MET 2019 Answer Key PDF

PHYSICS

1. (c) Acceleration of the system

$$a = \frac{F}{m} = \frac{10}{2+3+5}$$

$$= \frac{10}{10} = 1 \text{ m/s}^2$$

$\therefore$ $$T_1 = (3+5) \times a = 8 \times 1 = 8 \text{ N}$$

2. (c) Acceleration of the system

$$a = \frac{m_1 - m_2}{m_1 + m_2} g = \frac{10-6}{10+6} \times g$$

$$= \frac{4}{16} \times 10 = 2.5 \text{ m/s}^2$$

3. (a) $\because$ L / R is a time constant so the unit of L / R is second.

4. (b) According to the question, $F \propto v^2$

or $F = kv^2$

$$k = \frac{F}{v^2} \Rightarrow \frac{[MLT^{-2}]}{[L^2T^{-2}]} = [ML^{-1}\, T^0]$$

5. (b) Ionisation energy

$$E = 13.6\, Z^2$$

For He^+, Z = 2

$$E = 13.6\,(2)^2 = 13.6 \times 4 = 54.4 \text{ eV}$$

6. (d) Max. kinetic energy of photoelectrons = 4 eV

If stopping potential is V_0, then

$$eV_0 = 4 \text{ eV}$$

$\therefore$ $$V_0 = 4 \text{ volt}$$

7. (d) Out of the given choices, Poisson's ratio has not unit.

8. (a) In candescent electric lamp produces continuous emission spectrum whereas mercury and sodium vapour given line emission spectrum. Polyatomic substance such as H_2CO_2 and $KMNO_4$ produces band absorption spectrum.

9. (b) Let the two balls cross each other after time t.

Height covered by first ball

$$h_1 = \frac{1}{2} gt^2$$

Height covered by second ball

$$h_2 = ut - \frac{1}{2} gt^2$$

but $h_1 + h_2 = 100$

$$\frac{1}{2} gt^2 + ut - \frac{1}{2} gt^2 = 100$$

$$ut = 100$$

$$50 \times t = 100 \Rightarrow t = 2 \text{ s}$$

10. (d) Observed speed, $v = \sqrt{(10)^2 + (6)^2}$

$$= \sqrt{100+36} = \sqrt{136} \simeq 11 \text{ m/s}$$

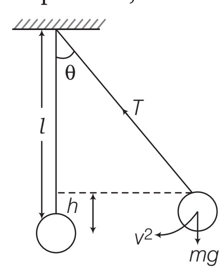
11. (b) Orbital speed of satellite near the earth surface is approximately 8 km/s.

12. (a) $g = \dfrac{GM_e}{R_e^2} = G\dfrac{\frac{4}{3}\pi R_e^3 d}{R_e^2} = \dfrac{4}{3}\pi G R_e d$

$$g \propto d$$

So, if density is doubled, then acceleration due to gravity will also be doubled.

13. (d) In mean position,



$$T = 3 \text{ mg}$$

Net force, $T - mg = \dfrac{mv^2}{l}$

$$3mg - mg = \frac{mv^2}{l}$$

$$\Rightarrow \quad v = \sqrt{2gl}$$

Let maximum displacement with respect to vertical is θ, then

$$\frac{1}{2}mv^2 = mgh$$

$$= mg(l - l\cos\theta)$$

$$\frac{1}{2}m(2gl) = mgl(1 - \cos\theta)$$

$$\cos\theta = 0$$

$$\Rightarrow \quad \theta = 90^\circ$$

14. (a) Force, $F = ma = m\frac{dv}{dt}$

$$3x = 8\frac{dv}{dx}\cdot\frac{dx}{dt}$$

$$3x = 8v\frac{dv}{dx} \qquad \left[\because \frac{dx}{dt} = v\right]$$

$$3x\,dx = 8v\,dv$$

Integrating both sides

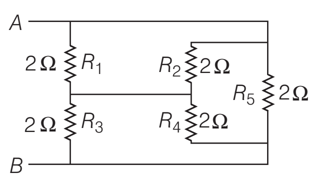
$$3\left[\frac{x^2}{2}\right]_2^{10} = 8\left[\frac{v^2}{2}\right]_0^v$$

$$\frac{3}{2}[10^2 - 2^2] = 4v^2$$

$$\frac{3}{2}\times 96 = 4v^2$$

$$v = 6\text{ m/s}$$

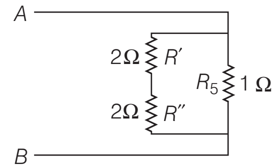
15. (a) R_1 and R_2 are in parallel order, so their equivalent resistance



$$R' = \frac{2\times 2}{2+2} = 1\,\Omega$$

Similarly, equivalent resistance of R_3 and R_4

$$R'' = 1\,\Omega$$



R' and R'' are in series order, so their equivalent resistance

$$R''' = R' + R''$$

$$= 1 + 1 = 2\,\Omega$$

R''' and R_5 are in parallel.

So, $\quad R_{AB} = \frac{2\times 2}{2+2} = 1\,\Omega$

16. (a) ${}_1H^1 + {}_1H^1 \longrightarrow {}_2He^4$

Mass defect

$\Delta m = 2\times$ mass of ${}_1H^1$ – mass of ${}_2He^4$

$= 2\times 2.0141 - 4.0024$

$= 0.0258$

Energy released $= \Delta m \times 931$ MeV

$= 0.0258\times 931 = 24$ MeV

17. (c) When a thin metal plate is placed between the plates of a parallel plate capacitor, then its capacitance increases so, out of the given choices, 4 C will be the right answer.

18. (a) Linear acceleration, $a = \dfrac{g\sin\theta}{1 + \dfrac{k^2}{r^2}}$

For solid sphere $\dfrac{k^2}{r^2} = \dfrac{2}{5}$

$$\therefore \quad a = \frac{g\sin\theta}{1 + \frac{2}{5}}$$

$$= \frac{5}{7}g\sin\theta = \frac{5}{7}g\sin 30^\circ$$

$$= \frac{5}{7}g\times\frac{1}{2} = \frac{5g}{14}$$

19. (d) From law of conservation of momentum

$MV = mv$

$$1\times 5 = 10\times 10^{-3}\times v$$

$$v = \frac{5}{10\times 10^{-3}} = 500\text{ m/s}$$

20. (a) Acceleration in circular motion

$$a = \frac{v^2}{r}$$

But, $$v = \frac{2\pi r}{T}$$

$$r = \frac{vT}{2\pi}$$

$\therefore$ $$a = \frac{v^2}{vT/2\pi}$$

$$= \frac{2\pi v}{T}$$

21. (b) The centre of mass will be slightly farther from C-atom (because it is lighter), so correct choice will be (b).

22. (d) Work done, $W = Fd\cos\theta$

$$25 = 10 \times 10 \cos\theta$$

$$\cos\theta = \frac{1}{4} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{4}\right)$$

23. (c) Ist case,

$$U = \frac{1}{2}kx_1^2$$

$$100 = \frac{1}{2}k\,(2 \times 10^{-2})^2$$

$$k = \frac{200}{4 \times 10^{-4}}$$

$$= 50 \times 10^4$$

IInd case

$$U = \frac{1}{2}kx_2^2$$

$$= \frac{1}{2} \times 50 \times 10^4\,(4 \times 10^{-2})^2$$

$$= 25 \times 10^4 \times 16 \times 10^{-4}$$

$$= 400\text{ J}$$

So, stored energy increases by

$400 - 100 = 300$ J

24. (d) Young's modulus, $Y = \dfrac{F \cdot L}{\pi r^2 \Delta L}$

Both wires are of same material.

So, $$Y_1 = Y_2$$

$$\frac{F_A \cdot L_A}{r_A^2 \Delta L_A} = \frac{F_B L_B}{r_B^2 \cdot \Delta L_B}$$

Here, $$\Delta L_A = \Delta L_B$$

$\therefore$ $$\frac{F_A}{F_B} = \frac{L_B}{L_A} \times \left(\frac{r_A}{r_B}\right)^2$$

$$= \frac{2}{1} \times \left(\frac{2}{1}\right)^2 = \frac{8}{1}$$

$$F_A : F_B = 8 : 1$$

25. (c) Radius of path of charged particle

$$r = \frac{\sqrt{2m\,E_k}}{Bq}$$

E_k remains same, so

$$\frac{r_1}{r_2} = \sqrt{\frac{m_1}{m_2}} \cdot \frac{q_2}{q_1}$$

$$= \sqrt{\frac{4}{16}} \times \frac{2}{1} = \frac{1}{2} \times \frac{2}{1} = \frac{1}{1}$$

So, correct choice is (c).

26. (a) $\beta = \dfrac{\lambda D}{d}$

$$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} \cdot \frac{d_2}{d_1} = \frac{7000}{3500} \times \frac{2d}{d}$$

$$\beta_2 = \frac{\beta_1}{4}$$

So, the width of fringes changes.

27. (c) Coherent sources of light are those sources of light which emit light waves of same wavelength, same frequency and are in same phase or having constant phase difference.

28. (a) In metal, either there is no energy gap between the condition band which is partially filled with electrons and valence band or the condition band and valence band overlap each other.

29. (c) The semiconductors in which majority charge carriers are holes i.e. positinely charged and minority charge are holes, so, in p-type semiconductor, the holes movement results in the formation of the current.

30. (b) A laser device produces amplification in the ultraviolet or visible region.

31. (a) Focal length of glass rod, $f = \frac{R}{2}$

$$= \frac{30}{2} = 15 \text{ cm}$$

Distance of object, $u = -15$ cm

From formula,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\mu_2 = 1.5 \quad \text{(for glass)}$$

$$\mu_1 = 1 \quad \text{(for air)}$$

$$\frac{1.3}{v} = -\frac{1}{-15} = \frac{1.5 - 1}{30}$$

$$\frac{1.5}{v} + \frac{1}{15} = \frac{0.5}{30}$$

$$\frac{1.5}{v} = \frac{1}{60} - \frac{1}{15}$$

$$\frac{1.5}{v} = \frac{-1}{20}$$

$$\Rightarrow \quad v = -30 \text{ cm}$$

32. (c) Rainbow occurs in the form of concentric colourd circular arcs in the sky, when the sun shines on rain drops during or after the shower. This phenomenon occurs due to combined effect of dispersion, refraction and total internal reflection of sunlight by spherical drop of water.

33. (b) Near the junction positive charge is built on n-side and negative charge on P-side. This sets up potential difference across the junction and internal electric field E_i directed from n-side to P-side. The equilibrium is established when the field E_i becomes strong enough to stop further diffusion of the majority charge carriers.

34. (a) At 45° total internal reflection takes place so, critical angle, C = 45°.

Refractive index, $\mu = \frac{1}{\sin C} = \frac{1}{\sin 45°}$

$$= \sqrt{2} = 1.414$$

$$\approx 1.5$$

35. (d) $\because$ $F \propto S^{-1/3}$ or $a \propto S^{-1/3}$

or $Vdv \propto S^{-1/3} ds \left(a = V \cdot \frac{dV}{ds} \right)$

Integrating we have, $V^2 \propto S^{2/3}$

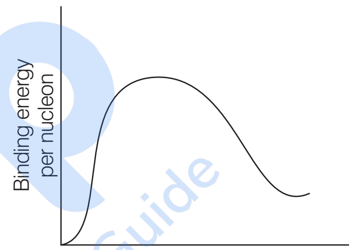
or $\quad V \propto S^{1/3}$

Now, power $P \propto F \cdot V \propto S^{-1/3} \cdot S^{1/3}$

or $\quad P \propto S^0$

36. (c) Binding Energy per nuclear

$$= \frac{\text{Total binding energy}}{\text{Number of nucleon (A)}}$$



37. (d) Velocity of light in fiber

$$v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m/s}$$

Straight length is 1 km,

Distance covered by light,

$$s = \frac{1 \text{ km}}{\sin 60°} = \frac{1000}{\sqrt{3}/2} = \frac{2000}{\sqrt{3}}$$

So, time taken $= \frac{s}{v} = \frac{2000/\sqrt{3}}{2 \times 10^8} = 3.85 \mu s$

38. (b) $W_{PQ} = P\Delta V$

$$= 100 \times 10^3 (300 - 100) \times 10^{-6}$$

$$= 100 \times 10^{-3} \times 200 = 20 \text{ J}$$

$W_{QR} = 0$ ($\because$ V is constant)

$W_{RS} = P\Delta V$

$$= 200 \times 10^3 (100 - 300) \times 10^{-6}$$

$$= -200 \times 10^{-3} \times 200 = -40 \text{ J}$$

$W_{SP} = 0$

Net work done

$$= 20 + (-40) = -20 \text{ J}$$

39. (a) Let each resistance is R

$$P = \frac{E^2}{R_{eq}}$$

In choice (a) $R_{eq} = \frac{R \times R}{R + R} = \frac{R}{2}$

$$\Rightarrow \qquad P_1 = \frac{E^2}{\frac{R}{2}} = \frac{2E^2}{R}$$

In choice (b)

$$R_{eq} = R + R = 2R$$

$$P_2 = \frac{E^2}{2R} = 0.5\frac{E^2}{R}$$

In choice (c) $R_{eq} = \frac{R}{2} + R = \frac{3R}{2}$

$$P_3 = \frac{2E^2}{3R} = 0.6\frac{E^2}{R}$$

In choice (d) $R = \frac{2R \times R}{2R + R} = \frac{2R}{3}$

$$P_4 = \frac{3E^2}{2R} = 1.5\frac{E^2}{R}$$

So, P_1 is maximum.

40. (b) Charge redistributes on the capacitors in the ratio of their capacitance.

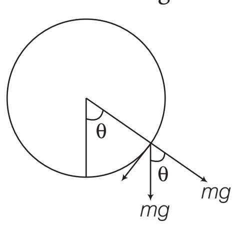
$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{C_1}{2C_1} = \frac{1}{2}$$

$$Q_1 : Q_2 = 1 : 2$$

$$\therefore \qquad Q_1 = \frac{1}{1+2} \times Q = \frac{Q}{3}$$

$$Q_2 = \frac{2}{1+2}Q = \frac{2}{3}Q$$

41. (c) Tension in the string



$$T = \frac{mv^2}{l} + mg\cos\theta$$

So, $\qquad T \propto \cos\theta$

$$\frac{T_1}{T_2} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{\cos 30°}{\cos 60°}$$

$$= \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{1}$$

$$\therefore \qquad T_1 > T_2$$

42. (c) $L = \frac{\phi}{nI} = \frac{10^{-5}}{100 \times 5 \times 10^{-3}} = \frac{10^{-2}}{100 \times 5}$

$$= \frac{10^{-4}}{5}\,H = \frac{10^{-1}}{5}\,mH$$

$$= 0.02\,mH$$

43. (d) Energy drawn from the battery

$$E = V\,I\,t$$

$$= 12 \times 300 \times 2 \times 60$$

$$= 432 \times 10^3\,J$$

$$= 432\,kJ$$

44. (d) From $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\frac{1 \times 10^5}{2 + 273} = \frac{h\,d\,g + 1 \times 10^5}{T_2}$$

[$\because P_1$ = atmospheric pressure]

h = 20 m, d = density of water

$$= 1 \times 10^3\,kg/m^3$$

$$\therefore \quad \frac{1 \times 10^5}{275} = \frac{20 \times 10^3 \times 10 + 1 \times 10^5}{T_2}$$

$$\frac{1 \times 10^5}{275} = \frac{3 \times 10^5}{T_2}$$

$$T_2 = 275 \times 3 = 825\,K$$

$$= 825 - 273$$

$$= 552°\,C$$

45. (b) $y_1 = 4\sin(\omega t + kx)$ ...(i)

$$y_2 = -4\cos(\omega t + kx)$$

$$= -4\sin(\omega t + kx + \pi/2)$$

$$= 4\sin\left(\omega t + kx + \frac{3\pi}{2}\right) \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

The phase difference, $\Delta\phi = \frac{3\pi}{2}$

46. (b) $m = 0.003 \text{ g} = 0.003 \times 10^{-3}$ kg

$$E = 6 \times 10^4 \text{ N/C}$$

The particle is stationary, so

Electric force = Weight of the particle

$$qE = mg$$

$$q = \frac{mg}{E} = \frac{0.003 \times 10^{-3} \times 10}{6 \times 10^4}$$

$$= \frac{3 \times 10^{-10} \times 10}{6} = 5 \times 10^{-10} \text{C}$$

47. (b) Force on each plate, $F = \frac{qE}{2}$

$$= \frac{1 \times 10^{-6} \times 10^5}{2} = \frac{10^{-1}}{2}$$

$$= 0.05 \text{ N}$$

48. (b) Given heat, $Q = mL = 1 \times 540$ cal

$$= 540 \times 4.18 \times 10^7 \text{ J}$$

Work done, $W = P\Delta V$

$P = 1 \text{ atm} = 1 \times 10^5 \text{ N/m}^2 = 10^6 \text{ dyne/cm}^2$

$W = 10^6 \times 1680$ J

Change in internal energy

$$\Delta U = Q - W$$

$$= 540 \times 4.18 \times 10^7 - 10^6 \times 1680$$

$$= \frac{540 \times 4.18 \times 10^7 - 10^6 \times 1680}{4.18 \times 10^7} \text{ cal}$$

$$= 500 \text{ cal}$$

49. (a) Induced emf

$$E = -\frac{\Delta\phi}{\Delta t} = -A\frac{\Delta B}{\Delta t}$$

$$= -0.01 \times \frac{1-2}{10^{-3}} = \frac{0.01}{10^{-3}}$$

$$= 10 \text{ V}$$

$\therefore$ Heat produced, $H = \frac{E^2}{R} t$

$$= \frac{(10)^2}{0.01} \times 10^{-3}$$

$$= \frac{100 \times 10^{-3}}{0.01}$$

$$= 10 \text{ J}$$

50. (b) $R_t = R_0(1 + \alpha\,\Delta T)$

$$2 = 1(1 + 0.00125 \times \Delta T)$$

$$\Delta T = \frac{1}{0.00125} \quad \text{(neglecting – ve sign)}$$

$$T = 800 + 300 = 1100 \text{ K}$$

CHEMISTRY

1. (b) $\underset{s}{CaF_2} \rightleftharpoons \underset{s}{Ca^{2+}} + \underset{2s}{2F^-}$

$$[Ca^{2+}] = s$$

$$[F^-] = 2s$$

$$K_{sp} = [Ca^{2+}] \cdot [F^-]^2 = s \cdot (2s)^2$$

$$K_{sp} = 4s^3$$

2. (a) If p, T, ρ and R represents pressure, temperature, density and universal gas constant then ideal gas equation

pV = nRT for n mole of gas

pV = RT for 1 mole of gas

We know that

$$M = V \times \rho$$

$$V = \frac{M}{\rho}$$

Put the value of V in gas equation

$$p \times \frac{M}{\rho} = RT$$

then molar mass of gas

$$M = \frac{\rho RT}{p}$$

3. (b) The kinetic energy of a gas molecule is directly proportional to absolute temperature. According to kinetic theory of gases, the pressure P exerted by a gas is given by

$$p = \frac{1}{3}\frac{Mu^2}{V}$$

or $pV = \frac{1}{3} Mu^2$

According to gas equation pV = RT

$\therefore \quad \frac{1}{3} Mu^2 = RT$

or $\frac{1}{3} \frac{Mu^2}{N} = \frac{RT}{N}$

We know that

$$\frac{R}{N} = K$$

K = Boltzmann constant

or $\frac{1}{2} Mu^2 = \frac{3}{2} KT$

But $\frac{1}{2} Mu^2$ is average kinetic energy per molecule of the gas.

$\therefore \quad$ Average K.E. $= \frac{3}{2} KT$

$$K.E. \propto T$$

Thus average kinetic energy of one mole of any gas is directly proportional to its absolute temperature.

4. (d) Molecular weight of $SO_2(M_1) = 64$

Molecular weight of $O_2(M_2) = 32$

According to Graham's law of diffusion

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

$$\frac{r_1}{r_2} = \sqrt{\frac{32}{64}}$$

$$= \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

or $= \frac{1}{1.414}$

$$r_1 : r_2 = 1 : 1.414$$

5. (a) Elevation in boiling point $\Delta T_b = K_b \times m$

Dipression in freezing point

$$\Delta T_f = K_f \times m$$

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b}{K_f}$$

$$\frac{\Delta T_b}{0.186} = \frac{0.512}{1.86}$$

$$\Delta T_b = 0.0512°C$$

6. (a) Species : $O_2^{2-}, O_2^-, O_2, O_2^+$

Bond Order : 1 1.5 2 2.5

7. (a) In hydrolysis of a salt of weak acid and strong base.

$$A^- + H_2O \rightleftharpoons HA + OH^-$$

the hydrolysis constant K_h

$$K_h = \frac{[HA][OH^-]}{[A^-]}$$

(According to law of mass action)

($\because$ conc. of water remains constant)

For the ionisation of weak acid HA,

we has $HA \rightleftharpoons H^+ + A^-$

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

Also we know that

$$\frac{K_w}{K_a} = \frac{[H^+][OH^-][HA]}{[H^+][A^-]}$$

$$\frac{K_w}{K_a} = \frac{[OH^-][HA]}{[A^-]}$$

or $\frac{K_w}{K_a} = K_h$

8. (c) $aA + bB \longrightarrow$ Products

Rate of reactant for A $= -\frac{1}{a}\frac{d[A]}{dt}$

Rate of reactant for B $= -\frac{1}{b}\frac{d[B]}{dt}$

the $-\frac{d[A]}{dt}$ is equal to $= -\frac{a}{b}\frac{d[B]}{dt}$

9. (a) Number of moles of hydrogen

$(H_2) = \frac{\text{Weight in gram of } H_2}{\text{Molecular weight of } H_2}$

Weight of H_2 in gram = 1g

molecular weight of H_2 = 2

Number of moles of $H_2 = \frac{1}{2} = 0.5$ moles

Number of moles of

$$O_2 = \frac{\text{Weight in g of } O_2}{\text{Molecular weight of } O_2} = \frac{8}{32}$$

$= 0.25$ moles

Mole fraction of

$$H_2 = \frac{\text{Number of moles of } H_2}{\text{Total number of moles}}$$

$$= \frac{0.5}{0.25 + 0.5} = 0.667$$

10. (c) $\overset{+7}{MnO_4^-} \rightarrow Mn^{+2}$ (Acid medium). Change in oxidation number $= +5$ Equivalent weight of $KMnO_4 = \dfrac{\text{Molecular weight of } KMnO_4}{\text{Change in oxidation number}}$

$$= \frac{M}{5}$$

11. (a) For the reaction

$$CH_3COOH\,(l) + 2O_2(g) \rightleftharpoons 2CO_2\,(g) + 2H_2O\,(l)$$

$$\Delta H = -874 \text{ kJ}$$

We know that $\Delta H = \Delta E + P\Delta V$

From ideal gas equation $P\Delta V = \Delta nRT$

Then $\Delta H = \Delta E + \Delta nRT$

Number of moles of reactant

$=$ Number of moles of products

$\therefore \quad \Delta n = 0$

Then $\quad \Delta H = \Delta E$

$\Delta E = -874$ kJ

12. (d) The electrophile involved in the sulphonation of benzene is SO_3

$$2H_2SO_4 \rightarrow SO_3 + H_3O^+ + HSO_4^-$$

13. (a) In heterogeneous catalysis, adsorption lowers the activation energy of the reaction.

14. (d) $C_2H_5Br + AgNO_2(\text{alc.}) \rightarrow \underset{\text{Nitro ethane}}{C_2H_5NO_2} + AgBr$

15. (b) Specific conductivity

$$(K) = \frac{1}{R} \times \text{Cell constant}$$

Cell constant $= K \times R$

$= 0.0129 \times 100 = 1.29$

16. (a) Cell $Zn\,|\,Zn^{2+}\,||\,Fe^{2+}\,|\,Fe$

Electrode potential for

$$Zn / Zn^{2+} = 0.763 \text{ V}$$

Electrode potential for

$$Fe^{2+}/Fe = -0.44 \text{ V}$$

EMF of cell $= E^\circ_{\text{oxidation (Fe)}} + E^\circ_{\text{reduction (Zn)}}$

$= -0.44 + 0.763$

$= 0.323$ V

17. (a) The electronic configuration of $_{24}C_r$ is $[Ar]_{18}\,3d_1^5 4s^1$. Chromium after loss of one electron gain stable configuration due to presence of half filled d-orbitals, therefore its second ionisation enthalphy is highest.

18. (a) If pH is less then 7, the solution is acidic but if the pH is more than 7 the solution becomes basic.

0 —Acid— 7 —Base— 14

If alkali is added in acid solution the pH increases.

19. (a) CO_2 is a gas, while SiO_2 is a solid but both are acidic in nature, both reacts with base and forms carbonate and silicates.

$$CO_2 + \underset{\text{Base}}{NaOH} \longrightarrow NaHCO_3$$

$$SiO_2 + \underset{\text{Base}}{2NaOH} \longrightarrow Na_2SiO_3 + H_2O$$

20. (c) HNO_3 is a strong oxidising agent. So it oxidise iron and form a layer of Fe_3O_4 that works as protective layer.

21. (b) O_3 (Ozone) works as oxidising agent. It reacts with potassium iodide (KI) solution and liberate iodine vapours that turns starch paper blue.

$$O_3 \longrightarrow O_2 + O$$

$$2KI + H_2O + O \longrightarrow 2KOH + I_2$$

$$2KI + H_2O + O_3 \longrightarrow 2KOH + I_2 + O_2$$

22. (c) It is the oxidation process.

$$3Fe + SO_2 \longrightarrow 2FeO + FeS$$

Fe is oxidised to form FeO and FeS.

23. (a) It is Wurtz reaction.

$$C_2H_5I + 2Na + IC_2H_5 \xrightarrow{\text{Dry ether}} C_2H_5—C_2H_5 + 2NaI$$

So, C_4H_{10} is formed.

24. (b) For the reaction

$$CH_3—CH=CH_2 + HOCl \longrightarrow A$$

This is addition reaction, this reaction involves in unsaturated compound or that compound which has multiple bonds ($=$ or $\equiv$ bonds)

$$CH_3—CH=CH_2 + HO\overset{+}{Cl} \longrightarrow CH_3—\underset{OH}{CH}—\underset{Cl}{CH_2}$$

According to Markownfikoff's rule, the –ve part of reagent is added to that carbon atom of the double bond which bears the least number of hydrogen atom.

25. (d) Ionic crystals have no directional bonds as the oppositely charged ions are attracted by electrostatic forces and arranged in a definite ratio.

26. (c) The sum of protons and neutrons give the mass number. The number of neutrons and protons are not equal.

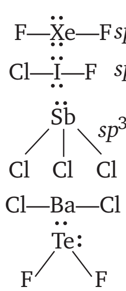
27. (a) According to the modern periodic table. If the last electron of element is placed in s-orbital, then elements are known as s-block element. Group IA has elements with ns^1configuration and IIA has elements with ns^2 configuration.

28. (b) Azimuthal quantum number l for d orbitals is 2. so the value of magnetic quantum number

is $\quad -l$ to $+l$

then $\quad -2$ to $+2 - 2, -1, 0, +1, +2$

29. (a)



sp^3d and linear

sp^3d and linear

sp^3 and pyramidal

sp and linear

sp^3 and Y-shape

30. (a) $O_2^+ \longrightarrow O_2^{2+} + e^-$

O_2^+ has $(15e^-) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2$

$\sigma 2p_x^2, \pi 2p_y^2, \pi 2p_z^2, \pi^* 2p_y^1$

So O_2^+ lost a electron to form O_2^{2+}, the e^- is from antibonding π orbital.

31. (b) Structure II shows A to be electropositive. Hence, It will have least contribution.

32. (b) $NaOH + 2S \longrightarrow \underset{\text{Sodium sulphite}}{Na_2SO_3} + Na_2S + H_2O$

33. (d) $SiF_4 + 4H_2O \longrightarrow \underset{\text{Silicic acid}}{Si(OH)_4} + 4HCl$

The trend towards hydrolysis decreases down the group. Silicon have vacant orbital to which water molecules can co-ordinate and hence their halides are hydrolysed by water.

34. (a) For extraction of metal only those minerals are used (ores) for which the process is economic.

35. (a) P_2O_5 is an acidic oxide, acting as an acidic flux.

36. (b) Basic Oxide of metals are basic in nature and form basic hydroxide with water.

37. (c) CO_2 gas is passed in $Ca(OH)_2$ solution the solution becomes milky white to form $CaCO_3$ and if the CO_2 is passed in excess the milky white solution become colourless due to the formation of $Ca(HCO_3)_2$.

$$Ca(OH)_2 + CO_2 \longrightarrow CaCO_3 \downarrow + H_2O$$

38. (b) $(NH_4)_2SO_4 \cdot Fe_2(SO_4)_3 \cdot 24H_2O$

39. (c) Transition elements have three series 3d, 4d and 5d.

3d series has elements 21 (Sc) to 30 (Zn) atomic number. 4d series has elements 39 (Y) to 48 (Cd) atomic number.

$[Kr]4d^{1-10}, 5s^{1-2}$ has IInd row series.

40. (d) This complex is of the type MA_4B_2 with co-ordination number 6. It shows geometrical isomerism. *Cis* form is violet coloured and *trans* form is green.

41. (d) $\overset{1}{CH_3}-\underset{\substack{| \\ OH}}{\overset{2}{CH}}-\overset{3}{CH_2}-\underset{\substack{| \\ CH_3}}{\overset{4}{CH}}-\overset{5}{CH_3}$

4-methyl pentane-2-ol

42. (d) $\underset{}{R-X} \xrightarrow[\text{Sealed tube } 100^\circ C]{NH_3} \underset{\text{Primary amine } 1^\circ}{R\cdot NH_2} + HX$

$RX + R\cdot NH_2 \longrightarrow \underset{2^\circ \text{ amine}}{R_2NH} + HX$

$R_2NH + RX \longrightarrow \underset{3^\circ \text{ amine}}{R_3N} + HX$

43. (c) Cannizaro reaction is possible in which aldehyde where the α-hydrogen atoms are absent.

e.g. $HCHO$, C_6H_5CHO

$2HCHO \xrightarrow[\Delta]{NaOH} HCOONa + CH_3OH$

$2C_6H_5CHO \xrightarrow[\Delta]{NaOH} C_6H_5CH_2OH + C_6H_5COONa$

44. (d) This is Hoffmann's bromide reaction. In this reaction, product compound has one less 'C' atom than reactant.

So it causes shortening of 'C' chain.

$-CONH_2 \longrightarrow -NH_2$

$CH_3CONH_2 + Br_2 + 4KOH \longrightarrow CH_3NH_2 + 2KBr + 2H_2O + K_2CO_3$

45. (c) $HCHO + CH_3MgI \longrightarrow A \xrightarrow{H_2O} B + Mg(OH)I$

$H-\overset{\substack{H \\ |}}{C}=O + CH_3MgI \longrightarrow H-\underset{\substack{| \\ CH_3}}{\overset{\substack{H \\ |}}{C}}-OMgI$ (A)

$\xrightarrow{H_2O} H-\underset{\substack{| \\ CH_3}}{\overset{\substack{H \\ |}}{C}}-OH + Mg(OH)I$ (B)

46. (a) $\underset{\text{Secondary amine}}{R_2NH} + \underset{\text{Acetyl chloride}}{CH_3COCl} \longrightarrow \underset{\text{N, N, dialkyl acetamide}}{R_2NCO\cdot CH_3}$

47. (b) Reducing agent Sn/HCl is used.

It changes $-NO_2$ group into $-NH_2$.

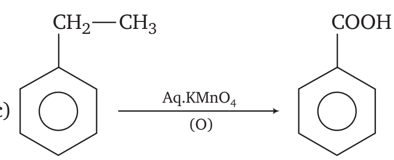
48. (b) Acetyl salicylic acid is used as analgesic drug. It is also known as aspirin.

49. (c) Benzene, $(4n+2)\pi$ electrons

$n = 1$ (6π electrons)

Naphthalene, $(4n+2)\pi$; $n = 2$, 10π electrons

Both obey Huckel's rule.

50. (c) 

Benzoic acid

MATHEMATICS

1. (a) Let θ be the angle between the given pair of straight lines

$$x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0 \quad \ldots(i)$$

$$\therefore \quad \theta = \tan^{-1}\frac{1}{3} \quad \text{(given)}$$

$$\text{or} \quad \tan\theta = \frac{1}{3}$$

We know that,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b} \quad \ldots(ii)$$

where, $h = -\frac{3}{2}, a = 1, b = \lambda$

put these value in Eq. (ii), we get

$$\frac{1}{3} = \frac{2\sqrt{\frac{9}{4} - \lambda \cdot 1}}{1+\lambda}$$

$$\Rightarrow \quad (1+\lambda) = 3\sqrt{9 - 4\lambda}$$

On squaring both sides, we get

$$(1+\lambda)^2 = (3\sqrt{9-4\lambda})^2$$

$$1 + \lambda^2 + 2\lambda = 81 - 36\lambda$$

$$\Rightarrow \quad \lambda^2 + 38\lambda - 80 = 0$$

$$\Rightarrow \quad \lambda^2 + 40\lambda - 2\lambda - 80 = 0$$

$$\Rightarrow \quad \lambda(\lambda + 40) - 2(\lambda + 40) = 0$$

$$\Rightarrow \quad (\lambda - 2)(\lambda + 40) = 0$$

$$\Rightarrow \quad \lambda = 2, -40 \quad \Rightarrow \quad \lambda = 2$$

($\because$ 'λ' is non-negative real number)

2. (a) Given that equation of lines are

$$2x - 3y = 4 \quad \ldots(i)$$

$$\text{and} \quad x + y = 1$$

$$\text{or} \quad x + y - 1 = 0 \quad \ldots(ii)$$

Now, let any line parallel to $x + y - 1 = 0$ is given by $x + y + \lambda = 0$ which passes through (1, 1).

$$\therefore \quad 1 + 1 + \lambda = 0 \Rightarrow \lambda = -2$$

$\therefore$ Required line is $x + y - 2 = 0$

$$\text{or} \quad x + y = 2 \quad \ldots(iii)$$

$\therefore$ Point of intersection of lines (i) and (iii) is (2, 0).

$\therefore$ Required distance

$$= \sqrt{(2-1)^2 + (0-1)^2} = \sqrt{1+1} = \sqrt{2}$$

3. (c) The equation of lines are $x + y = 0$ and $x - y = 0$

$\therefore$ The equation of bisectors of the angles between these lines are

$$\frac{x+y}{\sqrt{1+1}} = \pm\frac{x-y}{\sqrt{1+1}}$$

$$\Rightarrow \quad x + y = \pm(x - y) \quad \ldots(ii)$$

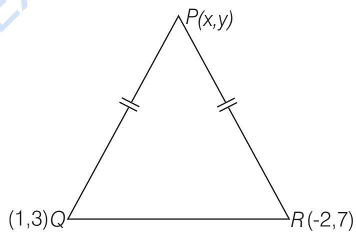
Taking + ve sign, we get $y = 0$.

Taking – ve sign, we get $x = 0$.

Hence the equation of bisectors are

$$x = 0, y = 0.$$

4. (c) Given that, the base of an isosceles triangle PQR are Q(1, 3) and R(– 2, 7). Let the co-ordinates of point P are (x, y).



$\because$ In an isosceles triangle PQ = PR

$$\text{i.e.} \quad \sqrt{(x-1)^2 + (y-3)^2} = \sqrt{(x+2)^2 + (y-7)^2}$$

$$\Rightarrow \quad (x-1)^2 + (y-3)^2 = (x+2)^2 + (y-7)^2$$

$$\Rightarrow \quad x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 + 4x + 4 + y^2 - 14y + 49$$

$$\Rightarrow -2x - 6y + 10 = 4x - 14y + 49 + 4$$

$$\Rightarrow \quad -6x + 8y = 43$$

Here, option 'c' (5/6, 6) satisfies the equation.

$\therefore$ Co-ordinate of P are (5/6, 6).

5. (b) Given that normal cuts the circle at points A(3, 4) and B(– 1, – 2). As we know, normal line is always passing through the centre of the circle and AB is the diameter of the circle. Therefore, A and B are the end points of a diameter.

$\therefore$ Required equation of circle is

$$(x-3)(x+1)+(y-4)(y+2)=0$$

$$\Rightarrow x^2-2x-3+y^2-2y-8=0$$

$$\Rightarrow \quad x^2+y^2-2x-2y-11=0$$

6. (b) Given that, $\cos P=\frac{1}{7}$, $\cos Q=\frac{13}{14}$

$$\because \ \sin P=\sqrt{1-\cos^2 P}=\sqrt{1-\frac{1}{49}}=\frac{1}{7}\sqrt{48}$$

Now, $\sin Q=\sqrt{1-\cos^2 Q}=\sqrt{1-\frac{169}{196}}=\frac{\sqrt{27}}{14}$

Now,

$$\cos(P-Q)=\cos P\cos Q+\sin P\sin Q$$

$$\cos(P-Q)=\frac{1}{7}\times\frac{13}{14}+\frac{1}{7}\sqrt{48}\times\frac{1}{14}\sqrt{27}$$

$$=\frac{13}{98}+\frac{36}{98}=\frac{49}{98}=\frac{1}{2}$$

$$\Rightarrow \quad \cos(P-Q)=\frac{1}{2}$$

$$\Rightarrow \quad \cos(P-Q)=\cos 60^\circ$$

$$\therefore \quad P-Q=60^\circ$$

7. (d) Given that $3\cos x+4\sin x=6$

Since the maximum value of $(3\cos x+4\sin x)=+\sqrt{3^2+4^2}=5$. Hence there is no 'x' satisfying $3\cos x+4\sin x=6$.

$\therefore$ No solution, as $|\sin x|\le 1, |\cos x|\le 1$ and both of them do not attain their maximum value for the same angle.

Alternate Method, $3\cos x+4\sin x=6$

Let $\quad r\cos\theta=3$

and $\quad r\sin\theta=4$

$$\Rightarrow \quad \theta=\tan^{-1}\left(\frac{4}{3}\right)$$

$$\Rightarrow \quad 5(\cos\theta\cos x+\sin\theta\sin x)=6$$

$$\Rightarrow \cos(\theta-x)=\frac{6}{5} \text{ or } \cos(x-\theta)=\frac{6}{5}$$

or $\cos(x-\theta)>1$ but $-1\le\cos x\le 1$.

$\therefore$ There exist no solution in $[-1, 1]$.

8. (d) Given that, $\sec^{-1}x=\operatorname{cosec}^{-1}y$

or $\quad \cos^{-1}\left(\frac{1}{x}\right)=\sin^{-1}\left(\frac{1}{y}\right)$

On adding $\cos^{-1}\left(\frac{1}{y}\right)$ on both sides, we get

$$\cos^{-1}\left(\frac{1}{x}\right)+\cos^{-1}\left(\frac{1}{y}\right)=\sin^{-1}\left(\frac{1}{y}\right)+\cos^{-1}\left(\frac{1}{y}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{1}{x}\right)+\cos^{-1}\left(\frac{1}{y}\right)=\frac{\pi}{2}$$

$$\left[\because \sin^{-1}\theta+\cos^{-1}\theta=\frac{\pi}{2}\right]$$

9. (b) Given that n be any integer, then n(n + 1) (2n + 1) always an integral multiple of even numbers (i.e. 6, 30, 84, ...)

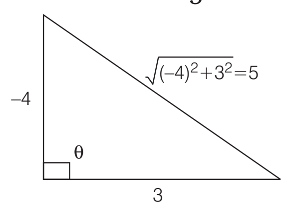
when

$n=1$, $n(n+1)(2n+1)=1(2)(3)=6$

$n=2$, $n(n+1)(2n+1)=2(3)(5)=30$

and so on.

10. (b) We have, $\tan\theta=-\frac{4}{3}$



$\because \tan\theta$ is less than 0 and $0<\theta<\frac{\pi}{2}$

or $\quad \frac{3\pi}{2}<\theta<2\pi.$

$\therefore \sin\theta=\frac{\text{perpendicular}}{\text{hypotenuse}}=-\frac{4}{5}$ (i.e. – ve)

But, θ lies also between $\frac{3\pi}{2}$ and 2π

$\therefore \quad \sin\theta=\frac{4}{5}$ also.

11. (d) Given that $c = 2\cos\theta$

$$\therefore \Delta = \begin{vmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 6 & 1 & c \end{vmatrix} = \begin{vmatrix} 2\cos\theta & 1 & 0 \\ 1 & 2\cos\theta & 1 \\ 6 & 1 & 2\cos\theta \end{vmatrix}$$

On expanding, we get

$$\Delta = 2\cos\theta\,[(4\cos^2\theta - 1)]$$
$$-1\,[(2\cos\theta - 6)] + 0\,[(1 - 12\cos\theta)]$$
$$= 8\cos^3\theta - 2\cos\theta - 2\cos\theta + 6$$
$$= 8\cos^3\theta - 4\cos\theta + 6$$

12. (a) The equation of parabola whose vertex is $(-1, -2)$ is

$$(x+1)^2 = 4a(y+2)$$

which passes through (3, 6)

i.e. $16 = 4a \times 8 \quad \Rightarrow \quad a = \dfrac{1}{2}$

$\therefore$ Required equation is

$(x+1)^2 = 2(y+2)$ or $x^2 + 2x - 2y - 3 = 0$

13. (c) Given that equation of conic is

$$9x^2 + 4y^2 - 6x + 4y + 1 = 0$$

$$\Rightarrow \quad 9x^2 - 6x + 4y^2 + 4y + 1 = 0$$

$$\Rightarrow \quad 9\left(x - \frac{1}{3}\right)^2 + 4\left(y + \frac{1}{2}\right)^2 = 1$$

or, $$\frac{\left(x - \frac{1}{3}\right)^2}{\frac{1}{9}} + \frac{\left(y + \frac{1}{2}\right)^2}{\frac{1}{4}} = 1$$

Comparing it with standard equation of ellipse $\dfrac{X^2}{a^2} + \dfrac{Y^2}{b^2} = 1$, we get $a = \dfrac{1}{3}$, $b = \dfrac{1}{2}$

Length of major axis $= 2b = 2 \times \dfrac{1}{2} = 1$

and length of minor axis $= 2a = 2 \times \dfrac{1}{3} = \dfrac{2}{3}$

14. (d) Given that, $f(x) = \cot^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$

and $$g(x) = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right)$$

Put $x = \tan\theta$ in $f(x)$, we get

$$f(x) = \cot^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$\Rightarrow \quad f(x) = \frac{\pi}{2} - 3\tan^{-1} x$$

Again put $x = \tan\theta$ in $g(x)$, we get

$$g(x) = \cos^{-1}\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right)$$

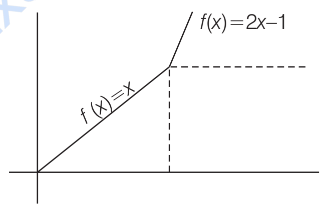
$$g(x) = 2\tan^{-1} x$$

$$\therefore \lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)}$$

$$= \lim_{x \to a} \frac{\left(\frac{\pi}{2} - 3\tan^{-1} x\right) - \frac{\pi}{2} + 3\tan^{-1} a}{2\tan^{-1} x - 2\tan^{-1} a}$$

$$= \lim_{x \to a} \frac{-3(\tan^{-1} a - \tan^{-1} x)}{2(\tan^{-1} a - \tan^{-1} x)} = -\frac{3}{2}$$

15. (c) Given that, $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2x - 1, & 1 < x \end{cases}$



From graph $f(x)$ is continuous, $\forall\ x \in R$.

i.e. $f'(x) = 1$ when $0 \le x \le 1$

and $f'(x) = 2 - 0$ when $x > 1$.

$\therefore$ Left hand limit $\ne$ Right hand limit

$\therefore$ At $x = 1$, $f(x)$ is continuous but not differentiable.

16. (c) We have, $y = \displaystyle\lim_{x \to -2} \frac{\sin^{-1}(x+2)}{x^2 + 2x}$

$$\Rightarrow \quad y = \lim_{x \to -2} \frac{\sin^{-1}(x+2)}{(x+2)} \times \frac{1}{x}$$

$$\Rightarrow \quad y = \lim_{x \to -2} (1) \times \frac{1}{x} \quad \left[\because \lim_{x \to \theta} \frac{\sin\theta}{\theta} = 1\right]$$

$$= -\frac{1}{2}$$

17. (b) Given that, $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

$f(x)$ will be differentiable at $x = 0$, if

$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0}$ exists finitely.

$\Rightarrow \lim_{x \to 0} \frac{x^p \cos\left(\frac{1}{x}\right)}{x}$ exists finitely

$\Rightarrow \lim_{x \to 0} x^{p-1} \cos\left(\frac{1}{x}\right)$ exists finitely

$\Rightarrow \quad p - 1 > 0 \quad \Rightarrow \quad p > 1$

18. (b) We have, $f(x) = 3|2 + x|$

as $f(x) = |x| \Rightarrow f'(x) = \frac{x}{|x|}$

Now, $f(x) = 3|x + 2|$

$\Rightarrow \quad f(x) = 3\frac{x+2}{|x+2|} = -3 \qquad (\text{as } x_0 = -3)$

19. (a) Given that, $f(x) = \log_5 (\log_7 x)$

or it can be written as

$$f(x) = \frac{\log_e (\log_7 x)}{\log_e 5} \quad \left[\because \log_a b = \frac{\log b}{\log a}\right]$$

On differentiating with respect to x, we get

$$\text{Now, } f'(x) = \frac{1}{\log_e 5}\left[\frac{1}{\log_7 x} \times \frac{d}{dx}(\log_7 x)\right]$$

$$= \frac{1}{\log_e 5} \times \frac{1}{\log_7 x} \times \frac{1}{\log_e 7} \cdot \frac{1}{x}$$

$$= \frac{1}{x(\log 5)(\log 7)(\log_7 x)}$$

20. (d) Here, $z^{1/3} = a - ib$

$\Rightarrow z = (a - ib)^3$

$\Rightarrow z = a^3 - 3a^2(ib) + 3a(ib)^2 - (ib)^3$

$\Rightarrow x + iy = (a^3 - 3b^2a) + i(-3a^2b + b^3)$

Equating real and imaginary parts, we get

$$x = a(a^2 - 3b^2) \text{ and } y = b(b^2 - 3a^2)$$

Now, $\frac{a(a^2 - 3b^2)}{a} - \frac{b(b^2 - 3a^2)}{b}$

$= k(a^2 - b^2)$

$\Rightarrow \quad 4a^2 - 4b^2 = k(a^2 - b^2) \quad \Rightarrow \quad k = 4$

21. (a) We have, $1 + |e^x - 1| = e^x(e^x - 2)$

$$1 + 1 + |e^x - 1| = e^{2x} - 2e^x + 1$$
$$= (e^x - 1)^2$$

$1 + 1 + |e^x - 1| = |e^x - 1|^2$

Let $(e^x - 1) = y$, then

$y^2 - y - 2 = 0 \quad \Rightarrow \quad y = 2, -1$

$\Rightarrow \quad |e^x - 1| = 2 \quad \Rightarrow \quad e^x - 1 = \pm 2$

$\Rightarrow \quad e^x = 1 \pm 2 \Rightarrow e^x = 3, -1$

or $\quad e^x = 3 \quad \Rightarrow x = \log_e 3$

$\therefore$ There is one real solution of the equation.

22. (b) Given that,

$$u = x^2 + y^2$$

and $\quad x = s + 3t, y = 2s - t$

Now, $\quad u = x^2 + y^2$

$$= (s + 3t)^2 + (2s - t)^2$$
$$= s^2 + 9t^2 + 6st + 4s^2 + t^2 - 4st$$

$\Rightarrow \quad u = 5s^2 + 2st + 10t^2$

On differentiating with respect to s, we get

$\frac{du}{ds} = 10s + 2t$

Again differentiating $\frac{d^2u}{ds^2} = 10$

23. (a) Given equations are

$$x^2 + px + q = 0 \qquad \text{...(i)}$$
$$x^2 + qx + p = 0 \qquad \text{...(ii)}$$

Let α be the common root, then

$\alpha^2 + p\alpha + q = 0$ and $\alpha^2 + q\alpha + p = 0$

$\Rightarrow \quad \frac{\alpha^2}{p^2 - q^2} = \frac{\alpha}{q - p} = \frac{1}{q - p}$

$\Rightarrow \quad \alpha^2 = \frac{(p - q)(p + q)}{-(p - q)} = -(p + q) \quad \text{...(iii)}$

and $\quad \alpha = \frac{q - p}{q - p} = 1 \qquad \text{...(iv)}$

$\therefore$ From Eq. (iii)

$$\alpha^2 = -p + q$$

$$\Rightarrow \quad (1)^2 = -(p+q) \quad \text{[from Eq. (iv)]}$$

$$\Rightarrow \quad p + q = -1 \Rightarrow p + q + 1 = 0$$

24. (a) Let α and β be the roots of the equation $x^2 - (a-2)x + (a-3) = 0$

$\therefore \quad \alpha + \beta = (a-2)$ and $\alpha\beta = a - 3$

Now, $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$$= (a-2)^3 - 3(a-3)(a-2)$$

$$= a^3 - 9a^2 + 27a - 26 = (a-3)^3 + 1$$

It assumes the least value, if $(a-3)^3 = 0$

$$\therefore \quad a = 3$$

25. (d) Given that, $|z| = \frac{1}{2}$ and $z_1 = \frac{1}{2} + \frac{\sqrt{3}\,i}{2}$

Here, one of the number must be a conjugate of $z_1 = \frac{1}{2} + \frac{\sqrt{3}\,i}{2}$ i.e. $z_2 = \frac{1}{2} - \frac{\sqrt{3}\,i}{2}$

and $\quad z_2 = z_1 e^{-i\,2\pi/3}$

$$\Rightarrow \quad z_2 = \left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left[\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right]$$

$$= -2 \times \frac{1}{2} = -1$$

$$\left[\because (1 + i\sqrt{3})\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -2\right]$$

26. (c) Given that, $z = \frac{-2}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$

$$= -\left(\frac{1 - i\sqrt{3}}{2}\right) = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{2} \times 2\right)$$

$$= \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\therefore \quad \arg(z) = \pi - \theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

27. (a) Given that,

$$1 + \omega + \omega^2 = 0 \text{ and } \omega^3 = 1$$

$$\therefore \; 2(1+\omega)(1+\omega^2) + 3(2\omega+1)(2\omega^2+1) + \ldots + (n+1)(n\omega+1)(n\omega^2+1)$$

$$= 2(1) + 3(3) + 4(7) + \ldots + (n^3 + 1)$$

Let $\quad T_n = n^3 + 1 \Rightarrow \Sigma T_n = \left[\frac{n(n+1)}{2}\right]^2 + n$

28. (a) We have, $\arg\left(\frac{z-1}{z+1}\right) = k, \; (k \neq 0)$

Put $\quad z = x + iy$, we get $\arg\left(\frac{x-1+iy}{x+1+iy}\right) = k$

$$\Rightarrow \; \arg\left[\frac{(x-1+iy)(x+1-iy)}{(x+1+iy)(x+1-iy)}\right] = k$$

$$\Rightarrow \; \arg\left[\frac{(x^2-1) + iy(2) + y^2}{(x+1)^2 + y^2}\right] = k$$

$$\Rightarrow \; \arg\left[\frac{x^2 + y^2 - 1 + 2iy}{(x+1)^2 + y^2}\right] = k$$

$$\Rightarrow \; \frac{2y}{x^2 + y^2 - 1} = k$$

$$\left[\because \arg(z) = \tan^{-1}\frac{y}{x}\right]$$

$$\Rightarrow \; (x^2 + y^2)k - k - 2y = 0$$

$$\Rightarrow \; kx^2 + ky^2 - 2y - k = 0$$

$$\Rightarrow \; x^2 + y^2 - \frac{2y}{k} - 1 = 0$$

Which represent a circle of centre $(0, 1/k)$ i.e. on y-axis.

29. (b) We have, $P(3, 4, 5), Q(4, 6, 3), R(-1, 2, 4)$ and $S(1, 0, 5)$.

$\therefore$ Direction ratios of PQ are $(1, 2, -2)$ and direction cosines of PQ are

$$\frac{1}{\sqrt{1^2 + 2^2 + (-2)^2}}, \frac{2}{\sqrt{1^2 + 2^2 + (-2)^2}}, \frac{-2}{\sqrt{1^2 + 2^2 + (-2)^2}}$$

$$= \frac{1}{\sqrt{9}}, \frac{2}{\sqrt{9}}, -\frac{2}{\sqrt{9}} = \frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$$

$\therefore$ Required projection of RS on PQ is

$l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

$$= \frac{1}{3}(2) + \left(\frac{2}{3}\right)(-2) + \left(-\frac{2}{3}\right)(1)$$

$$= \frac{2}{3} - \frac{4}{3} - \frac{2}{3} = -\frac{4}{3}$$

30. (d) Let α, β, γ be the angles which the line makes with the positive directions at the axes of x, y and z respectively, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosines of the line and these are usually denoted by

$$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$$

$$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = l^2 + m^2 + n^2 = 1$$

31. (a) Let $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

Here, $(a_1\hat{\mathbf{i}})\cdot\hat{\mathbf{i}} = a_1 = 2$; $(a_2\hat{\mathbf{j}})\cdot\hat{\mathbf{j}} = a_2 = 3$

and $(a_3\hat{\mathbf{k}})\cdot\hat{\mathbf{k}} = a_3 = 6$

$\therefore\quad \mathbf{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

Now, required length $= |\mathbf{a}| = \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

32. (a) $(10011.1)_2 = 2^4 + 0\times 2^3 + 0\times 2^2 + 1$

$\times 2^1 + 1\times 2^0 + 1\times 2^{-1}$

$= 16 + 2 + 1 + 0.5 = (19.5)_{10}$

33. (b)

2	60	
2	30	0
2	15	0
2	7	1
2	3	1
	1	1

$\therefore\quad (60)_{10} = (111100)_2$

34. (d) $(\sim p \wedge q) \cap (\sim p \vee p)$ is true.

35. (c) Given that, $f(x) = \sin\left(\dfrac{\pi x}{n-1}\right) + \cos\left(\dfrac{\pi x}{n}\right)$

Let $f(x) = g(x) + p(x),\ \forall\ n > 2$

$\therefore$ Period of $g(x) = \dfrac{2\pi\times(n-1)}{\pi} = 2(n-1)$

and period of $p(x) = \dfrac{2\pi n}{\pi} = 2n$

Period of $f(x) =$ LCM of $p(x)$ and $q(x)$

$= 2n(n-1)$

36. (d) We have, $f(\theta) = \sec^2\theta + \cos^2\theta$

Since AM $\geq$ GM

$$\frac{\sec^2\theta + \cos^2\theta}{2} \geq (\sec^2\theta\cdot\cos^2\theta)^{1/2}$$

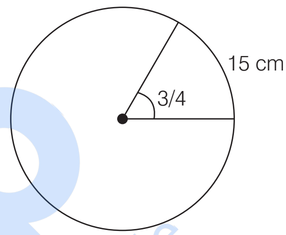
$$\Rightarrow\quad \frac{\sec^2\theta + \cos^2\theta}{2} \geq \sqrt{\sec^2\theta\cdot\cos^2\theta}$$

$$\Rightarrow\quad \frac{\sec^2\theta + \cos^2\theta}{2} \geq 1 \left[\because \sec\theta = \frac{1}{\cos\theta}\right]$$

$$\Rightarrow\quad \sec^2\theta + \cos^2\theta \geq 2 \quad\Rightarrow\quad f(\theta) \geq 2$$

$\therefore$ The value of $f(\theta)$ always lies in the interval $[2, \infty)$.

37. (b) Given that, arc = 15 cm.



$$\theta = \frac{3}{4} \text{ radian}$$

We know that, Angle $= \dfrac{\text{Arc}}{\text{Radius}}$

$\Rightarrow\quad$ Radius $= \dfrac{\text{Arc}}{\text{Angle}} = \dfrac{15\times 4}{3} = 20$ cm

38. (c) $\dfrac{d}{dx}\{f_n(x)\} = \dfrac{d}{dx}\{e^{f_{n-1}(x)}\}$

$= e^{f_{n-1}(x)}\dfrac{d}{dx}\{f_{n-1}(x)\} = f_n(x)\cdot\dfrac{d}{dx}\{e^{f_{n-1}(x)}\}$

$= f_n(x)\cdot e^{f_{n-2}(x)}\cdot\dfrac{d}{dx}\{f_{n-2}(x)\}$

$= f_n(x)\cdot f_{n-1}(x)\ldots f_2(x)\dfrac{d}{dx}\{f_1(x)\}$

$= f_n(x)\cdot f_{n-1}(x)\ldots f_2(x)\cdot f_1(x)\cdot\dfrac{d}{dx}e^{f_0(x)}$

$= f_n(x)\cdot f_{n-1}(x)\ldots f_2(x)\cdot f_1(x)\cdot\dfrac{d}{dx}(f_0(x))$

$= f_n(x)\cdot f_{n-1}(x)\ldots f_2(x)\cdot f_1(x)\cdot\dfrac{d}{dx}(x)$

$[\because f_0(x) = x \text{ (given)}]$

$= f_n(x)\cdot f_{n-1}(x)\ldots f_2(x)\cdot f_1(x).$

39. (a) We have, $3^x + 2^{2x} \geq 5^x$

$3^x + (2^2)^x \geq 5^x$; $3^x + 4^x \geq 5^x$

$$\Rightarrow\quad \left(\frac{3}{5}\right)^x + \left(\frac{4}{5}\right)^x \geq 1$$

$\Rightarrow \quad (\sin\theta)^x + (\cos\theta)^x \geq 1$

[by triangle inequality]

$\Rightarrow \quad x \leq 2$

$\therefore$ Solution set is $(-\infty, 2]$

40. (c) We have, $\dfrac{x+1}{x^2+2} > \dfrac{1}{4}$

$\Rightarrow \quad x^2 + 2 - 4x - 4 < 0 \Rightarrow x^2 - 4x - 2 < 0$

$\Rightarrow \quad (x-2)^2 - (\sqrt{6})^2 < 0$

$\Rightarrow \{(x-2) - \sqrt{6})\}\ (x - 2 + \sqrt{6}) < 0$

$\Rightarrow \quad 2 - \sqrt{6} < x < 2 + \sqrt{6}$

$\Rightarrow \quad 2 - 2.449 < x < 2 + 2.449$

$\Rightarrow \quad -0.449 < x < 4.449$

$\therefore$ In between -0.445 and 4.449 five values of x lies, i.e. 0, 1, 2, 3, 4.

$\therefore$ There are 5 integral solution of inequality.

41. (d) Let sides of a triangle are p, q, r.

Applying sine rule $\dfrac{p}{\sin P} = \dfrac{q}{\sin Q} = \dfrac{r}{\sin R} = \lambda$

$\because \quad \cos P = \dfrac{\sin Q}{2\sin R}$

$\Rightarrow \quad \dfrac{q^2 + r^2 - p^2}{2qr} = \dfrac{q}{2r}$

$\Rightarrow \quad q^2 + r^2 - p^2 = q^2 \Rightarrow r^2 = p^2 \Rightarrow r = p$

$\therefore$ Triangle is isosceles.

42. (c) Given that, $f(x) = (a - x^n)^{1/n}$

$\therefore \quad f[f(x)] = [a - \{f(x)\}^n]^{1/n}$

$= [a - (a - x^n)]^{1/n} = x$

43. (d) $f(x) = \dfrac{x^2 - 3x + 2}{x^2 + 2x - 3}$

On differentiating with respect to x, we get

$$f'(x) = \frac{(x^2 + 2x - 3)(2x - 3) - (x^2 - 3x + 2)(2x + 2)}{(x^2 + 2x - 3)^2}$$

$$= \frac{(2x^3 + 4x^2 - 6x - 3x^2 - 6x + 9) - (2x^3 - 6x^2 + 4x + 2x^2 - 6x + 4)}{(x^2 + 2x - 3)^2}$$

$$= \frac{5x^2 - 10x + 5}{(x^2 + 2x - 3)^2} > 0$$

$$= \frac{5(x-1)^2}{(x+3)^2(x-1)^2} > 0 = \frac{5}{(x+3)^2} > 0$$

$\therefore$ f(x) is increasing in its domain.

44. (b) Given that,

$\sqrt{(x-3)^2 + (y-1)^2} + \sqrt{(x+3)^2 + (y-1)^2} = 6$

$\Rightarrow \sqrt{(x-3)^2 + (y-1)^2} = 6 - \sqrt{(x+3)^2 + (y-1)^2}$

On squaring both sides, we get

$(x-3)^2 + (y-1)^2 = 36 + (x+3)^2 + (y-1)^2 - 12\sqrt{(x+3)^2 + (y-1)^2}$

$\Rightarrow x^2 - 6x + 9 + (y^2 - 2y + 1)$

$= 36 + x^2 + 6x + 9 + y^2 - 2y + 1 - 12\sqrt{(x+3)^2 + (y-1)^2}$

$\Rightarrow (x+3) = \sqrt{(x+3)^2 + (y-1)^2}$

On squaring both sides of above equation

$\Rightarrow (x+3)^2 = [(x+3)^2 + (y-1)^2]$

$\Rightarrow (y-1)^2 = 0$

$\Rightarrow$ Which represents a pair of straight lines.

45. (a) Since, $|z_1| = |z_2| = |z_3| = 1$

we get $z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$

Now, $\quad 1 = \left|\dfrac{1}{z_1} + \dfrac{1}{z_2} + \dfrac{1}{z_3}\right|$

$= |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = |\overline{z_1 + z_2 + z_3}|$

$= |z_1 + z_2 + z_3|$

46. (a) Unique solution of $x * a = b$ is taken by $x = b * a^{-1}$.

47. (d) We have, $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$ ...(i)

Put the value of $\pi = 3.141$

$\Rightarrow \quad \pi^2 =$ app. 9.6 in (i)

$\Rightarrow f(x) = \cos 9x + \cos 10x \qquad [\because \pi = 3.141]$

Put $\quad x = \dfrac{\pi}{4}$,

$f\left(\dfrac{\pi}{4}\right) = \cos\dfrac{9\pi}{4} + \cos\dfrac{10\pi}{4} = \dfrac{1}{\sqrt{2}} + 0 = \dfrac{1}{\sqrt{2}}$

Now, $f(-\pi) = \cos 9\pi + \cos 10\pi = -1 + 1 = 0$

and $f(\pi) = \cos 9\pi + \cos 10\pi = 0$

$$\therefore \quad f\left(\frac{\pi}{2}\right) = \cos\frac{9\pi}{2} + \cos\frac{10\pi}{2}$$
$$= \cos\frac{9\pi}{2} + \cos 5\pi = 0 + (-1) = -1$$

48. (a) Given that, $f(x) = \sec\left(\frac{\pi}{4}\cos^2 x\right)$

Since, $\cos^2 x$ lies between 0 to 1.

i.e. $0 \le \cos^2 x \le 1 \;\forall\; x \in (-\infty, \infty)$

$$\Rightarrow \quad 0 \le \frac{\pi}{4}\cos^2 x \le \frac{\pi}{4}$$
$$\Rightarrow \quad \sec 0 \le \sec\left(\frac{\pi}{4}\cos^2 x\right) \le \sec\frac{\pi}{4}$$
$$\Rightarrow \quad 1 \le \sec\left(\frac{\pi}{4}\cos^2 x\right) \le \sqrt{2}$$

49. (b) Given that, $f(x) = \dfrac{\sin^{-1}(3-x)}{\log(|x|-2)}$

From numerator of the function, we have

$\sin^{-1}(3-x)$, here $-1 \le 3-x \le 1$

$$\Rightarrow \quad -4 \le -x \le -2$$
$$\Rightarrow \quad 2 \le x \le 4 \qquad \ldots(i)$$

From denominator of the function, we have

$$|x| - 2 > 1 \text{ or } |x| > 3$$
$$\Rightarrow \quad x < -3 \cup x > 3 \qquad \ldots(ii)$$

From both cases, we get the domain of the given function.

i.e. $\quad 3 < x \le 4$ or $x \in (3, 4]$.

50. (c) $(1! + 2! + 3! + 4! + 5! + \ldots + 200!) \div 14$

$= 1 + 2 + 6 + 24 + 120 + 720 + (14M)$

$= 873 + 14M = 5 + 868 + 14M$

$= 5 + 14 \times 62 + 14M = 5 + 14(62 + M)$

Hence, when $1! + 2! + \ldots + 200!$ is divided by 14, remainder is 5.

51. (c) $\cos^{-1}[\cos\{2 \cdot \cot^{-1}(\sqrt{2}-1\}]$

$= \cos^{-1}[\cos\{2 \times 67.5\}]]$

$= \cos^{-1}[\cos 135°] = 135° = \dfrac{3\pi}{4}$

52. (c) We have $f(x) = [x]\cos\left[\dfrac{2x-1}{2}\right]\pi$

Since, $g(x) = [x]$ is always discontinuous at all integral values of points. Hence, $f(x)$ is discontinuous for all integral points.

53. (a) Obtain the foot of the perpendicular N(–4, 1, – 3).

Find the equations of the line passing thorugh P(2, 4, – 1) and N(–4, 1, – 3).

$$\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$

54. (b) Total number of mappings from a set A having elements into itself is n^n. And the total number of one to one mapping is $n!$.

$\therefore$ Required probability $= \dfrac{n!}{n^n}$.

55. (a) $\mathbf{a} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$,

$\mathbf{c} = -4\hat{\mathbf{i}} - \hat{\mathbf{k}}$

$\therefore \mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + (\mathbf{a} \cdot \mathbf{b})\mathbf{c} = (\mathbf{a} \cdot \mathbf{c})\mathbf{b}$

$= (4+1)(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) = 5\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 15\hat{\mathbf{k}}$

56. (c) Since, total number of numbers 1 to 5 digits are 99999 and total number of numbers 1 to 4 digits are 9999.

Hence, the total number of numbers of exactly 5 digits = 99999 – 9999 = 90000.

57. (b) The maximum value of $3\cos\theta + 4\sin\theta$

$$= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

58. (a) The vertex connectivity of any tree is one.

59. (a) Since, $x\mathbf{a} + y\mathbf{b} + z\mathbf{c} = 0$

Also, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are non-collinear vectors, then $x = y = z = 0$.

60. (b) Since, 10, 3, e, 2 are in decreasing order.

$\therefore$ $\log_{10}\alpha, \log_3\alpha, \log_e\alpha, \log_2\alpha$ are in increasing order.

61. (a) Now, $\dfrac{b-c}{r_1} + \dfrac{c-a}{r_2} + \dfrac{a-b}{r_3}$

$$= \frac{[(b-c)(s-a) + (c-a)(s-b) + (a-b)(s-c)]}{\Delta}$$

$= 0$

62. (d) Let $T = |\sin nx|$

Put $\quad n = 2$, then

$T = |\sin 2x| = |2\sin x\cos x| \le 2|\sin x|$

63. (c) Since, $(n + 1)\,p = \dfrac{101}{3}$ is not an integer.

Therefore, $p(X = r)$ is maximum when

$$r = \left[\frac{101}{3}\right] = 33.$$

64. (d) The equation of xy-plane is $z = 0$

$\therefore$ Direction cosines of its normal are 0, 0, 1.

65. (b) We have, $\int f(x)\,dx = F(x)$

$$\therefore \int x^3 f(x^2)dx = \frac{1}{2}\int x^2 f(x^2)\,d(x^2)$$

66. (d) $\int_1^5 x^2\,dx$

$$= \frac{4}{4}\left[\frac{1}{2}f(1) + f(2) + f(3) + f(4) + \frac{1}{2}f(5)\right]$$

$$= \frac{1}{2}[x^2F(x^2) - \int F(x^2)\,d(x^2)]$$

$$= \left[\frac{1}{2} + 4 + 9 + 16 + \frac{25}{2}\right] = 42 \text{ [using } f(x) = x^2]$$

67. (c) Here, $2ae = 6$ and $2b = 8 \Rightarrow b = 4$

$\Rightarrow b^2 = 16 \Rightarrow a^2(1 - e^2) = 16$

$\Rightarrow \dfrac{9}{e^2}(1 - e^2) = 16 \Rightarrow 9 - 9e^2 = 16e^2$

$\Rightarrow \quad 25e^2 = 9 \Rightarrow e = \dfrac{3}{5}$

68. (c) We know that, if $I_n = \int \sin^n x\,dx$, then

$$I_n = \frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$$

where n is a positive integer.

$\Rightarrow nI_n - (n - 1)\,I_{n-2} = -\sin^{n-1} x \cos x$

69. (a) The given graph is a connected graph.

70. (a) We have, $5^{124} = (5^3)^{41} \cdot 5$

Now, $\quad 5^3 = 1 \pmod{124}$

$\therefore \quad (5^3)^{41} = 1 \pmod{124}$

$\quad (5^3)^{41} \cdot 5 = 1 \pmod{124}$

$\Rightarrow \quad 5^{124} = 5 \pmod{124}$

GENERAL APTITUDE

16. (a) As 'kind' is related to 'judge', in the same way 'Throne' is related to 'Bench'.

17. (b) 'Roar' is the sound produced by 'Lion', similarly 'Bray' is the sound produced by 'Ass'.

18. (a) 'Frisk' is the name given to the movement of 'Lamb' similarly 'Scamper' is the name given to the movement of 'Mouse'.

19. (d) As 'Arc' is a part of 'Circle', in the same way 'Line' is a part of 'Square'.

20. (d) 'Brinjal' is a 'Vegetable', in the same way 'Orange' is a 'Fruit'.

21. (d) All except 'Reading' are related to motion.

22. (d) All except 'Bhagalpur' are in the state of Uttar Pradesh.

23. (d) Only Grapes grow in bunches and so it is odd one among others.

24. (a) Position of S from left = 19

$\therefore$ Position of S from right = 27 – 19 = 8

25. (a) From CFILORUX, Position of L from left = 12

26. (c) 7th letter from right = (27 – 7) th = 20th letter from left = T

27. (a) The pattern is as follows

$$6 \xrightarrow{+3} 9 \xrightarrow{+3} 12 \xrightarrow{+3} 15 \xrightarrow{+3} 18 \xrightarrow{+3} 21$$

$\therefore \quad ? = 20$

28. (b) The pattern is as follows

$$360 \xrightarrow{\div 2} 180 \xrightarrow{\div 2} 90 \xrightarrow{\div 2} 45 \xrightarrow{\div 2} 22.5 \xrightarrow{\div 2} 11.25$$

$\therefore \quad ? = 11.25$

29. (b) Moving anti-clockwise, the pattern is as follows 15 + 1 = 16; 16 + 2 = 18

18 + 3 = 21 and 21 + 4 = 25

So, missing number 25 + 5 = 30

30. (b) The pattern in clockwise direction

$$2 \xrightarrow{\times 2} \boxed{4} \xrightarrow{\times 4} 16 \xrightarrow{\times 16} 256 \quad \therefore \; ? = 4$$