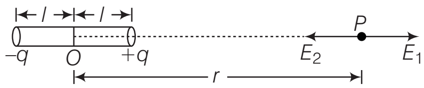
MET 2018 Answer Key PDF

Physics

1. *(b)* For a dipole of length $2l$



$$E = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{2pr}{(r^2 - l^2)^2}$$

When $(l << r)$, then l^2 may be neglected in comparison to r^2.

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0 K}\frac{2pr}{r^4} = \frac{1}{4\pi\varepsilon_0 K}\frac{2p}{r^3}\ \text{N/C}$$

$$\Rightarrow \quad E \propto \frac{1}{r^3}$$

2. *(b)* Force due to magnetic field is

$$F = qvB\sin\theta$$

when $\quad \theta = 90°$

$$F = qvB \qquad \ldots(\text{i})$$

Force due to electric field E is

$$F = qE \qquad \ldots(\text{ii})$$

Equating Eqs. (i) and (ii), we get

$$|\vec{\mathbf{E}}| = v|\vec{\mathbf{B}}|$$

3. *(b)* The magnetic field is given by

$$B = \mu_0 ni$$

where $\quad \mu_0 = 4\pi \times 10^{-7}\ \text{T mA}^{-1}$,

$$n = \frac{1000}{50 \times 10^{-2}},\ i = 5\ \text{A}$$

$$\therefore \quad B = 4\pi \times 10^{-7} \times \frac{1000 \times 10^{-2}}{50 \times 10^{-2}} \times 5$$

$$B = 1.26 \times 10^{-2}\ \text{T}$$

4. *(a)* From transformer ratio

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$$\Rightarrow \quad V_s = \frac{V_p \times N_s}{N_p}$$

$$= \frac{220 \times 40000}{200}$$

$$= 44000\ \text{V}$$

Potential difference per turn is

$$\frac{V_s}{N_s} = \frac{44000}{40000} = 1.1\ \text{V}$$

5. *(c)* Slit width, $w = \dfrac{D\lambda}{d}$

Given, $\quad D = 80\text{cm} = 80 \times 10^{-2}\ \text{m}$,

$$\lambda = 6000\text{Å} = 6000 \times 10^{-10}\ \text{m},$$

$$d = \frac{5}{2}\text{mm} = \frac{5 \times 10^{-3}}{2}\ \text{m}.$$

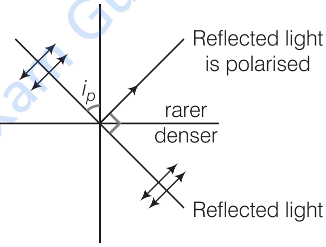
$$\therefore \quad w = \frac{80 \times 10^{-2} \times 6000 \times 10^{-10} \times 2}{5 \times 10^{-3}}$$

$$w = 0.192 \times 10^{-3}\text{m} = 0.192\ \text{mm}$$

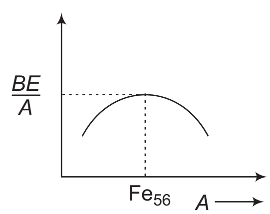
6. *(d)* According to Brewster's law, when unpolarised light is incident at polarising angle i_p on an interface separating a rarer medium from a denser medium of refractive index μ, such that

$$\mu = \tan i_p$$

then light reflected in the rarer medium is completely polarised.



7. *(b)* Binding energy per nucleon increases with atomic number and is maximum for iron, after that it decreases.



8. *(a)* From lens formula

$$\frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(\text{i})$$

Also, $\quad {}_l\mu_g = \dfrac{\mu_g}{\mu_l} = \dfrac{1.5}{1.6}$

$$\therefore \quad \frac{1}{f'} = ({}_l\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

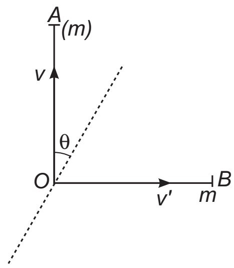
$$\frac{1}{f'} = \left(\frac{15}{16} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots\text{(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{f}{f'} = \frac{\left(\frac{15}{16} - 1\right)}{(1.5 - 1)} = -\frac{1}{16 \times 5}$$

$$\Rightarrow \quad f = -160 \text{ cm}$$

9. *(d)* From law of conservation of momentum



$$mv - mv'\cos\theta = 0 \quad \ldots\text{(i)}$$

$$mv - mv'\sin\theta = 0 \quad \ldots\text{(ii)}$$

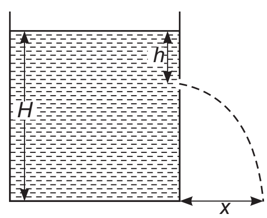
From Eqs. (i) and (ii), we get

$$\theta = 45°$$

$$v' = \frac{v}{\cos\theta} = v\sqrt{2}$$

10. *(b)* When two identical transverse or longitudinal progressive waves travel in a bounded medium with the same speed, but in opposite directions, then by their superposition a new type of wave is produced which appears stationary in the medium. Thus, is called stationary wave.

11. *(c)* From Bernoulli's theorem



$$p + 0 + \rho g H = p + \frac{1}{2}\rho v^2 + \rho g (H - h)$$

$$\frac{1}{2}\rho v^2 = \rho g h$$

$$\Rightarrow \quad v = \sqrt{2gh}$$

12. *(b)* In N forces of equal magnitude works on a single point and their resultant is zero, then angle between any two forces is

$$\theta = \frac{360°}{N} = \frac{360°}{3} = 120°$$

If these 3 vectors are represented by three sides of triangle, then they form equilateral triangle.

13. *(c)* $\quad F = Bev$ and $B = \dfrac{\mu_0 i}{2\pi r}$

$$\therefore \quad F = \frac{\mu_0 i}{2\pi r} \times ev$$

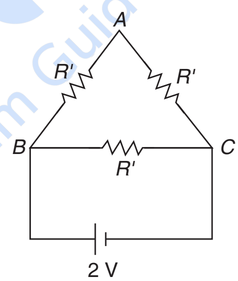
$$= \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.1} \times 1.6 \times 10^{-19} \times 5 \times 10^6$$

$$= 8 \times 10^{-18} \text{ N}$$

14. *(c)* $\quad R' = \dfrac{9}{3} = 3\Omega$

$$R_{net} = 2\,\Omega$$

PD across AB



$$V' = i \times R' = \frac{V}{R_{net}} \times R = \frac{2}{2} \times 3 = 3\text{ V}$$

15. *(c)* $E_1 + E_2 = k \times 58 \quad \ldots\text{(i)}$

$$E_1 - E_2 = k \times 29 \quad \ldots\text{(ii)}$$

$$\therefore \quad \frac{E_1 + E_2}{E_1 - E_2} = 2$$

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{3}{1}$$

16. *(b)* When the lift is stationary $w = mg$

$$\Rightarrow \quad 49 = m \times 9.8$$

$$m = 5 \text{ kg}$$

When the lift is moving downward with an acceleration

$$R = m(9.8 - a) = 5(9.8 - 5) = 24 \text{ N}$$

17. *(c)* From Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = \left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

Therefore,

$$\frac{60^\circ - 40^\circ}{7} \propto \left(\frac{60^\circ + 40^\circ}{2} - 10^\circ\right) \quad \ldots\text{(i)}$$

$$\frac{40^\circ - 28^\circ}{t} \propto \left(\frac{40^\circ + 28^\circ}{2} - 10^\circ\right) \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$t = 7\text{ s}$$

18. *(b)* Einstein's equation is

$$V = \frac{h\nu}{e} - \frac{h\nu_0}{e}$$

Here, slope of line $= \dfrac{h}{e}$

$\Rightarrow h =$ Planck's constant = slope $\times\, e$

19. *(d)* Width of central maximum

$$\beta = \frac{D\lambda}{d}, d \text{ being slit width.}$$

If the slit width is increased the central maximum becomes narrower and more light reaches the region.

20. *(d)* Since, collision is elastic hence, velocities of bobs A and B get interchanged. Thus, A will come to rest and B will move with the velocity of A.

21. *(a)* $I = k\tan\theta$, k being reduction factor and $I \propto \dfrac{1}{R}$

$$\Rightarrow \quad \frac{R + r}{(R + r) + R'} = \frac{\tan\theta'}{\tan\theta}$$

$$\Rightarrow \quad \frac{4 + 2}{6 + R'} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$\Rightarrow \quad R' = 12\,\Omega$$

22. *(d)*

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.9}{1 - 0.9} = 9$$

$$\Rightarrow \quad \frac{\Delta I_c}{\Delta I_b} = 9$$

$$\Rightarrow \quad I_c = 9 \times 2$$

$$\Rightarrow \quad \Delta I_c = 18\,\mu\text{A}$$

23. *(a)* Energy needed = Increment in surface energy

= (surface energy of n small drops)

− (surface energy of one big drop)

$$= n\, 4\pi r^2 T - 4\pi R^2 T = 4\pi T(nr^2 - R^2)$$

24. *(a)*

$$p \propto T$$

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \quad \frac{p_2 - p_1}{p_1} = \frac{T_2 - T_1}{T_2}$$

$$\Rightarrow \quad \left[\frac{\Delta p}{p}\right]\% = \left[\frac{251 - 250}{250}\right] \times 100 = 0.4\%$$

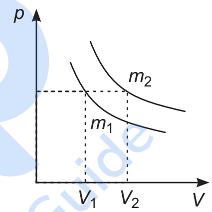
25. *(c)* Net phase difference between current and voltage

$$= \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \frac{\pi}{3} = 60^\circ$$

Hence, current leads the voltage by 60°.

26. *(b)* According to Lenz's law acceleration of the magnet will be less than the acceleration due to gravity due to the fact that emf induced in the ring opposes the motion of magnet.

27. *(c)* $pV = nRT = \dfrac{m}{M}RT$



For 1st graph, $p = \dfrac{m_1}{M}\dfrac{RT}{V_1}$

For 2nd graph, $p = \dfrac{m_2}{M}\dfrac{RT}{V_2}$

Equating the two, we get

$$\frac{m_1}{m_2} = \frac{V_1}{V_2}$$

$$\Rightarrow \quad m \propto \frac{1}{V}$$

As $V_2 > V_1 \Rightarrow m_1 < m_2$

28. *(d)* $F_{BA} = \dfrac{\mu_0 I^2}{2\pi x}$ (attractive force)

and $\quad F_{CA} = \dfrac{\mu_0 (I)(2I)}{2\pi(2x)} = \dfrac{\mu_0 I^2}{2\pi x}$ (repulsive force)

Hence, $\quad F_{BA} = -F_{CA}$

$\Rightarrow \quad F_1 = -F_2$

29. *(d)* From the problem it is clear that $\angle e = \angle r_2 = 0$

From $A = r_1 + r_2$

$$\Rightarrow \quad r_1 = A = 45^\circ$$

$$\therefore \quad \mu = \frac{\sin i}{\sin r_1} = \frac{\sin 60^\circ}{\sin 45^\circ} = \sqrt{\frac{3}{2}}$$

Also, from $i + e = A + \delta$

$$\Rightarrow \quad 60 + 0 = 45 + \delta$$

$$\Rightarrow \quad \delta = 15^\circ$$

30. *(c)* $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$= (1.6 - 1)\left[\frac{1}{60} - \frac{1}{\infty}\right] = \frac{1}{100}$$

$f = 100$ cm

31. *(b)* Conservation of linear momentum gives

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$\Rightarrow \quad m_2v_2 = \frac{3u}{2} \quad \ldots(i)$

Conservation of kinetic energy gives

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$\Rightarrow \quad m_2v_2^2 = \frac{15u^2}{8} \quad \ldots(ii)$

Hence, on solving Eqs. (i) and (ii), we get

$$m_2 = 1.2 \text{ kg}$$

32. *(b)* Given,

$$10 = \frac{G \times 6 \times 10^{24} \times 1}{(6.4 \times 10^6)^2}$$

$\Rightarrow \quad G = 6.82 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

Therefore,

$$F_{\text{satellite}} = \frac{6.82 \times 10^{-11} \times 6 \times 10^{24} \times 200}{\left(\frac{3 \times 6.4 \times 10^6}{2}\right)^2} = 889 \text{ N}$$

33. *(a)* Electron concentration is greater than hole concentration in *n*-type semiconductor.

34. *(b)* *p-n* junction diode will be forward biased, if greater positive potential is given to *p*-side. So, diodes 2, 4, 5 are forward biased.

35. *(b)* $y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$ and

$$y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \phi\right)$$

$$= a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \phi + \frac{\pi}{2}\right)$$

So, phase difference $= \phi + \frac{\pi}{2}$

and path difference $\Delta = \frac{\lambda}{2\pi}\left(\phi + \frac{\pi}{2}\right)$

36. *(c)* $I = \frac{E - e}{R} \quad \ldots(i)$

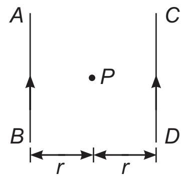
and $\eta \times 100 = \left(\frac{e}{E}\right) \times 100 \quad \ldots(ii)$

$\therefore \quad 30 = \frac{e}{50} \times 100$ [from Eqs. (ii)]

$\Rightarrow \quad e = 15$ V

Thus, $R = \frac{50 - 15}{12} = 2.9\ \Omega$

37. *(a)* At point *P*,



$$B_{\text{net}} = B_{AB} + B_{CD}$$

$$= \frac{\mu_0 i}{2r} + \frac{\mu_0 i}{2r} = \frac{\mu_0 i}{r}$$

38. *(c)* $S = \frac{I_gG}{I - I_g} = \frac{10^{-3} \times 20}{1 - 1 \times 10^{-3}} = 0.02\ \Omega$

39. *(d)* $\frac{V_s}{V_p} = \frac{I_s}{I_p}$

$\Rightarrow \quad I_s = I_p \times \frac{V_s}{V_p} = 5 \times \frac{220}{22000} = 0.05$ A

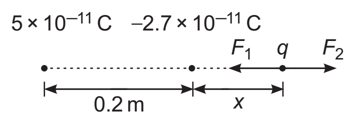
40. *(a)* $\frac{I_{\text{max}}}{I_{\text{min}}} = \frac{(\sqrt{2I} + \sqrt{I})^2}{(\sqrt{2I} - \sqrt{I})^2} = \left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right)^2$

$$= \left[\frac{(\sqrt{2} + 1)(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}\right]^2$$

$$= \left(\frac{2 + 1 + 2\sqrt{2}}{2 - 1}\right)^2$$

$$= \frac{9 + 8 + 12\sqrt{2}}{1} \approx 34$$

41. *(c)* From the question $F_1 = F_2$



$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \times \frac{5 \times 10^{-11} \times q}{(0.2 + x)^2} = \frac{1}{4\pi\varepsilon_0} \times \frac{2.7 \times 10^{-11} \times q}{x^2}$$

$\Rightarrow \quad x = 0.556$ m from IInd charge.

42. *(b)* $\frac{4}{3}\pi r^3 \times n = \frac{4}{3}\pi R^3$

$\Rightarrow \quad R = n^{1/3}\, r$

$$\therefore \quad V_{\text{big}} = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \times \frac{nq}{n^{1/3}\, r} = n^{2/3}\, V$$

43. *(a)* The charge will flow from the outer surface of smaller one to bigger one only in case (a).

44. *(c)* $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\dfrac{A}{2}}$

$$\sqrt{3} = \dfrac{\sin\left(\dfrac{60^\circ+\delta_m}{2}\right)}{\sin\dfrac{60^\circ}{2}} \Rightarrow \dfrac{\sqrt{3}}{2} = \sin\left[30^\circ + \dfrac{\delta_m}{2}\right]$$

$\Rightarrow \delta_m = 60^\circ$

45. *(c)* Magnetic field due to solenoid is independent of diameter (Because $B = \mu_0 ni$).

46. *(c)* Resistance per unit length of wire

$$x = \frac{3}{100} = 3 \times 10^{-2}\ \Omega\ \text{cm}^{-1}$$

Equivalent length of wire $l' = \dfrac{R+3}{3\times 10^{-2}}$,

R being additional resistance

$$\frac{E}{k} = \frac{R+3}{3\times 10^{-2}}$$

$$\Rightarrow \quad \frac{2}{1\times 10^{-3}} = \frac{R+3}{3\times 10^{-2}}$$

$$\Rightarrow \quad R = 57\ \Omega$$

47. *(d)*

White	Brown	Red	Silver
9	1	10^2	10%

$\therefore \quad R = 91 \times 10^2 \pm 10\%\ \Omega$

ie, $\quad R = 9.1\ \text{k}\Omega$ with tolerance $\pm 10\%$

48. *(d)*

$$E = \Delta mc^2$$

$$E = \frac{0.3}{1000} \times (3\times 10^8)^2$$

$$= 2.7 \times 10^{13}\ \text{J}$$

$$= \frac{2.7\times 10^{13}}{3.6\times 10^6} = 7.5 \times 10^6\ \text{kWh}$$

49. *(b)*

$$\lambda_{\text{photon}} = \frac{hc}{E}$$

$$\lambda_{\text{photon}} = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow \quad \frac{\lambda_{\text{photon}}}{\lambda_{\text{electron}}} = c\sqrt{\frac{2m}{E}}$$

$$\frac{\lambda_{\text{photon}}}{\lambda_{\text{electron}}} \propto \frac{1}{\sqrt{E}}$$

50. *(a)* Applying Kirchhoff's law in the given figure.

At junction A

$$i + i_1 + i_2 = 1 \quad \text{...(i)}$$

For loop (1)

$$-60i + (15+5)\, i_1 = 0$$

$$\Rightarrow \quad i_1 = 3i \quad \text{...(ii)}$$

For loop (2)

$$-(15+5)\, i_1 + 10 i_2 = 0$$

$$\Rightarrow \quad i_2 = 2 i_1 = 6\, i \quad \text{...(iii)}$$

On solving Eqs. (i), (ii) and (iii), we get

$$i = 0.1\ \text{A}$$

Chemistry

1. *(b)* $CaCO_3 + 2HCl \longrightarrow CaCl_2 + CO_2 + H_2O$
100 g $\quad$ 73 g $\quad\quad$ 44 g

100 mL of 20% HCl solution = 20 g HCl

Here $CaCO_3$ is the limiting reactant.

100 g of $CaCO_3$ gives = 44 g CO_2

$\therefore$ 20 g of $CaCO_3$ will give $= \dfrac{44}{100} \times 20$

$= 8.80$ g of CO_2

2. *(a)* Mass of $H_2 = \dfrac{0.7}{22.4} \times 2 = \dfrac{14}{224}\ \text{g} = 0.0625\ \text{g}$

$\therefore$ 0.0625 g of hydrogen is displaced by x g of metal.

$\therefore$ 1 g of hydrogen is displaced by $\dfrac{x}{0.0625}$ g of metal.

$$\therefore \quad \frac{x}{0.0625} = 28$$

$\therefore$ Mass of metal $(x) = 28 \times 0.0625 = 1.75$ g

3. *(b)* In Balmer series electrons transfer from higher orbits to 2nd orbit.

$$n_2 = 3, 4, 5, 6, \ldots.$$

$$n_1 = 2$$

4. *(d)* The electron affinity of Be is almost similar to that of Ne because they have stable (fully filled subshells) configuration.

5. *(a)* For a spontaneous process the value of ΔG must be negative.

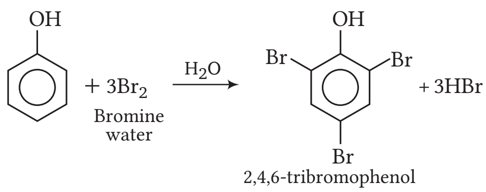
6. *(a)* pH of an acid buffer is given by Henderson equation.

$$\text{pH} = -\log K_a + \log\frac{[\text{Salt}]}{[\text{Acid}]}$$

or $\quad \text{pH} = \text{p}K_a + \log\dfrac{[\text{Salt}]}{[\text{Acid}]}$

7. *(d)* In S_N1 reaction the racemization takes place due to inversion and retention of configuration.

8. *(d)* Phenol reacts with excess of bromine water to give 2, 4, 6-tribromophenol.



9. *(c)* $\underset{\text{Acetic acid}}{CH_3COOH} + NH_3 \longrightarrow \underset{\text{Ammonium acetate}}{CH_3COONH_4}$

$\xrightarrow{\Delta} \underset{\text{Acetamide}}{CH_3CONH_2} + H_2O$

10. *(a)* In ZnS, S^- ions adopt fcc lattice in which Zn^{2+} ions are present at alternate tetrahedral sites. Hence, coordination number of S^{2-} is equal to coordination number of Zn^{2+} *i.e.* 4.

11. *(b)* $\because$ Rate $= k[A]^{3/2}[B]^{-1}$

$\therefore$ Order of reaction $= \frac{3}{2} - 1 = \frac{1}{2}$

12. *(d)* $k = \frac{2.303}{32} \log \frac{a}{a - 0.99a} = 0.1439 \text{ min}^{-1}$

$t = \frac{2.303}{0.1439} \log \frac{a}{a - 0.999a} = 48 \text{ min}$

13. *(b)* Greater the number of ions, greater will be elevation in boiling point. Hence, 0.1m $FeCl_3$ exhibits the maximum elevation in boiling point among these.

14. *(a)* The balanced equation is as

$$5I^- + IO_3^- + 6H^+ \longrightarrow 3I_2 + 3H_2O$$

15. *(c)* Lyophilic sols are more stable than lyophobic sols because the colloidal particles are solvated.

16. *(c)* The compounds which contain either $CH_3{-}\underset{|}{C}O$ or $CH_3{-}\underset{\underset{OH}{|}}{CH}^-$ group exhibit positive iodoform test. $(CH_3)_2CO$ due to presence of $CH_3{-}\underset{|}{C}{=}O$ group gives positive iodoform test

$$\begin{matrix} H_3C \diagdown \\ H_3C \diagup \end{matrix} C{=}O + NaOH + I_2 \longrightarrow CHI_3$$

$$+ CH_3COONa + H_2O$$

17. *(a)* Starch is the main storage polysaccharide of plants. It is a polymer of α-D-glucose.

$$\underset{\text{Starch}}{(C_6H_{10}O_5)_n} \xrightarrow{H^+} \underset{\text{Maltose}}{C_{12}H_{22}O_{11}} \xrightarrow{H^+} \underset{\text{D-glucose}}{C_6H_{12}O_6}$$

18. *(c)* The nitro group can attach to metal through nitrogen as $(-NO_2)$ or through oxygen as nitro $(-ONO)$, hence nitropentamine chromium (III) chloride shows linkage isomerism.

19. *(c)* $2Ag^+(aq) + 2e^- \longrightarrow 2Ag(s); E° = 0.80$ V

$Cd(s) \longrightarrow Cd^{2+}(aq) + 2e^-; E° = 0.40$ V

$2Ag^+(aq) + Cd(s) \longrightarrow Cd^{2+}(aq) + 2Ag(s)$

$E° = 1.20$ V

$\Delta G = -nFE°$

$= -2 \times 96500 \times 1.20$

$= -231.6$ kJ

20. *(d)* $\mu = \sqrt{n(n+2)}$

where, n = no. of unpaired electrons. Hence, the ion which has the maximum number of unpaired electron, will show the highest magnetic moment. Mn^{2+} has maximum 5 unpaired electrons, hence it shows the highest magnetic moment.

21. *(b)* Cane sugar reacts with conc. H_2SO_4 to give charrs. It is because H_2SO_4 acts as a dehydrating agent.

$C_{12}H_{22}O_{11} + H_2SO_4(\text{conc.}) \longrightarrow 12C + 11H_2O + H_2SO_4$

22. *(d)* For the value $n = 3$, the value of l are 0,1 and 2. Hence, m cannot have value 3.

23. *(c)* The formation of oxide ion $O^{2-}(g)$ require first an exothermic and then an endothermic step because the O^- ion will tend to resist the addition of another electron.

24. *(b)* Potassium dichromate acts like an oxidising agent in the presence of dilute H_2SO_4 as :

$Cr_2O_7^{2-} + 14H^+ + 6e^- \longrightarrow 2Cr^{3+} + 7H_2O$

$\therefore$ Equivalent mass of $K_2Cr_2O_7$

$= \frac{\text{molecular mass of } K_2Cr_2O_7}{6}$

25. *(a)* The number of atoms of the ligands that are directly bound to the central metal atom or ion by coordinate bond is known as the coordination number of the metal atom or ion.

Coordination number of metal = number of σ bonds formed by metal with ligands.

26. *(b)* Ethylene diamine is an example of bidentate ligand. Its structure is as :

$$\begin{array}{l} CH_2—NH_2 \\ | \\ CH_2—NH_2 \end{array}$$

27. *(b)* Deviation from ideal gas behaviour of a gas is expressed in terms of compressibility factor, Z

$$Z = \frac{pV}{nRT}$$

28. *(b)* Total moles of NaOH $= 4 \times 0.2 + 2 \times 0.5$

$$= 0.8 + 1.0 = 1.8$$

Total volume $= 4 + 2 = 6$ L

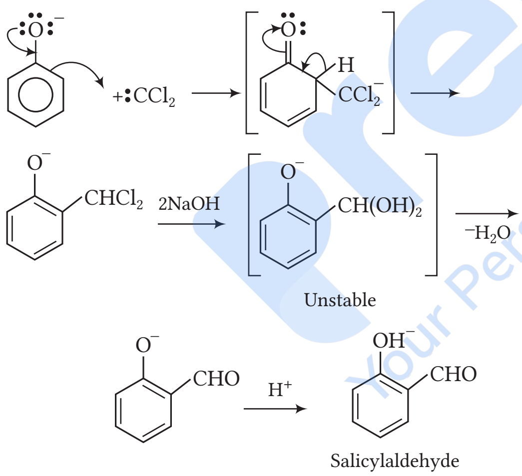
$\therefore$ Molarity $= \frac{1.8}{6}$

$= 0.3$ M

29. *(b)* In the Reimer-Tiemann reaction carbene intermediate is formed

$$CHCl_3 + OH^- \longrightarrow H_2O + CCl_3^-$$

$$\longrightarrow Cl^- + \underset{\text{Dichlorocarbene}}{:CCl_2}$$



30. *(a)* Chloral (CCl_3CHO) does not contain any α-hydrogen atom, hence it will not give aldol condensation reaction.

31. *(a)* $\Rightarrow \Delta S_{fus} = \frac{\Delta H_{fus}}{T}$

Given, $T = 27 + 273$

$= 300$ K

$\therefore \quad \Delta S_{fus} = \frac{600}{300} = 2 \text{ cal deg}^{-1}$

32. *(d)* $Al_2(SO_4)_3 \rightleftharpoons 2Al^{3+} + 3SO_4^{2-}$

$$K_{sp} = [Al^{3+}]^2 [SO_4^{2-}]^3$$

33. *(b)* Bulky groups on the carbon atom attached to the halogen atom will hinder attack by S_N2 mechanism due to steric hindrance and hence, would favour S_N1 mechanism.

34. *(b)* Bond strength depends on bond order. More is the bond order, more is the strength.

Species	**Bond order**
O_2	2
O_2^-	1.5
O_2^{2-}	1.0

Hence, the order of bond strength is as :

$$O_2 > O_2^- > O_2^{2-}$$

35. *(a)* $\because$ 24 g of Mg produce MgO $= 1$ mol

$\therefore$ 1.2 g of Mg will produce MgO $= \frac{1 \times 1.2}{24}$

$= 0.05$ mol

36. *(a)* $\Delta S_{vap} = \frac{\Delta H_{vap}}{T}$

$\because \Delta H_{vap} = 40.8 \text{ kJ mol}^{-1}, T = 373 \text{ K}$

$$\Delta S_{vap} = \frac{40.8 \times 10^3}{373} = 109.4 \text{ JK}^{-1} \text{ mol}^{-1}$$

37. *(c)* The function of $AlCl_3$ in Friedel-Craft's reaction is to produce electrophile.

$$R—Cl + AlCl_3 \longrightarrow \underset{\text{Electrophile}}{R^+} + AlCl_4^-$$

38. *(a)* O_2 has two unpaired electrons in $\overset{*}{\pi}$ molecular orbitals, hence it is paramagnetic in nature.

39. *(b)* Pent-3-en-2-ol shows both optical as well as geometrical isomerism and hence, four stereoisomers are possible.

$$CH_3—\underset{\underset{OH}{|}}{\overset{*}{C}H}—CH=CH—CH_3$$

40. *(b)* When A–B attractions are greater than A–A and B–B, then less vapours are formed and hence, solution shows negative deviation.

41. *(c)* Radon is radioactive with half-life period of 3.8 days and is used for the treatment of cancer.

42. *(c)* $He_2 : (\sigma 1s)^2\ (\overset{*}{\sigma} 1s)^2$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{2-2}{2} = 0$$

43. *(d)* 0.16 g of dibasic acid is neutralised by

$$= 25\text{mL of } 0.1\text{M NaOH}$$

$$H_2X + 2NaOH = Na_2X + H_2O$$

$$\text{Millimoles of acid} = \frac{1}{2} \times \text{millimoles of NaOH}$$

$$= \frac{1}{2} \times 25 \times 0.1$$

$$= 1.25$$

$$\because \quad \text{Millimoles} = \frac{W\ (\text{mg})}{\text{mol. mass}}$$

$$\therefore \text{Mol. mass} = \frac{\text{W (mg)}}{\text{millimoles}} = \frac{0.16 \times 1000}{1.25} = 128$$

44. *(d)* The relationship between the standard free energy ($\Delta G°$) and equilibrium constant (K) is as

$$\Delta G° = -\ 2.303\ RT \log K$$

45. *(a)* Number of moles of HCl $= \dfrac{0.192}{36.5} = 0.0053$

$$\text{Volume} = 0.5\text{L}$$

$$\text{Molarity of HCl} = \frac{0.0053}{0.5} = 1.06 \times 10^{-2}\ \text{M}$$

$$[H^+] \text{ in HCl solution} = 1.06 \times 10^{-2} \times \frac{95}{100} \approx 1 \times 10^{-2}$$

$$\text{pH} = -\log\ [H^+] = -\log\ (1 \times 10^{-2}) = 2$$

46. *(a)* Molisch test is given by all carbohydrates.
Molisch test : Carbohydrate solution + 10% alco. solution of α-naphthol + conc. H_2SO_4 along the side of test tube $\longrightarrow$ A reddish violet ring at the junction of two layers shows the presence of carbohydrate.

47. *(c)* Insulin is a peptide hormone. It is secreted from pancreas. It has 51 amino acids.

48. *(d)* As the crystal field splitting in case of complexes of metal ions in $4d$ and $5d$ transition series are much larger, the electrons of the metal ion are paired up first in lower energy set of orbitals before going to the higher energy set within the same d-subshell causing maximum pairing and thus, low spin (or spin paired) complexes are formed.

49. *(d)* $CH_2{=}CH{-}\overset{*}{C}H(CH_3){-}CH_2CH_3 \xrightarrow{H_2/Pt} CH_3CH_2{-}CH(CH_3){-}CH_2CH_3$

optically active 3-methyl-1-pentene $\qquad$ optically inactive

50. *(a)* $\because$

$$\Delta x = \Delta v$$

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \times m \cdot \Delta v = \frac{h}{4\pi}$$

$$(\Delta v)^2 = \frac{h}{4\pi m} \qquad (\because \Delta x = \Delta v)$$

$$\therefore \qquad \Delta p = m \cdot \Delta v = m\sqrt{\frac{h}{4\pi m}} = \sqrt{\frac{mh}{4\pi}}$$

$$\Delta p = \frac{1}{2}\sqrt{\frac{mh}{\pi}}$$

Mathematics

1. *(d)* $3^{|3x+4|} = 9^{2x-2}, x > 0$

When $x > 0$, then

$$|3x + 4| = 3x + 4$$

$$\therefore \quad 3^{3x+4} = 3^{2(2x-2)}$$

$$\Rightarrow \quad 3x + 4 = 4x - 4$$

$$\Rightarrow \quad x = 8$$

2. *(c)* Given, $\vec{\mathbf{a}} = 3\vec{\alpha} - \vec{\beta}, \vec{\mathbf{b}} = \vec{\alpha} + 3\vec{\beta}$ and

$|\vec{\alpha}| = |\vec{\beta}| = 2$

$$\therefore |\vec{\mathbf{a}} + \vec{\mathbf{b}}|^2 = |4\vec{\alpha} + 2\vec{\beta}|^2$$

$$= 16|\vec{\alpha}|^2 + 4|\vec{\beta}|^2 + 16|\vec{\alpha}||\vec{\beta}|\cos\frac{\pi}{3}$$

$$= 16(4) + 4(4) + 16(2 \times 2) \times \frac{1}{2}$$

$$= 64 + 16 + 32 = 112$$

$$\Rightarrow \quad |\vec{\mathbf{a}} + \vec{\mathbf{b}}| = 4\sqrt{7}$$

3. *(c)* Since, every group of prime order is cyclic. Here, given number is prime. So, it is cyclic.

4. *(a)* Given, $U_n = 2 + 2^3 + 2^5 + 2^7 + \ldots + 2^{2n+1}$

and $V_n = 1 + 4 + 4^2 + \ldots + 4^{n-1}$

$$\therefore \quad U_n = \frac{lr - a}{r - 1} = \frac{2^{2n+1} \cdot 4 - 2}{4 - 1} = \frac{2^{2n} \cdot 8 - 2}{3}$$

$$\text{and} \quad V_n = \frac{lr - a}{r - 1} = \frac{4^{n-1} \cdot 4 - 1}{4 - 1} = \frac{2^{2n} - 1}{3}$$

$$\text{Now,} \quad \frac{U_n}{V_n} = \frac{2^{2n} \cdot 8 - 2}{2^{2n} - 1} = \frac{8 - \frac{2}{2^{2n}}}{1 - \frac{1}{2^{2n}}}$$

$$\therefore \quad \lim_{n \to \infty} \frac{U_n}{V_n} = \lim_{n \to \infty} \frac{8 - \frac{2}{2^{2n}}}{1 - \frac{1}{2^{2n}}} = \frac{8 - 0}{1 - 0} = 8$$

5. *(a)* Since, $P(1) : 1 = 1$ (true)

Let $P(k) \equiv 1 + 3 + 5 + \ldots + (2k - 1) = k^2$

$$\therefore P(k+1) \equiv 1 + 3 + 5 + \ldots + (2k - 1) + 2k + 1$$

$$= k^2 + 2k + 1 = (k + 1)^2$$

So, it holds for all n.

6. *(a)* Let $f(x) = x^2 + \dfrac{1}{1 + x^2}$

On differentiating w.r.t. x, we get

$$f'(x) = 2x - \frac{1}{(1 + x^2)^2} 2x$$

For a minimum, put $f'(x) = 0$

$$\Rightarrow \quad x = 0$$

So, the function has minimum value at $x = 0$.

7. *(b)* Let the coefficient of x^{-9} is in the $(r + 1)$th term in the expansion of $\left(\dfrac{x^2}{2} - \dfrac{2}{x}\right)^9$, then

$$T_{r+1} = {}^9C_r \left(\frac{x^2}{2}\right)^{9-r} \left(-\frac{2}{x}\right)^r$$

$$= {}^9C_r \frac{x^{18-2r}}{2^{9-r}} \cdot \frac{(-1)^r \cdot 2^r}{x^r}$$

$$= {}^9C_r \frac{x^{18-3r}}{2^{9-2r}} (-1)^r$$

For coefficient of x^{-9}, put

$$x^{18-3r} = x^{-9}$$

$$\Rightarrow \quad 18 - 3r = -9 \Rightarrow 27 = 3r$$

$$\Rightarrow \quad r = 9$$

$$\therefore \text{Coefficient of } x^{-9} = {}^9C_9 \frac{1}{2^{-9}} (-1)^9 = -2^9 = -512$$

8. *(d)* $\dfrac{1}{2} + \dfrac{3}{4} + \dfrac{7}{8} + \dfrac{15}{16} + \ldots$upto n terms

$$= \left(1 - \frac{1}{2}\right) + \left(1 - \frac{1}{4}\right) + \left(1 - \frac{1}{8}\right) \ldots \text{upto } n \text{ terms}$$

$$= (1 + 1 + 1 + \ldots n \text{ terms}) - \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots n \text{ terms}\right)$$

$$= n - \frac{\frac{1}{2}\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = n - 1 + \frac{1}{2^n} = n - 1 + 2^{-n}$$

9. *(b)* $\begin{vmatrix} a & a + b & a + 2b \\ a + 2b & a & a + b \\ a + b & a + 2b & a \end{vmatrix}$

$$= \begin{vmatrix} 3(a + b) & 3(a + b) & 3(a + b) \\ a + 2b & a & a + b \\ a + b & a + 2b & a \end{vmatrix} (R_1 \to R_1 + R_2 + R_3)$$

$$= 3(a + b) \begin{vmatrix} 1 & 0 & 0 \\ a + 2b & -2b & -b \\ a + 2b & b & -b \end{vmatrix} \quad \begin{pmatrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{pmatrix}$$

$$= 3(a + b) \begin{vmatrix} -2b & -b \\ b & -b \end{vmatrix}$$

$$= 3b^2 (a + b) \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 3b^2 (a + b)(2 + 1) = 9b^2 (a + b)$$

10. (a) $\sim[p \vee (\sim p \vee q)] \equiv \sim p \wedge \sim(\sim p \vee q)$

$\equiv \sim p \wedge (\sim(\sim p) \wedge \sim q)$

$\equiv \sim p \wedge (p \wedge \sim q)$

11. (b) Since, $2 \times_5 2^{-1}$ = Identity element 1

Therefore, $(2 \times_5 2^{-1})^{-1} = 1$

12. (a) Given, $P(n) = 9^n - 8^n$

$\therefore \quad P(1) = 9 - 8 = 1$

$\therefore \; P(1) - 1 = 0$, which is divisible by 8.

Now, $P(2) = 9^2 - 8^2 = 17$

When divided by 8, it leaves the remainder 1.

13. (a) $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$

$$\Rightarrow \quad \tan^{-1}\left\{\frac{x + y + z - xyz}{1 - xy - yz - zx}\right\} = \pi$$

$\Rightarrow x + y + z - xyz = 0 \Rightarrow x + y + z = xyz$

14. (a) Since, $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line, then $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + n^2 = 1 \Rightarrow \frac{1}{4} + \frac{1}{9} + n^2 = 1$

$$\Rightarrow n^2 = 1 - \frac{1}{4} - \frac{1}{9} = \frac{23}{36} \Rightarrow n = \frac{\sqrt{23}}{6}$$

15. (c) Let $y = x^x$

Taking log on both sides, we get

$$\log y = x \log x$$

On differentiating w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \log x$$

$$\Rightarrow \quad \frac{dy}{dx} = y\,(1 + \log x) = x^x\,(1 + \log x)$$

16. (b) $(3 - x) \equiv (2x - 5) \pmod 4$

$\Rightarrow \quad 3 - x - 2x + 5$ is divisible by 4

$\Rightarrow \quad 8 - 3x$ is divisible by 4

$\therefore \quad x = 4$

17. (b) $2x^2 - 5xy + 2y^2 + 3x - 3y + 1 = 0$

$\Rightarrow \quad (x - 2y + 1)(2x - y + 1) = 0$

$\therefore$ Two equations are $x - 2y + 1 = 0$ and $2x - y + 1 = 0$.

Length of perpendiculars from $(-1, 2)$ are

$$p_1 = \left|\frac{-1 - 4 + 1}{\sqrt{1 + 4}}\right| = \frac{4}{\sqrt{5}} \text{ and } p_2 = \left|\frac{-2 - 2 + 1}{\sqrt{5}}\right| = \frac{3}{\sqrt{5}}$$

$\therefore \quad \text{Product} = p_1 \cdot p_2 = \frac{12}{5}$

18. (a) $\mathbf{a} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$,

$\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$, $\mathbf{c} = -4\hat{\mathbf{i}} - \hat{\mathbf{k}}$

$\therefore \quad \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + (\mathbf{a} \cdot \mathbf{b})\,\mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b}$

$= (4 + 1)(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 15\hat{\mathbf{k}}$

19. (c) Work done = (Force · displacement)

$= (4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \cdot (-\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$= (-4 + 9 + 10) = 15$ unit

20. (c) $f(x) = a \sin x + \frac{1}{3} \sin 3x$

On differentiating w.r.t. x, we get

$$f'(x) = a \cos x + \frac{3}{3}\cos 3x = 0, \text{ at } x = \frac{\pi}{3},$$

$$a \cos\frac{\pi}{3} + \cos\frac{\pi}{3} \cdot 3 = 0 \Rightarrow a \cdot \frac{1}{2} + (-1) = 0$$

$\therefore \quad a = 2$

21. (d) $|f(x)|$ is continuous at $x = a$, $f(x)$ is not be continuous at $x = a$.

$$\therefore \quad |f(x)| = \begin{cases} f(x), & x > a \\ -f(x), & x < a \end{cases}$$

22. (a) Given equation can be rewritten as

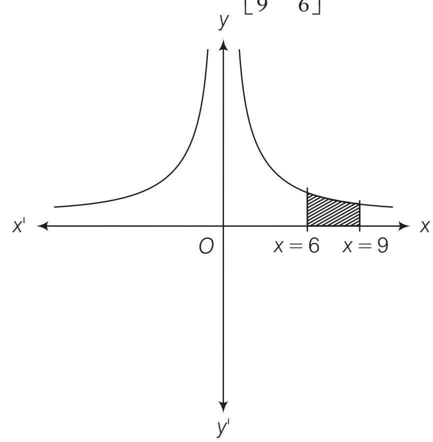
$$\frac{x}{(1 + x^2)^{1/2}}\,dx + \frac{y}{(1 + y^2)^{1/2}}\,dy = 0$$

On integrating, we get

$$\sqrt{1 + x^2} + \sqrt{1 + y^2} = c$$

23. (d) Required area $= \int_6^9 \frac{36}{x^2}\,dx = \left[-\frac{36}{x}\right]_6^9$

$$= -36\left[\frac{1}{9} - \frac{1}{6}\right] = 3 \text{ sq unit}$$



24. *(d)* Let $I = \int_0^{\pi/2} \frac{\sin^n \theta}{\sin^n \theta + \cos^n \theta} d\theta$ ...(i)

$$= \int_0^{\pi/2} \frac{\sin^n(\pi/2 - \theta)}{\sin^n(\pi/2 - \theta) + \cos^n(\pi/2 - \theta)} d\theta$$

$$= \int_0^{\pi/2} \frac{\cos^n \theta}{\cos^n \theta + \sin^n \theta} d\theta \quad \text{...(ii)}$$

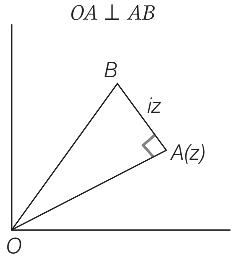
On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^n \theta + \cos^n \theta}{\cos^n \theta + \sin^n \theta} d\theta = \int_0^{\pi/2} d\theta = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

25. *(d)* Since, $iz = ze^{i\pi/2}$

This implies that iz is the vector obtained by rotating vector z in anticlock wise direction through 90°.

$\therefore \quad OA \perp AB$



So, area of $\Delta OAB = \frac{1}{2} OA \times OB = \frac{1}{2}|z||iz| = \frac{1}{2}|z|^2$

26. *(a)* Given, $f(x) = \frac{\log(1 + ax) - \log(1 - bx)}{x}$

$f(x)$ is continuous at $x = k$ and $f(0) = k$.

$$\therefore \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\log(1 + ax) - \log(1 - bx)}{x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$= \lim_{x \to 0} \left(\frac{1}{1 + ax} \cdot a + \frac{b}{1 - bx}\right) = a + b$$

$\therefore \quad a + b = f(0) = k$

27. *(c)* Since, $123 = -1(\text{mod}124)$

$125 = 1(\text{mod}124)$

$127 = 3(\text{mod}124)$

$\therefore \quad 123 \times 125 \times 127 = (-1)(1)(3) \text{ mod } (124)$

$= -3 \text{ mod } (124) = 121 \text{ mod}(124)$

28. *(b)* Given, $y = Ax^2 + Bx$ ...(i)

On differentiating, we get

$$\frac{dy}{dx} = 2Ax + B$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = 2A$$

$$\therefore \quad \frac{dy}{dx} - \frac{d^2y}{dx^2} x = B$$

From Eq. (i),

$$y = \frac{1}{2}\frac{d^2y}{dx^2}x^2 + x\left[\frac{dy}{dx} - \frac{d^2y}{dx^2}x\right]$$

$$\Rightarrow \quad y = -\frac{1}{2}x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx}$$

$$\therefore \quad x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

29. *(a)* Since, the given equation has real roots.

$\therefore$ Discriminant,

$$D = \cos^2 p - 4(\cos p - 1)\sin p \geq 0$$

$$\Rightarrow \quad D = (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0$$

$$\Rightarrow \quad 0 < p < \pi$$

30. *(a)* Given,

$$3\sin^{-1}\left(\frac{2x}{1 + x^2}\right) - 4\cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1 - x^2}\right) = \frac{\pi}{3}$$

$$\Rightarrow 6\tan^{-1} x - 8\tan^{-1} x + 4\tan^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow \quad 2\tan^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow \quad \tan^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow \quad x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

31. *(c)* We have, $\sin(\pi \cos\theta) = \cos(\pi\sin\theta)$

$$\Rightarrow \quad \cos\left(\frac{\pi}{2} - \pi\cos\theta\right) = \cos(\pi\sin\theta)$$

$$\Rightarrow \quad \frac{\pi}{2} - \pi\cos\theta = 2n\pi \pm \pi\sin\theta$$

$$\Rightarrow \quad \frac{1}{2} - \cos\theta = 2n \pm \sin\theta$$

Taking '+' sign,

$$\Rightarrow \quad \frac{1}{2} - \cos\theta = 2n + \sin\theta$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \frac{1}{2\sqrt{2}} - \sqrt{2}n$$

$$\Rightarrow \quad \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}} \quad (\text{for } n = 0)$$

32. *(a)* Since, $f(x)$ is continuous at $x = 1$.

$$\therefore \quad \lim_{h \to 0^+} f(1 + h) = f(1)$$

$$\Rightarrow \quad \lim_{h \to 0} 3 - a(1 + h)^2 = 2$$

$\Rightarrow \quad 3 - a(1+0)^2 = 2$

$\Rightarrow \quad a = 1$

33. *(b)* Required area $= \int_2^4 \left(1 + \frac{8}{x^2}\right) dx$

$$= \left[x - \frac{8}{x}\right]_2^4 = 4 - \frac{8}{4} - 2 + \frac{8}{2}$$

$$= 4 - 2 - 2 + 4$$

$$= 4 \text{ sq unit}$$

34. *(b)* Let the number of persons in the room be n, then

$$^nC_2 = 66$$

$\Rightarrow \quad \frac{n(n-1)}{2} = 66$

$\Rightarrow \quad n^2 - n - 132 = 0$

$\Rightarrow \quad (n-12)(n+11) = 0$

$\Rightarrow \quad n = 12 \qquad (\because n \neq -11)$

35. *(a)* Let one root of equation $ax^2 + bx + c = 0$ is α, then another root will be α^2.

$\therefore \quad \alpha + \alpha^2 = -\frac{b}{a} \qquad \ldots(i)$

and $\quad \alpha \cdot \alpha^2 = \frac{c}{a}$

$\Rightarrow \quad \alpha^3 = \frac{c}{a} \qquad \ldots(ii)$

From Eq. (i),

$$(\alpha + \alpha^2)^3 = -\frac{b^3}{a^3}$$

$\Rightarrow \quad \alpha^3 + \alpha^6 + 3\alpha^3(\alpha + \alpha^2) = -\frac{b^3}{a^3}$

$\Rightarrow \quad \frac{c}{a} + \frac{c^2}{a^2} + 3\frac{c}{a}\left(-\frac{b}{a}\right) = -\frac{b^3}{a^3}$

$\Rightarrow \quad a^2c + ac^2 - 3abc + b^3 = 0$

36. *(d)* $\sum_{r=1}^{\infty} [3 \cdot 2^{-r} - 2 \cdot 3^{1-r}]$

$$= 3\sum_{r=1}^{\infty} \frac{1}{2^r} - 2\sum_{r=1}^{n} 3^{1-r}$$

$$= 3\left[\frac{1}{2} + \frac{1}{2^2} + \ldots \infty\right] - 2\left[1 + \frac{1}{3} + \frac{1}{3^2} + \ldots \infty\right]$$

$$= \frac{3}{2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\right] - 2\left[1 + \frac{1}{3} + \frac{1}{3^2} + \ldots \infty\right]$$

$$= \frac{3}{2}\left(\frac{1}{1 - \frac{1}{2}}\right) - 2\left(\frac{1}{1 - \frac{1}{3}}\right)$$

$$= \frac{3}{2} \times 2 - \frac{2 \times 3}{2} = 0$$

37. *(b)* Given that,

$$x(1 + y^2)\,dx + y(1 + x^2)\,dy = 0,\ y(0) = 1$$

$\Rightarrow \quad \frac{x}{1+x^2}dx + \frac{y}{1+y^2}dy = 0$

On integrating both sides, we get

$$\frac{1}{2}\log(1 + x^2) + \frac{1}{2}\log(1 + y^2) = c$$

Now,

$$y(0) = 1 \Rightarrow c = \frac{1}{2}\log(1 + 1) = \frac{1}{2}\log 2$$

$\therefore \quad \sqrt{1 + x^2}\sqrt{1 + y^2} = \sqrt{2}$

$\Rightarrow \quad (1 + x^2)(1 + y^2) = 2$

$\Rightarrow \quad x^2 + y^2 + x^2y = 1$

38. *(d)* Given, $f(x) = \frac{x + \cos x}{x - \cos x}$

On differentiating w.r.t. x, we get

$$f'(x) = \frac{\left[\begin{array}{r}(x - \cos x)(1 - \sin x) \\ -(x + \cos x)(1 + \sin x)\end{array}\right]}{(x - \cos x)^2}$$

$$= \frac{-2x\sin x - 2\cos x}{(x - \cos x)^2}$$

39. *(b)* Diagonals of parallelogram bisects each other.

$\Rightarrow$ Diagonal BD passes through mid point of AC.

i.e. $\quad \left(\frac{5}{2}, \frac{7}{2}\right)$.

$\therefore$ Equation of diagonal passing through $(5, 1)$ and $\left(\frac{5}{2}, \frac{7}{2}\right)$ is

$$y - 1 = \frac{\frac{7}{2} - 1}{\frac{5}{2} - 5}(x - 5)$$

$\Rightarrow \quad y - 1 = -1(x - 5)$

$\Rightarrow \quad y + x - 6 = 0$

40. *(d)* $\int_0^1 x \sin \pi x \, dx$

$$= \left[-x\frac{\cos \pi x}{\pi}\right]_0^1 + \int_0^1 \frac{\cos \pi x}{\pi} dx$$

$$= \left[-x\frac{\cos \pi x}{\pi} + \frac{\sin \pi x}{\pi^2}\right]_0^1 = \left[\frac{-(-1)}{\pi}\right] = \frac{1}{\pi}$$

41. *(b)* Since, $(a, b) = c$

$\therefore$ c is the GCD of a and b.

$\therefore \quad \frac{c}{b}$

42. *(c)* Given, $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

Applying, $C_1 \to C_1 + C_2 + C_3$ and taking common from C_1

$$\Delta = (a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$$= -(a+b+c)\,[a^2+b^2+c^2-ab-bc-ca]$$

$$= -\frac{(a+b+c)}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$$

> 0 as $a, b, c < 0$

43. *(a)* The product of 'n' positive integers is unity. Then their sum is a positive integer.

44. *(d)* Let $z = \sin x\,(1+\cos x)$

or $\quad z = \sin x + \frac{1}{2}\sin 2x$

On differentiating w.r.t. x, we get

$$\frac{dz}{dx} = \cos x + \cos 2x$$

For maximum or minimum, put $\frac{dz}{dx} = 0$

$\Rightarrow \quad \cos x + \cos 2x = 0$

$\Rightarrow \quad 2\cos^2 x - 1 + \cos x = 0$

$\Rightarrow \quad (\cos x + 1)(2\cos x - 1) = 0$

$\Rightarrow \quad \cos x = -1, \frac{1}{2}$

$\Rightarrow \quad x = \frac{\pi}{3}, \pi$

But $0 \le x \le \frac{\pi}{2}$, therefore we take only $x = \frac{\pi}{3}$.

$\therefore \frac{d^2z}{dx^2} = -\sin x - 2\sin 2x = -$ ve, for $x = \pi/3$

$\therefore$ It is maximum at $x = \pi/3$.

45. *(c)* Given,

$$y = a\cos \mu x + b\sin \mu x \qquad \ldots(\text{i})$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -a\mu \sin \mu x + b\mu \cos \mu x$$

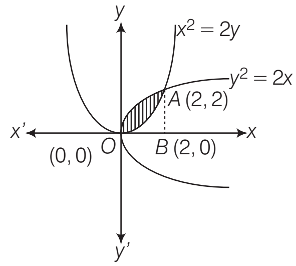
Again, differentiating, we get

$$\frac{d^2y}{dx^2} = -a\mu^2 \cos \mu x - b^2\mu \sin \mu x$$

$$= -\mu^2 y \qquad \text{[from Eq. (i)]}$$

$\Rightarrow \quad \frac{d^2y}{dx^2} + \mu^2 y = 0$

46. *(b)* The point of intersection of the curves $y^2 = 2x$ and $x^2 = 2y$ are $O\,(0, 0)$, $A\,(2, 2)$.



$\therefore$ Required area $= \int_0^2 \left(\sqrt{2x} - \frac{x^2}{2}\right) dx$

$$= \left[\sqrt{2}\,\frac{x^{3/2}}{3/2} - \frac{x^3}{6}\right]_0^2$$

$$= \left[\frac{2}{3}\,.\,2^2 - \frac{2^3}{6}\right] = \left[\frac{8}{3} - \frac{4}{3}\right]$$

$$= \frac{4}{3} \text{ sq unit}$$

47. *(a)* $A \cdot \text{adj}\,(A) = O \Rightarrow |A|\,I_n = O$

where n is order of A

$\Rightarrow \quad |A| = 0$

48. *(b)* Given, $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$

$\Rightarrow \quad |\mathbf{a}+\mathbf{b}|^2 = |\mathbf{a}-\mathbf{b}|^2$

$\Rightarrow \quad |\mathbf{a}|^2 + |\mathbf{b}|^2 + 2\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2\mathbf{a}\cdot\mathbf{b}$

$\Rightarrow \quad 4\mathbf{a}\cdot\mathbf{b} = 0$

Hence, $\mathbf{a}$ is perpendicular to $\mathbf{b}$.

49. *(d)* $\lim_{x \to 0} \frac{(2+x)\sin(2+x) - 2\sin 2}{x}\left(\frac{0}{0} \text{ form}\right)$

$$= \lim_{x \to 0} \frac{\sin(2+x) + (2+x)\cos(2+x)}{1}$$

$$= \sin 2 + 2\cos 2$$

50. *(a)* Let $I = \int \frac{2dx}{(e^x + e^{-x})^2} = \int \frac{2dx}{e^{2x} + e^{-2x} + 2}$

$$= \int \frac{2e^{2x}\,dx}{e^{4x} + 2e^{2x} + 1}$$

Put $\quad e^{2x} = t$

$\Rightarrow \quad 2e^{2x}\,dx = dt$

$\therefore \quad I = \int \frac{dt}{t^2 + 2t + 1} = \int \frac{dt}{(t+1)^2} = -\frac{1}{t+1} + c$

$$= -\frac{1}{e^{2x}+1} = -\frac{e^{-x}}{e^x + e^{-x}} + c$$

51. *(a)* Circumference of circle, $C = 2\pi r$

$$\Rightarrow \quad \frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

$$\Rightarrow \quad \frac{0.3}{2\pi} = \frac{dr}{dt} \qquad \left(\because \frac{dC}{dt} = 0.3 \text{ cm / s given}\right)$$

Now, $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt} \Rightarrow \frac{dA}{dt} = r \times 0.3$$

$$\Rightarrow \quad \left[\frac{dA}{dt}\right]_{r=5} = 5 \times 0.3 = 1.5 \text{ sq cm/s}$$

52. *(d)* If ω is a cube root of unity, then

$1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$.

$$\therefore (1 + \omega - \omega^2)(1 - \omega + \omega^2) = (-\omega^2 - \omega^2)(-\omega - \omega)$$

$$= 2\omega^2 \cdot 2\omega = 4$$

53. *(b)* Let $z = x + iy$

$\therefore \quad i + z = x + i(1 + y)$

and $\quad i - z = -x + i(1 - y)$

$$\because \quad \left|\frac{i+z}{i-z}\right| = 1 \Rightarrow |i + z| = |i - z|$$

$$\Rightarrow \quad x^2 + (1 + y)^2 = x^2 + (1 - y)^2$$

$$\Rightarrow x^2 + y^2 + 2y + 1 = x^2 + y^2 + 1 - 2y$$

$$\Rightarrow \quad 4y = 0$$

$$\Rightarrow \quad y = 0$$

i.e. x-axis.

54. *(c)* In a triangle ABC,

$$A + B + C = \pi,$$

$$\Rightarrow \sin(A + B + C) = 0, \cos(A + B) = -\cos C$$

$$\therefore \begin{vmatrix} \sin(A + B + C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A + B) & -\tan A & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$$

$$= 0$$

($\because$ It is the determinant of a skew symmetric matrix of odd order.)

55. *(c)* Let $y = x \log_e x$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x = (1 + \log x)$$

Again, differentiating, we get

$$\frac{d^2y}{dx^2} = \frac{1}{x}$$

Put, $\frac{dy}{dx} = 0$ for maxima or minima.

$$\Rightarrow \quad 1 + \log x = 0$$

$$\Rightarrow \quad x = \frac{1}{e}$$

$$\therefore \quad \left(\frac{d^2y}{dx^2}\right)_{(x = 1/e)} = e$$

$\therefore \quad y$ is minimum at $x = \frac{1}{e}$

$$\therefore \quad y_{\min} = \frac{1}{e} \log_e\left(\frac{1}{e}\right) = -\frac{1}{e}$$

56. *(c)* $\because f''(0) = k$ and

$$\lim_{x \to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$$

Using L'Hospital's rule,

$$= \lim_{x \to 0} \frac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x}$$

Again, using L'Hospital's rule,

$$= \lim_{x \to 0} \frac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$$

$$= \frac{1}{2}[2f''(0) - 12f''(0) + 16f''(0)]$$

$$= \frac{1}{2}[2k - 12k + 16k] = 3k$$

57. *(c)* $\int_2^4 [3 - f(x)]\, dx = 7$

$$\Rightarrow \quad [3x]_2^4 - \int_2^4 f(x)\, dx = 7$$

$$\Rightarrow \quad 6 - \int_2^4 f(x)\, dx = 7$$

$$\Rightarrow \quad \int_2^4 f(x)\, dx = -1 \qquad \ldots\text{(i)}$$

and $$\int_{-1}^4 f(x)\, dx = 4 \qquad \ldots\text{(ii)}$$

$$\int_{-1}^4 f(x)\, dx = \int_{-1}^2 f(x)\, dx + \int_2^4 f(x)\, dx$$

$$\Rightarrow \quad 4 = \int_{-1}^2 f(x)\, dx - 1$$

$$\Rightarrow \quad \int_{-1}^2 f(x)\, dx = 5$$

58. *(a)* The given differential equation is

$$\frac{dy}{dx} = (x - y)^2$$

Put $\quad x - y = t$

$$\Rightarrow \quad 1 - \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 - \frac{dt}{dx}$$

$\therefore \quad 1 - \frac{dt}{dx} = t^2$

$\Rightarrow \quad (1 - t^2) = \frac{dt}{dx}$

On integrating both sides, we get

$$\int \frac{1}{1-t^2}\,dt = \int 1\,dx$$

$\Rightarrow \quad \frac{1}{2}\log\left(\frac{1+t}{1-t}\right) = x + c$

$\Rightarrow \quad \left(\frac{1+t}{1-t}\right) = e^{2x+2c}$

$\Rightarrow \quad \frac{1+x-y}{1-x+y} = Ae^{2x}$

$\because$ It passes through origin.

$\therefore \quad A = 1$

$\therefore$ Required curve is

$$(1 + x - y) = e^{2x}\,(1 - x + y)$$

59. *(d)* $\because \quad (\sqrt{3} - i) = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$\therefore \quad (\sqrt{3} - i)^{50} = 2^{50}\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^{50}$

$= 2^{50}\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$

$= 2^{50}\left(\frac{1}{2} - i\,\frac{\sqrt{3}}{2}\right)$

But $\quad (\sqrt{3} - i)^{50} = 2^{48}(x - iy)$

$\therefore \quad x = 2$ and $y = 2\sqrt{3}$

$\therefore \quad x^2 + y^2 = 4 + 12 = 16$

60. *(b)* $T_r = T_{(r-1)+1} = {}^{20}C_{r-1}\cdot x^{r-1}$

$\therefore$ Coefficient of the rth term $= {}^{20}C_{r-1}$

and coefficient of the $(r+1)$th term $= {}^{20}C_r$

$\therefore \quad \frac{{}^{20}C_{r-1}}{{}^{20}C_r} = \frac{1}{2}$

$\Rightarrow \quad \frac{r}{21-r} = \frac{1}{2}$

$\Rightarrow \quad r = 7$

61. *(c)* Now, $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$= \hat{\mathbf{i}}\,(4 - 6) - \hat{\mathbf{j}}\,(12 + 2) + \hat{\mathbf{k}}\,(-9 - 1)$

$= -2\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$

$\therefore$ Area of required parallelogram

$= \frac{1}{2}|\,\mathbf{a}\times\mathbf{b}\,| = \frac{1}{2}\sqrt{4 + 196 + 100} = \frac{10\sqrt{3}}{2} = 5\sqrt{3}$

62. *(a)* In the given series

$$a = \frac{\sqrt{2}+1}{\sqrt{2}-1} \text{ and } r = \frac{1}{2+\sqrt{2}} = \frac{2-\sqrt{2}}{2}$$

$\therefore \quad S_\infty = \frac{a}{1-r} = \frac{\frac{\sqrt{2}+1}{\sqrt{2}-1}}{1 - \frac{2-\sqrt{2}}{2}}$

$= \frac{\frac{\sqrt{2}+1}{\sqrt{2}-1}}{\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}\,(\sqrt{2}+1)}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$

$= \sqrt{2}\,(\sqrt{2}+1)^2$

63. *(c)* $\because (1 + ax)^n = 1 + axn + \frac{n(n-1)}{2!}(ax)^2 + \dots$

Given that,

$axn = 6x$

$\Rightarrow \quad an = 6 \quad \dots(i)$

and $\quad \frac{a^2 n(n-1)}{2} = 16 \quad \dots(ii)$

On solving Eqs. (i) and (ii), we get

$$a = \frac{2}{3} \text{ and } n = 9$$

64. *(b)* Since, $f(x)$ is continuous at $x = 3$, then

$\lim_{x\to 3} f(x) = f(3) \quad \dots(i)$

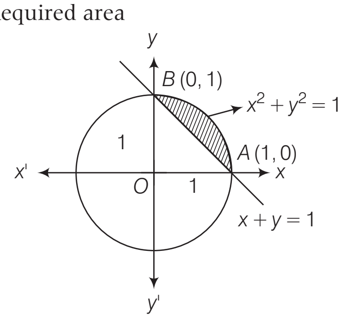
$\therefore \lim_{x\to 3}\frac{x^2 - 9}{x - 3} = \lim_{x\to 3} x + 3 = 3 + 3 = 6$

and $\quad f(3) = 2\times 3 + k = k + 6$

$\therefore$ From Eq. (i),

$6 = k + 6 \Rightarrow k = 0$

65. *(c)* Required area



$= \frac{1}{4}$ (Area of circle) $-$ (Area of $\Delta\,AOB$)

$$= \frac{\pi (1)^2}{4} - \frac{1}{2} \times OA \times OB$$
$$= \left(\frac{\pi}{4} - \frac{1}{2}\right) \text{sq unit}$$

66. (a) $\left|\frac{x}{2} - 1\right| < 3 \Rightarrow -3 < \frac{x}{2} - 1 < 3$

$$\Rightarrow \quad -3 + 1 < \frac{x}{2} < 3 + 1$$
$$\Rightarrow \quad -2 < \frac{x}{2} < 4$$
$$\Rightarrow \quad -4 < x < 8$$
$$\therefore \quad x \in (-4, 8)$$

67. (b) $\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\frac{3}{\sqrt{10}}$

Now, $\sin^{-1}\frac{1}{\sqrt{5}} = \tan^{-1}\left(\frac{1/\sqrt{5}}{\sqrt{1 - \frac{1}{5}}}\right) = \tan^{-1}\frac{1}{2}$

and $\cos^{-1}\frac{3}{\sqrt{10}} = \tan^{-1}\left(\frac{\sqrt{1 - \frac{9}{10}}}{\frac{3}{\sqrt{10}}}\right) = \tan^{-1}\frac{1}{3}$

$$\therefore \quad \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

68. (b) Given, $f(x) = \sec x - \cot x$

$$\therefore \quad f'(x) = \sec x \tan x + \operatorname{cosec}^2 x$$

$$\Rightarrow \quad f'\left(\frac{\pi}{6}\right) = \sec\frac{\pi}{6}\tan\frac{\pi}{6} + \operatorname{cosec}^2\frac{\pi}{6}$$
$$= \frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + 4 = \frac{14}{3}$$

69. (a) Let C divides AB in the ratio $m:1$. Then

$$\mathbf{c} = \frac{m\mathbf{b} + \mathbf{a}}{m+1}$$
$$\Rightarrow \quad m\mathbf{c} + \mathbf{c} = m\mathbf{b} + \mathbf{a}$$
$$\Rightarrow \quad \mathbf{a} + m\mathbf{b} - m\mathbf{c} - \mathbf{c} = \mathbf{0}$$
$$\Rightarrow \quad \mathbf{a} + m\mathbf{b} + (-m-1)\,\mathbf{c} = \mathbf{0} \quad \ldots\text{(i)}$$

Given, $3\mathbf{a} + 4\mathbf{b} - 7\mathbf{c} = \mathbf{0}$

$$\Rightarrow \quad \mathbf{a} + \frac{4}{3}\mathbf{b} - \frac{7}{3}\mathbf{c} = \mathbf{0} \quad \ldots\text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$m = \frac{4}{3}$$

$\therefore$ Required ratio is 4 : 3

70. (d) $\begin{bmatrix} a & 2 & 3 \\ b & 5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix}$

$$= \begin{bmatrix} a + 6 - 3 & 2a + 8 + 3 \\ b + 15 + 1 & 2b + 20 - 1 \end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix} a + 3 & 2a + 11 \\ b + 16 & 2b + 19 \end{bmatrix} = \begin{bmatrix} 4 & 13 \\ 12 & 11 \end{bmatrix} \quad \text{(given)}$$
$$\Rightarrow \quad a + 3 = 4 \Rightarrow a = 1$$
$$\text{and} \quad b + 16 = 12 \Rightarrow b = 12 - 16 = -4$$
$$\therefore \quad (a, b) = (1, -4)$$

English & General Aptitude

16. (b) Indira and Nehru were the Prime Ministers of India, while Benazir was the Prime Minister of Pakistan.

17. (d) All are former Indian cricketers.

18. (c) 'Folketing' is the parliament of 'Denmark'; 'Stortling' is the parliament of 'Norway', 'Knesset' is the parliament of Israel.

19. (b) They all are modes of transport.

20. (c) 'PTI' is an Indian news agency; 'Irna' is a news agency in Iran while 'Xin-Era' is a news agency in China.

21. (c) 'Pitcher' is a term used in Baseball; 'Dusra' is a term used in cricket and 'Bunker' is a term used in polo.

22. (d) These all are snakes.

23. (c) These all are units of distance.

24. (d) These all are water animals.