MET 2020 Answer Key PDF

Physics

1. *(c)* Increase in potential energy

= Final potential energy – Initial potential energy

$$= -\frac{GMm}{\left(R+\frac{R}{5}\right)} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{5}{6}\frac{GMm}{R} + \frac{GMm}{R}$$

$$= \frac{1}{6}\frac{GMm}{R} = \frac{1}{6}\frac{GMmR}{R^2}$$

$$= \frac{1}{6}mgR \qquad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{5}{6}mgh \qquad \left[\because h = \frac{R}{5}\right]$$

2. *(b)* To break the covalent bond in a semiconductor, energy equal to forbidden energy gap is required.

3. *(a)* Given, $A = 60°$ and $\delta_m = 30°$

$$\text{So, } \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{60° + 30°}{2}\right)}{\sin\frac{60°}{2}}$$

$$= \frac{\sin 45°}{\sin 30°} = \frac{1}{\sqrt{2}} \times 2 = \frac{2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Wo $\quad \mu = \dfrac{1}{\sin C}$

or $\quad \sqrt{2} = \dfrac{1}{\sin C}$

or $\quad C = \sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = 45°$

4. *(c)* Faraday constant is a universal constant.

5. *(c)* If a comb is run through wet hair on rainy day, then it will not attract small bits of paper.

6. *(b)* Given, $x = 9t^2$

$$\therefore \quad v = \frac{dx}{dt} = \frac{d}{dt}(9t^2) = \frac{9d}{dt}(t^2) = 18\,t$$

also, $\quad a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = \dfrac{d}{dt}(18t) = 18$

So, the graph between acceleration and time is a line parallel to time axis as acceleration is constant.

7. *(d)* The minimum negative potential V_0 is called stopping potential.

Here, $\quad K_{max} = 10$ eV

So, the stopping potential, $eV_0 = K_{m}$

$\Rightarrow \quad eV_0 = 10$ eV

or $\quad V_0 = -10$ V

8. *(b)* Thermal radiations exists in infrared part of the electromagnetic spectrum

9. *(c)* For a thin uniform rod, moment of inertia about an axis through its centre perpendicular to length

If $\quad I = \dfrac{1}{12}ML^2$

Here, $\quad M = (m_v + m_b)$

$\therefore \quad I = \dfrac{1}{12}(m_w + m_b)L^2$

10. *(d)* The time period of a particle executing SHM is given as

$$T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$$= 2\pi\sqrt{\frac{3 \times 10^{-2}}{12 \times 10^{-2}}} = 2\pi \times \frac{1}{2}$$

$$= \pi = 3.14 \text{ s}$$

11. *(d)* A transformer is essentially an AC device, it does not work on DC. So, voltage developed across secondary is zero.

12. $\quad \text{eye} = \left(\dfrac{1}{60}\right)^\circ = \dfrac{1}{60} \times \dfrac{\pi}{180}$

Let the minimum distance between the poles be d,

then, $\quad \dfrac{d}{11000} = \dfrac{1}{60} \times \dfrac{\pi}{180}$

or $\quad d = 11000 \times \dfrac{1}{60} \times \dfrac{\pi}{180} = 3$ m

13. *(b)* Stefan's constant, $\sigma = \frac{E}{T^4}$ (where, E is energy/s area)

$$= \frac{[ML^2 T^{-2}]}{[T][L^2][\theta^4]} = [MT^{-3}\theta^{-4}]$$

14. *(b)* Time period of oscillation of mass m suspended from a spring

$$T = 2\pi\sqrt{\frac{m}{k}}$$

If the spring is cut into two halves, then the new time period.

$$T' = 2\pi\sqrt{\frac{m}{2k}} = 2\frac{\pi}{\sqrt{2}}\sqrt{\frac{m}{k}} = \frac{T}{\sqrt{2}}$$

15. *(a)* Given, $I = 2.0$ A

$$R = 25\ \Omega$$
$$t = 1\text{ min } = 60\text{ s}$$

$\therefore$ Heat produced, $H = I^2 Rt$

$$= (2)^2 \times 25 \times 60$$
$$= 4 \times 25 \times 60\text{ J}$$
$$= 6 \times 10^3\text{ J}$$

16. *(b)* Frequency for a mass m executing SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{...(i)}$$

When the mass is increased by 4 times, then the new frequency

$$f' = \frac{1}{2\pi}\sqrt{\frac{k}{4m}} = \frac{1}{2}\cdot\frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{f}{2}$$

17. *(a)* For isothermal process, $dU = 0$,

i.e., internal energy remains unchanged.

For a cyclic process, $dU = 0$,

i.e., internal energy remains unchanged.

18. *(c)* An electric charge in uniform motion produces both electric field and magnetic field.

19. *(a)* For photoelectric effect to take place, the incident wavelength (λ) should be equal to or smaller than the threshold wavelength (λ_0) *i.e.*,

$$\lambda \le \lambda_0$$

20. *(b)* $_7X^{15} + {}_2He^4 \longrightarrow {}_9Y^{19}$

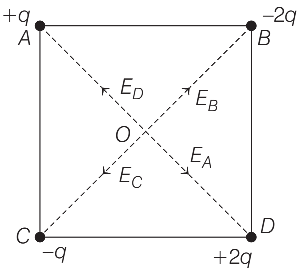
$$_9Y^{19} \longrightarrow {}_8Z^{18} + {}_1H^1$$

According to the conservation of mass number

$$19 = 18 + 1$$

So, the mass number of changed atom = 18

21. *(a)* $AD = BC = \sqrt{(5)^2 + (5)^2} = \sqrt{25 + 25} = \sqrt{2} \times 5$ cm



$\Rightarrow AO = BO = CO = OD = \frac{5\sqrt{2}}{2}$ cm

The electric field,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

So, $E_A = \frac{9 \times 10^9 \times 10 \times 10^{-9} \times 4}{25 \times 2}$

$= 7.2\ NC^{-1}$ along OD

$E_B = \frac{9 \times 10^9 \times 2 \times 10 \times 10^{-9} \times 4}{25 \times 2}$

$= 14.4\ NC^{-1}$ along OB

$E_C = \frac{9 \times 10^9 \times 10 \times 10^{-9} \times 4}{25 \times 2}$

$= 7.2$ along OC

$E_D = \frac{9 \times 10^9 \times 2 \times 10 \times 10^{-9} \times 4}{25 \times 2}$

$= 14.4$ along OA

Resultant of E_A and E_D, $E_1 = (14.4 - 7.2)$

$= 7.2\ NC^{-1}$ along OA.

Resultant of E_B and E_C, $E_2 = (14.4 - 7.2)$

$= 7.2\ NC^{-1}$ along OB.

Since, E_1 and E_2 are perpendicular to each other.

$\therefore E = \sqrt{E_1^2 + E_2^2}$ is along 45° to OA upward.

22. *(a)* For an adiabatic process, $TV^{\gamma - 1}$ = constant.

23. *(c)* The correct relation is $v_{rms} > \bar{v} > v_{mp}$

As $\quad \bar{v} = 1.6\sqrt{\frac{RT}{M}}, v_{rms} = 1.73\sqrt{\frac{RT}{M}}$

and $\quad v_{mp} = 1.41\sqrt{\frac{RT}{M}}$

24. *(a)* Due to reverse biasing, in a junction diode the potential barrier increases.

25. *(a)* Given, $Vp^2 = $ constant

or $V\left[\dfrac{RT}{V}\right]^2 = $ constant or $\dfrac{T^2}{V} = $ constant

or $\dfrac{T'^2}{T^2} = \dfrac{V'}{V} \Rightarrow \dfrac{T'^2}{T^2} = \dfrac{2V}{V} \Rightarrow T' = \sqrt{2}T$

26. *(c)* Warming of glass bulb due to filament is an example of heat transfer due to radiation.

27. *(a)* Vertical component of earth's magnetic field is

$$B_V = B_H \tan\delta$$

where, B_H = horizontal component of earth's magnetic field

δ = angle of dip.

Total intensity of earth's magnetic field

$$B = \sqrt{B_H^2 + B_V^2}$$

$$= \sqrt{B_H^2 + B_H^2} \quad [\because \tan\delta = \tan 45° = 1]$$

$$= \sqrt{2}\, B_H$$

$$= 1.44 \times 0.4 \times 10^{-4}\ \text{T}$$

$$= 5.656 \times 10^{-5}\ \text{T}$$

$$= 0.5 \times 10^{-4}\ \text{T}$$

28. *(a)* Generalised formula for apparent frequency is

$$f_a = \left(\frac{v \pm v_o}{v \mp v_s}\right) f_0$$

According to the question source is stationary,

$$v_s = 0$$

$$f_a = f' = \left(\frac{v \pm v_o}{v}\right) f_0$$

Observer is moving towards the source

$$\Rightarrow \quad f_a = f'\left(\frac{v + v_o}{v}\right) f$$

where, v = speed of the wave

v_o = speed of the observer

f = original frequency of the source

29. *(c)* Magnetic flux linked with the secondary due to current in the primary.

$$\phi_2 = MI_1$$

$$\Rightarrow \quad \frac{d\phi_2}{dt} = M\frac{dI_1}{dt}$$

$\Rightarrow$ Induced emf in the secondary

$$\varepsilon_2 = \frac{d\phi_2}{dt} = M\frac{dI_1}{dt}$$

where, M = mutual inductance = 1.5 H

$$\frac{dI_1}{dt} = \frac{5}{1 \times 10^{-3}} = 5000\ \text{A/s}$$

$$\therefore \quad \varepsilon_2 = -1.5 \times 5000\ \text{V}$$

$$= 7.5 \times 10^3 V = 7500\ \text{V}$$

30. *(c)* Magnetic field at the centre of a toroid is given by

$$B = \frac{\mu_0 Ni}{2\pi r}$$

where, $n = \dfrac{N}{2\pi r}$ = number of turns per unit length of the toroid.

31. *(a)* Magnification produced by the lens

$$m = \frac{v}{u} = -3 \Rightarrow v = -3u$$

Applying lens of formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-3u} - \frac{1}{u} = \frac{1}{0.12}$$

or $\quad \dfrac{-1-3}{3u} = \dfrac{100}{12} = \dfrac{25}{3}$

or $\quad -\dfrac{4}{3u} = \dfrac{25}{3}$

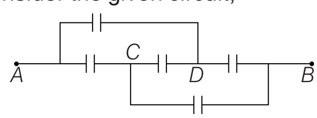
or $\quad u = -\dfrac{4}{25}\text{m} = 0.16\ \text{m}$

32. *(b)* Rate of change of momentum of the object,

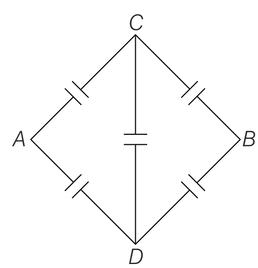
$$\frac{\Delta p}{\Delta t} = F_{\text{ext}} = \frac{mv^2}{r}$$

$$\frac{\Delta p}{\Delta t} \propto v^2 \quad \text{(where, } m \text{ and } r \text{ are constant)}$$

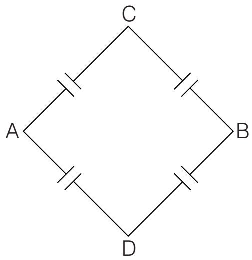
33. *(d)* Consider the given circuit,



The circuit can be redrawn as shown below.



The circuit is a balanced bridge. We can remove the capacitor between C and D.



Thus, equivalent capacitance between A and B is

$$C_{AB} = \frac{C}{2} + \frac{C}{2} = C$$

34. *(b)* Time period of a suspended magnet is given by

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where, I = moment of inertia of the magnet about the point of suspension.

M = magnetic moment

B = magnetic field

When the magnet is broken in length

$$T^{2'} = 2\pi\sqrt{\frac{I'}{M'B}}$$

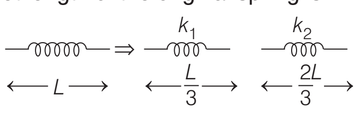
$$\Rightarrow \quad \frac{T}{T^{2'}} = \sqrt{\frac{I}{I'}} \times \sqrt{\frac{M'}{M}} = \sqrt{\frac{m L^2}{m' L'^2}} \times \sqrt{\frac{L'}{L}}$$

Here, m and m' are the masses of the magnet and L and L's are their lengths.

$$= \sqrt{\frac{m}{m'}} \times \sqrt{\frac{L}{L'}} = \sqrt{\frac{m}{m/2}} \times \sqrt{\frac{L}{L/2}} = \sqrt{2} \times \sqrt{2} = 2$$

$$\Rightarrow \quad T' = \frac{T}{2} = \frac{4}{2} = 2 \text{ s}$$

35. *(b)* Let length of the original spring is L.



As we know that,

Spring constant, $k \propto \dfrac{1}{\text{length}}$

$$\Rightarrow \quad k \propto \frac{1}{L}$$

$$k_1 \propto \frac{3}{L}$$

$$k_2 \propto \frac{3}{2L}$$

$$\Rightarrow \quad \frac{k_2}{k} = \frac{3/2L}{1/L} = \frac{3}{2}$$

$$\Rightarrow \quad k_2 = \frac{3}{2}k$$

36. *(d)* Number of half-lives in 40 min

$$n = \frac{40}{T_{1/2}} = \frac{40}{10} = 4$$

Amount of the substance remaining after n half- lives

$$N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^4 = \frac{N_0}{16}$$

where, N_0 is original amount of the substance.

Amount of the substance decayed in 40 min.

$$N' = N_0 - N = N_0 - \frac{N_0}{16} = \frac{15}{16}N_0$$

$$= \left(\frac{15}{16} \times 100\%\right) \text{ of } N_0$$

$$= 15 \times 6.25\ \% \text{ of } N_0$$

$$= 93.75\ \% \text{ of } N_0$$

37. *(c)* For isothermal expansion of an ideal gas

$$pV = \text{constant}$$

$$\Rightarrow \quad p\Delta V + V\Delta p = 0$$

For container A,

$$p_{iA}(V) + V(\Delta p) = 0$$

$$\Rightarrow \quad p_{iA} = -\frac{V\Delta P}{V} = -\Delta p \qquad \text{...(i)}$$

where, p_i is initial pressure.

For container B,

$$p_{iB}(V) + V(1.5\ \Delta p) = V$$

$$\Rightarrow \quad p_{iB} = -1.5\ \Delta p \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$p_{iA} = \frac{p_{iB}}{1.5} = \frac{2}{3}p_{iB}$$

$$\Rightarrow \quad m_A = \frac{2}{3}mB$$

$$\Rightarrow \quad 3m_A = 2mB \qquad [\because p \propto m]$$

38. *(b)* For monoatomic gas,

$$\gamma_1 = \frac{C_{p_1}}{C_{V_1}} = \frac{5}{3} \quad \text{and} \quad C_{p_1} - C_{V_1} = R$$

this gives $C_{p_1} = \dfrac{5}{2}R$ and $C_{V_1} = \dfrac{3}{2}R$

For diatomic gas,

$$\gamma_2 = \frac{C_{p_2}}{C_{V_2}} = 7/5 \quad \text{and} \quad C_{p_2} - C_{V_2} = R$$

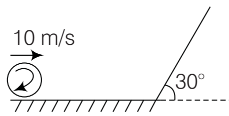
$$\Rightarrow \quad C_{p_2} = \frac{7}{2}R \quad \text{and} \quad C_{V_2} = \frac{5}{2}R$$

Now, we can write,

$$\gamma_{mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$$= \frac{C_{p_1} + C_{p_2}}{C_{V_1} + C_{V_2}} = \frac{\frac{5}{2}R + \frac{7}{2}R}{\frac{3}{2}R + \frac{5}{2}R} = \frac{12}{8} = \frac{3}{2} = 1.5$$

39. *(d)* Applying conservation of mechanical energy, we can write



$$\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = mgh$$

where, h is maximum height reached by the sphere, v is linear velocity and ω is angular velocity of the sphere.

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh \qquad [\because v = \omega r]$$

$$\Rightarrow \frac{1}{2} \times \frac{2}{5}mr^2 \times \frac{v^2}{r^2} + \frac{1}{2}mv^2 = mgh$$

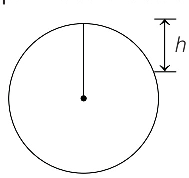
$$\frac{1}{5}mv^2 + \frac{1}{2}mv^2 = mgh$$

$$\frac{7}{10}mv^2 = mgh$$

$$h = \frac{7}{10} \times \frac{v^2}{g} = \frac{7}{10} \times \frac{10^2}{9.8} = \frac{70}{9.8}$$

$$= \frac{100}{14} = \frac{50}{7} = 7.1 \text{ m.}$$

40. *(b)* Suppose depth inside the earth is h.



As we know that, inside the earth

$$g_i \propto \left(\frac{R-h}{R^3}\right) \qquad \ldots\text{(i)}$$

On the surface of the earth

$$g_s \propto \frac{1}{R^2} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{g_i}{g_s} = \frac{(R-h)R^2}{R^3} = \frac{R-h}{R}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{R-h}{R}$$

$$\Rightarrow \quad R = 4R - 4h \Rightarrow 3R = 4h$$

$$\Rightarrow \quad h = \frac{3}{4}R.$$

41. *(c)* Range of a projectile is maximum for angle of projection of 45°.

The nearest value is 43°. Thus, among given values of angles of projection, range will be maximum for 43°.

42. *(c)* Resistance of a wire is given by

$$R = \frac{\rho L}{A}$$

where, $\rho =$ resistivity, $L =$ length of the wire

$A =$ area of cross- section of the wire

Equating initial and finial volumes of the wire, we can write

$$AL = A'L' = A' \times (2L) \Rightarrow A' = \frac{A}{2}$$

$$\therefore \quad \frac{R'}{R} = \frac{\rho L'/A'}{\rho L/A} = \left(\frac{L'}{L}\right)\left(\frac{A}{A'}\right)$$

$$= \left(\frac{2L}{L}\right)\left(\frac{A}{A/2}\right) = 4$$

$$\Rightarrow \quad R' = 4R = 4(5) = 20\ \Omega.$$

43. *(d)* Magnifying power of a telescope is given by

$$m = \frac{f_o}{f_e}$$

If the focal length of the eye piece is doubled, the new magnifying power is given by

$$m' = \frac{f_o}{f_e'} = \frac{f_o}{2f_e}$$

$$\therefore \quad m' = \frac{m}{2}$$

44. *(d)* Given $V = 250$ V and $P = 500$ W

As we know that,

$$P = \frac{V^2}{R}$$

$$\Rightarrow \qquad R = V^2/P = \frac{250 \times 250}{500}$$

$$= \frac{625}{5} = 125\ \Omega.$$

45. *(a)* As the battery remains connected. potential difference (V) of the capacitor remains constant. Suppose dielectric of dielectric constant K is introduced between the plates. Now, capcitance of the capacitor is

$$C' = KC_0$$

where, C_0 is original capacitance.

Initial energy stored in the capacitor

$$U_i = \frac{1}{2} C_0 V^2$$

Final energy stored in the capacitor

$$U_f = \frac{1}{2} C' V^2 = \frac{1}{2} (K C_0) V^2$$

Clearly, $\dfrac{U_f}{U_i} = K > 1 \quad \Rightarrow \quad U_f > U_i$

As, $\qquad Q = CV$

$\therefore$ V remains constant and C increases.

$\therefore$ Q increases .

46. *(a)* Power of the combination

$$P = P_1 + P_2$$

$$= P_{\text{concave}} + P_{\text{convex}}$$

$$\Rightarrow \qquad \frac{100}{50} = P_{\text{concave}} + \frac{100}{20}$$

or $\qquad 2 = P_{\text{concave}} + 5$

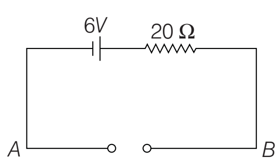
or $\qquad P_{\text{concave}} = 2 - 5 = -3.0$ D

47. *(a)* Critical angle (θ_c) for glass water interface can be written as

$$\text{Sin } \theta_c = \frac{{}_a\mu_w}{{}_a\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$$

$$\Rightarrow \qquad \theta_c = \sin^{-1} (8/9)$$

48. *(a)* In the given circuit, the diode is reverse biased. Only drift currant flows through the diode.



Consider the circuit in which by applying KVL, we can write

$$V_A - 6 + I \times 20 = V_B$$

$$\Rightarrow \quad V_A - V_B = 6 - I \times 20 = 6 - 30 \times 10^{-6} \times 20$$

$$= 6 - 0.006 = -5.9994 \text{ V}$$

49. *(d)* Given, power $P = 3$D

$$f = \frac{1}{P} = \frac{1}{3} \text{ m} = \frac{100}{3} \text{ cm.}$$

$$u = -25 \text{ cm.}$$

Applying lens formula, we can write

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \qquad \frac{1}{v} + \frac{1}{25} = \frac{3}{100}$$

$$\Rightarrow \qquad \frac{1}{v} = \frac{3}{100} - \frac{1}{25} = \frac{3-4}{100} = \frac{-1}{100}$$

$$v = -100 \text{ cm.}$$

Now, again using lens formula we can write

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

As, focal length of a far sighted person deceases

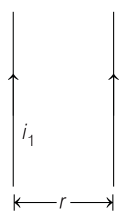
$$\Rightarrow \qquad \frac{1}{-100} - \frac{1}{u} > \frac{1}{1.7} = \frac{10}{17} \qquad [\because f < 1.7 \text{ cm}]$$

$$\Rightarrow \qquad \frac{1}{u} < \frac{-1}{100} - \frac{10}{17}$$

$$= \frac{-17 - 1000}{1700} = \frac{-1017}{1700}$$

$$\Rightarrow \ u > \frac{1700}{1000} \approx -1.7 = -1 \text{ m (nearest option)}$$

50. *(a)* Force per unit length between the two parallel current carrying wires

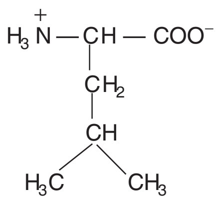


$$\frac{F}{L} = \frac{\mu_0}{2\pi r} i_1 i_2$$

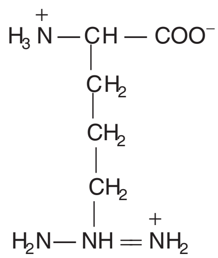
$$= \frac{\mu_0}{2\pi (2d)} i_1 i_2 = \frac{\mu_0\, i_1\, i_2}{4\pi d} \qquad [\because r = 2d]$$

Chemistry

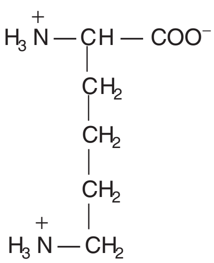
1. *(a)*



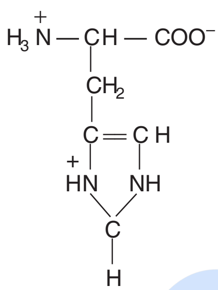
Leucine



Arginine



Lusine



Histidine

Leucine is not a basic amino acid while others three i.e., arginine, lysine and histidine are basic amino acids as they have more number of amino groups than carboxyl groups.

2. *(c)* In qualitative analysis, NH_4Cl is added before NH_4OH to decrease $[OH^-]$ concentration. NH_4Cl suppresses the ionisation of NH_4OH due to common ion effect. Otherwise basic radicals of group V and group VI will be precipitated along with IIIrd group basic radicals.

3. *(a)* 11.2 L H_2 gas at NTP = 0.5 mole of H_2 gas =1 g of H_2 g

$\frac{w_1}{w_2} = \frac{E_1}{E_2} = \frac{1}{w_2} = \frac{1}{108}$ or $w_2 = 108$g silver is deposited in half an hour.

Therefore, $108 \times 2 = 216$ g silver will be deposited in one hour.

4. *(c)* A. $[Co(NH_3)_6]^{+3}$ inner orbital (d^2sp^3) complex

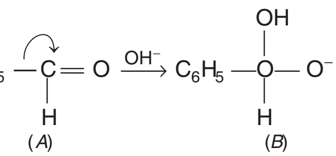
B. $[Fe(CN)_6]^{4-}$ inner orbital (d^2sp^3) complex

C. $[Ni(NH_3)_6]^{2+}$ outer orbital (sp^3d^2) complex

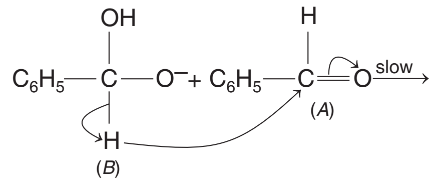
D. $[Mn(CN)_6]^{2-}$ inner orbital (d^2sp^3) complex

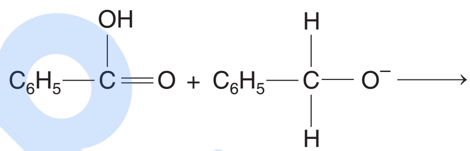
5. *(a)* For a cyclic process, $\Delta E = 0$. In cyclic process, system returns to its original state after a number of reactions.

6. *(b)* **Step I.**



Step II.





$$C_6H_5COO^- + C_6H_5CH_2OH$$

H^- is obtained by fission of C — H bond of (*B*). It is transferred to (II). This is the slowest step.

7. *(d)* $R = k[A]^x$ ...(i)

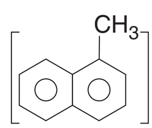
$$2R = k[4A]^x \quad ...(ii)$$

Divide Eq. (ii) by (i)

$$2 = [4]^x$$
$$R = k[A]^{1/2}$$
$$x = \frac{1}{2} = 0.5$$
$$R = k(400\,A)^{1/2}$$
$$\text{Rate} = 20k[A]^{1/2}$$

Rate becomes 20 times.

8. *(a)* Diesel is fraction of petroleum having $C_{15} - C_{18}$ composition and boiling point in the range of 573 – 673 K. Generally cetane $[CH_3 - (CH_2)_{14} - CH_3]$ and α-methyl naphthalene



are its main constituents.

9. *(b)* Organometalllic compounds must contain, at least one metal-carbon bond. Methyl lithium, CH_3 Li is an organometallic compound.

10. *(b)*

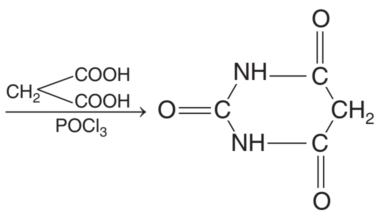
	N_2O_4	$\rightleftharpoons$	$2NO_2$
Initial	1		0
At. equil.	1		0

Total number of moles at equilibrium

$$= (1-\alpha) + 2\alpha = 1 + \alpha$$

11. *(b)*

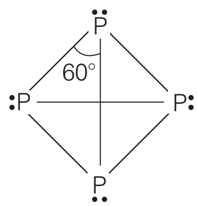
$$CHCl_3 \xrightarrow[h\nu]{[O]} COCl_2 \xrightarrow{NH_3} NH_2CONH_2$$



Barbituric acid

Barbituric acid and its derivatives are used in medicines as hypnotics and sedatives.

12. *(a)*



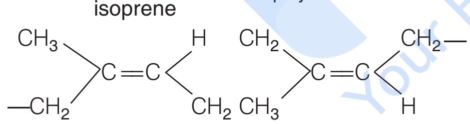
White phosphorus (P_4)

It has six P — P single bonds.

13. *(a)* Natural rubber is a linear polymers of isoprene and is also called as *cis*-1, 4-polyisoprene.

It is a linear 1, 4-polymer of isoprene (2-methyl-1,3-butadiene).

$$nCH_2=\underset{\text{isoprene}}{\overset{CH_3}{\overset{|}{C}}-CH=CH_2} \xrightarrow[\text{polymerisation}]{\text{1, 4 addition}}$$



Natural rubber

14. *(a)*

A.	XeF_4	sp^3d^2
B.	H_2O	sp^3
C.	PCl_5	sp^3d
D.	$[Pt(NH_3)_4]^{2+}$	dsp^2

15. *(d)* A new carbon- carbon bond formation takes place in Friedel-Craft reaction and Reimer-Tiemann reaction.

16. *(b)* $C_6H_5CHO + (CH_3CO)_2O \xrightarrow[453\text{ K}]{CH_3COONa}$

$$C_6H_5CH=CH\overset{O}{\overset{\|}{C}}-O-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow[-CH_3COOH]{H_2O/H^+} \underset{\text{Cinnamic acid}}{C_6H_5CH=CHCOOH}$$

17. *(a)* $\lambda = \dfrac{h}{mv} = \dfrac{h}{\sqrt{2m\times(KE)}}$

$$\lambda_1 = \frac{h}{\sqrt{2m\times 200}}$$

$$\lambda_2 = \frac{h}{\sqrt{2m\times 50}}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{1\times\sqrt{50}}{\sqrt{200}\times 1} = \frac{1}{2}$$

$$\lambda_1 : \lambda_2 = 1:2$$

18. *(c)* $CuCO_3\cdot Cu(OH)_2 \longrightarrow \underset{(A)}{Cu_2O} + CO_2 + H_2O$

$$Cu_2O + C \longrightarrow \underset{(B)}{2Cu} + CO$$

19. *(b)*

	Formula	Name of the compound
A.	$NaNO_3$	Chile salt petre
B.	$Na(NH_4)HPO_4$	Microcosmic salt
C.	$NaHCO_3$	Baking soda
D.	$Na_2CO_3\cdot 10H_2O$	Washing soda

20. *(b)* $2MnO_2 + 4HNO_3 \longrightarrow 2Mn(NO_3)_2 + 2H_2O + O_2$

$$2Mn(NO_3)_2 + 5\,PbO_2 + 6HNO_3 \longrightarrow \underset{\substack{\text{Permanganic}\\ \text{acid (violet pink)}}}{2HMnO_4} + 5Pb\,(NO_3)_2 + 2\,H_2O$$

21. *(c)* Edge length of (NaCl crystal) unit cell

$$= 2(r_{Na^+} + r_{Br^-})$$

22. *(d)* Magnetic moment $\propto$ no. of unpaired electron.

$Zn^{2+} = 3d^{10}4s^0$ unpaired electron=0;

$\mu = 0$

$Ti^{3+} = 3d^{10}4s^0$ unpaired electron=1

$\mu = \sqrt{n(n+2)} = 1.73$

$Sc^{3+} = 3d^0 4s^0$ unpaired electron=0;

$\mu = 0$

$Mn^{2+} = 3d^5 4s^0$ unpaired electrons =5

$\mu = \sqrt{5(5+2)} = 5.91$

23. *(a)* Order of solubility of fluorides of alkaline earth metals in water is

Halides of alkaline earth metals (except Be) are ionic solids and are therefore are water soluble and their solubility in water decreases from Mg to Ba due to the decrease in the hydration energy.

However, fluorides of alkanline earth metals excepts BeF_2 are almost insoluble in water.

24. *(c)*

Element	% amount	Atomic mass	Mole ratio	Simplest ratio
C	20	12	$\frac{20}{12} = 1.66$	$\frac{1.66}{1.66} = 1$
N	46.66	14	$\frac{46.66}{14} = 3.33$	$\frac{3.33}{1.66} = 2$
H	6.66	1	$\frac{6.66}{1} = 6.66$	$\frac{6.66}{1.66} = 4$
O	26.68	16	$\frac{26.68}{16} = 1.66$	$\frac{1.66}{1.66} = 1$

$\therefore$ Empirical formula $= CN_2H_4O$

25. *(a)* $KI + I_2 \longrightarrow KI_3$ or $\underset{\text{from KI}}{I^- + I_2} \longrightarrow I_3^-$

On increasing concentration of KI and I_2 two fold and three fold respectively, the concentration of KI_3 becomes two fold.

26. *(b)* $\underset{(C_{12}H_{22}O_{11})}{(+)\text{- Sucrose}} \xrightarrow{HOH/H^+} \underset{(C_6H_{12}O_6)}{D\ (+)\text{- glucose}} + \underset{(C_6H_{12}O_6)}{D\ (-)\text{- fructose}}$

$\underset{(C_{12}H_{22}O_{11})}{\text{Maltose}} \xrightarrow{HOH/H^+} \text{Glucose} + \text{Glucose}$

$\underset{(C_{12}H_{22}O_{11})}{\text{Lactose}} \xrightarrow{HOH/H^+} D\ (+)\text{- glucose} + D(+)\text{-galactose}$

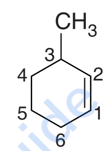
Hence, $A = $ Sucrose, $B = $ Maltose, $C = $ Lactose

27. *(a)* A decay process is possible only if accompanied by the release of energy. There are three different decay processes that increase the *n/p* ratio. These are : (i) electron capture, (ii) position emission (β^+- emission) and (iii) alpha particle decay.

28. *(b)* Kerosene is a thin, clear liquid formed from hydro carbons obtained from the fractional distillation of petroleum between 150-275°C resulting in a mixture with a density of 0.78-0.81 g/cm^3 composed of carbon chains that typically contain between 6 and 16 carbon atoms per molecule.

29. *(c)* Poly tetrafluoro ethylene (PTFE) or teflon $\text{-}\!\!(CF_2 - CF_2)\!\!\text{-}_n$ is an addition homopolymer. It is used as lubricant, insulator and in making cooking wares.

30. *(a)* The IUPAC name of the given compound is 3-methyl cyclohexene.



31. *(d)* The correct representation of a complexionian is $[Co(NH_3)_4(H_2O)Cl]^{2+}$.

32. *(d)* Atoms and ions having inert gas configuration (such as helium), i.e. stable (half-filled) configuration have high ionisation potential, i.e. it is relatively easy to remove an electron from a partially filled valence shell, where Z_{eff} is lower but it is relatively difficult to remove an electron from an atom or ion that has a filled valence shell, where Z_{eff} is higher.

33. *(c)* $\Delta H = \Delta E + \Delta n_g RT$

where $\Delta n_g = $ number of moles of gaseous product – number of moles of gaseous reactant

For $\quad H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

$$\Delta n_g = 2 - 2 = 0$$

Thus, $\quad \Delta H = \Delta E + 0 \times RT$ or $\Delta H = \Delta E$

34. *(b)* Calcination is used in metallurgy proceeds only with the expulsion of some small molecules like water, CO_2, SO_2 etc. without any chemical changes. e.g.

$$2Al(OH)_3 \xrightarrow{1500°C} Al_2O_3 + 3H_2O\uparrow$$

$$FeCO_3 \xrightarrow{\Delta} FeO + CO_2\uparrow$$

35. *(b)* $[Ni(CN)_4)]^x \equiv [Ni(CN)_4]^{2-}$

In this complex, the oxidation state of nickel is $+2$ and that of (CN) is -1.

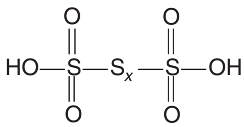
$\therefore \quad x = +2 + y(-1) = -2$

36. *(d)* From Arrhenius equation,

$$\log k = \log A - \frac{E_a}{2.303\,RT}$$

As the activation energy of a reaction is zero, the rate constant of this reaction becomes zero. It shows that the reaction is independent of temperature.

37. *(c)* I. $H_2S_3O_6$– trithionic acid is an example of polythionic acid.



III. H_2S_4 or HS—S—S—SH is an hydrosulfane.

The compounds (II) and (IV) do not exist.

38. *(b)* $^{13}_{7}N \xrightarrow{-{}_2He^4} {}^{10}_{5}B$

$^{242}_{96}Cm \xrightarrow{-{}_0n^1} {}^{244}_{94}Pu$

$^{14}_{7}N \xrightarrow{-{}_0n^1} {}^{14}_{6}C$

$^{30}_{15}P \xrightarrow[-g]{-{}_1H^1} {}^{28}_{14}Si$

39. *(b)* When cold H_2O_2 is added in cold mixture of $K_2Cr_2O_7$ and conc. H_2SO_4 (i.e. $K_2Cr_2O_7$ in acidic medium), a deep blue solution of blue perchromate is obtained as,

$$K_2Cr_2O_7 + H_2SO_4 \longrightarrow K_2SO_4 + H_2Cr_2O_7$$

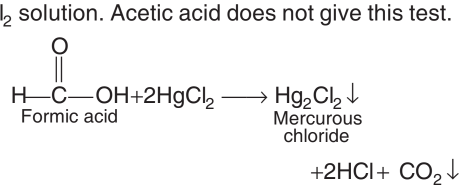
$$H_2Cr_2O_7 + 4\,H_2O_2 \longrightarrow \underset{\text{Blue perchromate}}{2CrO_5} + 5\,H_2O$$

40. *(d)* In blast furnace the highest temperature is in combustion zone. In the first stage, coke burns to produce CO_2 which rises temperature.

$$C + O_2 \longrightarrow CO_2 + \text{Heat}$$

This reaction is exothermic and hence temperature is 2170 K. It is known as combustion zone.

41. *(b)* $HgCl_2$ distinguishes formic acid and acetic acid. Formic acid produces a white precipitate with $HgCl_2$ solution. Acetic acid does not give this test.



42. *(d)* (i) Fluorine required energy when an electron is removed, i.e. ionisation energy.

(ii) Fluorine releases energy to add one electron, i.e. electron affinity ($-\,328\ \text{kJ mol}^{-1}$)

43. *(b)* Liquid hydrocarbon is converted into mixture of gaseous hydrocarbons by oxidation oxidation can be complete or partial. Complete oxidation is called combustion which is shown by the generalised reaction as

$$C_nH_{2n+2} + \left(\frac{3n+1}{2}\right)O_2 \xrightarrow{\text{Flame}} nCO_2 + (n+1)\,H_2O + \text{Heat}$$

44. *(b)* A → 2 B → 1 C → 3 D → 4

Cannizaro reaction – NaOH

Stephan's reaction – $SnCl_2$ / HCl

Clemmensen reduction – Zn/Hg –Conc. HCl

Rosenmund's method – $\dfrac{\text{Pd - BaSO}_4}{\text{Boiling xylene}}$

45. *(a)* Atomic raddii as well as ionic radii increases down the group. All the element of the nitrogen family contain same charge, i.e. – 3.

Thus, the correct order of ionic radius of nitrogen family is $N^{3-} < P^{3-} < As^{3-} < Sb^{3-} < Bi^{3-}$

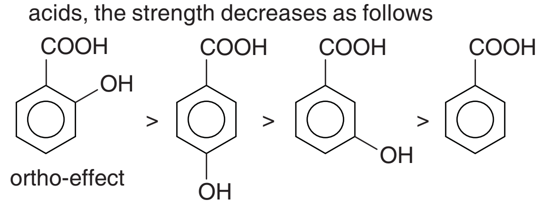
46. *(d)* Chlorophyll is responsible for the synthesis of glucose (carbohydrate) in plants.

In presence of oxygen, haemoglobin forms oxyhemoglobin.

Acetyl salicylic acid is known as aspirin

Vitamin B_{12} (Cyanocobalamin, $C_{63}H_{88}O_4N_{14}PCo$) contains Co^{2+} ion.

47. *(a)* Strength of acid is indicated by pK_a value. Higher the value of K_a or lower the value of pK_a, stronger is the acid. Among the given aromatic acids, the strength decreases as follows

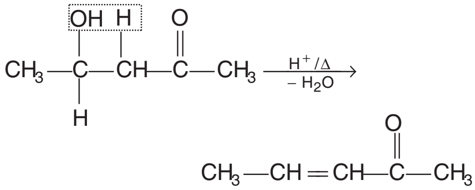


ortho-effect

(lowest pK_a value)

***Ortho*-effect** The effect of a group is maximum at *ortho*-position due to nearness, is called *ortho* effect.

48. *(a)*

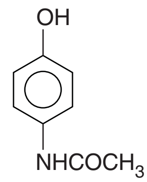


α, β-unsaturated ketones

Among the given compounds, the compound present in option (a) is readily dehydrated in acidic medium due to presence of acidic hydrogen with — OH group, In option (d), the strength of acidic hydrogen is less.

49. *(a)* The chemical substances which are used to bring down body temperature during high fever are called antipyretics,

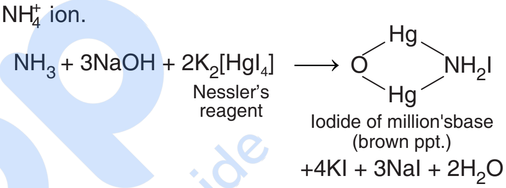
e.g. paracetamol, aspirin, phenacetin etc.



Paracetamol
(4-acetamidophenol)

50. *(d)* Analysis of NH_4^+ cation

Take small amount of salt in the test tube and add NaOH and heat. Add Nessler's reagent $[K_2H_gI_4]$ to the solution, brown ppt is obtained. It confirmed NH_4^+ ion.



Mathematics

1. *(c)* Given, $P(A \cup B) = P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) - P(A \cap B) = P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = 2P(A \cap B)$

$\Rightarrow \quad P(A) + P(B) = 2 \times P(A)\, P(B/A)$

$$\left[\because P(B/A) = \frac{P(A \cap B)}{P(A)}\right]$$

2. *(*)* Let the equation of tangent be

$$y = mx + c \quad \ldots\text{(i)}$$

Given equation of circle is

$$x^2 + y^2 - 2x - 2y + 1 = 0$$

Its centre is (1, 1)

and radius, $r = \sqrt{1^2 + 1^2 - 1} = \sqrt{1 + 1 - 1} = 1$

We know that, if line $y = mx + c$ be a tangent to the circle, then $c = \pm r\sqrt{1 + m^2}$

$\therefore \quad c = \pm 1\sqrt{1 + m^2} \quad \ldots$(ii)

Since, the tangent line is perpendicular to $y = x$.

$\therefore \quad m \times 1 = -1 \quad (\because m_1 m_2 = -1)$

$\Rightarrow \quad m = -1$

On putting $m = -1$ in Eq. (ii), we get

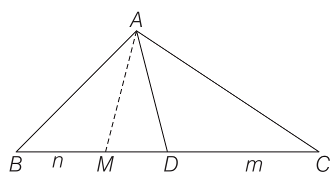
$$c = \pm 1\sqrt{1 + (-1)^2} = \pm\sqrt{1 + 1} = \pm\sqrt{2}$$

On putting the values of $m = -1$ and $c = \pm\sqrt{2}$ in Eq. (i), we get

$$y = -x \pm \sqrt{2}$$

3. *(c)* $mBD^2 + nCD^2 + (m + n)AD^2$

$= mBD^2 + nCD^2 + mAD^2 + nAD^2$



$= m(BD^2 + AD^2) + n(CD^2 + AD^2)$

$= m(AB^2 + 2DM \cdot DB) + n(AC^2 - 2DM \cdot DC)$

$= mAB^2 + m \cdot 2DM \cdot DB + nAC^2 - n \cdot 2DM \cdot DC$

$= mAB^2 + nAC^2 + 2DM(mDB - nDC)$

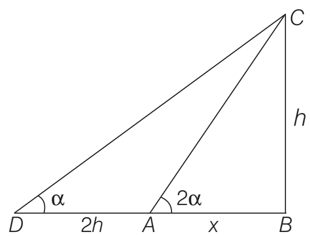
$= mAB^2 + nAC^2 + 2DM(mn - nm)$

$= mAB^2 + nAC^2$

4. *(a)* Let the height of the pole be $BC = h$ m.

In ΔABC,

$$\tan 2\alpha = \frac{h}{x} \quad \ldots\text{(i)}$$



and in ΔDBC,

$$\tan\alpha = \frac{h}{2h + x}$$

$$\Rightarrow \quad \tan\alpha = \frac{\frac{h}{x}}{2\left(\frac{h}{x}\right) + 1}$$

$$\Rightarrow \quad \tan\alpha = \frac{\tan 2\alpha}{2\tan 2\alpha + 1}$$

$$\Rightarrow \quad \tan\alpha\,(2\tan 2\alpha + 1) = \tan 2\alpha$$

$$\Rightarrow \tan\alpha\left[2 \times \frac{2\tan\alpha}{1 - \tan^2\alpha} + 1\right] = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$\Rightarrow \quad [4\tan\alpha + 1 - \tan^2\alpha] = 2$$

$$\Rightarrow \quad \tan^2\alpha - 4\tan\alpha + 1 = 0$$

$$\Rightarrow \quad \tan\alpha = \frac{+4 \pm \sqrt{16 - 4}}{2 \times 1}$$

$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$\Rightarrow \quad \tan\alpha = 2 \pm \sqrt{3}$$

Taking '–' sign, we get

$$\Rightarrow \quad \tan\alpha = 2 - \sqrt{3}$$

$$\Rightarrow \quad \tan\alpha = \tan 15^\circ$$

$$\Rightarrow \quad \alpha = 15^\circ \quad \text{or} \quad \frac{\pi}{12}$$

5. *(d)* Given, $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]} & \text{for}[x] \neq 0 \\ 0 & \text{for}[x] = 0 \end{cases}$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin(1 + [x])}{[x]}$$

$$= \frac{\sin[1 - 1]}{-1} = \frac{\sin 0}{-1} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sin(1 + [x])}{[x]}$$

$$= \frac{\sin(1 + 0)}{0} = \infty$$

Hence, limit does not exist.

6. *(b)* Given equation of straight lines are

$$y = (2 - \sqrt{3})x + 5$$

and $\quad y = (2 + \sqrt{3})x - 7$

On comparing with $y = mx + c$, we get

$$m_1 = 2 - \sqrt{3}$$

and $\quad m_2 = 2 + \sqrt{3}$

$$\therefore \quad \tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{2 + \sqrt{3} - (2 - \sqrt{3})}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}$$

$$= \frac{2\sqrt{3}}{1 + (4 - 3)} = \frac{2\sqrt{3}}{1 + 4 - 3}$$

$$\Rightarrow \quad \tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60^\circ$$

7. *(c)* Equation of plane passing through $(-1, 1, 1)$ is

$$a(x + 1) + b(y - 1) + c(z - 1) = 0 \quad \ldots\text{(i)}$$

Also, it is passing through $(1, -1, 1)$.

$$\therefore \quad a(1 + 1) + b(-1 - 1) + c(1 - 1) = 0$$

$$\Rightarrow \quad 2a - 2b + 0c = 0 \quad \ldots\text{(ii)}$$

Also, required equation of plane (i) is perpendicular to $x + 2y + 2z = 5$.

$$\therefore \quad a \times 1 + b \times 2 + c \times 2 = 0$$

$$\Rightarrow \quad a + 2b + 2c = 0 \quad \ldots\text{(iii)}$$

Eqs. (ii) and (iii) are identical.

$$\therefore \quad \frac{a}{-4 - 0} = \frac{-b}{4 - 0} = \frac{c}{4 + 2}$$

$$\Rightarrow \quad \frac{a}{-4} = \frac{b}{-4} = \frac{c}{6}$$

$$\Rightarrow \quad \frac{a}{-2} = \frac{b}{-2} = \frac{c}{3} = \lambda \quad \text{(say)}$$

$$\Rightarrow \quad a = -2\lambda, b = -2\lambda, c = 3\lambda$$

On putting the values of a, b and c in Eq. (i), we get

$$-2\lambda(x + 1) - 2\lambda(y - 1) + 3\lambda(z - 1) = 0$$

$$\Rightarrow \quad \lambda[-2x - 2 - 2y + 2 + 3z - 3] = 0$$

$$\Rightarrow \quad -2x - 2y + 3z - 3 = 0$$

$$\Rightarrow \quad 2x + 2y - 3z + 3 = 0$$

8. *(d)* Given, $g(x) = f(\tan^2 x - 2\tan x + 4)$

On differentiating w.r.t. x, we get

$$g'(x) = f'(\tan^2 x - 2\tan x + 4) \times (2\tan x \sec^2 x - 2\sec^2 x)$$

$$= f'(\tan^2 x - 2\tan x + 4)\, 2\sec^2 x\,(\tan x - 1)$$

$\because \quad f'(x) > 0 \; \forall x \in \left(0, \frac{\pi}{2}\right)$

Also, $2\sec^2 x > 0 \; \forall \, x \in \left(0, \frac{\pi}{2}\right)\left[\because x \in \left(0, \frac{\pi}{2}\right), \text{given}\right]$

But $\tan x - 1 > 0 \; \forall \, x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\therefore \quad g'(x) > 0 \; \forall \, x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

Hence, $g(x)$ is increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

9. *(a)* The total number of sample points in a sample space of single throw of two dice, $n(S) = 36$

Let E_1 = Event of getting a sum 7

E_2 = Event of getting a sum 9

$\therefore \quad E_1 = \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$

$\Rightarrow \quad n(E_1) = 6$

and $\quad E_2 = \{(3, 6), (6, 3), (4, 5), (5, 4)\}$

$\Rightarrow \quad n(E_2) = 4$

$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

and $\quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Now, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$= \frac{1}{6} + \frac{1}{9} = \frac{6+4}{36} = \frac{10}{36} = \frac{5}{18}$

10. *(b)* Compiler is a system software which transforms source code written in a programming language into another computer language.

11. *(a)* Given, $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Using L' Hospital's rule, we get

$$\lim_{t \to x} \frac{t^2 f'(x) - 2x f(t)}{-1} = 1$$

$\Rightarrow \quad x^2 f'(x) - 2x\, f(x) + 1 = 0$

$\Rightarrow \quad \frac{x^2 f'(x) - 2x f(x)}{(x^2)^2} + \frac{1}{x^4} = 0$

$\Rightarrow \quad \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = -\frac{1}{x^4}$

On integrating both sides, we get

$$\frac{f(x)}{x^2} = +\frac{1}{3x^3} + c \quad \Rightarrow \quad f(x) = \frac{1}{3x} + cx^2$$

Also, $\quad f(1) = 1 \quad \Rightarrow \quad 1 = \frac{1}{3 \times 1} + c(1)^2$

$\Rightarrow \quad 1 = \frac{1}{3} + c \quad \Rightarrow \quad \frac{2}{3} = c$

$\therefore \quad f(x) = \frac{1}{3x} + \frac{2}{3x^2}$

12. *(c)* We have,

$$\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx = A\sin^{-1}x + Bx\sqrt{1-x^2} + C$$

Let $\quad I = \int \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$

Put $\quad x = \cos 2\theta \quad \Rightarrow \quad dx = -2\sin 2\theta \, d\theta$

$\therefore \quad I = -\int \sin\left\{2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right\} 2\sin 2\theta \, d\theta$

$= -\int \sin\left\{2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right\} 2\sin 2\theta \, d\theta$

$= -\int \sin\{2\tan^{-1}\sqrt{\tan^2\theta}\} 2\sin 2\theta \, d\theta$

$= -\int \sin\{2\tan^{-1}\tan\theta\} 2\sin 2\theta \, d\theta$

$= -\int [\sin(2\theta)] \, 2\sin 2\theta \, d\theta$

$= -\int 2\sin^2 2\theta \, d\theta = -\int (1 - \cos 4\theta) \, d\theta$

$= -\left[\theta - \frac{\sin 4\theta}{4}\right] + C$

$= \left[-\frac{1}{2}\cos^{-1}x + \frac{1}{4} \times 2\sin 2\theta \cos 2\theta\right] + C$

$= \left[-\frac{1}{2}\left(\frac{\pi}{2} - \sin^{-1}x\right) + \frac{1}{2}\sqrt{1-x^2} \times x\right] + C$

$= \frac{1}{2}\sin^{-1}x + \frac{x}{2}\sqrt{1-x^2} + \left(C - \frac{\pi}{4}\right)$

But $\quad I = A\sin^{-1}x + Bx\sqrt{1-x^2} + C$

$\therefore \quad A = \frac{1}{2} \quad \text{and} \quad B = \frac{1}{2}$

Hence, $\quad A + B = \frac{1}{2} + \frac{1}{2} = 1$

13. *(c)* Let $E = (1 + x + x^3 + x^4)^{10}$

$= [1 + x + x^3(1 + x)]^{10} = (1 + x)^{10}(1 + x^3)^{10}$

$= [{}^{10}C_0 + {}^{10}C_1 x^1 + {}^{10}C_2 x^2 + \ldots + {}^{10}C_9 x^9$

$+ {}^{10}C_{10} x^{10}] \times [{}^{10}C_0 1 + {}^{10}C_1 1 \times (x^3)$

$+ {}^{10}C_2 (x^3)^2 + {}^{10}C_3 (x^3)^3 + \ldots]$

$\therefore$ Coefficient of x^4 in $E = {}^{10}C_4 \times {}^{10}C_0 + {}^{10}C_1 \times {}^{10}C_1$

$$= \frac{10!}{6! \times 4!} \times 1 + 10 \times 10$$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} + 100$$

$$= 210 + 100 = 310$$

14. *(a)* Let equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Given equation of circle is

$$x^2 + y^2 - 4x + 8 = 0$$

The centres of above circles are $(-g, -f)$ and $(2, 0)$.

Condition of orthogonality is

$$2(g_1 g_2 + f_1 f_2) = c_1 + c_2$$

$\therefore \quad 2(g \times (-2) + (f) \times 0) = c + 8$

$\Rightarrow \quad -4g = c + 8 \quad \ldots(i)$

Also, the assume circle touch the line $x + 1 = 0$.

$\therefore$ The perpendicular drawn from centre to the line is equal to radius.

$\therefore \quad \dfrac{-g+1}{\sqrt{1^2}} = \sqrt{g^2 + f^2 - c}$

$\Rightarrow \quad -g + 1 = \sqrt{g^2 + f^2 - c}$

On squaring both sides, we get

$$g^2 + 1 - 2g = g^2 + f^2 - c$$

$\Rightarrow \quad c = f^2 + 2g - 1$

Putting the value of c in Eq. (i), we get

$$-4g = f^2 + 2g - 1 + 8$$

$\Rightarrow \quad f^2 + 2g + 4g + 7 = 0$

$\Rightarrow \quad f^2 + 6g + 7 = 0$

$\therefore$ Locus of centre of circle is $y^2 + 6x + 7 = 0$.

15. *(c)* Given,

$$f(x) = \begin{vmatrix} \sin 3x & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & 4 & 1 + 2\tan 3x \end{vmatrix}$$

On differentiating w.r.t. x, we get

$$f'(x) = \begin{vmatrix} \frac{d}{dx}(\sin 3x) & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \frac{d}{dx}(\cos 3x) & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \frac{d}{dx}(\tan 3x) & 4 & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & \frac{d}{dx}(1) & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & \frac{d}{dx}(-1) & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & \frac{d}{dx}(4) & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & 1 & 2\frac{d}{dx}\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & -1 & 2\frac{d}{dx}\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & 4 & \frac{d}{dx}(1 + 2\tan 3x) \end{vmatrix}$$

$$= \begin{vmatrix} 3\cos 3x & 1 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ 3\sin 3x & -1 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ 3\sec^2 3x & 4 & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & 0 & 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right)^2 \\ \cos 3x & 0 & 2\left(\cos^2\frac{3x}{2} - \sin^2\frac{3x}{2}\right) \\ \tan 3x & 0 & 1 + 2\tan 3x \end{vmatrix}$$

$$+ \begin{vmatrix} \sin 3x & 1 & 2 \times 2\left(\cos\frac{3x}{2} + \sin\frac{3x}{2}\right) \times \left(\frac{-3}{2}\sin\frac{3x}{2} + \frac{3}{2}\cos\frac{3x}{2}\right) \\ \cos 3x & -1 & 2\left(-2\cos\frac{3x}{2} \times \frac{3}{2}\sin\frac{3x}{2} - 2\sin\frac{3x}{2} \times \frac{3}{2}\cos\frac{3x}{2}\right) \\ \tan 3x & 4 & (0 + 2 \times 3\sec^2 3x) \end{vmatrix}$$

At $x=(2n+1)\pi$,

$$f'(x)=\begin{vmatrix} 3(-1) & 1 & 2(1) \\ 0 & -1 & 2(-1) \\ 3 & 4 & 1+0 \end{vmatrix}+0$$

$$+\begin{vmatrix} 0 & 1 & 4(0-1)\times\left[-\frac{3}{2}(-1)+\frac{3}{2}\times 0\right] \\ -1 & -1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$=\begin{vmatrix} -3 & 1 & 2 \\ 0 & -1 & -2 \\ 3 & 4 & 1 \end{vmatrix}+0+\begin{vmatrix} 0 & 1 & -6 \\ -1 & -1 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$$=[-3(-1+8)-1(0+6)+2(0+3)]$$
$$+[0-1(0-0)-6(-4)]$$

$$=-21+24=3$$

16. *(b)* Given equation of line is $lx+my+n=0$

and equation of ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1.$

$\therefore$ The equation of any normal to the ellipse is

$$5x\sec\theta-3y\operatorname{cosec}\theta=25-9$$

$$\left(\begin{array}{l}\because \text{ the equation of any normal to the ellipse} \\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ is } ax\sec\theta-by\operatorname{cosec}\theta=a^2-b^2\end{array}\right)$$

$$\Rightarrow \quad 5x\sec\theta-3y\operatorname{cosec}\theta-16=0 \qquad \ldots\text{(i)}$$

As the Eq. (i) is the normal to the ellipse.

$$\therefore \quad \frac{5\sec\theta}{l}=\frac{-3\operatorname{cosec}\theta}{m}=\frac{-16}{n}$$

$$\Rightarrow \quad \cos\theta=\frac{5n}{-16l} \quad\text{and}\quad \sin\theta=\frac{3n}{16m}$$

$$\because \quad \cos^2\theta+\sin^2\theta=1$$

$$\therefore \quad \left(\frac{5n}{16l}\right)^2+\left(\frac{3n}{16m}\right)^2=1$$

$$\Rightarrow \quad \frac{25n^2}{256l^2}+\frac{9n^2}{256m^2}=1$$

$$\Rightarrow \quad \frac{25}{l^2}+\frac{9}{m^2}=\frac{256}{n^2}$$

17. *(a)* Given, $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$

$$\Rightarrow \quad 4(1-\sin x)+\sin x=a\sin x(1-\sin x)$$

$$\Rightarrow \quad 4-4\sin x+\sin x=a\sin x-a\sin^2 x$$

$$\Rightarrow \quad a\sin^2 x-(3+a)\sin x+4=0 \qquad \ldots\text{(i)}$$

It is a quadratic equation in $\sin x$, so

$$D\geq 0$$

$$\Rightarrow \quad (3+a)^2-4\times 4a\geq 0$$

$$\Rightarrow \quad 9+a^2+6a-16a\geq 0$$

$$\Rightarrow \quad a^2-10a+9\geq 0$$

$$\Rightarrow \quad (a-1)(a-9)\geq 0$$

$$\Rightarrow \quad a\geq 9 \quad\text{or}\quad a\leq -9$$

Now, at $a=9$, Eq. (i) becomes

$$9\sin^2 x-12\sin x+4=0$$

$$\Rightarrow \quad (3\sin x-2)^2=0$$

$$\Rightarrow \quad \sin x=\frac{2}{3}<0 \quad\Rightarrow\quad x\in\left(0,\frac{\pi}{2}\right)$$

Hence, least value of a is 9.

18. *(c)* If one line of regression coefficient is less than unity, then other will be greater than unity.

19. *(a)* Given edges of a parallelopiped are

$\mathrm{a}=2\hat{\mathrm{i}}-3\hat{\mathrm{j}}+\hat{\mathrm{k}}, \mathrm{b}=\hat{\mathrm{i}}-\hat{\mathrm{j}}+2\hat{\mathrm{k}}$ and $\mathrm{c}=2\hat{\mathrm{i}}+\hat{\mathrm{j}}-\hat{\mathrm{k}}$

$\therefore$ Volume of parallelopiped $=[\mathrm{a}\ \mathrm{b}\ \mathrm{c}]$

$$=\left\|\begin{matrix} 2 & -3 & 1 \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{matrix}\right\|$$

$$=|2(1-2)+3(-1-4)+1(1+2)|$$

$$=|-2-15+3|=14 \text{ cu units}$$

20. *(a)* Let $f(x)=x^3-6x+1$

Now, $\quad f(2)=(2)^3-6\times 2+1$

$$=8-12+1=-3$$

and $\quad f(3)=(3)^3-6\times 3+1$

$$=27-18+1=10$$

Here, we see that $f(2)$ and $f(3)$ have opposite signs, so one of the roots lies in $(2,3)$.

21. *(b)* Given, $\lim\limits_{x\to 0}\frac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=-\frac{1}{12} \qquad \ldots\text{(i)}$

$$\Rightarrow \quad \frac{\sin\sin 0-\sin 0}{a(0)^3+b(0)^5+c}=-\frac{1}{12}$$

$$\Rightarrow \quad \frac{0}{c}=-\frac{1}{12} \quad\Rightarrow\quad c=0$$

Applying L' Hospital's rule in Eq.(i),we get

$$\lim_{x\to 0}\frac{\cos(\sin x)\cos x-\cos x}{3ax^2+5bx^4+0}=-\frac{1}{12}$$

$$\Rightarrow \quad \lim_{x\to 0}\frac{\cos x(\cos(\sin x)-1)}{x^2(3a+5bx)}=-\frac{1}{12}$$

$$\Rightarrow \quad \lim_{x\to 0}\cos x\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{x^2(3a+5bx)}=-\frac{1}{12}$$

$$\Rightarrow \frac{\cos 0}{3a + 5b \times 0} \times \lim_{x \to 0} \frac{2\sin^2\left(\frac{\sin x}{2}\right)}{4\left(\frac{\sin x}{2}\right)^2 \times \left(\frac{x}{\sin x}\right)^2} = -\frac{1}{12}$$

$$\Rightarrow \frac{1}{3a + 0} \times \frac{1}{2} \times \frac{1}{1} = -\frac{1}{12} \quad \left(\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$$

$$\Rightarrow \frac{1}{6a} = -\frac{1}{12} \Rightarrow 6a = -12$$

$$\therefore \quad a = -2$$

Hence, $a = -2$, $b \in R$ and $c = 0$

22. *(d)* Let $x = \sqrt{12}$

$$\Rightarrow \quad x^2 = 12$$

$$\Rightarrow \quad x^2 - 12 = 0$$

Let $\quad f(x) = x^2 - 12$

The first approximation in the Newton-Raphson method is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{x_0^2 - 12}{2x_0} = \frac{x_0^2 + 12}{2x_0}$$

$\because \quad 3 < \sqrt{12} < 3.5$

We can take $x_0 = 3.5$

$$\therefore \quad x_1 = \frac{(3.5)^2 + 12}{2 \times 3.5} = \frac{12.25 + 12}{7}$$

$$= \frac{24.25}{7} = 3.464$$

23. *(b)* Given equation is

$$\frac{(3 - i)^2}{2 + i} = A + iB$$

$$\Rightarrow \frac{9 - 1 - 6i}{2 + i} = A + iB$$

$$\Rightarrow \frac{8 - 6i}{2 + i} = A + iB$$

$$\Rightarrow \frac{2(4 - 3i)}{2 + i} \times \frac{2 - i}{2 - i} = A + iB$$

$$\Rightarrow \frac{2[8 - 4i - 6i - 3]}{4 + 1} = A + iB$$

$$\Rightarrow \frac{2[5 - 10i]}{5} = A + iB$$

$$\Rightarrow 2 - 4i = A + iB$$

On equating the real and imginary parts from both sides, we get

$$A = 2 \text{ and } B = -4$$

24. *(a)* Given system of circles is

$$x^2 + y^2 + 4x + 7 = 0 \quad \ldots(\text{i})$$

$$2(x^2 + y^2) + 3x + 5y + 9 = 0$$

or $$x^2 + y^2 + \frac{3}{2}x + \frac{5}{2}y + \frac{9}{2} = 0 \quad \ldots(\text{ii})$$

and $$x^2 + y^2 + y = 0 \quad \ldots(\text{iii})$$

The radical centre can be obtained by solving the Eqs. (i), (ii) and (iii).

On subtracting Eq. (ii) from Eq. (i), we get

$$4x - \frac{3}{2}x - \frac{5}{2}y + 7 - \frac{9}{2} = 0$$

$$\Rightarrow \frac{5}{2}x - \frac{5}{2}y + \frac{5}{2} = 0$$

$$\Rightarrow x - y + 1 = 0 \quad \ldots(\text{iv})$$

On subtracting Eq. (iii) from Eq. (i), we get

$$4x - y + 7 = 0 \quad \ldots(\text{v})$$

On solving Eqs. (iv) and (v), we get

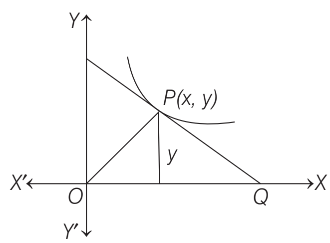
$$x = -2 \text{ and } y = -1$$

Hence, radical centre is $(-2, -1)$.

25. *(b)* Tangent drawn at any point (x, y) is

$$Y - y = \frac{dy}{dx}(X - x)$$

When $Y = 0$, $X = x - y\dfrac{dx}{dy}$



$\because \quad$ Area of $\Delta OPQ = a^2$ (given)

$$\therefore \quad \left|\frac{1}{2}X \cdot y\right| = a^2$$

$$\Rightarrow \left|\left(x - y\frac{dx}{dy}\right)y\right| = 2a^2$$

$$\Rightarrow xy - y^2\frac{dx}{dy} = \pm 2a^2$$

$$\Rightarrow \frac{dx}{dy} - \frac{x}{y} = \pm \frac{2a^2}{y^2}$$

Here, $P = -\dfrac{1}{y}$ and $Q = \pm \dfrac{2a^2}{y^2}$

$\therefore \quad IF = e^{\int P\,dy} = e^{\int -\frac{1}{y}dy}$

$= e^{-\log y} = e^{\log\frac{1}{y}} = \frac{1}{y}$

Hence, required solution is

$$x \times \frac{1}{y} = \int \pm \frac{2a^2}{y^2} \times \frac{1}{y}\,dy$$

$$\Rightarrow \quad \frac{x}{y} = \pm \frac{2a^2 y^{-2}}{-2} + C$$

$$\Rightarrow \quad x = Cy \pm \frac{a^2}{y}$$

which is the required curve.

26. *(d)* Given differential equation can be written as

$$\frac{dy}{dx} - \left(\frac{\tan 2x}{\cos^2 x}\right) y = \cos^2 x$$

Here, $P = -\dfrac{\tan 2x}{\cos^2 x} = \dfrac{-\sin 2x}{\cos 2x\left(\dfrac{\cos 2x + 1}{2}\right)}$

and $\quad Q = \cos^2 x$

$\because \quad IF = e^{\int P\,dx} = e^{-\int \frac{2\sin 2x}{\cos 2x(\cos 2x + 1)}dx}$

Put $\cos 2x = t \Rightarrow -2\sin 2x\,dx = dt$

$\therefore \quad IF = e^{\int \frac{1}{t}\left(\frac{1}{t+1}\right)dt}$

$= e^{\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt}$

$= e^{[\log t - \log(t+1)]} = e^{\log\frac{t}{t+1}}$

$= e^{\log\frac{\cos 2x}{\cos 2x + 1}} = \dfrac{\cos 2x}{\cos 2x + 1}$

Now, solution is

$$y \times \frac{\cos 2x}{\cos 2x + 1} = \int \frac{\cos 2x}{\cos 2x + 1} \times \cos^2 x\,dx + C$$

$$= \int \frac{\cos 2x}{2\cos^2 x} \times \cos^2 x\,dx + C$$

$$= \frac{1}{2}\int \cos 2x\,dx + C$$

$$= \frac{1}{2} \times \frac{\sin 2x}{2} + C$$

$$\Rightarrow \quad y\frac{\cos 2x}{\cos 2x + 1} = \frac{1}{4}\sin 2x + C \qquad \ldots(i)$$

But $\quad y\left(\dfrac{\pi}{6}\right) = \dfrac{3\sqrt{3}}{8}$

$$\therefore \quad \frac{3\sqrt{3}}{8} \times \frac{\cos\left(2 \times \frac{\pi}{6}\right)}{\cos\left(2 \times \frac{\pi}{6}\right) + 1} = \frac{1}{4}\sin 2\left(\frac{\pi}{6}\right) + C$$

$$\Rightarrow \quad \frac{\frac{3\sqrt{3}}{8} \times \frac{1}{2}}{\frac{1}{2} + 1} = \frac{1}{4} \times \frac{\sqrt{3}}{2} + C$$

$$\Rightarrow \quad \frac{3\sqrt{3}}{2 \times 8 \times \frac{3}{2}} = \frac{\sqrt{3}}{8} + C$$

$$\Rightarrow \quad \frac{\sqrt{3}}{8} = \frac{\sqrt{3}}{8} + C \Rightarrow C = 0$$

From Eq. (i), we get

$$y\frac{\cos 2x}{\cos 2x + 1} = \frac{1}{4}\sin 2x + 0$$

$$\Rightarrow \quad y = \frac{1}{4}\frac{\sin 2x}{\frac{\cos 2x}{\cos 2x + 1}} = \frac{1}{4}\frac{\sin 2x}{\frac{\cos^2 x - \sin^2 x}{2\cos^2 x}}$$

$$= \frac{1}{2} \cdot \frac{\sin 2x}{1 - \tan^2 x}$$

27. *(a)* $(1 - \omega + \omega^2)(1 + \omega - \omega^2)$

$= (1 + \omega^2 - \omega)(1 + \omega - \omega^2)$

$= (-\omega - \omega)(-\omega^2 - \omega^2) \quad (\because 1 + \omega + \omega^2 = 0)$

$= (-2\omega)(-2\omega^2)$

$= 4(\omega^3) = 4 \times 1 \qquad (\because \omega^3 = 1)$

$= 4$

28. *(*)* Given, $\dfrac{dy}{dx} = \dfrac{y - 1}{x^2 + x}$

$$\Rightarrow \quad \frac{1}{y - 1}dy = \frac{dx}{x(x + 1)}$$

On integrating both sides, we get

$$\int \frac{1}{(y - 1)}dy = \int\left(\frac{1}{x} - \frac{1}{x + 1}\right)dx$$

$$\Rightarrow \quad \log|y - 1| = [\log|x| - \log|x + 1|] + C$$

$$\Rightarrow \quad \log|y - 1| = \log\left|\frac{x}{x + 1}\right| + C \qquad \ldots(i)$$

At point (1, 0), we get

$$\log|0 - 1| = \log\left(\frac{1}{2}\right) + C$$

$\Rightarrow \quad C = -\log\frac{1}{2} + 0$

From Eq. (i), we get

$$\log|y-1| = \log\left|\frac{x}{x+1}\right| - \log\frac{1}{2}$$

$\Rightarrow \quad \log(y-1) = \log 2\left|\frac{x}{x+1}\right|$

$\Rightarrow \quad (y-1) = \frac{2x}{x+1}$

$\Rightarrow \quad (y-1)(x+1) = 2x$

$\Rightarrow \quad (y-1)(x+1) - 2x = 0$

29. *(c)* Given, $f(x) = [\sin x + \cos x]$

$$= \left[\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right)\right]$$

$$= \left[\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)\right]$$

We know that, greatest integer function is discontinuous on integer values.

Function $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ will gives integer values at

$x = 90°, 135°, 180°, 270°, 315°,$

Hence, there are five points in the given interval, in which $f(x)$ is not continuous.

30. *(d)* Let $I = \cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right)$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} + \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2} - \sqrt{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}\right]$$

$$= \cot^{-1}\left(\frac{\sin\frac{x}{2} + \cos\frac{x}{2} + \cos\frac{x}{2} - \sin\frac{x}{2}}{\sin\frac{x}{2} + \cos\frac{x}{2} - \cos\frac{x}{2} + \sin\frac{x}{2}}\right)$$

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right) = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$$

31. *(d)* Given vertices of a ΔABC are $A(-1, 3, 2), B(2, 3, 5)$ and $C(3, 5, -2)$.

Now DR's of $AB = (2+1, 3-3, 5-2) = (3, 0, 3)$

DR's of $BC = (3-2, 5-3, -2-5) = (1, 2, -7)$

and DR's of $CA = (-1-3, 3-5, 2+2)$

$= (-4, -2, 4)$

Now, the angle between AB and BC,

$$\cos B = \frac{|3\times 1 + 0\times 2 + 3\times(-7)|}{\sqrt{3^2+0^2+3^2}\sqrt{1^2+2^2+(-7)^2}}$$

$$= \frac{|3+0-21|}{\sqrt{9+0+9}\sqrt{1+4+49}}$$

$$= \frac{18}{3\sqrt{2}\times 3\sqrt{6}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

angle between BC and CA,

$$\cos C = \frac{|1\times(-4) + 2(-2) + (-7)(4)|}{\sqrt{1^2+2^2+(-7)^2}\sqrt{(-4)^2+(-2)^2+(4)^2}}$$

$$= \frac{|-4-4-28|}{\sqrt{1+4+49}\sqrt{16+4+16}}$$

$$= \frac{36}{\sqrt{54}\sqrt{36}} = \frac{36}{3\sqrt{6}\times 6}$$

$$= \frac{2}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$$

and angle between AC and AB,

$$\cos A = \frac{|-4\times 3 + (-2)\times 0 + 4\times 3|}{\sqrt{(-4)^2+(-2)^2+(4)^2}\sqrt{3^2+0^2+3^2}}$$

$= |0|$

$\Rightarrow \quad A = 90°$

32. *(a)* $\lim_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}$ $\quad\left(\text{form}\frac{0}{0}\right)$

$$= \lim_{x\to 0}\frac{\sin\sqrt{x^2}\times 2x}{3x^2}$$

(using L' Hospital's rule)

$$= \lim_{x\to 0}\frac{2}{3}\frac{\sin x}{x} = \frac{2}{3}\times 1 = \frac{2}{3}$$

33. *(b)* $[\mathrm{a}\times\mathrm{b}\ \ \mathrm{b}\times\mathrm{c}\ \ \mathrm{c}\times\mathrm{a}]$

$= (\mathrm{a}\times\mathrm{b})\cdot[(\mathrm{b}\times\mathrm{c})\times(\mathrm{c}\times\mathrm{a})]$

$= (\mathrm{a}\times\mathrm{b})\cdot[((\mathrm{b}\times\mathrm{c})\cdot\mathrm{a})\,\mathrm{c} - ((\mathrm{b}\times\mathrm{c})\cdot\mathrm{c})\,\mathrm{a}]$

$= (\mathrm{a}\times\mathrm{b})\cdot([\mathrm{b}\ \mathrm{c}\ \mathrm{a}]\,\mathrm{c} - [\mathrm{b}\ \mathrm{c}\ \mathrm{c}]\,\mathrm{a})$

$= (\mathrm{a}\times\mathrm{b}\cdot\mathrm{c})[\mathrm{b}\ \mathrm{c}\ \mathrm{a}] - [\mathrm{a}\times\mathrm{b}\cdot\mathrm{a}]\ 0$

$= [\mathrm{a}\ \mathrm{b}\ \mathrm{c}]\,[\mathrm{a}\ \mathrm{b}\ \mathrm{c}] - 0 = [\mathrm{a}\ \mathrm{b}\ \mathrm{c}]^2$

34. *(a)* Geometric mean of a and $b = \sqrt{ab}$

$\Rightarrow \quad \sqrt{ab} = 16 \quad$ (given)

$\Rightarrow \quad ab = 256 \quad$ ...(i)

And harmonic mean of a and $b = \frac{2ab}{a+b}$

$\therefore \quad \dfrac{2ab}{a+b} = \dfrac{64}{5}$ (given)

$\Rightarrow \quad \dfrac{2 \times 256}{a+b} = \dfrac{64}{5}$ [from Eq. (i)]

$\Rightarrow \quad a + b = 40$ ...(ii)

Now, $(a-b) = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{(40)^2 - 4 \times 256}$

$= \sqrt{1600 - 1024}$

$= \sqrt{576}$

$\Rightarrow \quad a - b = 24$ ...(iii)

On solving Eqs. (ii) and (iii), we get

$a = 32$ and $b = 8$

$\therefore \quad a : b = 32 : 8$

$= 4 : 1$

35. *(b)* Let four terms in a GP be ar^3, ar, $\dfrac{a}{r}$ and $\dfrac{a}{r^3}$.

According to the given condition,

$ar^3 + ar + \dfrac{a}{r} + \dfrac{a}{r^3} = 60$ ...(i)

and $\dfrac{ar^3 + \dfrac{a}{r^3}}{2} = 18$

$\Rightarrow \quad ar^3 + \dfrac{a}{r^3} = 36$ ...(ii)

Now, from Eq. (i), we have

$\left(ar + \dfrac{a}{r}\right) + ar^3 + \dfrac{a}{r^3} = 60$

$\Rightarrow \quad a\left(r + \dfrac{1}{r}\right) + 36 = 60$ [from Eq. (ii)]

$\Rightarrow \quad a\left(r + \dfrac{1}{r}\right) = 24$ ...(iii)

On dividing Eq. (iii) by Eq. (ii), we get

$$\frac{a\left(r^3 + \dfrac{1}{r^3}\right)}{a\left(r + \dfrac{1}{r}\right)} = \frac{36}{24}$$

$$\Rightarrow \quad \frac{\left(r + \dfrac{1}{r}\right)\left(r^2 + \dfrac{1}{r^2} - 1\right)}{r + \dfrac{1}{r}} = \frac{3}{2}$$

$$\Rightarrow \quad 2\left(r^2 + \frac{1}{r^2} - 1\right) = 3$$

$$\Rightarrow \quad \frac{2(r^4 + 1 - r^2)}{r^2} = 3$$

$\Rightarrow \quad 2r^4 + 2 - 2r^2 = 3r^2$

$\Rightarrow \quad 2r^4 - 5r^2 + 2 = 0$

$\Rightarrow \quad 2r^4 - 4r^2 - r^2 + 2 = 0$

$\Rightarrow \quad 2r^2(r^2 - 2) - 1(r^2 - 2) = 0$

$\Rightarrow \quad (r^2 - 2)(2r^2 - 1) = 0$

$\Rightarrow \quad r^2 = 2, 2r^2 = 1$

$\Rightarrow \quad r = \pm\sqrt{2}, r = \pm\dfrac{1}{\sqrt{2}}$

On putting $r = \sqrt{2}$ in Eq. (iii), we get

$a\left(\sqrt{2} + \dfrac{1}{\sqrt{2}}\right) = 24$

$\Rightarrow \quad a\left(\dfrac{2+1}{\sqrt{2}}\right) = 24$

$\Rightarrow \quad 3a = 24\sqrt{2}$

$\Rightarrow \quad a = 8\sqrt{2}$

$\therefore$ Series becomes $8\sqrt{2}(\sqrt{2})^3$, $8\sqrt{2}(\sqrt{2})$, $\dfrac{8\sqrt{2}}{\sqrt{2}}$ and $\dfrac{8\sqrt{2}}{(\sqrt{2})^3}$ *i.e.*, 32, 16, 8 and 4.

If we take $r = \dfrac{1}{\sqrt{2}}$, we get the series 4, 8, 16 and 32.

36. *(d)* Given, $2|x|^2 + 51 = |1 + 20x|$

Then, following two possible cases arise:

***Case* I** When $1 + 20x > 0$

$\Rightarrow \quad x > -\dfrac{1}{20}$

$\therefore \quad 2x^2 + 51 = (1 + 20x)$

$\Rightarrow \quad 2x^2 - 20x + 50 = 0$

$\Rightarrow \quad x^2 - 10x + 25 = 0$ $[\because 2 \neq 0]$

$\Rightarrow \quad (x-5)^2 = 0 \Rightarrow x = 5, 5$

***Case* II** When $1 + 20x < 0$

$\Rightarrow \quad x < -\dfrac{1}{20}$

$\therefore \quad 2x^2 + 51 = -(1 + 20x)$

$\Rightarrow \quad 2x^2 - 20x + 52 = 0$

$\Rightarrow \quad x^2 - 10x + 26 = 0$

Here, $D < 0$

Thus, roots are imaginary.

Hence, sum of real roots $= 5 + 5 = 10$

37. *(a)* Given roots are $\frac{1}{3+\sqrt{2}}$ and $\frac{1}{3-\sqrt{2}}$

i.e. $\frac{1}{(3+\sqrt{2})} \times \frac{(3-\sqrt{2})}{(3-\sqrt{2})}$

and $\frac{1}{(3-\sqrt{2})} \times \frac{(3+\sqrt{2})}{(3+\sqrt{2})}$

$\Rightarrow \frac{3-\sqrt{2}}{7}$ and $\frac{3+\sqrt{2}}{7}$

So, the required quadratic equation is

$$x^2 - (\text{Sum of the roots})\, x + \text{Product of the roots} = 0$$

$$\Rightarrow x^2 - \left[\frac{3-\sqrt{2}}{7} + \frac{3+\sqrt{2}}{7}\right]x + \left(\frac{3-\sqrt{2}}{7}\right)\left(\frac{3+\sqrt{2}}{7}\right) = 0$$

$$\Rightarrow x^2 - \left(\frac{6}{7}\right)x + \frac{9-2}{49} = 0$$

$$\Rightarrow x^2 - \frac{6}{7}x + \frac{7}{49} = 0 \Rightarrow 7x^2 - 6x + 1 = 0$$

38. *(d)* Gorden Moore

39. *(c)* Vector which is perpendicular to vertors **a** and **b**, is $\mathbf{a} \times \mathbf{b}$ or $\mathbf{b} \times \mathbf{a}$.

Hence, two vectors are possible which are perpendicular to **a** and **b**.

40. *(b)* Given that,

$$I = \int_0^6 \frac{dx}{1+x^2} \qquad \ldots\text{(i)}$$

From Eq. (i), $f(x) = \frac{1}{1+x^2}$

Now, divide the interval [0, 6] into six parts each of width

$$h = \frac{6-0}{6} = 1.$$

The value of $f(x)$ are given below

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

The trapezoidal rule is

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1})]$$

$$\therefore \int_0^6 \frac{1}{1+x^2}\,dx = \frac{1}{2}[(1+0.027) + 2\{0.5 + 0.2 + 0.1 + 0.0588 + 0.0385\}]$$

$$= \frac{1}{2}[1.027 + 2(0.8973)]$$

$$= \frac{1}{2}[1.027 + 1.7946]$$

$$= \frac{1}{2}[2.8216] = 1.4108$$

41. *(b)* Given,

$$\frac{dr}{dt} = 2\text{ m/s}, \quad \frac{dh}{dt} = -3\text{ m/s} \qquad \ldots\text{(i)}$$

$\because$ Volume of cylinder, $V = \pi r^2 h$

$$\therefore \frac{dV}{dt} = \pi\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right]$$

At $r = 3$ m and $h = 5$ m,

$$\frac{dV}{dt} = \pi\,[2 \times 3 \times 5 \times 2 - 9 \times 3] \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \frac{dV}{dt} = \pi\,[60 - 27] = 33\pi$$

$$\Rightarrow \frac{dV}{dt} = 33\,\pi\text{ m}^3/\text{s}$$

42. *(c)* Let required point be (α, β) on the straight line $y = 2x + 11$, which is nearest to the circle

$$16(x^2 + y^2) + 32x - 8y - 50 = 0$$

$$\Rightarrow x^2 + y^2 + 2x - \frac{1}{2}y - \frac{50}{16} = 0$$

$\therefore$ Centre of circle $= \left(-1, \frac{1}{4}\right)$

and radius, $r = \sqrt{1 + \frac{1}{16} + \frac{50}{16}} = \frac{\sqrt{67}}{4}$

Now, equation of straight line passing through centre $\left(-1, \frac{1}{4}\right)$ and (α, β) is

$$y - \frac{1}{4} = \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right)(x+1) \qquad \ldots\text{(i)}$$

Now, gradient of this straight line $= \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right)$

Since, straight line (i) is perpendicular to the line

$$y = 2x + 11$$

$$\therefore \quad \left(\frac{\beta - \frac{1}{4}}{\alpha + 1}\right) \times 2 = -1 \qquad [\because m_1 \cdot m_2 = -1]$$

$$\Rightarrow \quad 2\beta - \frac{1}{2} = -\alpha - 1$$

$$\Rightarrow \quad 2\beta + \alpha = -1 + \frac{1}{2} = \frac{-1}{2}$$

$$\Rightarrow \quad 4\beta + 2\alpha = -1 \qquad \ldots\text{(ii)}$$

$\because$ Point (α, β) lies on straight line

$$y = 2x + 11$$

$$\therefore \quad \beta = 2\alpha + 11$$

$$\Rightarrow \quad \beta - 2\alpha = 11 \qquad \ldots\text{(iii)}$$

On solving Eqs. (ii) and (iii), we get

$$5\beta = 10 \Rightarrow \beta = 2$$

and $\quad 2\alpha = 2 - 11 = -9 \Rightarrow \alpha = \dfrac{-9}{2}$

$\therefore$ Required point is $\left(\dfrac{-9}{2}, 2\right)$.

43. *(a)* Given equation is

$$b^2x^2 + y^2 = a^2b^2$$

$$\Rightarrow \quad \frac{x^2}{a^2} + \frac{y^2}{a^2b^2} = 1 \qquad \ldots\text{(i)}$$

Above equation of an ellipse with semi-major axis (a) and semi-minor axis (ab).

Now, ecentricity,

$$e = 1 - \frac{a^2b^2}{a^2} \Rightarrow b^2 = 1 - e^2 \qquad \ldots\text{(ii)}$$

Let (x, y) be extrimities of latusrectum, then

$$x = ae \text{ and } y = \pm\frac{a^2b^2}{a}$$

$$\Rightarrow \quad \frac{x}{a} = e \text{ and } \frac{y}{a} = \pm b^2$$

From Eq. (ii), we get

$$\pm\frac{y}{a} = 1 - \frac{x^2}{a^2} \Rightarrow \pm ay + x^2 = a^2$$

Hence, locus of latusrectum is $x^2 \pm ay = a^2$.

44. *(d)* Given differential equation is

$$(y\log x - 1)\, y\, dx = x dy$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(y\log x - 1)y}{x}$$

$$= \frac{y^2 \log x}{x} - \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \frac{y^2 \log x}{x}$$

$$\Rightarrow \quad \frac{1}{y^2}\frac{dy}{dx} + \frac{y^{-1}}{x} = \frac{\log x}{x} \qquad \ldots\text{(i)}$$

Put $\quad y^{-1} = v \Rightarrow -y^{-2}\dfrac{dy}{dx} = \dfrac{dv}{dx}$

$$\Rightarrow \quad y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

From Eq. (i), we have

$$-\frac{dv}{dx} + \frac{v}{x} = \frac{\log x}{x}$$

$$\Rightarrow \quad \frac{dv}{dx} - \frac{v}{x} = -\frac{\log x}{x} \qquad \ldots\text{(ii)}$$

This is linear differential equation.

Here, $\quad \text{IF} = e^{\int\left(-\frac{1}{x}\right)dx} = e^{-\log x} = \dfrac{1}{x}$

So, solution is $\quad v \cdot \text{IF} = \int \text{IF} \cdot Q\, dx + C$

$$v \cdot \frac{1}{x} = \int \frac{1}{x}\left(-\frac{\log x}{x}\right)dx + C$$

$$\Rightarrow \quad v \cdot \frac{1}{x} = -\int \frac{\log x}{x^2}\, dx + C$$

$$= -\left[\log x\left(\frac{-1}{x}\right) + \int \frac{1}{x} \cdot \frac{1}{x}\, dx\right] + C$$

$$\Rightarrow \quad \frac{1}{x \cdot y} = \frac{\log x}{x} + \frac{1}{x} + C \qquad \left[\because v = \frac{1}{y}\right]$$

$$\Rightarrow \quad 1 = y[\log x + 1 + Cx]$$

$$\Rightarrow \quad 1 = y[\log x + \log e + Cx] \qquad [\because 1 = \log e]$$

$$\Rightarrow \quad 1 = y[\log e \cdot x + Cx]$$

45. *(b)* A particular thing is received by a man with probability, $p = \dfrac{a}{a+b}$ and by a woman with probability, $q = \dfrac{b}{a+b}$.

Now, this experiment is repeated m times. Since, probability in each trial remains same for the men or women. Thus, we can apply binomial distribution.

Since, men should get odd number of things, so the random variable x would have $x = 1, 3, 5, \ldots$

$\therefore$ Required probability

$$= P(X = 1) + P(X = 3) + P(X = 5) + \ldots$$

$$= {}^mC_1 pq^{m-1} + {}^mC_3 p^3 q^{m-3} + {}^mC_5 p^5 q^{m-5} + \ldots$$

$$= \left[\frac{(q+p)^m - (q-p)^m}{2}\right]$$

$$= \left[\frac{1 - \left(\frac{b}{a+b} - \frac{a}{a+b}\right)^m}{2}\right] \quad [\because p + q = 1]$$

$$= \frac{(a+b)^m - (b-a)^m}{2(a+b)^m}$$

46. *(a)* Given differential equation is

$$\sqrt{a+x}\,\frac{dy}{dx} + xy = 0$$

$$\Rightarrow \quad \sqrt{a+x}\,\frac{dy}{dx} = -xy$$

$$\Rightarrow \quad \frac{dy}{y} = \frac{-x}{\sqrt{a+x}}\,dx$$

On integrating both sides, we get

$$\log y - \log C = \int\left[\frac{-x-a}{\sqrt{a+x}} + \frac{a}{\sqrt{a+x}}\right]dx$$

$$\Rightarrow \quad \log\left(\frac{y}{C}\right) = -\int\sqrt{x+a}\,dx + a\int\frac{1}{\sqrt{a+x}}\,dx$$

$$= -\frac{(x+a)^{3/2}}{3/2} + a\,\frac{(a+x)^{1/2}}{1/2}$$

$$\Rightarrow \quad \log\left(\frac{y}{C}\right) = -\frac{2}{3}(x+a)\sqrt{x+a} + 2a\sqrt{x+a}$$

$$= \frac{2}{3}(2a - x)\sqrt{x+a}$$

$$\Rightarrow \quad \frac{y}{C} = e^{\frac{2}{3}(2a-x)\sqrt{x+a}}$$

$$\Rightarrow \quad y = C \cdot e^{\frac{2}{3}(2a-x)\sqrt{x+a}}$$

which is the required solution.

47. *(a)* Given function is

$$f(x) = 2e^x - ae^{-x} + (2a+1)x - 3$$

For increasing function, $f'(x) > 0, \forall x$

$$\Rightarrow f'(x) = 2e^x + ae^{-x} + (2a+1) > 0$$

$$\Rightarrow \quad (2e^x + 1) + a(e^{-x} + 2) > 0$$

$$\Rightarrow \quad a(e^{-x} + 2) > -(1 + 2e^x)$$

$$\Rightarrow \quad a > \frac{-(1+2e^x)}{(e^{-x}+2)}$$

$$\Rightarrow \quad a > \frac{-(1+2e^x)}{(1+2e^x)} \cdot e^x$$

$$\Rightarrow \quad a > -e^x$$

$$\because \quad e^x \in (0, \infty) \Rightarrow -e^x \in (-\infty, 0)$$

and $\quad a > -e^x$

$$\therefore \quad a \in [0, \infty)$$

48. *(a)* Given, $f(x) = x^4 - x - 10$

We assume $x_0 = 2$ is the approximate root of $f(x)$.

Then, $\quad h = -\frac{f(x_0)}{f'(x_0)} = -\frac{f(2)}{f'(2)}$

$$\Rightarrow \quad h = -\left[\frac{(2)^4 - 2 - 10}{4(2)^3 - 1}\right]$$

$$\Rightarrow \quad h = -\left[\frac{16 - 12}{31}\right] = \frac{-4}{31}$$

$$\Rightarrow \quad h = -0.129$$

$\therefore$ Positive square root of $f(x)$ by Newton-Raphson method,

$$x_1 = x_0 + h = 2 + (-0.129)$$

$$= 2 - 0.129 = 1.871$$

49. *(b)* Given, $3a = b + c$

$$\because \quad \cot\left(\frac{B}{2}\right) = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$$

and $\quad \cot\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$$\therefore \cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right) = \sqrt{\frac{s(s-b)}{(s-a)(s-c)}} \times \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$= \frac{s}{(s-a)} = \frac{2s}{2s - 2a}$$

On putting $2s = a + b + c$, we get

$$\cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right) = \frac{a+b+c}{a+b+c-2a}$$

$$\Rightarrow \quad \cot\left(\frac{B}{2}\right) \cdot \cot\left(\frac{C}{2}\right) = \frac{a+3a}{a+3a-2a} = \frac{4a}{2a} = 2$$

50. *(b)* Given, $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$$\Rightarrow \quad \sin^{-1}x = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{2}\right)$$

$$\Rightarrow \qquad \sin^{-1} x = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\left[\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right]$$

$$\Rightarrow \qquad x = \sin\left\{\tan^{-1}\left(\frac{1}{2}\right)\right\}$$

$$\Rightarrow \qquad x = \sin\left\{\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right\}$$

$$\Rightarrow \qquad x = \frac{1}{\sqrt{5}}$$

51. *(c)* Given, $I = \int_0^{\sqrt{\ln(\pi/2)}} \cos(e^{x^2}) \cdot 2xe^{x^2}\, dx$

Put $e^{x^2} = t \Rightarrow 2xe^{x^2}\, dx = dt$

Now, lower limit, $t = 1$

Upper limit, $t = e^{\ln \pi/2} = \dfrac{\pi}{2}$

$$\therefore \quad I = \int_1^{\pi/2} \cos(t)\, dt = [\sin t]_1^{\pi/2}$$

$$= \sin\left(\frac{\pi}{2}\right) - \sin 1$$

$$\Rightarrow \qquad I = 1 - \sin 1$$

52. *(c)* Application Programming Interface.

53. *(d)* Given, $Z = i\log(2 - \sqrt{3})$

$$\Rightarrow \cos Z = \cos[i\log(2 - \sqrt{3})]$$

$$= \cos h\{\log(2 - \sqrt{3})\}$$

$$[\because \cos ix = \cosh x, \text{ for } x \in R]$$

$$= \frac{e^{\log(2-\sqrt{3})} + e^{-\log(2-\sqrt{3})}}{2}$$

$$\left[\because \cos hx = \frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{e^{\log(2-\sqrt{3})} + e^{\log\left(\frac{1}{2-\sqrt{3}}\right)}}{2}$$

$$= \frac{2 - \sqrt{3} + \dfrac{1}{2-\sqrt{3}}}{2}$$

$$= \frac{(2-\sqrt{3})^2 + 1}{2(2-\sqrt{3})} = \frac{4 + 3 - 4\sqrt{3} + 1}{2(2-\sqrt{3})}$$

$$= \frac{8 - 4\sqrt{3}}{2(2-\sqrt{3})} = \frac{4(2-\sqrt{3})}{2(2-\sqrt{3})} = 2$$

54. *(c)* Given, $\lim_{x \to 2} \dfrac{2 - \sqrt{2+x}}{2^{1/3} - (4-x)^{1/3}}$ $\left[\dfrac{0}{0}\text{ form}\right]$

$$= \lim_{x \to 2} \frac{0 - \dfrac{1}{2\sqrt{2+x}}}{0 - \dfrac{1}{3}\cdot\dfrac{(-1)}{(4-x)^{2/3}}}$$

$$= \lim_{x \to 2} \frac{-3(4-x)^{2/3}}{2\sqrt{2+x}}$$

$$= \frac{-3}{2} \times \frac{(4-2)^{2/3}}{\sqrt{2+2}} = \frac{-3}{2} \times \frac{2^{2/3}}{2}$$

$$= -3 \cdot 2^{\frac{2}{3} - 2} = -3 \cdot 2^{-4/3}$$

55. *(d)* Three lines of triangle are given by

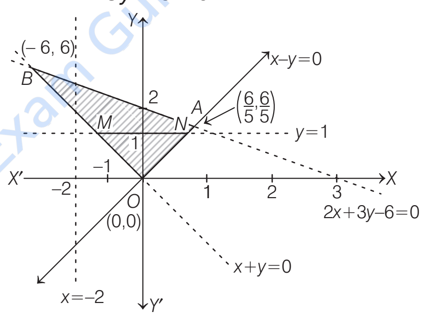
$$(x^2 - y^2)(2x + 3y - 6) = 0$$

$$\Rightarrow (x - y)(x + y)(2x + 3y - 6) = 0$$

$\therefore$ The three lines of triangle are

$$x - y = 0, x + y = 0$$

and $\qquad 2x + 3y - 6 = 0$



From given lines of triangle, the required ΔOAB is formed.

$\because (-2, \lambda)$ lies inside the triangle.

$\therefore \; 2(-2) + 3(\lambda) - 6 < 0$

and $\qquad -2 + \lambda > 0$

$\Rightarrow \qquad -4 + 3\lambda - 6 < 0$ and $\lambda > 2$

$\Rightarrow \qquad 3\lambda < 10$ and $\lambda > 2$

$\Rightarrow \qquad \lambda < \dfrac{10}{3}$ and $\lambda > 2$

$$\therefore \qquad \lambda \in \left(2, \frac{10}{3}\right) \qquad \ldots\text{(i)}$$

Now, $(\mu, 1)$ lies outside the triangle.

To find value of μ, we find the interval $[M, N]$ for values of x.

$$x + 1 \geq 0 \text{ and } x - 1 \leq 0$$

$\Rightarrow \qquad x \geq -1$ and $x \leq 1$

$\therefore \qquad x \in [-1, 1]$

$\because$ $(\mu, 1)$ lies outside the triangle.

$\therefore \qquad \mu \in (-\infty, -1) \cup (1, \infty)$

or $\qquad \mu \in R - [-1, 1]$

56. *(a)* System software : Utility software :: Operating system : Arti-virus

Utility software-Anti-virus

57. *(c)* Given equations of circles are

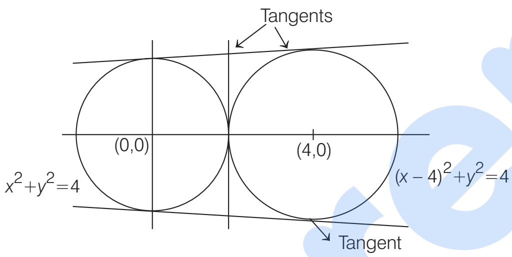
$$x^2 + y^2 = 4$$

and $\qquad x^2 + y^2 - 8x + 12 = 0$

or $\qquad x^2 + y^2 = (2)^2 \qquad \ldots(i)$

$$(x-4)^2 + (y-0)^2 = (2)^2 \qquad \ldots(ii)$$

The figure of both the circles are shown below :



From figure, we see that there is exactly three common tangents.

58. *(a)* Let $f(x) = \frac{1}{3}ax^3 + \frac{1}{2}bx^2 + cx$

Then, $f(x)$ is a polynomial.

So, it is continuous in R.

Now, $\qquad f(0) = 0$

and $\qquad f(1) = \frac{a}{3} + \frac{b}{2} + c = \frac{2a + 3b + 6c}{6}$

$\Rightarrow \qquad f(1) = 0 \qquad [\because\ 2a + 3b + 6c = 0$, given$]$

$\because f(x)$ is a polynomial, so it is differentiable in R, so in $(0, 1)$.

Hence, by Rolle's theorem, there exists atleast one point $x \in (0, 1)$, there exists such that

$$f'(x) = 0$$

$\Rightarrow \qquad ax^2 + bx + c = 0$

Hence, required interval is $(0, 1)$.

59. *(a)* Given, for $x, y \in N$,

$$f(x + y) = f(x) \cdot f(y)$$

Then, function will be of the form

$$f(x) = a^x, \text{ where } a \in N \qquad [\because a \neq 1]$$

$\because \qquad f(1) = 3$

$\Rightarrow \qquad f(1) = a^1 = 3$

$\Rightarrow \qquad a = 3$

$\therefore$ Function is $f(x) = 3^x$.

Now, $\qquad \sum_{x=1}^{n} f(x) = 120$

$\Rightarrow \qquad \sum_{x=1}^{n} 3^x = 120$

$\Rightarrow 3 + 3^2 + 3^3 + \ldots + 3^n = 120$

$\Rightarrow \qquad \frac{3(3^n - 1)}{3 - 1} = 120$

$\Rightarrow \qquad 3^n = 1 + \frac{120 \times 2}{3}$

$\Rightarrow \qquad 3^n = 81 = 3^4$

So, $\qquad n = 4$

60. *(b)* Given function is

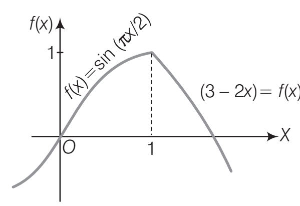
$$f(x) = \begin{cases} \sin\left(\frac{\pi x}{2}\right), & \text{if } x < 1 \\ 3 - 2x, & \text{if } x \leq 1 \end{cases}$$

$\because$ LHL and RHL of $f(x)$ is 1, when $x \to 1$

and $\qquad f(1) = 3 - 2 = 1$

$\therefore \quad f(x)$ is continuous at $x = 1$.

Now, graph of function $f(x)$ is



From the above graph of function, we see that $f(x)$ has local maxima at $x = 1$.

61. *(c)* Given differential equation is

$$\frac{dy}{dx} = y\tan x - y^2 \sec x$$

$\Rightarrow \qquad \frac{dy}{dx} - y\tan x = -y^2 \sec x$

$\Rightarrow \qquad \frac{1}{y^2}\frac{dy}{dx} - \frac{\tan x}{y} = -\sec x \qquad \ldots(i)$

Put $\qquad \frac{-1}{y} = u \Rightarrow \frac{1}{y^2}\frac{dy}{dx} = \frac{du}{dx}$

From Eq. (i),

$$\frac{du}{dx} + \tan x \cdot u = -\sec x \quad \ldots(ii)$$

This is linear differential equation of the form

$\frac{du}{dx} + P \cdot u = Q$, where $P = \tan x$, $Q = -\sec x$

$$\therefore \quad \text{IF} = e^{\int \tan x \, dx} = e^{\log \sec x} = \sec x$$

Hence, general solution is

$$u \cdot \text{IF} = \int \text{IF} \cdot Q \, dx + C_1$$

$$\Rightarrow \quad u \cdot \sec x = \int (\sec x) \cdot (-\sec x) \, dx + C_1$$

$$\Rightarrow \quad u \cdot \sec x = -\int \sec^2 x \, dx + C_1$$

$$\Rightarrow \quad u \sec x = -\tan x + C_1$$

$$\Rightarrow \quad \frac{-\sec x}{y} = -\tan x + C_1 \quad \left[\because u = \frac{-1}{y}\right]$$

$$\Rightarrow \quad \sec x = y(\tan x - C_1)$$

$$\Rightarrow \quad \sec x = y(\tan x + C) \quad [\text{where, } C = -C_1]$$

62. *(c)* Given,

$$(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = \lambda(\hat{i}x + \hat{j}y + \hat{k}z)$$

On equating the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$ both sides, we have

$$x + 3y - 4z = \lambda x,$$
$$x - 3y + 5z = \lambda y$$
and
$$3x + y + 0 = \lambda z$$

Above three equations can be rewritten as

$$(1 - \lambda)x + 3y - 4z = 0$$
$$x - (3 + \lambda)y + 5z = 0$$
$$3x + y - \lambda z = 0$$

This is homogeneous system of equations in three variables x, y and z.

It is consistent and have non-zero solution.

i.e. $(x, y, z) \neq (0, 0, 0)$, if determinant of coefficient matrix is zero.

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

On expanding along first row, we have

$$(1 - \lambda)[\lambda(3 + \lambda) - 5] - 3(-\lambda - 15) - 4(1 + 9 + 3\lambda) = 0$$

$$\Rightarrow (1 - \lambda)(\lambda^2 + 3\lambda - 5) + 3\lambda + 45 - 40 - 12\lambda = 0$$

$$\Rightarrow \lambda^2 + 3\lambda - 5 - \lambda^3 - 3\lambda^2 + 5\lambda - 9\lambda + 5 = 0$$

$$\Rightarrow \quad -\lambda^3 - 2\lambda^2 - \lambda = 0$$

$$\Rightarrow \quad \lambda(\lambda^2 + 2\lambda + 1) = 0$$

$$\Rightarrow \quad \lambda(\lambda + 1)^2 = 0$$

$$\Rightarrow \quad \lambda = 0, -1$$

63. *(b)* It is given, $\cot A$, $\cot B$ and $\cot C$ are in AP.

$$\Rightarrow \quad 2\cot B = \cot A + \cot C$$

$$\Rightarrow \quad 2\frac{\cos B}{\sin B} = \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C}$$

$$\because \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$$\Rightarrow \quad \frac{2\cos B}{kb} = \frac{\cos A}{ak} + \frac{\cos C}{ck}$$

$$\Rightarrow \quad \frac{2\cos B}{b} = \frac{\cos A}{a} + \frac{\cos C}{c}$$

$$\Rightarrow \quad \frac{2}{b}\left[\frac{a^2 + c^2 - b^2}{2ac}\right] = \frac{1}{a}\left[\frac{b^2 + c^2 - a^2}{2bc}\right] + \frac{1}{c}\left[\frac{a^2 + b^2 - c^2}{2ab}\right]$$

$$\Rightarrow \quad \frac{2(a^2 + c^2 - b^2)}{2abc} = \frac{1}{2abc}[(b^2 + c^2 - a^2) + (a^2 + b^2 - c^2)]$$

$$\Rightarrow \quad 2(a^2 + c^2 - b^2) = 2b^2$$

$$\Rightarrow \quad a^2 + c^2 = 2b^2$$

$\therefore$ a^2, b^2 and c^2 are in AP.

64. *(c)* Given integral is $\int_1^2 e^{-x/2} \, dx$

On dividing interval [1, 2] in four parts, we have

$$h = \frac{2-1}{4} = \frac{1}{4}$$

Now, value of $f(x) = e^{-x/2}$ is given below

x	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	e
$f(x)$	0.6065	0.5352	0.4724	0.4168	0.3679

Now, Simpson's $\frac{1}{3}$ rule is

$$\int_{x_0}^{x_0 + nh} f(x) dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})]$$

$$\therefore \int_1^2 e^{-x/2} dx = \frac{1}{12}[(0.6065 + 0.3679) + 4(0.5352 + 0.4168) + 2(0.4724)]$$

$$= \frac{1}{12}[0.9744 + 3.808 + 0.9448]$$

$$= \frac{1}{12} \times 5.7272 = 0.477$$

65. *(b)* When a die is rolled three times.

Then, cardinality of sample space $= 6^3$

Now, according to question, the favourable outcomes

$= \{(1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 2, 6), (1, 3, 4), (1, 3, 5), (1, 3, 6), (1, 4, 5), (1, 4, 6), (1, 5, 6), (2, 3, 4), (2, 3, 5), (2, 3, 6), (2, 4, 5), (2, 4, 6), (3, 4, 5), (3, 4, 6), (4, 5, 6), (2, 5, 6), (3, 5, 6)\}$

$\therefore$ Required probability $= \dfrac{20}{6^3} = \dfrac{5}{54}$

66. *(d)* Given, $f(x) = \log_e(6 - |x^2 + x - 6|)$

The function $f(x)$ is defined, if

$$(6 - |x^2 + x - 6|) > 0$$

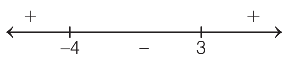
$$\Rightarrow \quad |x^2 + x - 6| < 6$$

$$\Rightarrow \quad -6 < x^2 + x - 6 < 6$$

If $\quad x^2 + x - 6 < 6$

$$\Rightarrow \quad x^2 + x - 12 < 0$$

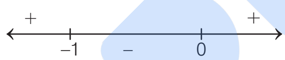
$$\Rightarrow \quad (x + 4)(x - 3) < 0$$



$\therefore \quad x \in (-4, 3) \quad \ldots(i)$

Now, if $-6 < x^2 + x - 6$

$$\Rightarrow \quad x^2 + x > 0 \Rightarrow x(x + 1) > 0$$



$\therefore \quad x \in (0, \infty) \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$x \in (0, 3)$$

$\because f(x)$ has only two integral values.

$\therefore \quad x = 1, 2$

67. *(a)* The given equation is

$$y^2 + z^2 = 0$$

Above equation represents a point circle in the *YZ*-plane.

68. *(d)* Given complex numbers are

$$Z_1 = (1 + i) \equiv (1, 1) \equiv (a_1, b_1)$$

$$Z_2 = (-2 + 3i) \equiv (-2, 3) \equiv (a_2, b_2)$$

and $\quad Z_3 = \dfrac{ai}{3} \equiv \left(0, \dfrac{a}{3}\right) \equiv (a_3, b_3)$

These three points will be collinear, if

$$\frac{1}{2}[a_1(b_2 - b_3) + a_2(b_3 - b_1) + a_3(b_1 - b_2)] = 0$$

$$\Rightarrow \quad 1\left(3 - \frac{a}{3}\right) - 2\left(\frac{a}{3} - 1\right) + 0(1 - 3) = 0$$

$$\Rightarrow \quad \frac{9 - a}{3} - 2\left(\frac{a - 3}{3}\right) + 0 = 0$$

$$\Rightarrow \quad 9 - a - 2a + 6 = 0$$

$$\Rightarrow \quad 3a = 15$$

$$\Rightarrow \quad a = 5$$

69. *(b)* Given that, a, b and c are in HP.

$\therefore$ Harmonic mean of a and c is b.

and geometric mean of a and c is $\sqrt{ac}$.

$\because$ Geometric mean > Harmonic mean

$$\Rightarrow \quad \sqrt{ac} > b \quad \ldots(i)$$

Now, for the positive numbers a^n and c^n, we have

$$\text{Geometric mean} = \sqrt{a^n \cdot c^n}$$

$$\text{and arithmetic mean} = \frac{a^n + c^n}{2}$$

$\because \quad$ AM > GM

$$\therefore \quad \frac{a^n + c^n}{2} > \sqrt{a^n \cdot c^n}$$

$$\Rightarrow \quad a^n + c^n > 2\sqrt{a^n \cdot c^n}$$

$$\Rightarrow \quad a^n + c^n > 2(\sqrt{ac})^n > 2b^n \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad a^n + c^n > 2b^n$$

70. *(d)* $\tan\left[2\tan^{-1}\left(\dfrac{1}{5}\right) - \dfrac{\pi}{4}\right] = \tan\left[\tan^{-1}\left(\dfrac{\frac{2}{5}}{1 - \frac{1}{25}}\right) - \dfrac{\pi}{4}\right]$

$$\left[\because 2\tan^{-1}\theta = \tan^{-1}\left(\frac{2\theta}{1 - \theta^2}\right)\right]$$

$$= \tan\left[\tan^{-1}\left(\frac{10}{24}\right) - \frac{\pi}{4}\right]$$

$$= \frac{\tan\left\{\tan^{-1}\left(\frac{10}{24}\right)\right\} - \tan\left(\frac{\pi}{4}\right)}{1 + \tan\left\{\tan^{-1}\left(\frac{10}{24}\right)\right\} \cdot \tan\left(\frac{\pi}{4}\right)}$$

$$= \frac{\frac{10}{24} - 1}{1 + \frac{10}{24} \cdot 1}$$

$$= \frac{10 - 24}{24 + 10}$$

$$= \frac{-14}{34} = \frac{-7}{17}$$

General Aptitude

16. *(b)* 'Brays' is the sound produced by Donkey, similarly howls is the sound produced by 'Wolf'.

17. *(d)* As 'astronauts' related to space, in the same manner Argonauts is related to 'sea'.

18. *(b)* As orthopaedic is the specialist of Bones, in the same manner dermatologist is the specialist of Skin.

19. *(a)* 'Basilica' is an early Christian church building, in the same manner, 'dormer' is a window that projects vertically from a sloping roof.

20. *(c)* As 'although' means nevertheless in the same manner, 'Though' means 'however'.

21. *(a)* Except Sun, all other are planets.

22. *(b)* Hesitant is different as all others are synonyms of each other.

23. *(b)* Except Tulip, all others are flowers whereas Tulip is a kind of plant.

24. *(d)* Position of P from left = 16

$\therefore$ Position of P from right $= 27 - 16 = 11$

25. *(d)* 14th to the right of 6th letter from the left

$= (6 + 14)$th letter from the left

$= 20$th letter from the left

$= T$

26. *(a)* In backward order of alphabet,

3rd letter from right $= C$

13th letter to the left of 3rd letter from right $= (13 + 3)$th letter from the right

$= 16$th letter from the right

$= P$

27. *(d)* The pattern is as follows

$$23 \xrightarrow{+5} 28 \xrightarrow{+6} 34 \xrightarrow{+7} 41 \xrightarrow{+8} 49 \xrightarrow{+9} 58$$

$\therefore ? = 58$

28. *(a)* The pattern is as follows

$$11 \xrightarrow{+6} 17 \xrightarrow{+6} 23 \xrightarrow{+6} 29$$

$$13 \xrightarrow{+6} 19 \xrightarrow{+6} 25$$

11 13 17 19 23 25 29

$\therefore ? = 29$

29. *(c)* In figure I, $2^2 + 4^2 = 4 + 16 = 20$

In figure II, $3^2 + 9^2 = 9 + 81 = 90$

Similarly in figure III, $1^2 + 7^2 = 1 + 49 = 50$

30. *(b)* In figure-I

$$\sqrt{64} + \sqrt{36} + \sqrt{49} = 8 + 6 + 7 = 21$$

Similarly, in figure II.

$$\sqrt{121} + \sqrt{81} + \sqrt{100} = 11 + 9 + 10 = 30$$