$$

Second case : $k = 4$

$$F' = \frac{1}{4\pi\epsilon_0 \times 4} \cdot \frac{q^2}{r^2} \quad \dots (ii)$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{F}{F'} = 4 \Rightarrow F' = \frac{F}{4}$$

7. (d) The height (h) to which water rises in a capillary tube is given by

$$h = \frac{2T \cos \theta}{r \rho g} \text{ where } \theta \text{ is angle of contact, } r \text{ the radius, } \rho \text{ the density and } g \text{ acceleration due to gravity.}$$

When lift moves down with constant acceleration, height is less than h , because effective value of acceleration due to gravity increases hence h decreases.

8. (a) From Stefan's law

$$E = \sigma T^4 A$$

Given,

$$T = 727^\circ\text{C} = 727 + 273 = 1000 \text{ K},$$

$$A = 5 \times 10^{-4} \text{ m}^2$$

$$\therefore \text{Energy} = 5.67 \times 10^{-8} (1000)^4 (5 \times 10^{-4}) 60$$

$$\therefore E = 1.7 \times 10^3 \text{ J}$$

9. (a) NAND gate is obtained when the output of AND gate is made as the input of NOT gate
Boolean expression for NAND gate is



$$Y = \overline{A \cdot B}$$

A	B	Y
0	0	1
1	0	1
0	1	1
1	1	0

10. (d) Torque on dipole is

$$\tau = pE \sin \theta$$

$$\tau = q \times 2lE \sin \theta$$

$$\Rightarrow q = \frac{\tau}{2El \sin \theta}$$

$$\text{Given, } \tau = 4 \text{ Nm}, E = 2 \times 10^5 \text{ NC}^{-1},$$

$$l = 2 \times 10^{-2} \text{ m}, \theta = 30^\circ$$

$$\therefore q = \frac{4}{2 \times 2 \times 10^5 \times 2 \times 10^{-2} \times \sin 30^\circ}$$

$$\text{or } q = 2 \text{ mC}$$

11. (b) The velocity (v) is given by

$$v = f\lambda$$

$$\Rightarrow f = \frac{v}{\lambda}$$

Number of beats = difference in frequencies

$$\therefore f_1 - f_2 = v \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

$$\frac{10}{3} = v \left(\frac{1}{1} - \frac{1}{1.01} \right)$$

$$\frac{10}{3} = v \left(\frac{0.01}{1.01} \right)$$

$$\Rightarrow v = \frac{10 \times 1.01}{3 \times 0.01} = 336.7 \text{ m/s}$$

12. (b) The relation between α and β is

$$\beta = \frac{\alpha}{1 - \alpha}$$

Putting this value in the given equation, we have

$$\frac{\beta - \alpha}{\alpha\beta} = \frac{\frac{\alpha}{1 - \alpha} - \alpha}{\alpha \cdot \frac{\alpha}{1 - \alpha}} = \frac{\alpha^2}{\alpha^2} = 1$$

13. (a) From Doppler's effect, perceived frequency

$$f' = f \left(\frac{v - v_o}{v - v_s} \right)$$

$$\frac{9}{8} = \frac{340}{340 - v_s}$$

$$\Rightarrow 9(340 - v_s) = 8 \times 340$$

$$\therefore v_s = 37.7 \text{ ms}^{-1} \approx 40 \text{ ms}^{-1}$$

14. (b) The angular distance (θ) is given by

$$\theta = \frac{\lambda}{d}$$

$$\text{Given, } \theta = 2^\circ = \frac{\pi}{180} \times 2, \lambda = 6980 \text{ \AA}$$

$$= 6980 \times 10^{-10} \text{ m}$$

$$\therefore d = \frac{\lambda}{\theta} = \frac{6980 \times 10^{-10} \times 180}{3.14 \times 2}$$

$$\Rightarrow d \approx 2 \times 10^{-5} \text{ mm}$$

15. (d) The resistance at temperature t is

$$R_t = R_0 (1 + \alpha t)$$

$$R'_t = R_0 (1 + \alpha t')$$

$$\Rightarrow 4 = 1 + 15\alpha \quad \dots(i)$$

$$5 = 1 + 37.5\alpha \quad \dots(ii)$$

$$\therefore \frac{4}{5} = \frac{1 + 15\alpha}{1 + 37.5\alpha}$$

$$\Rightarrow 4(1 + 37.5\alpha) = 5(1 + 15\alpha)$$

$$\Rightarrow 75\alpha = 1 \Rightarrow \alpha = \frac{1}{75} ^\circ\text{C}^{-1}$$

16. (c) The relation between K and p is

$$p = \sqrt{2Km}$$

when increased by 25%, we have

$$p' = p + \frac{25}{100} p = \frac{5p}{4}$$

$$\Rightarrow \frac{4p}{5p} = \sqrt{\frac{K_1}{K_2}}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{16}{25}$$

Hence, percentage increase is

$$= \frac{25-16}{16} \times 100 = \frac{9}{16} \times 100 = 56.25\%$$

- 17. (a)** The focal length of the combination is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Given, $f_1 = 50$ cm, $f_2 = 50$ cm

$$\therefore \frac{1}{F} = \frac{1}{50} + \frac{1}{50} = \frac{2}{50}$$

$$\Rightarrow F = \frac{50}{2} = 25 \text{ cm}$$

Object when placed at centre of curvature forms a real, inverted image of same size as object = ($2 \times 25 = 50$ cm).

18. (a) $M_{21} = \frac{\mu_0 N_1 N_2 A_2}{l_1}$

$$(4 \times 3.14 \times 10^{-7}) \times 1500 \times 100$$

$$\therefore M_{21} = \frac{\times \{3.14 (2 \times 10^{-2})^2\}}{80 \times 10^{-4}}$$

$$M_{21} = 2.96 \times 10^{-4} \text{ H}$$

$$\Rightarrow M_{12} = M_{21} = 2.96 \times 10^{-4} \text{ H}$$

19. (b) Refractive index, $\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

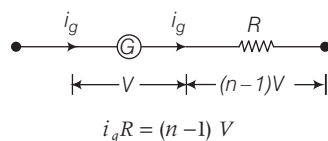
$$\Rightarrow \sqrt{2} = \frac{\sin\left(\frac{60^\circ + \delta_m}{2}\right)}{\sin\left(\frac{60^\circ}{2}\right)}$$

$$\Rightarrow \sqrt{2} \times \sin 30^\circ = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\Rightarrow \sin 45^\circ = \sin\left(\frac{60^\circ + \delta_m}{2}\right)$$

$$\Rightarrow \delta_m = 30^\circ$$

- 20. (b)** Suppose resistance R is connected in series with voltmeter as shown. By Ohm's law



$$\Rightarrow R = (n-1)G$$

where $i_g = \frac{V}{G}$

21. (d) Fraction = $\frac{N}{N_0}$

$$= \left(\frac{1}{2}\right)^{\frac{6400}{1600}} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

- 22. (d)** As we know $Q \propto T^4$

$$\Rightarrow \frac{H_A}{H_B} = \left[\frac{273 + 727}{273 + 327}\right]^4 = \frac{625}{81}$$

- 23. (c)** Heat emitted by copper = Heat gained by ice

$$mc\Delta\theta = m'L$$

$$\Rightarrow m' = \frac{mc\Delta\theta}{L}$$

Given, $m = 2$ kg, $c = 400 \text{ J}\cdot\text{kg}^{-1} \cdot \text{C}^{-1}$,

$$\Delta\theta = 500, L = 3.5 \times 10^5 \text{ J}\cdot\text{kg}^{-1}$$

$$\therefore m' = \frac{2 \times 400 \times 500}{3.5 \times 10^5} = \frac{8}{7} \text{ kg}$$

- 24. (c)** The resultant of tension and weight given acceleration to the man, hence

$$mg - T = ma$$

$$\Rightarrow T = mg - ma = m(g - a)$$

Given, $m = 100$ kg,

$$g = 9.8 \text{ m/s}^2, a = 1.8 \text{ ms}^{-2}.$$

$$\therefore T = 100(9.8 - 1.8) = 800 \text{ N}$$

25. (a) $F = \frac{dp}{dt} \Rightarrow p = Fdt$

where p = momentum = mv

$$\therefore mv = Fdt$$

$$\Rightarrow v = \frac{Fdt}{m}$$

Given, $F = 20 \text{ N}$, $dt = 2 \text{ s}$, $m = 2 \text{ kg}$

$$\therefore v = \frac{20 \times 2}{2} = 20 \text{ m/s}^2$$

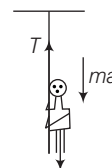
$$\text{Work done} = \text{KE} = \frac{1}{2} \times 2 \times (20)^2 = 400 \text{ J}$$

- 26. (d)** Force on a particle with charge q in electric field E is

$$F = qE = ma$$

$$\therefore a = \frac{qE}{m}$$

here $a = \frac{v}{t}$



$$\therefore \frac{v}{t} = \frac{qE}{m} \Rightarrow v = \frac{qEt}{m}$$

Also, kinetic energy is due to velocity (v) is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qEt}{m}\right)^2 = \frac{q^2E^2t^2}{2m}$$

27. (a) $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

Given, $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$, $V = +100$ volt,

$$r = 10 \text{ cm} = 0.10 \text{ m.}$$

$$\therefore 100 = (9 \times 10^9) \times \frac{q}{0.10}$$

$$\Rightarrow q = \frac{100 \times 0.10}{9 \times 10^9} = 1.1 \times 10^{-9} \text{ C}$$

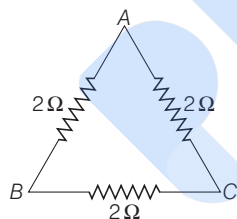
28. (a) Internal resistance (r) is given by

$$r = \frac{E - V}{i}$$

Also, $i = \frac{V}{R}$, we have

$$r = \frac{E - V}{V/R} \Rightarrow r = R\left(\frac{E}{V} - 1\right)$$

29. (a) In triangle formation, two resistors are in series and their sum is connected in parallel. Therefore,

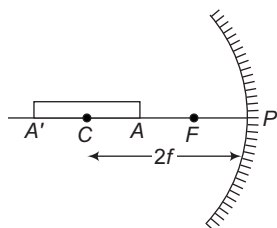


$$R' = 2\Omega + 2\Omega = 4\Omega$$

$$\frac{1}{R''} = \frac{1}{4} + \frac{1}{2} = \frac{1+2}{4} = \frac{3}{4} \Omega$$

$$R'' = \frac{4}{3} \Omega$$

30. (a) $u = PA = PC - AC = 2f - \frac{f}{3} = \frac{5f}{3}$



From mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{f} - \frac{3}{5f} = \frac{2}{5f}$$

Length of image

$$CA' = PA' - PC = \frac{5f}{2} - 2f = \frac{f}{2}$$

$$\therefore \text{Magnification } (m) = \frac{CA'}{CA} = \frac{f/2}{f/3} = 1.5$$

31. (d) Intensity ($I = ka^2$), where a is amplitude and k is a constant.

$$\therefore I_1 = ka_1^2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{9}{4}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{3}{2}$$

Maximum intensity

$$I_{\max} = (a_1 + a_2)^2 = (3 + 2)^2 = 25$$

Minimum intensity

$$I_{\min} = (a_1 - a_2)^2 = (3 - 2)^2 = 1$$

$$\therefore \frac{I_{\max}}{I_{\min}} = \frac{25}{1}$$

32. (d) Refractive index is given by

$$n = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin \frac{A}{2}}$$

Given, $\delta_m = 180^\circ - 2A$

$$\therefore n = \frac{\sin\left(\frac{A + 180^\circ - 2A}{2}\right)}{\sin \frac{A}{2}}$$

$$n = \frac{\sin\left(\frac{180^\circ - A}{2}\right)}{\sin \frac{A}{2}}$$

$$\Rightarrow n = \frac{\cos \frac{A}{2}}{\sin \frac{A}{2}} = \cot \frac{A}{2}$$

33. (a) $\frac{1}{2}mv^2 = eV$

$$\Rightarrow \frac{e}{m} = \frac{v^2}{2V}$$

Given, $V = 200$ volt, $v = 8.4 \times 10^8$ cm/s

$$= 8.4 \times 10^6 \text{ m/s.}$$

$$\therefore \frac{e}{m} = \frac{(8.4 \times 10^6)^2}{2 \times 200} = 1.76 \times 10^{11} \text{ C/kg}$$

$$34. (a) \frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For Lyman series $n = 1$

For Balmer series $n = 2$

$$\therefore \frac{1}{\lambda_L} = R \left(\frac{1}{1^2} \right), \frac{1}{\lambda_B} = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \Rightarrow \frac{\lambda_B}{\lambda_L} = \frac{3}{1}$$

35. (b) When emf of cell is E , the potential drop is

$$V = E - ir$$

where i is current and r the internal resistance.

Given, $i = 10 \text{ mA} = 10 \times 10^{-3} \text{ A}$, $R = 200 \Omega$,

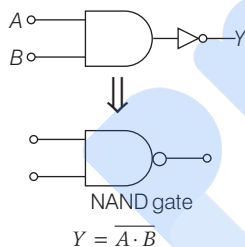
$$V = 0.5 \text{ volt.}$$

$$\Rightarrow E = V + ir$$

$$E = 0.5 + 10 \times 10^{-3} \times 200$$

$$E = 1.5 \text{ V}$$

36. (d) In the following figure, output of AND gate is made as input of NOT gate, we get NAND gate



37. (d) Distance of the satellite from the centre are $7R$ and $3.5 R$ respectively.

$$\frac{T_2}{T_1} = \left(\frac{R_2}{R_1} \right)^{3/2}$$

$$\Rightarrow T_2 = 24 \left[\frac{3.5 R}{7R} \right]^{3/2} = 6\sqrt{2} \text{ h}$$

38. (b) We know that $v \propto \lambda$

$$\Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

$$\therefore v_2 = \frac{v_1}{\lambda_1} \times \lambda_2$$

$$= 3 \times 10^8 \times \frac{4500}{6000} = 2.25 \times 10^8 \text{ m/s}$$

39. (b) The angle of minimum deviation is given by

$$\delta_m = (n - 1) A$$

$$\delta_m = (\sqrt{2} - 1) 60$$

$$\text{Angle of incidence, } i = \frac{A + \delta_m}{2}$$

$$i = \frac{60 + (\sqrt{2} - 1) 60}{2}$$

$$i = \frac{60\sqrt{2}}{2} = 30\sqrt{2} = 42^\circ$$

40. (c) From Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{q \times \frac{q}{2}}{(a+x)^2} - \frac{1}{4\pi\epsilon_0} \frac{q \times \frac{q}{2}}{(a-x)^2} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{2} \left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{q^2}{2} \left[-\frac{4ax}{(a^2-x^2)^2} \right] \end{aligned}$$

When $x \ll a$, then

$$F = -\frac{2q^2}{4\pi\epsilon_0 a^3} x$$

$$\Rightarrow F \propto -x$$

Hence, SHM along x -axis.

41. (b) From Coulomb's law

$$F = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{2a^2}$$

F_1 and F_2 will be directed as shown, for this both q should be negative

$$F_1 = F_2 = \frac{1}{4\pi\epsilon_0} \frac{Qq}{a^2}$$

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{\sqrt{2} Qq}{a^2}$$

For equilibrium of Q , we have $\begin{matrix} q & & q/2 & & q \\ & \leftarrow x \rightarrow & & & \end{matrix}$
 $(-a, 0) \quad (-a, 0) \quad (-a, 0) \quad (+a, 0)$

$$F = -F_{12}$$

$$\frac{Q^2}{2a^2} = -\frac{\sqrt{2} Qq}{a^2}$$

$$\Rightarrow q = \frac{Q}{2\sqrt{2}}$$

Given $Q = +\sqrt{2} \mu\text{C}$

$$\therefore q = -\frac{\sqrt{2} \mu\text{C}}{2\sqrt{2}} = -0.5 \mu\text{C}$$

42. (c) Let $p_1 = p$, $p_2 = p_1 + 50\%$ of p_1

$$= p_1 + \frac{p_1}{2} = \frac{3p_1}{2}$$

$$E \propto p^2$$

$$\Rightarrow \frac{E_2}{E_1} = \left(\frac{p_2}{p_1}\right)^2 = \left(\frac{3p_1/2}{p_1}\right)^2 = \frac{9}{4}$$

$$\Rightarrow E_2 = 2.25E_1 = E_1 + 1.25E_1$$

$$\therefore E_2 = E_1 + 125\% \text{ of } E_1$$

ie, kinetic energy will increase by 125%

43. (c) The energy stored in capacitor is given by

$$E = \frac{1}{2} CV^2$$

Resultant capacitance

$$C' = C_1 + C_2 = 4 + 6 = 10 \text{ pF}$$

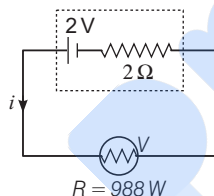
$$E = \frac{1}{2} \times 10 \times 10^{-12} \times (100)^2$$

$$(1 \text{ pF} = 10^{-12} \text{ F})$$

$$= 5 \times 10^{-8} \text{ J}$$

44. (a) From Ohm's law

$$i = \frac{E}{R + r} = \frac{2}{998 + 2} = 2 \times 10^{-3} \text{ A}$$



Potential difference across the voltmeter is

$$V = iR = (2 \times 10^{-3}) \times 998 = 1.996 \text{ V}$$

45. (d) When no current passes through the galvanometer, the bridge is balanced hence,

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \frac{24 + X}{10} = \frac{84 + Y}{30}$$

Also, applying Kirchhoff's law

$$\Sigma iR = \Sigma i'R'$$

(parallel)

$$(X + 24) + (84) + Y = 3(10 + 30)$$

$$\Rightarrow X + Y = 12$$

...(i)

$$\text{and } 3X - Y = 12$$

...(ii)

From Eqs. (i) and (ii), we get

$X = 6$, $Y = 6$ (neglecting negative value)

46. (b) The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L = 400 \times 20 \times 10^{-3} = 8 \text{ H}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \times 625 \times 10^{-6}} = 4 \text{ C}$$

$$Z = \sqrt{(3)^2 + (8 - 4)^2} = 5$$

$$i = \frac{E}{Z} = \frac{300}{5} = 60 \text{ A}$$

47. (a) If stopping potential is V_0 , then maximum kinetic energy of photoelectrons is given by

$$E_k = eV$$

Given, $V = 1.5 \text{ volt}$

$$E_k = 1.5 \text{ eV}$$

48. (b) $a \sin \theta = 1 \times \lambda$

$$\Rightarrow \lambda = a \sin \theta = 10^{-4} \times \frac{1}{2} = \frac{10 \times 10^{-5}}{2}$$

$$= 5 \times 10^{-5} \text{ cm} = 5000 \text{ Å}$$

49. (c) NOR gate is obtained when the output of OR gate is made as the input of NOT gate, Boolean expression for NOR gate is

$$Y = A + B$$

Symbol of NOR gate



Truth Table

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

50. (c) From Doppler's effect the perceived frequency f' is given by

$$f' = f \left(\frac{v + v_o}{v} \right)$$

Given, $f = 990 \text{ Hz}$, $v = 300 \text{ m/s}$, $v_o = 33 \text{ m/s}$

$$\therefore f' = 990 \left(\frac{330 + 33}{330} \right)$$

$$= 990 \times \frac{363}{330} = 1089 \text{ Hz}$$

Chemistry

1. (b) For f orbital, $l = 3$

Hence, $m = -3, -2, -1, 0, +1, +2, +3$

2. (b) $3C + \frac{3}{2}O_2 \longrightarrow 3CO$



1 mole of $Fe_2O_3 \equiv 3$ mole of $CO \equiv \frac{3}{2}$ mole of O_2

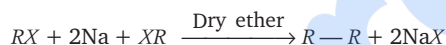
$\therefore 160$ g of Fe_2O_3 require $O_2 = \frac{3}{2} \times 32 = 48$ g

$\therefore 1.60$ kg of Fe_2O_3 will require $O_2 = 480$ g

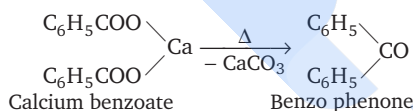
3. (c) The phenoxide ion left from o -hydroxybenzaldehyde and p -hydroxybenzaldehyde are stabilized by $-I$ and $-R$ effect of the $-CHO$ group. But in o -isomer due to chelation it is difficult to remove the H-atom. Hence, p -hydroxybenzaldehyde is the strongest acid.

4. (a) Purification of NaCl by passage of hydrogen chloride through brine is based on common ion effect.

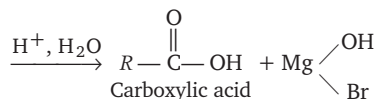
5. (d) Wurtz reaction involves the reduction of alkyl halide with Na in ether.



6. (d) Dry distillation of calcium benzoate gives benzophenone.



7. (b) $RMgBr + \overset{\overset{O}{\parallel}}{C} = O \longrightarrow R - \overset{\overset{O}{\parallel}}{C} - OMgBr$
Grignard reagent



8. (a) The dilute aqueous solution (7 to 8%) of ethanoic acid (acetic acid) is called vinegar.

9. (a) $2(r^+ + r^-) = \sqrt{3}a$
 $2(1.69 + 1.81) = \sqrt{3}a$
 $2(3.5) = \sqrt{3}a$
 $7.0 = \sqrt{3}a$

$$a = \frac{7.0}{\sqrt{3}} = 4.04 \text{ \AA}$$

10. (a) $CaCl_2 + 2AgNO_3 \longrightarrow Ca(NO_3)_2 + 2AgCl$

111 g 2 \times 43.5 g

$CaCl_2$ required to produce 2×143.5 g of $AgCl$

$$= 111 \text{ g}$$

$CaCl_2$ required to produce 14.35 g of $AgCl$

$$= \frac{111 \times 14.35}{2 \times 143.5} = 5.55 \text{ g}$$

11. (b) In the hydrogen spectrum the Lyman series of lines appears in ultra violet region of electromagnetic spectrum.

12. (b) The first ionisation potential (IE_1) decreases from top to bottom in a group.

Hence, the order of ionisation potential is as :

$$Be > Mg > Ca$$

13. (c) Fog is a liquid in gas colloidal solution.

14. (b) $NOClO_4 \equiv (NO^+) (ClO_4^-)$

Let the O.N. of N in NO^+ is x .

$$x + (-2) = +1$$

$$x = 3$$

Let O.N. of Cl in ClO_4^- is y .

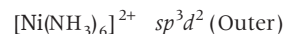
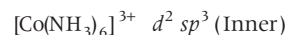
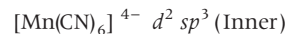
$$y + 4(-2) = -1$$

$$y - 8 = -1$$

$$y = +7$$

15. (a) Slag formed during the extraction of iron is always lighter than the molten metal. Its melting point should also be less than the molten metal.

16. (d) Complex ion Hybridisation on central atom



17. (b) Sugar : Lactose Glucose Sucrose Fructose
Relative

sweetness 16 74 100 173

18. (c) $\Rightarrow KE = \frac{3}{2} RT$ (for 1 mole)

Given, $T = 47 + 273 = 320$ K

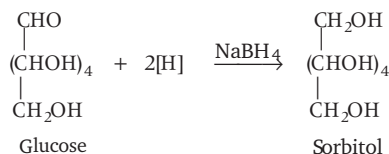
$$KE = \frac{3}{2} \times 8.314 \times 320 = 3990.7 \text{ J mol}^{-1}$$

Molar mass of $O_2 = 32 \text{ g mol}^{-1}$

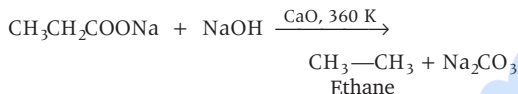
$$\begin{aligned}\text{KE (per gram of O}_2\text{)} &= \frac{3990.7}{32} \text{ J g}^{-1} \\ &= 124.7 \text{ J g}^{-1} \\ &= 1.24 \times 10^2 \text{ J g}^{-1}\end{aligned}$$

$$19. (a) \pi = \frac{n}{V} RT = \frac{0.2 \times 0.082 \times 300}{1.0} = 4.92 \text{ atm}$$

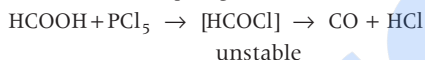
20. (a) Reduction of glucose with NaBH_4 gives sorbitol.



21. (d) Decarboxylation of sodium propionate leads to the formation of ethane.



22. (d) Formic acid reacts with PCl_5 to give carbon monoxide and hydrogen chloride.



23. (c) $\Delta H < 0$ and $\Delta S > 0$ are always favourable conditions for spontaneity at all temperatures.

24. (b) $\Delta G^\circ = -RT \ln K$

$$\begin{aligned}\Delta G^\circ &= -2.303 RT \log K \\ &= -2.303 \times 8 \times 300 \times \log 10 \\ &= -2.303 \times 8 \times 300 \times 1 \\ &= -5527 \text{ J mol}^{-1} = -5.527 \text{ kJ mol}^{-1}\end{aligned}$$

25. (a) Due to $-R$ effect of the $\text{C}=\text{O}$ group the electron density on the N-atom decreases, hence amides are weaker base than NH_3 and aliphatic amines. Further due to $-I$ effect of NH_2 group, the electron density on the other N-atom decreases. As a result, NH_2NH_2 is a much weaker base than NH_3 and aliphatic amines. Therefore, ethylamine is most basic among these.

26. (a) $\text{CaCO}_3 + 2\text{HCl} \longrightarrow \text{CaCl}_2 + \text{CO}_2 + \text{H}_2\text{O}$

$$25 \text{ mL of } 0.75 \text{ M HCl} = \frac{25}{1000} \times 0.75 = 0.01875 \text{ mol}$$

$$\begin{aligned}\text{Moles of CaCO}_3 \text{ required} &= \frac{\text{moles of HCl}}{2} \\ &= \frac{0.01875}{2} = 9.375 \times 10^{-3} \text{ mol}\end{aligned}$$

Mass of CaCO_3 required

$$\begin{aligned}&= 9.375 \times 10^{-3} \text{ mol} \times 100 \text{ g mol}^{-1} \\ &= 0.9375 \text{ g} = 0.94 \text{ g}\end{aligned}$$

27. (d) There are $(2l + 1)$ values of m for the each value of l .

$$\therefore l = 3$$

$$\therefore \text{Total values of } m = (2 \times 3 + 1) = 7$$

28. (c) The ionisation energy of nitrogen is larger than that of oxygen because half-filled p -orbitals of nitrogen have extra stability while oxygen has $2p^4$ electronic configuration of the outermost shell, hence its electrons can be easily removed.

29. (a) For two third of a reaction

$$\begin{aligned}[A]_0 &= a, [A] = a - \frac{2}{3}a = \frac{a}{3} \\ t_{2/3} &= \frac{2.303}{k} \log \frac{[A]_0}{[A]} \\ &= \frac{2.303}{5.48 \times 10^{-14}} \log \frac{a}{a/3} \\ &= \frac{2.303}{5.48 \times 10^{-14}} \log 3\end{aligned}$$

30. (a) Nitric oxide (NO) has one unpaired electron and it is paramagnetic in nature. It has 15 electrons.

$$\begin{aligned}\text{NO} : & \text{KK } \sigma(2s)^2 \sigma^*(2s)^2 \sigma(2p_z)^2 \\ & \pi(2p_x)^2 \pi(2p_y)^2 \pi^*(2p_x)^1\end{aligned}$$

31. (c) Oxygen has an oxidation state of $+2$ in OF_2 because oxidation number of fluorine is always -1 .

32. (b) Total volume after mixing = $50 + 30 = 80 \text{ cc}$

$$\text{Moles of HCl after mixing} = \frac{50}{80} \text{ M}$$

$$\text{Moles of NaOH after mixing} = \frac{30}{80} \text{ M}$$

$$\begin{aligned}\text{Remaining number of moles of HCl after mixing} \\ &= \frac{50}{80} - \frac{30}{80} = 0.25 \text{ M}\end{aligned}$$

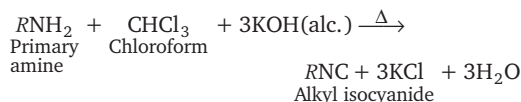
$$[\text{H}^+] = 0.25 = 2.5 \times 10^{-1}$$

$$\text{pH} = -\log [2.5 \times 10^{-1}]$$

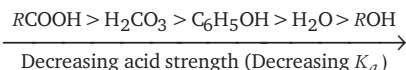
$$= 1 - 0.3979 = 0.6021$$

33. (a) Noble gases are monoatomic. The value of C_p / C_v for noble gases is 1.66.

- 34. (b)** Carbylamine test is performed in alcoholic KOH by heating a mixture of trihalogenated methane and a primary amine.



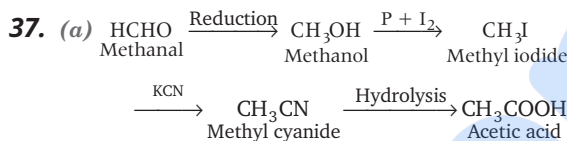
- 35. (a)** The order of acidity is as :



- 36. (d)** The order of reactivity of alcohols towards Lucas reagent is as

Tertiary > Secondary > Primary

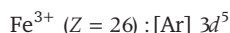
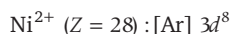
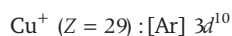
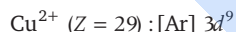
Since 2-methyl propan-2-ol is a tertiary alcohol, hence it reacts fastest with Lucas reagent at room temperature.



- 38. (a)** The conjugate base of the acid H_2S is HS^- .

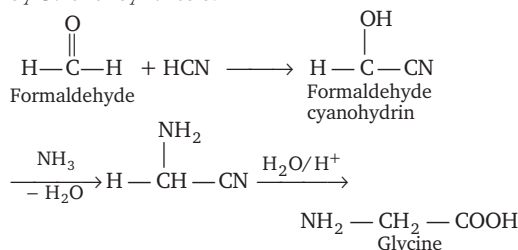


- 39. (b)** Transition metal ion having electronic configuration $(n-1)d^{1-9}$ forms coloured ion.



Hence, Cu^+ ion is not coloured.

- 40. (a)** Glycine can be obtained from formaldehyde by Strecker synthesis.



- 41. (c)** Polar aprotic solvents such as DMSO increase the rate of $\text{S}_{\text{N}}2$ reactions.

- 42. (c)** EAN of Cu in $[\text{Cu}(\text{NH}_3)_4] \text{SO}_4$ is as :

$$\text{EAN} = Z - (\text{O.N.}) + 2 (\text{C.N.})$$

$$\text{EAN} = 29 - 2 + 4 \times 2 = 35$$

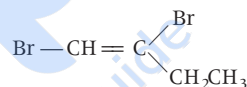
- 43. (a)** For a reaction to be spontaneous ΔG should be negative.

$$\Delta G = -nFE^\circ_{\text{cell}}$$

From this reaction, ΔG will be negative when E°_{cell} is positive.

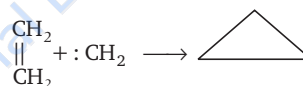
- 44. (c)** In a face centred cubic lattice, a unit cell is shared equally by six unit cells.

- 45. (b)** 1, 2-dibromobutene has different substituents on each C-atom of the double bond, hence it shows geometrical isomerism.



- 46. (a)** α -amino acids are essential building units of proteins.

- 47. (d)** $\text{CH}_2\text{N}_2 \xrightarrow{-\text{N}_2} \text{:CH}_2$
Carbene



- 48. (b)** The first ever compound of noble gas $\text{Xe}^+ [\text{PtF}_6]^-$ was prepared by Neil Bartlett in 1962.

- 49. (a)** $\text{K}_2\text{Cr}_2\text{O}_7$ on heating with aqueous alkalis gives chromates
 $\text{K}_2\text{Cr}_2\text{O}_7 + 2\text{NaOH} \longrightarrow \text{K}_2\text{CrO}_4 + \text{Na}_2\text{CrO}_4 + \text{H}_2\text{O}$

- 50. (d)** Energy of electron in n th orbit of hydrogen,

$$E_n = -\frac{E}{n^2}$$

where, E is a constant.

$$\Rightarrow \frac{E_2}{E_4} = \frac{(n_4)^2}{(n_2)^2}$$

$$\frac{-328}{E_4} = \frac{(4)^2}{(2)^2}$$

$$\therefore E_4 = \frac{-328 \times 4}{16} = -82 \text{ kJ mol}^{-1}$$

Mathematics

1. (c) Since, roots are complex.

$$\begin{aligned} \therefore a^2 - 36 &< 0 & (\because \text{discriminant} < 0) \\ \Rightarrow a^2 &< 36 \\ \Rightarrow |a| &< 6 \end{aligned}$$

2. (c) Since, projection of $\mathbf{PQ}$ on OX, OY, OZ are 12, 3 and 4 respectively, then $\mathbf{PQ} = 12\hat{i} + 3\hat{j} + 4\hat{k}$

$$\begin{aligned} \therefore |\mathbf{PQ}| &= \sqrt{12^2 + 3^2 + 4^2} = \sqrt{169} \\ &= 13 \end{aligned}$$

3. (a) Let e be the identity.

$$\begin{aligned} \therefore a * e &= a \\ \Rightarrow \frac{ab}{6} &= a \\ \Rightarrow e &= 6 \\ \therefore a * a^{-1} &= e \\ \therefore a * a^{-1} &= 6 \\ \Rightarrow \frac{a a^{-1}}{6} &= 6 \Rightarrow a^{-1} = \frac{6 \times 6}{a} \\ \therefore 9^{-1} &= \frac{6 \times 6}{9} = 4 \end{aligned}$$

4. (c) Let $I = \int_0^1 \frac{d}{dx} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) \right] dx$

$$\begin{aligned} \therefore I &= \int_0^1 \frac{d}{dx} [\sin^{-1} (\sin 2\theta)] dx \quad (\text{put } x = \tan \theta) \\ &= 2 \int_0^1 \frac{d}{dx} [\tan^{-1} x] dx \\ &= 2 [\tan^{-1} (1) - \tan^{-1} (0)] \\ &= 2 \cdot \frac{\pi}{4} = \frac{\pi}{2} \end{aligned}$$

5. (a) Given, $4\sin^{-1} x + \cos^{-1} x = \pi$

$$\begin{aligned} \Rightarrow 4\sin^{-1} x + \frac{\pi}{2} - \sin^{-1} x &= \pi \\ \Rightarrow 3\sin^{-1} x &= \frac{\pi}{2} \\ \Rightarrow x &= \sin \left(\frac{\pi}{6} \right) = \frac{1}{2} \end{aligned}$$

6. (b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$

$$\begin{aligned} &= -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty \\ &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty - 1 = \log 2 - 1 \end{aligned}$$

7. (b) Given, $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$

$$\begin{aligned} \Rightarrow \tan^{-1} \left(\frac{x+y+z-xyz}{1-xy-yz-zx} \right) &= \pi \\ \Rightarrow x+y+z-xyz &= 0 \end{aligned}$$

8. (a)

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

Clearly, $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

9. (d) $(1+x+x^2+x^3)^{-3} = \left[\frac{(x^4-1)}{x-1} \right]^{-3}$
- $$\begin{aligned} &= (x^4-1)^{-3} (x-1)^3 \\ &= (-1)^{-3} (1-x^4)^{-3} (x-1)^3 \\ &= (-1) (1+3x^4+\dots) (x^3-3x^2+3x-1) \\ \therefore \text{Coefficient of } x \text{ in } (1+x+x^2+x^3)^{-3} &\text{ is } -3 \end{aligned}$$

10. (a) $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$
- $$= \lim_{h \rightarrow 0} \frac{f(3) f(h) - f(3)}{h} \quad [\because f(x+y) = f(x) f(y)]$$

$$\text{Now, } f(x+0) = f(x) f(0)$$

$$\Rightarrow f(x) [f(0) - 1] = 0$$

Either $f(x) = 0$ or $f(0) = 1$

$$\begin{aligned} \therefore f'(3) &= f(3) \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} \\ &= f(3) f'(0) = f(3) \cdot 11 \\ &= 3 \times 11 = 33 \end{aligned}$$

11. (c) The equation of given line is

$$\frac{x}{a} - \frac{y}{b} = 1$$

$$\Rightarrow bx - ay = ab$$

A line perpendicular to given line is

$$ax + by = \lambda$$

$\therefore$ In the given options, option (c) is correct.

12. (c) Given, $\cos^{-1} x + \cos^{-1} y = 2\pi$

$$\Rightarrow \frac{\pi}{2} - \sin^{-1} x + \frac{\pi}{2} - \sin^{-1} y = 2\pi$$

$$\Rightarrow \pi - 2\pi = \sin^{-1} x + \sin^{-1} y$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = -\pi$$

13. (b) $\log_3 2, \log_3 (2^x - 5), \log_3 \left(2^x - \frac{7}{2}\right)$ are in AP.

$$\Rightarrow \log_3 (2^x - 5) = \frac{\log_3 \left(2^x - \frac{7}{2}\right) + \log_3 2}{2}$$

$$\Rightarrow (2^x - 5)^2 = 2 \cdot \left(2^x - \frac{7}{2}\right)$$

$$\Rightarrow (2^x)^2 + 25 - 10 \cdot 2^x = 2 \cdot 2^x - 7$$

$$\Rightarrow (2^x)^2 - 12 \cdot 2^x + 25 + 7 = 0$$

$$\Rightarrow (2^x)^2 - 12 \cdot 2^x + 32 = 0$$

$$\Rightarrow 2^x = 8 \text{ or } 2^x = 4$$

$$\Rightarrow x = 3 \text{ or } 2$$

$$\Rightarrow x = 3 (\because x = 2 \text{ does not satisfy the given series})$$

14. (c) $\int_0^1 x(1-x)^{12} dx$

$$= \left[\frac{x(1-x)^{13}}{-13} + \int_0^1 \frac{(1-x)^{13}}{13} dx \right]_0^1$$

$$= \left[\frac{x(1-x)^{13}}{-13} \right]_0^1 + \left[\frac{(1-x)^{14}}{-14 \times 13} \right]_0^1$$

$$= [0 - 0] + \left[0 - \frac{1}{-14 \times 13} \right] = \frac{1}{182}$$

15. (a) Middle term is $\left(\frac{n}{2} + 1\right)$ th term for n even.

$$\therefore \text{Middle term} = \left(\frac{6}{2} + 1\right) = 4 \text{th term}$$

$$\therefore T_4 = {}^6C_3 (\sqrt{x})^3 \left(-\frac{1}{\sqrt{x}}\right)^{6-3}$$

$$= \frac{6!}{3!3!} (\sqrt{x})^3 \frac{(-1)^3}{(\sqrt{x})^3}$$

$$= \frac{-6 \times 5 \times 4}{6}$$

$$= -20$$

16. (d) Let a and b be any element of group G .

$$\therefore a = a^{-1}, b = b^{-1} \quad \forall a, b \in G$$

$$\text{Now, } (ab)^{-1} = ab^{-1} \because a, b \in G \Rightarrow ab \in G$$

$$\Rightarrow b^{-1}a^{-1} = ab \Rightarrow ba = ab \quad \forall a, b \in G$$

Hence, commutative law holds in group G .

Therefore, G is abelian group.

17. (b) Since, $P(3)$ is true.

Assume $P(k)$ is true $\Rightarrow P(k+1)$ is true means, if $P(3)$ is true $\Rightarrow P(4)$ is true $\Rightarrow P(5)$ is true and so on. So, statement is true for all $n \geq 3$.

$$18. (b) \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

(using $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$)

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

$$= (b-a)(c-a)(c+a-b-a)$$

$$\Rightarrow (b-a)(c-a)(c-b)$$

$$= k(a-b)(b-c)(c-a) \quad (\text{given})$$

$$\Rightarrow k = 1$$

19. (c) Number of required ways = ${}^{22-4-2}C_{11-2}$
 $= {}^{16}C_9$

$$20. (b) \text{ Given, } \Delta_1 = \begin{vmatrix} x & a & b \\ b & x & a \\ a & b & x \end{vmatrix}, \Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$$

$$\therefore \frac{d}{dx}(\Delta_1) = \begin{vmatrix} 1 & 0 & 0 \\ b & x & a \\ a & b & x \end{vmatrix} + \begin{vmatrix} x & a & b \\ 0 & 1 & 0 \\ a & b & x \end{vmatrix} + \begin{vmatrix} x & a & b \\ b & x & a \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} x & a \\ b & x \end{vmatrix} + \begin{vmatrix} x & b \\ a & x \end{vmatrix} + \begin{vmatrix} x & a \\ b & x \end{vmatrix} = 3\Delta_2$$

$$\therefore \frac{d}{dx}(\Delta_1) = 3\Delta_2$$

21. (a) Let $I = \int_{-1/2}^{1/2} \frac{dx}{(1-x^2)^{1/2}}$

$$\text{Again, let } f(x) = \frac{1}{(1-x^2)^{1/2}}$$

$$f(-x) = \frac{1}{(1-x^2)^{1/2}} = f(x)$$

$$\therefore I = 2 \int_0^{1/2} \frac{dx}{(1-x^2)^{1/2}} = \left[2 \sin^{-1} \frac{x}{1} \right]_0^{1/2}$$

$$= 2 \sin^{-1} \frac{1}{2} = 2 \times \frac{\pi}{6} = \frac{\pi}{3}$$

$$22. (c) \lim_{n \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+5n} \right]$$

$$= \lim_{n \rightarrow \infty} \sum_{r=0}^{5n} \frac{1}{n+r} = \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{r=0}^{5n} \frac{n}{n+r} \right]$$

$$= \lim_{n \rightarrow \infty} \left[\frac{1}{n} \sum_{r=0}^{5n} \frac{1}{1+(r/n)} \right]$$

$$= \int_0^5 \frac{1}{1+x} dx = [\log(1+x)]_0^5$$

$$= \log 6 - \log 1 = \log 6$$

$$23. (b) \left[5 + \left(\frac{dy}{dx} \right)^2 \right]^{5/3} = x^5 \left[\frac{d^2y}{dx^2} \right]$$

$$\Rightarrow \left[5 + \left(\frac{dy}{dx} \right)^2 \right]^5 = x^{15} \left[\frac{d^2y}{dx^2} \right]^3$$

$$\therefore \text{Degree} = 3$$

24. (c) The difference of the focal distance at any point on the hyperbola is same as length of transverse axis.

25. (b) Since, a ray makes angles α, β, γ with the positive direction of the axes.

$$\therefore \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\Rightarrow 1 - \sin^2 \alpha + 1 - \sin^2 \beta + 1 - \sin^2 \gamma = 1$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

26. (b) Given, $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \leq x \leq 2 \\ 37 - x, & 2 \leq x \leq 3 \end{cases}$

On differentiating, we get

$$f'(x) = \begin{cases} 6x + 12, & -1 \leq x \leq 2 \\ -1, & 2 \leq x \leq 3 \end{cases}$$

$$\Rightarrow f'(x) > 0 \text{ for } -1 \leq x \leq 2$$

$$\therefore f(x) \text{ is increasing in } [-1, 2]$$

$$\lim_{x \rightarrow 2^+} f'(x) = -1 \text{ and } \lim_{x \rightarrow 2^-} f'(x) = 24$$

$$\therefore f'(2) \text{ does not exist.}$$

27. (c) Given,

$$\mathbf{a} = \frac{1}{7} (2\hat{i} + 3\hat{j} + 6\hat{k}), \mathbf{b} = \frac{1}{7} (3\hat{i} - \lambda\hat{j} + 2\hat{k})$$

Since, the vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.

$$\therefore \mathbf{a} \cdot \mathbf{b} = 0$$

$$\Rightarrow \frac{1}{7} (2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot \frac{1}{7} (3\hat{i} - \lambda\hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow \frac{1}{49} (6 - 3\lambda + 12) = 0$$

$$\Rightarrow 3\lambda = 18$$

$$\Rightarrow \lambda = 6$$

28. (c) The locus of the point of intersection of two perpendicular tangents to a circle is called director circle.

29. (b) The equation of given curve is

$$y = x^4 - 2x^3 + x^2 + 3$$

On differentiating w.r.t. x , we get

$$\frac{dy}{dx} = 4x^3 - 6x^2 + 2x$$

Again, differentiating, we get

$$\frac{d^2y}{dx^2} = 12x^2 - 12x + 2$$

Put, $\frac{dy}{dx} = 0$ for maxima or minima.

$$\Rightarrow 2x(2x^2 - 3x + 1) = 0$$

$$\Rightarrow x = 0, 1, \frac{1}{2}$$

$$\therefore \left(\frac{d^2y}{dx^2} \right)_{(x=0)} = 2$$

$\therefore$ Function is minimum at $x = 0$

$$\text{and } \left(\frac{d^2y}{dx^2} \right)_{(x=1)} = 12 - 12 + 2 = 2$$

$\therefore$ Function is minimum at $x = 1$

$$\text{Also, } \left(\frac{d^2y}{dx^2} \right)_{(x=\frac{1}{2})} = -1 < 0$$

$\therefore$ Function is maximum at $x = \frac{1}{2}$

$$\therefore \text{Required area} = \int_0^1 (x^4 - 2x^3 + x^2 + 3) dx$$

$$= \left[\frac{x^5}{5} - \frac{2x^4}{4} + \frac{x^3}{3} + 3x \right]_0^1$$

$$= \frac{1}{5} - \frac{1}{2} + \frac{1}{3} + 3$$

$$= \frac{91}{30} \text{ sq unit}$$

$$30. (b) \int (e^{a \log x} + e^{x \log a}) dx = \int (e^{\log x^a} + e^{\log a^x}) dx$$

$$= \int (x^a + a^x) dx = \frac{x^{a+1}}{a+1} + \frac{a^x}{\log_e a} + c$$

$$31. (d) \lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{4n^2 - 1}} + \frac{1}{\sqrt{4n^2 - 2^2}} + \dots + \frac{1}{\sqrt{4n^2}} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{1}{\sqrt{4 - \left(\frac{1}{n}\right)^2}} + \frac{1}{\sqrt{4 - \left(\frac{2}{n}\right)^2}} + \dots + \frac{1}{\sqrt{4 - \left(\frac{n}{n}\right)^2}} \right]$$

$$= \lim_{h \rightarrow 0} h \left[\frac{1}{\sqrt{4 - h^2}} + \frac{1}{\sqrt{4 - (2h)^2}} + \dots + \frac{1}{\sqrt{4 - (nh)^2}} \right]$$

$$= \int_0^1 \frac{dx}{\sqrt{4 - x^2}} = \left[\sin^{-1} \frac{x}{2} \right]_0^1$$

$$= \sin^{-1} \left(\frac{1}{2} \right) - \sin^{-1} (0) = \frac{\pi}{6}$$

32. (b) Let $I = \int \frac{dx}{x^2 + 4x + 13}$

$$= \int \frac{dx}{x^2 + 4x + 4 + 3^2} = \int \frac{dx}{(x+2)^2 + 3^2}$$

$$= \frac{1}{3} \tan^{-1} \left(\frac{x+2}{3} \right) + c$$

33. (b) Equation of tangent at $(a, 0)$ is $x = a$ and the point of intersection of $x = a$ and the asymptotes will be obtained by solving $x = a$ and the equation of asymptotes $bx \pm ay = 0$

$$\therefore ab \pm ay = 0$$

$$\Rightarrow y = \pm b$$

Now, point of intersections are (a, b) and $(a, -b)$.

$$\therefore \text{Area of triangle} = \frac{1}{2}(ab \times 2) = ab \text{ sq unit}$$

34. (c) The equation of line passing through the origin is given by

$$y = mx$$

On differentiating w.r.t. x , we get

$$\frac{dy}{dx} = m \Rightarrow \frac{dy}{dx} = \frac{y}{x} \quad [\text{from Eq. (i)}]$$

$$\Rightarrow x \frac{dy}{dx} - y = 0$$

35. (b) $\therefore PQ = \begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix} \begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$

$$= \begin{bmatrix} -i^2 - i^2 & i^2 + i^2 \\ 0 + 0 + i^2 & -i^2 \\ i^2 & -i^2 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$$

36. (a) $x = \sin^{-1} (3t - 4t^3)$

and $y = \cos^{-1} (\sqrt{1-t^2})$

Let $t = \sin \theta$

$$\therefore x = \sin^{-1} (3 \sin \theta - 4 \sin^3 \theta)$$

$$= \sin^{-1} (\sin 3\theta) = 3 \sin^{-1} t$$

$$\therefore \frac{dx}{dt} = \frac{3}{\sqrt{1-t^2}}$$

and $y = \cos^{-1} (\sqrt{1-\sin^2 \theta})$

$$= \cos^{-1} (\sqrt{\cos^2 \theta}) = \sin^{-1} t$$

$$\therefore \frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1/\sqrt{1-t^2}}{3/\sqrt{1-t^2}} = \frac{1}{3}$$

37. (d) Since, a and b are the roots of equation

$$x^2 - 3x + 1 = 0$$

$$\therefore a + b = 3 \text{ and } ab = 1$$

Now, the given roots are $a - 2$ and $b - 2$

$$\therefore \text{Sum of roots} = a - 2 + b - 2$$

$$= 3 - 4 = -1$$

$$\text{and products of roots} = (a - 2)(b - 2)$$

$$= ab - 2(a + b) + 4$$

$$= 1 - 6 + 4 = -1$$

$$\therefore \text{Required equation is } x^2 + x - 1 = 0$$

$$\text{Which is equivalent to } x^2 + px + q = 0$$

$$\therefore p = 1 \text{ and } q = -1$$

38. (a) Since, $\mathbf{AB} = \hat{i} + 5\hat{j} - \hat{k}$

and $\mathbf{BC} = -3\hat{j} - \hat{k}$

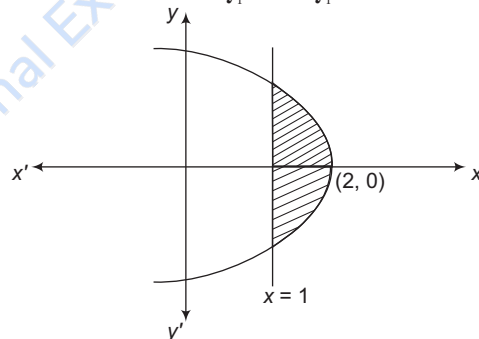
$$\therefore \text{Projection of } \mathbf{AB} \text{ on } \mathbf{BC} = \frac{\mathbf{AB} \cdot \mathbf{BC}}{|\mathbf{BC}|}$$

$$= \frac{(\hat{i} + 5\hat{j} - \hat{k}) \cdot (-3\hat{j} - \hat{k})}{\sqrt{(-3)^2 + (-1)^2}} = \frac{-15 + 1}{\sqrt{10}} = -\frac{14}{\sqrt{10}}$$

39. (a) Given equation can be rewritten as

$$y^2 = -4(x - 2)$$

$$\therefore \text{Required area} = 2 \int_1^2 y \, dx = \int_1^2 \sqrt{8 - 4x} \, dx$$



$$= \left[\frac{(8 - 4x)^{3/2}}{\frac{3}{2}(-4)} \right]_1^2 = -\frac{1}{6} [0 - 8]$$

$$= \frac{4}{3} \text{ sq unit}$$

40. (b) The given equation is

$$(2y - 1) \, dx - (2x + 3) \, dy = 0$$

or it can be rewritten as

$$\frac{dx}{(2x + 3)} - \frac{dy}{(2y - 1)} = 0$$

On integrating both sides, we get

$$\int \frac{1}{(2x+3)} dx - \int \frac{1}{(2y-1)} dy = 0$$

$$\Rightarrow \frac{1}{2} \log(2x+3) - \frac{1}{2} \log(2y-1) = \frac{1}{2} \log c$$

$$\Rightarrow \log \left(\frac{2x+3}{2y-1} \right) = \log c \Rightarrow \frac{2x+3}{2y-1} = c$$

41. (a) According to the question,

$$5 \left(-\frac{1}{2} \right)^{n-1} = \frac{5}{1024}$$

$$\Rightarrow \left(-\frac{1}{2} \right)^{n-1} = \left(-\frac{1}{2} \right)^{10}$$

$$\Rightarrow n-1=10$$

$$\Rightarrow n=11$$

42. (a) Give that, $77x \equiv 88 \pmod{5}$

$\Rightarrow 77x - 88$ is divisible by 5.

$$\therefore x = 4$$

$$[\because 77 \times 4 - 88 = 308 - 88 = 220 \text{ is divisible by } 5]$$

43. (b) Let p : Paris is in France.

q : London is in England.

$\therefore$ We have, $p \wedge q$

Its negation is $\sim(p \wedge q) = \sim p \vee \sim q$

ie, Paris is not in France or London is not in England.

44. (a) Given, ${}^{2n}C_3 : {}^nC_2 = \frac{44}{3}$

$$\Rightarrow \frac{2n!}{(2n-3)!3!} \times \frac{2!(n-2)!}{n!} = \frac{44}{3}$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{44}{3}$$

$$\Rightarrow \frac{2}{3} \times 2(2n-1) = \frac{44}{3}$$

$$\Rightarrow 2n-1=11 \Rightarrow 2n=12 \Rightarrow n=6$$

45. (a) Now, $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{3x + \tan^2 x}{x}$

$$= \lim_{x \rightarrow 0} \frac{3 + 2 \tan x \sec^2 x}{1}$$

$$= 3$$

Since, $f(x)$ is continuous at $x = 0$

$$\therefore f(0) = 3$$

46. (c) $0.2 + 0.22 + 0.222 + \dots n \text{ terms}$

$$= 2(0.1 + 0.11 + 0.111 + \dots n \text{ terms})$$

$$= 2 \left(\frac{1}{10} + \frac{11}{100} + \frac{111}{1000} + \dots n \text{ terms} \right)$$

$$= \frac{2}{9} \left(\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \dots n \text{ terms} \right)$$

$$= \frac{2}{9} \left(1 - \frac{1}{10} + 1 - \frac{1}{100} + 1 - \frac{1}{1000} + \dots n \text{ terms} \right)$$

$$= \frac{2}{9} \left[n - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots n \right) \right]$$

$$= \frac{2}{9} \left[n - \frac{1}{10} \left\{ 1 - \left(\frac{1}{10} \right)^n \right\} \right]$$

$$= \frac{2}{9} \left[n - \frac{1}{10} \times \frac{10}{9} \cdot \left(\frac{10^n - 1}{10^n} \right) \right]$$

$$= \frac{2}{9} \left[n - \frac{1}{9} (1 - 10^{-n}) \right]$$

47. (d) $(1 + \omega - \omega^2)^5 + (1 - \omega + \omega^2)^5$

$$= (-\omega^2 - \omega^2)^5 + (-\omega - \omega)^5$$

$$= -32(\omega^2)^5 + (-32)\omega^5$$

$$= -32(\omega + \omega^2) = 32$$

48. (d) Lines, $x + y = 6$, $2x + y = 4$ and $x + 2y = 5$ intersect at points $(-2, 8)$, $(7, -1)$ and $(1, 2)$.

Now, all these points lie on

$$x^2 + y^2 - 17x - 19y + 50 = 0$$

49. (b) $\lim_{x \rightarrow 0} \frac{\sin 3x - \sin x}{\sin x}$

$$= \lim_{x \rightarrow 0} \frac{2 \cos 2x \sin x}{\sin x} = 2$$

50. (c) Product of roots of equation is

$$\frac{2m-1}{m} = (-1) \text{ given}$$

$$\Rightarrow 2m-1 = -m$$

$$\Rightarrow 3m=1 \Rightarrow m = \frac{1}{3}$$

51. (a) Let $I = \int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$... (i)

$$I = \int_0^{2a} \frac{f(2a-x)}{f(2a-x)+f(x)} dx$$
 ... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{2a} \frac{f(2a-x) + f(x)}{f(2a-x) + f(x)} dx = \int_0^{2a} dx = 2a$$

$$\Rightarrow I = a$$

$$52. (c) \text{ Let } I = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int \frac{\sqrt{\tan x}}{\tan x} \sec^2 x dx$$

$$= \int \frac{1}{\sqrt{\tan x}} \cdot \sec^2 x dx = \frac{\sqrt{\tan x}}{1/2} = 2\sqrt{\tan x} + c$$

53. (a) The given equation of curve is $(1+x^2)y = 2-x$, it meets x -axis at $(2, 0)$
or it can be rewritten as $y = \frac{2-x}{1+x^2}$

On differentiating w.r.t. x , we get

$$\frac{dy}{dx} = \frac{(1+x^2)(-1) - (2-x)(2x)}{(1+x^2)^2}$$

$$= \frac{-1-x^2-4x+2x^2}{(1+x^2)^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,0)} = \frac{-1-4-8+8}{25} = -\frac{1}{5}$$

$\therefore$ Required equation of tangent is

$$(y-0) = -\frac{1}{5}(x-2) \Rightarrow x+5y=2$$

54. (b) $\lim_{x \rightarrow 0} \left(\frac{1 - \cos mx}{1 - \cos nx} \right)$

$$= \lim_{x \rightarrow 0} \frac{2\sin^2(mx/2)}{(mx/2)^2} \cdot \frac{(n x/2)^2}{2\sin^2(n x/2)} \cdot \frac{m^2}{n^2} = \frac{m^2}{n^2}$$

55. (c) $|\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2$
 $+ 2(\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a})$
 $= a^2 + a^2 + a^2$
 $(\because \mathbf{a}, \mathbf{b} \text{ and } \mathbf{c} \text{ are mutually perpendicular})$
 $\Rightarrow |\mathbf{a} + \mathbf{b} + \mathbf{c}| = \sqrt{3}a$

56. (a) We know that K_n , the complete graph of n vertices is a connected graph in which degree of each vertex is $n-1$. Since, a graph is Eulerian if and only if it is connected and degree of each vertex is even, we conclude that K_n is an Euler graph if and only if n is odd.

57. (a) Now, $5^2 \equiv 4 \pmod{7}$
 $5^4 \equiv 16 \pmod{7} = 2 \pmod{7}$
 $(5^4)^5 \equiv 2^5 \pmod{7}$
i.e. $5^{20} \equiv 32 \pmod{7} = 4 \pmod{7}$
 $\therefore$ Remainder is 4.

58. (c) Since, $a \cdot b = (a * b) + 3$
 $\therefore 4 \cdot 7 = (4 * 7) + 3 = 7 + 3 = 10$

59. (b) Total degree $= 2(n-1) = 2n-2$

Let p be the number of pendant vertices. Then $(n-p-1)$ are vertices with degree 3, one vertex of degree 2. Total degree $= 2 + p + 3(n-p-1)$

$$\Rightarrow 2n-2 = -1-2p+3n$$

$$\Rightarrow p = \frac{n+1}{2} = \frac{9+1}{2} \quad (\because n=9 \text{ given})$$

$$= 5$$

60. (d) $\frac{1}{\log_{25} 10} + \frac{1}{\log_4 10} + \frac{1}{\log_{\sqrt{2}} 10} + \frac{1}{\log_{\sqrt{5}} 10}$

$$= \frac{\log 25}{\log 10} + \frac{\log 4}{\log 10} + \frac{\log \sqrt{2}}{\log 10} + \frac{\log \sqrt{5}}{\log 10}$$

$$= \frac{2 \log 5 + \frac{1}{2} \log 5}{\log 10} + \frac{2 \log 2 + \frac{1}{2} \log 2}{\log 10}$$

$$= \frac{5 \left(\frac{\log 5 + \log 2}{\log 10} \right)}{2} = \frac{5 \log 10}{2 \log 10} = \frac{5}{2}$$

61. (a) Here, $a=2, b=3, h=\frac{5}{2}$

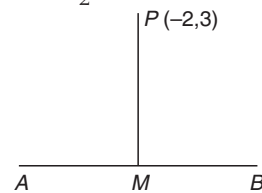
$$\therefore \tan \theta = \frac{2\sqrt{\left(\frac{5}{2}\right)^2 - 6}}{2+3}$$

$$\Rightarrow m = \frac{1}{5}$$

62. (b) Let AB be the line $2x - y - 3 = 0$... (i)

Its slope $= 2$

$$\text{Slope of } PM = -\frac{1}{2}$$



Equation of PM is

$$y-3 = -\frac{1}{2}(x+2)$$

$$\Rightarrow x+2y-4=0$$

... (ii)

On solving Eqs. (i) and (ii), we get

$$x=2, y=1$$

$\therefore$ Foot of perpendicular is $(2, 1)$.

63. (a) $\int_{-\pi/2}^{\pi/2} |\sin x| dx = 2 \int_0^{\pi/2} \sin x dx$

$$= 2[-\cos x]_0^{\pi/2} = 2$$

64. (b) $\therefore \begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix.

$$\therefore \begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$$

$$\Rightarrow (2+x)(5-2) - 3(-5-2x) + 4(1+x) = 0$$

$$\Rightarrow 6 + 3x + 15 + 6x + 4 + 4x = 0$$

$$\Rightarrow 13x + 25 = 0$$

$$\Rightarrow x = -\frac{25}{13}$$

65. (c) $(b^2 + c^2) a \cos A + (c^2 + a^2) b \cos B$
 $+ (a^2 + b^2) c \cos C$
 $= ab(b \cos A + a \cos B) + bc(b \cos C + c \cos B)$
 $+ ca(a \cos C + c \cos A)$
 $= abc + abc + abc = 3abc$

66. (a) Given, $y = a(1 - \cos x)$

On differentiating w.r.t. x , we get

$$\frac{dy}{dx} = a \sin x$$

Again differentiating w.r.t. x , we get

$$\frac{d^2y}{dx^2} = a \cos x$$

Put $\frac{dy}{dx} = 0$ for maxima or minima

$$\Rightarrow a \sin x = 0 \Rightarrow x = \pi$$

$$\therefore \left(\frac{d^2y}{dx^2} \right)_{x=\pi} = a \cos \pi = -a$$

$\therefore$ Function is maximum at $x = \pi$.

67. (b) Volume of parallelopiped = $\begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$

$$= 2(-1) + 3(-1+3) = -2 + 6 = 4 \text{ cu unit.}$$

68. (a) It is not possible for wheel W_n ($n \geq 3$) to be bipartite.

69. (c)
$$\begin{array}{r} 364 \overline{)462(1} \\ \underline{364} \\ 98 \\ \underline{98} \\ 0 \end{array}$$

$\therefore$ GCD of 364 and 462 is 14.

70. (d) Given system of equations are

$$x + 4y - 2z = 3$$

$$3x + y + 5z = 7$$

and $2x + 3y + z = 5$

$$\therefore \Delta = \begin{vmatrix} 1 & 4 & -2 \\ 3 & 1 & 5 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= 1(1-15) - 4(3-10) - 2(9-2)$$

$$= -14 + 28 - 14 = 0$$

$$\text{and } \Delta_2 = \begin{vmatrix} 1 & 3 & -2 \\ 3 & 7 & 5 \\ 2 & 5 & 1 \end{vmatrix}$$

$$= 1 \neq 0$$

$\therefore$ No solution will exist.

English & General Aptitude

16. (a) All are the names of rivers.

17. (c) 'Leap' is the movement of rabbit; 'Frisk' is the movement of 'Lamb' and 'Trot' is the movement of 'Donkey'.

18. (b) All are stationery goods.

19. (c) All are week days.

20. (a) They have two vowels 'a' and 'e'.

21. (d) None of the options is correct as they are the names of currencies.

22. (d) 'Jaipur' is the capital of 'Rajasthan'; 'Bengaluru' is the capital of 'Karnataka' and 'Mumbai' is the capital of 'Maharashtra'.

23. (c) 'Colombo' is the capital of Sri Lanka; 'Kathmandu' is the capital of 'Nepal' and 'Havana' is the capital of 'Cuba'.

24. (d) 'Squeak' is the sound produced by Mice; Hiss is the sound produced by Snake and Howl is the sound produced by Jackal.

25. (b) 'Kathak' is the classical dance of North India; 'Bharatnatyam' is the classical dance of Tamilnadu while 'Odissi' is the classical dance of Orissa.