MET 2021 Answer Key PDF

Physics

Ans 1

Correct Option. B

Solution:

$y = A\sin[Bx + Ct + D]$

As each term inside the bracket is dimension less,

$\therefore \quad [A] = [y] = [L], \quad [B] = \frac{1}{[x]} = [L^{-1}]$

$[C] = \frac{1}{[t]} = [1^{-1}]$ and D is dimensionless.

$\therefore [ABCD] = [\mathrm{L}]\left[\mathrm{L}^{-1}\right][\mathrm{T}^{-1}][1] = \left[\mathrm{N}^0\,\mathrm{L}^0\,\mathrm{T}^{-1}\right]$

Ans 2

Correct Option. C

Solution:

Average velocity $= \frac{\text{displacement}}{\text{time}} = \frac{2r}{t} = \frac{2\times 40}{40}$

= 2 m/s

Ans 3

Correct Option. B

Solution:

Given, $a = \frac{dv}{dt} = 6t + 5$ Integrating it, $\int_0^v dv = \int_0^t (6t+5)dt$

$v = \frac{6t^2}{2} + 5t$

As $\quad v = \frac{ds}{dt}$, so $ds = \left(\frac{6t^2}{2} + 5t\right)dt$ Integrating it, $\int_0^s ds = \int_0^t \left(\frac{6t^2}{2} + 5t\right)dt$

$s = \frac{3t^3}{3} + \frac{5t^2}{2}$

When $t = 2\ \mathrm{s}, s = 3\frac{(2)^3}{3} + \frac{5(2)^2}{2}$

$= 8 + 10 = 18\ \mathrm{m}$

Ans 4

Correct Option. B

Solution:

Centripetal force $= \frac{mv^2}{R} = \frac{k}{R^2}$ (in magnitude)

$\therefore \quad \mathrm{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{k}{R}$

$PE = -\int F dR = -\int \frac{-k}{R^2} dR = -\frac{k}{R}$

$\therefore$ Total energy $= \mathrm{KE} + \mathrm{PE}$

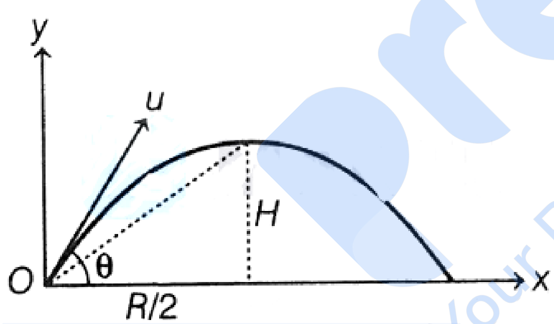
$= \frac{k}{2R} - \frac{k}{R} = -\frac{k}{2R}$

Ans 5

Correct Option. D

Solution:

Refer figure as shown, the average velocity



$= \frac{\text{displacement}}{\text{time taken}}$

$v_{\mathrm{av}} = \frac{\sqrt{H^2 + R^2/4}}{T/2}$ ...(i)

Here, $H = \frac{u^2 \sin^2 \theta}{2g}$

$R = \frac{u^2 \sin 2\theta}{g}$

Putting these values in Eq. (i), we get

$v_{\mathrm{av}} = \frac{u}{2}\sqrt{1 + 3\cos^2 \theta}$

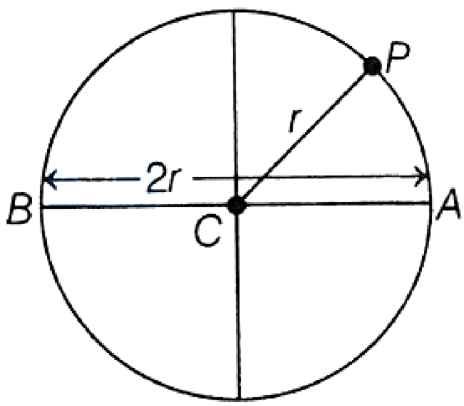
Ans 6

Correct Option. B

Solution:

Angular velocity about A,

$\omega_1 = \frac{v}{2r}$



Angular velocity about C,

$$\omega_2 = \frac{v}{r}$$
$$\therefore \frac{\omega_1}{\omega_2}$$
$$= \frac{v/2r}{v/r} = 1:2$$

Ans 7

Correct Option. C

Solution:

Solution:

The mass of block A = 4 kg

Force of friction between A and B = 12 N Mass of block A and B = 4 + 8 = 12 kg ie, mass is increased 3 times.

The force of friction also increases 3 times

$= 12 \times 3 = 36\text{N}$

Hence, maximum horizontal force required = 36 N

Ans 8

Correct Option. C

Solution:

Force of friction =720 N

If aa is the acceleration of fireman, sliding down the pole, then

$$mg - F = ma$$
$$a = \frac{mg - F}{m}$$
$$= \frac{(80 \times 10 - 720)}{80}$$
$$= 1 \mathrm{~m/s^2}$$

Ans 9

Correct Option. B

Solution:

Since $L_1 + nL_2$ where n is an integer, so the spring is made of $(n+1)$ equal parts in length each of length I_2

$\therefore \quad \| \quad \frac{1}{k} = \frac{(n+1)}{k'}$

or $k' = (n+1)k$

The spring of length $l_1 (= nl_2)$ will be equivalent to n springs connected in series where spring constant

$k'' = \frac{k'}{n} = \frac{(n+1)k}{n}$

Ans 10

Correct Option. B

Solution:

Mass $M_1 = \frac{5}{8}$ kg, Mass $M_2 = \frac{5}{24}$ kg

Total maks $= M_1 + M_2 = \frac{5}{8} + \frac{5}{24} = \frac{20}{24}$ kg

Therefore, total acceleration in two masses

$$= \frac{F}{M_1 + M_2} = \frac{5}{20/24}$$
$$= \frac{5 \times 24}{20} = 6 \mathrm{~ms^{-2}}$$

Ans 11

Correct Option. C

Solution:

Energy at y = total KE - work done from x to y

= 800 – 100 × 2 = 600 J

Ans 12

Correct Option. B

Solution:

$t = \sqrt{x} + 3$

or $x = (t-3)^2$

Now, $v = \frac{dx}{dt} = 2(t-3)$

At $t = 0, v_1 = 2(-3) = -6$

At $t = 6, v_2 = 2(6-3) = 6$

Work done = change in KE

$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = 0$

Ans 13

Correct Option. C

Solution:

When θ = 180^0, the particle will be at diametrically opposite point. Where its velocity is opposite to the direction of motion.

The change in momentum = mv − (−mv) + 2mv (maximum).

When θ = 360^0, the particle is at the initial position with momentum mv change in momentum = mv – mv = 0 (minimum).

Ans 14

Correct Option. A

Solution:

While firing a bullet from a rifle, there will be equal back thrust on light rifle as well as heavy rifle. Due to this, the light rifle will move backwards with greater speed than heavy rifle. That is why light rifle will cause more injury.

Ans 15

Correct Option. C

Solution:

When a particle undergoes normal collision with a floor or a wall, with coefficient of restitution e, the speed after collision is e times the speed before collision. Therefore, change in momentum after Ist impact is

$\mathrm{ep} - (-\mathrm{p}) = \mathrm{p}(1 + \mathrm{e})$

After the second impact, change in momentum would be $\mathrm{e(ep)} - (-\mathrm{ep}) = \mathrm{ep}(1 + \mathrm{e})$ and soon.

Therefore, total change in momentum of ball

$$= p(1+e)\left[1+e+e^2+\ldots\right]$$
$$= p\left(\frac{1+e}{1-e}\right)$$

Ans 16

Correct Option. B

Solution:

Maximum height

$$H = \frac{v^2 \sin^2 45^\circ}{2g} = \frac{v^2}{2g} \times \frac{1}{2} = \frac{v^2}{4g}$$

Momentum of particle at the highest point

$$p = mv\cos 45^\circ = mv/\sqrt{2}$$

Angular momentum $= \mathrm{pH}$

$$= \frac{mv}{\sqrt{2}} \times \frac{v^2}{4g} = \frac{mv^3}{4\sqrt{2}g}$$

Ans 17

Correct Option. B

Solution:

For a solid sphere,

$$I = \frac{2}{5} MR^2$$

For a spherical shell,

$$I' = \frac{2}{3} MR^2$$
$$= \frac{5}{3} \times \frac{2}{5} MR^2 = \frac{5}{3} I$$

Ans 18

Correct Option. D

Solution:

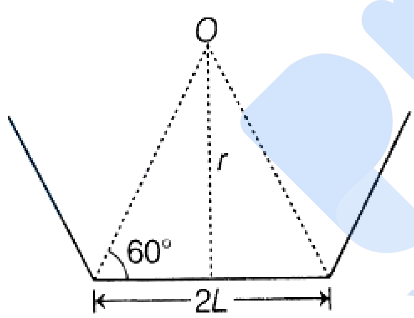
Angular momentum w.r.t. origin (L = mvr) will remain constant.

Ans 19

Correct Option. A

Solution:

Length of each side of hexagon =2L and mass of each side = m



Let O be the centre of mass of hexagon. Therefore, perpendicular distance of O from each side, $r = L \tan 60^0 = L\sqrt{3}$

The desired moment of inertia of hexagon about O is

$$I = 6\left[I_{\text{one side}}\right] = 6\left[\frac{m(2L)^2}{12} + mr^2\right]$$
$$= 6\left[\frac{mL^2}{3} + m(L\sqrt{3})^2\right]$$
$$= 20mL^2$$

Ans 20

Correct Option. B

Solution:

Gravitational force $\left(=\frac{GMm}{R^{5/2}}\right)$ provides the necessary centripetal force $\left(ie, mR\omega^2\right)$. So,

$$\frac{GMm}{R^{5/2}}=mR\omega^2=mR\left(\frac{2\pi}{T}\right)^2=\frac{4\pi^2mR}{T^2}$$

or

$$T^2=\frac{4\pi^2R^{7/2}}{GM}$$

i.e,

$$T^2\propto R^{7/2}$$

Ans 21

Correct Option. A

Solution:

Areal velocity $=\frac{dA}{dt}=\frac{L}{2m}$

$$=\frac{mvr}{2m}=\frac{vr}{2}=\frac{r}{2}\sqrt{\frac{GM}{r}}=\frac{1}{2}\sqrt{GMr}$$

So, $\frac{dA}{dt}\propto\sqrt{r}$

Ans 22

Correct Option. C

Solution:

$v_e=\sqrt{2gR}$ and $v_o=\sqrt{gR}$

$$\therefore\quad\frac{\mathrm{KE}_1}{\mathrm{KE}_2}=\frac{\frac{1}{2}mv_e^2}{\frac{1}{2}mv_o^2}=\frac{v_e^2}{v_o^2}=2$$

Ans 23

Correct Option. B

Solution:

$$\text{Strain} = \frac{\Delta l}{l} = \frac{2\pi R - 2\pi r}{2\pi r} = \left(\frac{R-r}{r}\right)$$

$$\text{Stress} = Y \times \text{strain} = \frac{F}{A}$$

$$\therefore \quad F = Y \times A \times \text{strain} = AY \times \left(\frac{R-r}{r}\right)$$

Ans 24

Correct Option. A

Solution:

If h is the height of liquid column in cylinder and I is the length of each side of base. Then

$$F_B = (h\rho g)(I \times l) = h\rho g l^2$$
$$F_S = \left(\frac{h\rho g}{2}\right)(4I \times h) = 2h^2 \rho g l$$

$$\therefore \quad h\rho g l^2 = 2h^2 \rho g l$$

$$\text{or } h = \frac{1}{2} = \frac{10}{2} = 5 \text{ cm}$$

Ans 25

Correct Option. C

Solution:

Velocity of liquid flowing out of orifice,

V = $\sqrt{2}$gh

Average velocity of liquid flowing out of orifice till the whole vessel is emptied

$$v_{av} = \frac{v+0}{2} = \frac{1}{2}\sqrt{2gh}$$

If A is area of cross-section of the orifice, then the time taken by liquid of volume V to flow out of tank through orifice is

$$t = \frac{V}{A \times \frac{1}{2}\sqrt{2gh}}$$

In the second case, volume is V/2 and height of liquid in vessel is $h/2$. Therefore, time t is :

$$t' = \frac{V/2}{A \times \frac{1}{2}\sqrt{2gh/2}} = \frac{t}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7 \text{ min}$$

Ans 26

Correct Option. A

Solution:

$$A = 10L \times 2L = 20L^2$$
$$\frac{\Delta A}{A} \times 100 = 2\frac{\Delta L}{L} \times 100 = 2 \times 1 = 2\%$$

Ans 27

Correct Option. A

Solution:

Mass of ice melted per minute $= \frac{10}{10} = 1\text{ g}$

$$\begin{aligned}&\text{Quantity of heat used}\\ &\quad\text{Area of lens } = \pi r^2\\ &\qquad\qquad\quad = 19.625\text{ cm}^2\end{aligned}$$

$\therefore$ Amount of heat received from sun

$$= \frac{80}{19.625} = 4\text{cal/cm}^2\text{ min}$$

Latent heat of fusion $= 80$

Ans 28

Correct Option. C

Solution:

Slope of isothermal curve (AB) is smaller than slope of adiabatic curve (BC).

Ans 29

Correct Option. C

Solution:

$$P=\frac{n_1RT+n_2RT+n_3RT}{V}$$
$$=(n_1+n_2+n_3)\frac{RT}{V}$$
$$=\left(\frac{8}{16}+\frac{14}{28}+\frac{22}{44}\right)\times\frac{0.082\times 300}{10}$$
$$=\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)\times 2.46$$
$$=\frac{3}{2}\times 2.46=3.69\text{ atm}\approx 3.7\text{ atm}$$

Ans 30

Correct Option. C

Solution:

$$K_{\text{adia}}=\gamma P=-\frac{\Delta P}{\Delta V/V}$$
$$\therefore\quad \frac{\Delta V}{V}=-\frac{\Delta P}{\gamma P}$$

Ans 31

Correct Option. D

Solution:

$$\eta=\frac{W}{Q_1}$$
$$\Rightarrow\quad w=\eta Q_1=\frac{1}{3}\times 1000\text{cal}$$
$$=\frac{1000}{3}\times 4.2\text{ J}=1400\text{ J}$$

Ans 32

Correct Option. A

Solution:

Total pressure, when the two gases are filled in one cylinder is equal to sum of the pressures of the individual gases.

Thus, total pressure $=2.5+1=3.5$ atm

Ans 33

Correct Option. D

Solution:

$$y = 3\sin 314t + 4\cos 314t$$
$$= r\cos\theta \sin 314t + r\sin\theta \cos 314t$$

$= r\sin(314t + \theta)$...(i)

$\therefore \quad r^2\left(\cos^2\theta + \sin^2\theta\right) = 3^2 + 4^2 = 25$

or

$r = 5\ \mathrm{cm}$

Ans 34

Correct Option. A

Solution:

$$y = A\sin(kx - \omega t)$$
$$\frac{dy}{dt} = A\cos(kx - \omega t) \times (\omega)$$
$$\left(\frac{dy}{dt}\right)_{\max} = A(1)(\omega) = A\omega$$

Ans 35

Correct Option. A

Solution:

$T = 2 = 2\pi\sqrt{\frac{M}{k}}$

and $2 + 1 = 2\pi\sqrt{\frac{M+4}{k}}$

or $3 = 2\pi\sqrt{\frac{M+4}{k}}$

$\therefore \quad \frac{4}{9} = \frac{M}{M+4}$

or $4M + 16 = 9M$

or $M = \frac{16}{5} = 3.2\ \mathrm{kg}$

Ans 36

Correct Option. B

Solution:

Let frequency of tuning fork be n_1

$\frac{n_2}{n_1} = \frac{100}{101}$

As $n_1 - n_2 = 5$

So, $n_1 = 505\ \mathrm{Hz}$

Ans 37

Correct Option. B

Solution:

Let $m =$ mass per unit length of rope

$T =$ tension in the rope at a distance x from the lower end

$\therefore T = (\mathrm{mg})\mathrm{x} =$ weight of x metre of rope

As $v = \sqrt{\frac{T}{m}}$

$\therefore \quad v = \sqrt{\frac{mgx}{m}} = \sqrt{gx}$

i.e., $\quad v \propto \sqrt{x}$

Ans 38

Correct Option. D

Solution:

The path is a parabola, because initial velocity can be resolved into two rectangular components, one along $\vec{E}$ and other perpendicular to $\vec{E}$. The former decreases at a constant rate and later is unaffected. The resultant path is, therefore a parabola.

Ans 39

Correct Option. C

Solution:

$\vec{E}$ is parallel to dipole axis, at both the points P and Q, i.e. on axial line as well as equatorial line.

Ans 40

Correct Option. A

Solution:

$$\text{As } V = \frac{q}{4\pi\varepsilon_0 R}$$
$$\therefore \quad \frac{q}{4\pi\varepsilon_0} = VR \text{ and } E = \frac{q}{4\pi\varepsilon_0 r^2}$$
$$\therefore \quad E = \frac{VR}{r^2}$$

Ans 41

Correct Option. B

Solution:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{q_1(q-q_1)}{4\pi\varepsilon_0 r^2}$$

For maximum force between q_1 and

$$\frac{dF}{dq_1} = 0$$
$$\therefore \quad q - 2q_1 = 0$$
$$\text{or} \quad q_1 = \frac{q}{2} \quad \text{or } \frac{q_1}{q} = 0.5$$

Ans 42

Correct Option. D

Solution:

$$\frac{1}{C_S} = \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore \quad C_S = \frac{3}{2}\mu\,\mathrm{F}$$

In combination charge on each capacitor in same.

So, $Q = C_S V = \frac{3}{2} \times 800 = 1200\mu\mathrm{C}$

Ans 43

Correct Option. C

Solution:

$$I = \frac{q}{T} = qv$$

$$= 2 \times 10^{-2} \times 30 = 0.60\ \mathrm{A}$$

Ans 44

Correct Option. B

Solution:

Resistance between Q and S,

$R' = \frac{5\times10}{5+10} = \frac{50}{15} = \frac{10}{3}\Omega$

Potential difference across Q and S,

$V_Q - V_S = 2 \times \frac{10}{3} = \frac{20}{3}$ V

Current through arm QPS,

$$I_1 = \frac{20}{3\times5} = \frac{4}{3}\ \mathrm{A}$$
$$V_Q - V_P = \frac{4}{3}\times2 = \frac{8}{3}\ \mathrm{V}$$

Current through arm QRS,

$$I_2 = \frac{20}{3\times10} = \frac{2}{3}\ \mathrm{A}$$
$$V_Q - V_R = \frac{2}{3}\times3 = 2\ \mathrm{V}$$

Hence,

$$V_P - V_R = (V_Q - V_R) - (V_Q - V_P)$$
$$= 2 - \frac{8}{3} = -\frac{2}{3} \approx -1\ \mathrm{V}$$

Ans 45

Correct Option. B

Solution:

$$\text{Heat produced } = \frac{V^2t}{4.2R} = mL$$
$$\text{or } m = \frac{V^2t}{4.2RL} = \frac{(210)^2\times1)}{4.2\times50\times80} = 2.62\ \mathrm{g}$$

Ans 46

Correct Option. C

Solution:

$q=\frac{m}{z}=\frac{1\times10^{-3}}{1.044\times10^{-8}}=\frac{10^5}{1.044}C$

Given, $\mathrm{H}=34\mathrm{k}-\mathrm{cal}=34\times10^3\times4.2\ \mathrm{J}$

Heat produced, $H=Vq$

$$\therefore \quad 34\times10^3\times4.2=\frac{V\times10^5}{1.044}$$

$$\therefore \quad V=\frac{34\times10^3\times4.2\times1.044}{10^5}=1.5\ \mathrm{V}$$

Ans 47

Correct Option. C

Solution:

Maximum magnetic field due to wire carrying current is at surface and is given by

$$B=\frac{\mu_0}{4\pi}\frac{2I}{R}$$

$$\text{or} \quad I=\frac{BR}{2\times(\mu_0/4\pi)}$$

$$\therefore \quad I=\frac{5\times10^{-4}\times\left(\frac{1.6\times10^{-3}}{2}\right)}{2\times10^{-7}}=2\ \mathrm{A}$$

Ans 48

Correct Option. D

Solution:

Perpendicular of O from PQ or QR, $a=r\sin\theta/2$ magnetic field induction at O due to current through PQ and QR is

$$B=\frac{\mu_0}{4\pi}\cdot\frac{1}{a}[\sin(90^\circ-\theta/2)+\sin90^\circ]\times2$$

$$=\frac{\mu_0}{2\pi}\cdot\frac{1}{r\sin\theta/2}(\cos\theta/2+1)$$

$$=\frac{\mu_0}{2\pi}\cdot\frac{l}{r}\frac{(1+\cos\theta/2)}{\sin\theta/2}$$

Ans 49

Correct Option. B

Solution:

The SI unit of magnetic moment is $\mathrm{A-m^2}$, but the magnetic moment is given in $\mathrm{Wb-m^2}$, which is the unit of $\mu_0 M$.

$$\therefore \quad \mu_0 M = 5\times10^{-5}$$
$$M = \frac{5\times10^{-5}}{\mu_0}\ \mathrm{Wb-m}$$
$$= \frac{5\times10^{-5}}{4\pi\times10^{-7}} = \frac{125}{\pi}$$

As

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

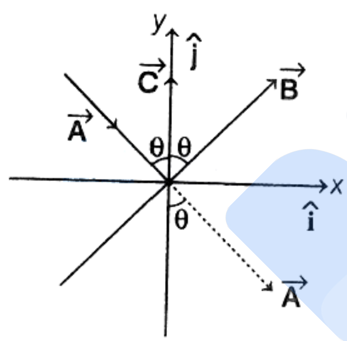
$$\therefore \quad I = \frac{T^2MB}{4\pi^2} = \frac{(15)^2\left(\frac{125}{\pi}\right)\times 8\pi\times10^{-4}}{4\pi^2}$$
$$= 0.57\ \mathrm{kg\ m^2}$$

Ans 50

Correct Option. D

Solution:

Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ be as shown in figure. Let θ be the angle of incidence, which is equal to the angle of reflection. Resolving these vectors in rectangular components, we have



$$\vec{\mathbf{A}} = \sin\theta\hat{\mathbf{i}} - \cos\theta\hat{\mathbf{j}}$$
$$\vec{\mathbf{B}} = \sin\theta\hat{\mathbf{i}} + \cos\theta\hat{\mathbf{j}}$$
$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = 2\cos\theta\hat{\mathbf{j}}$$

or $\vec{\mathbf{B}} = \vec{\mathbf{A}} + 2\cos\theta\hat{\mathbf{j}}$

Now, $\vec{\mathbf{A}}\cdot\vec{\mathbf{C}} = (1)(1)\cos(180^\circ - \theta) = -\cos\theta$

$$\therefore \quad \vec{B} = \vec{A} - 2(\vec{A}\cdot\vec{C})\hat{j}$$

or $\vec{B} = \vec{\mathbf{A}} - 2(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}})\vec{\mathbf{C}}(\text{ as } \hat{j} = \vec{\mathbf{C}})$

Chemistry

Ans 1

Correct Option. D

Solution:

The volumes of the reacting gases and those of gaseous products bear the simple ratio. This is called the law of gaseous volumes or Gay-Lussac's law.

Ans 2

Correct Option. B

Solution:

$Na_2CO_3 + 2HCl - 2NaCl + H_2O + CO_2$
Meq of $HCl = 1.0 \times 100$

Meq of $Na_2CO_3 = 2 \times 1.0 \times 75 = 150$

Thus, Na_2CO_3 is in excess, and being the salt of weak acid (H_2CO_3) and strong base $(NaOH)$, undergoes hydrolysis to produce basic character.

$Na_2CO_3 + 2H_2O \longrightarrow 2Na^+ + 2OH^- + H_2CO_3$

Ans 3

Correct Option. B

Solution:

According to aufbau principle. The order of increasing energies may be summed up as below. 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p.5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d and 7p

So, the correct order is $5p < 6s < 4f < 5d$

Ans 4

Correct Option. C

Solution:

When $n=3, l=0,1,2$ and

For

$l=0, m=0$
$l=1, m=-1,0,+1$
$l=2, m=-2,-1,0,+1,+2$

So, for the value $l=0, m$ cannot be -1

Ans 5

Correct Option. C

Solution:

$$\lambda=\frac{h}{m v}=\frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 10^{6}}$$
$$=0.727 \times 10^{-9}=0.727 \mathrm{~nm}$$

Ans 6

Correct Option. A

Solution:

For a given n value possible l values are 0 to $(n-1)$ possible m values for a given l value is

$-l, 0,+I$ and value of s for an electron $=+\frac{1}{2}$ or $-\frac{1}{2}$. So for $n=1, I=0$ not 1 and $m=0$ not 1 .

Ans 7

Correct Option. C

Solution:

Bohr's atom model is applicable only to one electron splcies, He^{2+} does not contain any electron all other species have one electron each.

Ans 8

Correct Option. C

Solution:

Prout's gave unitary theory for the development of periodic table. He suggested that the atoms of all the element are made up of hydrogen atom.

Newlands gave law of octaves: According to him, "If eleр̂ents are arranged in the increasing order of their atomic weights, the eighth element starting from the given one is the repetition of the first like the eighth node in an octave of music. Lother Meyer's plotted the atomic volumes (i.e., at. wt/density) of various elements against their atomic weight and pointed out that the similar element occupy similar position on the curve.

Rutherford is associated with structure of atom. He bombarded high speed α-particles on nitrogen and proved that nuclei of all atoms have protons.

Ans 9

Correct Option. A

Solution:

The required reaction is

$$C(s)+\frac{1}{2}O_2(g) \longrightarrow CO(g)$$

$$CO_2(g) \longrightarrow CO(g)+\frac{1}{2}O_2(g)$$

$\Delta H=\frac{+135.2}{2}\text{kcal}$...(i)

$$C(s)+O_2(g) \longrightarrow CO_2(g)$$

$\Delta H=-94\text{kcal}$...(ii)

The required equation is come out by adding Eq. (i) and (ii)

So, $\Delta H_{f(CO)}=-94+\frac{135.2}{2}=-26.4\text{kcal}$

Ans 10

Correct Option. B

Solution:

$$\Delta G_1=\Delta H-T\Delta S \text{ or } T=\frac{\Delta H-\Delta G}{\Delta S}$$

$$=\frac{[145.6-(-5.2)]\times 10^3}{216}=698.1\text{ K}$$

Ans 11

Correct Option. B

Solution:

$$\text{pOH}=\text{p}K_b+\log\frac{[\text{ salt }]}{[\text{ base }]}=pK_b+0$$

$$\text{p}K_b=\text{pOH}=14-\text{pH}=14-9.25=4.75$$

Ans 12

Correct Option. D

Solution:

The b.p. and viscosity of D_2O is higher than water. Solubility of NaCl in it is less than H_2O because reaction velocity is slightly less in D_2O due to the that deuterium bond is strong than protium bond.

Ans 13

Correct Option. A

Solution:

In ice each O-atom is bonded with 4 H -atoms in a tetrahedral fashion. Two bonds are normal covalent bonds, while two are H-bonds (weaker and larger in length) and hence give rise to loose cage structure.

Ans 14

Correct Option. C

Solution:

Ice floats on water because its density is less than that of water which in turn is due to empty space in its crystal structure. Crystal structure of ice is regular hexagon with empty space at the centre.

Ans 15

Correct Option. C

Solution:

Na_2CO_3 is manufactured by Solvay process.
Lowing process is for the manufacture of NaOH Leblanc process is for the manufacture of K_2CO_3 Haber's process is for the manufacture of NH_3

Ans 16

Correct Option. A

Solution:

NaOH when react with sand SiO_2, it forms sod. silicate which is generally called glass.

$$2NaOH + SiO_2 \longrightarrow \underset{\text{Sod. silicate (glass)}}{Na_2SiO_3} + H_2O$$

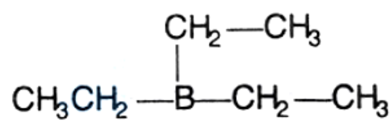
Ans 17

Correct Option. C

Solution:

Diborane readily adds at 0⁰C to 25⁰C to the olefin or acetylenic compound to form trialkyl boranes. This reaction is known as hydro-boration.

$$\underset{\text{Ethylene}}{6CH_2 = CH_2} + B_2H_6 \xrightarrow{\text{ether}}$$



Triethyl borane

Ans 18

Correct Option. B

Solution:

Presence of -/ group in a carbocation decreases delocalisation of the positive charge and hence decreases stability of the carbocation and vice versa. Thus the carbocation (iii) having $-COCH_3$ grouping is the least stable, followed by (ii) having $-OCH_3$ grouping, followed by (i) having only alkyl group.

Ans 19

Correct Option. A

Solution:

Alkanes have a general formula C_nH_{2n+2}

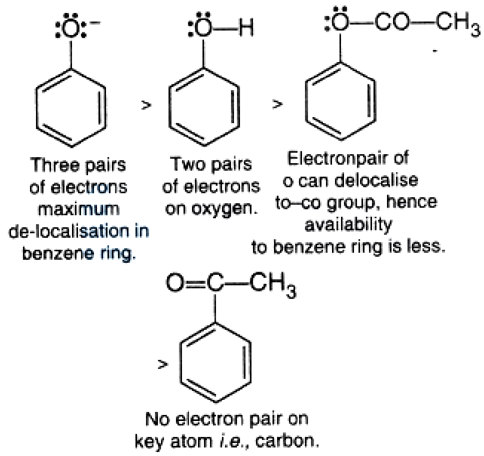
So, when

$n = 1, C_1H_{2\times1+2} = CH_4$
$n = 2, C_2H_{2\times2} + 2 = C_2H_6$
$n = 3, C_3H_{2\times3+2} = C_3H_8$
So, the all have a difference of CH_2.

Ans 20

Correct Option. A

Solution:

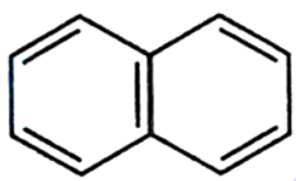


Ans 21

Correct Option. C

Solution:

Naphthalene has a structure,

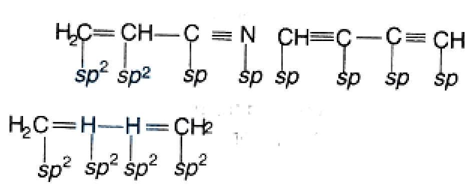


It has 5π bonds and thus the number of ππ electrons is $5 \times 2 = 10$

Ans 22

Correct Option. A

Solution:

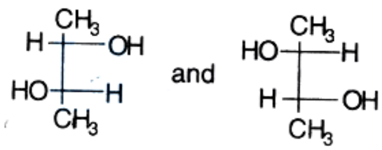


Ans 23

Correct Option. D

Solution:

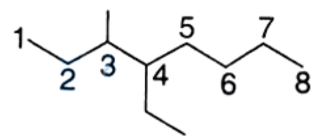
A pair of molecule which cannot be super imposed on its mirror image (i.e., asymmetrical) related to each other as an object to its mirror image are knowns as enantiomers, So,



Ans 24

Correct Option. D

Solution:



4-ethyl 3-methyloctane

Ans 25

Correct Option. C

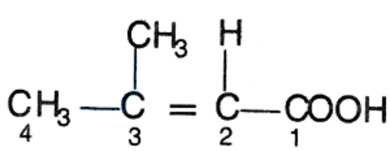
Solution:

Benzene-hexachloride exists in eight isomeric forms namely α,β,γ and δ etc. Out of which γ-isomer is most active and used as insecticide under the name of gammexene.

Ans 26

Correct Option. C

Solution:



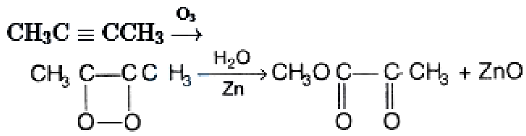
3-methyl-2-butanoic acid.

Ans 27

Correct Option. C

Solution:

This reaction is ozonolysis. O_3 is added to triple bond and then on hydrolysis form aldehyde or ketones



Ans 28

Correct Option. A

Solution:

Semiconductor are derived from compounds of p-block elements.

Ans 29

Correct Option. D

Solution:

Since, As is added from outside, it is an extrinsic semiconductor.

Ans 30

Correct Option. D

Solution:

For the cell reaction, Fe acts as cathode and Sn as anode

Hence,

$$\begin{aligned} E^{\circ}_{\text{cell}} &= E^{\circ}_{\text{cathode}} - E^{\circ}_{\text{anode}} \\ &= -0.44 - (-0.14) = -0.30 \text{ V} \end{aligned}$$

The negative emf suggests that the reaction goes spontaneously in reversed direction.

Ans 31

Correct Option. D

Solution:

On reducing the volume 1/4 th of the initial volume, each concentration will be increased by 4 times.

$$(\text{ Rate })_1 = k[A][B]$$
$$(\text{ Rate })_2 = k[4[\text{ A}][4[\text{ B}]]$$

Hence, $\frac{[\text{ Rate }]_2}{[\text{ Rate }]} = 4 \times 4 = 16$

Ans 32

Correct Option. D

Solution:

Substances which resist high temperatures and do not become soft are called refractories material. The neutral refractories are graphite, chromite and carboren, dum (SiC). Acidic refractories are SiO_2, basic refractories are CaO and MgO.

Ans 33

Correct Option. C

Solution:

When more electropositive metal displace less electropositive metals from their salt solution, this is reduction by precipitation and the process is called hydrometallurgy.

Ans 34

Correct Option. C

Solution:

Cold conc. HNO_3 makes it passive in which state it fails to exhibit the properties of normal iron. Passivity is said to be due to the deposit of a thin layer of iron oxide (Fe_3O_4) on the surface.

Ans 35

Correct Option. B

Solution:

When potassium iodide react with ozone it oxidise Kl and liberate I_2 it turns starch paper blue.

Ans 36

Correct Option. B

Solution:

Red hot iron absorbs SO_2 and give FeO and FeS.

$Fe + SO_2 \longrightarrow FeO + FeS$

Ans 37

Correct Option. A

Solution:

In group number 15 elements shows -III and III oxidation state (by gain or loss of three p electrons). These shows V oxidation state due to loss of $np^3 + ns^2$ electrons. But nitrogen is unique in having a large number of oxidation states including a fractional one (i.e., $-1/3$ in N_3H)

Ans 38

Correct Option. D

Solution:

Coinage metals are Cu,Ag and Au . They are transition metal and present in group II or IB. The valence shell configuration is $(n-1)d^{10}ns^1$.

Ans 39

Correct Option. A

Solution:

Normality of salt solution $= \frac{3.92}{392} \times \frac{1}{100} \times 1000$

$= 0.1\ \mathrm{N}$

20 mL of 0.1 N salt solution $\equiv 18\ \mathrm{mL}$ of $\mathrm{KMnO_4}$ solution

20 mL of 0.1 N salt solution $\equiv 18\ \mathrm{mL}$ of $\mathrm{KMnO_4}$ solution

$\therefore$ Normality of $\mathrm{KMnO_4}$ solution $= \frac{20 \times 0.1}{18}$

$$\therefore \text{ Normality of } \mathrm{KMnO_4} \text{ solution } = \frac{20 \times 0.1}{18} = \frac{1}{9}\ \mathrm{N}$$

$$\therefore \text{ Strength of } \mathrm{KMnO_4} \text{ solution } = \frac{1}{9} \times 31.6 = 3.5\ \mathrm{g\ L^{-1}}$$

Ans 40

Correct Option. D

Solution:

Stability of corresponding carbocation

$CH_2 = CH - \overset{+}{C}H_2 > CH_2 = \overset{+}{C}H > C_6H_5^+$

So, $ClCH_2 - CH = CH_2$ is most reactive towards nucleophilic substitution reaction.

Ans 41

Correct Option. A

Solution:

Stability of corresponding carbocation

$CH_3CH_2 > CH_2 = \overset{+}{C}H > C_6H_5^+$.

So, the correct order of reactivity of halogen atom is chlorobenzene, vinyl chloride, chloroethane.

Ans 42

Correct Option. B

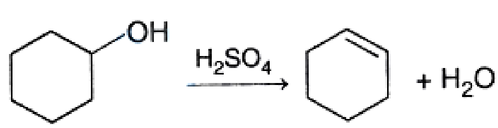
Solution:

The prefix neo means a carbon atom having four alkyl groups, position of alcohol group should be at C_1.

Ans 43

Correct Option. A

Solution:



The weight of cyclohexene formed

$= \frac{82 \times 100}{100} \times \frac{75}{100} = 61.5\ g$

Ans 44

Correct Option. C

Solution:

$1.56\ \mathrm{mL}^{-1} CH_4$ at STP is liberated by 4.12 mg of alcohol. 22400 mL of CH_4 at STP is liberated by

$$= \frac{4.12}{1.56} \times 22400\mathrm{mg\ of}\ ROH$$
$$= 61800\mathrm{mg\ of}\ ROH = 61.8\ \mathrm{g\ of\ ROH}$$

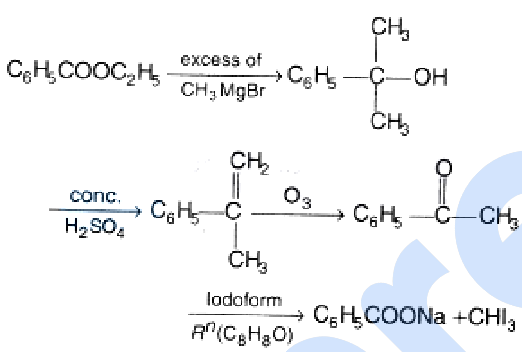
Thus, the molecular wt. of $ROH = 61.8$. This coincides with the molecular weight of $C_3H_7OH = 60$.

Ans 45

Correct Option. B

Solution:

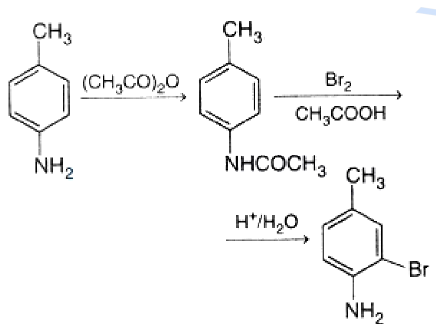
Only $C_6H_5COOC_2H_5$ corresponds with the molecular formula of the ester A. Further it explains all the given reactions.



Ans 46

Correct Option. B

Solution:

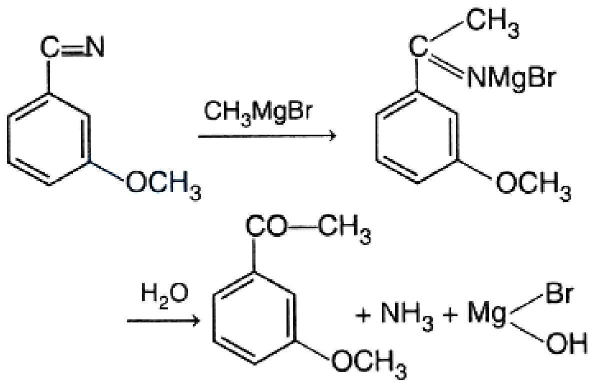


Ans 47

Correct Option. B

Solution:

Cyanide with Grignard reagent forms ketone.



Ans 48

Correct Option. B

Solution:

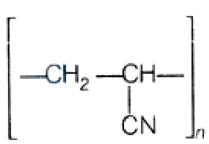
Carbohydrates are optically active polyhydroxy aldehyde ketones it must contain at least three carbon atom. Hydroxyaldehyde with, 2 carbon atom i.e., $CH_2OH \cdot CHO$ is not carbohydrate as it is optically inactive.

Ans 49

Correct Option. D

Solution:

Acrilan is a cyanide compound. It has a structure



Ans 50

Correct Option. B

Solution:

NH_4Cl and aqueous $NH_3(NH_4OH)$ precipitates basic radicals of 3rd group of qualitative analysis hydroxides of this group have very low value of solubility product. So, this reagent will precipitate Al^{3+} because the radicals present in 3rd group are Al^{3+}, Cr^{3+}, and Fe^{3+}.

Mathematics

Ans 1

Correct Option. B

Solution:

Now, $\dfrac{5^{\frac{3}{2}}-2^{\frac{3}{2}}}{\sqrt{5}-\sqrt{2}}+\dfrac{5^{\frac{3}{2}}+2^{\frac{3}{2}}}{\sqrt{5}+\sqrt{2}}$

$=\dfrac{\sqrt{5}^3-\sqrt{2}^3}{\sqrt{5}-\sqrt{2}}+\dfrac{\sqrt{5}^3+\sqrt{2}^3}{\sqrt{5}+\sqrt{2}}$

$=(5+2+\sqrt{10})+(5+2-\sqrt{10})$

$=7+7=14$

Ans 2

Correct Option. A

Solution:

Given, $9^{-x}=\frac{1}{27\sqrt{27}}=(81)^{-y}$

$\Rightarrow \quad (3)^{-2x}=(3)^{-3}(3)^{-\frac{3}{2}}=(3)^{-4y}$

$\Rightarrow \quad -2x=-\frac{9}{2} \text{ and } -4y=-\frac{9}{2}$

$\Rightarrow \quad x=\frac{9}{4} \text{ and } y=\frac{9}{8}$

Ans 3

Correct Option. C

Solution:

$x^{4/3}+x^{-1/3}=1 \Rightarrow x^{4/3}\cdot x^{1/3}+1=x^{1/3}$

$\Rightarrow \quad x^{5/3}-x^{1/3}=-1$

On cubing both sides, we get

$x^5-x-3x^2\left(x^{5/3}-x^{1/3}\right)=-1$

$\Rightarrow \quad x^5-x+3x^2=-1$

Ans 4

Correct Option. C

Solution:

Given, $x=\frac{2\sqrt{2}-\sqrt{7}}{2\sqrt{2}+\sqrt{7}}$

$$\begin{aligned}\therefore \quad x+\frac{1}{x} &= \frac{2\sqrt{2}-\sqrt{7}}{2\sqrt{2}+\sqrt{7}}+\frac{2\sqrt{2}+\sqrt{7}}{2\sqrt{2}-\sqrt{7}} \\ &= \frac{8+7-4\sqrt{14}+8+7+4\sqrt{14}}{8-7}=30\end{aligned}$$

Ans 5

Correct Option. A

Solution:

$$\begin{aligned}\left|\frac{x}{2}-1\right|<3 \Rightarrow -3<\frac{x}{2}-1<3 \\ -3+1<\frac{x}{2}<3+1\end{aligned}$$

$$\begin{aligned}&\Rightarrow \quad -2<\frac{x}{2}<4 \\ &\Rightarrow \quad -4<x<8 \\ &\therefore \quad x\in(-4,8)\end{aligned}$$

Ans 6

Correct Option. B

Solution:

$y=a+kx+p/x$

$$\begin{aligned}\therefore 6 &= a+k+p \\ -4 &= a-k-p \\ \text{and } 8 &= a+3k+p/3 \\ \Rightarrow 6-1 &= k+p \Rightarrow 5=k+p\end{aligned}$$

On solving these equations, we get

$a=1, k=2, p=3$

$\therefore$ Required equation is $y=1+2x+\frac{3}{x}$

Ans 7

Correct Option. C

Solution:

$\log x+\log x^3+\log x^5+\ldots+\log x^{2n-1}$

$$\begin{aligned}&=\log\left(x\cdot x^3\cdot x^5\ldots x^{2n-1}\right) \\ &=\log\left(x^{1+3+5+\ldots+2n-1}\right) \\ &=\log x^{\frac{n}{2}[2+(n-1)2]}=\log x^{n^2}=n^2\log x\end{aligned}$$

Ans 8

Correct Option. D

Solution:

$$\frac{1}{\log_{25} 10}+\frac{1}{\log_4 10}+\frac{1}{\log_{\sqrt{2}} 10}+\frac{1}{\log_{\sqrt{5}} 10}$$
$$=\frac{\log 25}{\log 10}+\frac{\log 4}{\log 10}+\frac{\log \sqrt{2}}{\log 10}+\frac{\log \sqrt{5}}{\log 10}$$
$$=\frac{2\log 5+\frac{1}{2}\log 5}{\log 10}+\frac{2\log 2+\frac{1}{2}\log 2}{\log 10}$$
$$=\frac{5}{2}\left(\frac{\log 5+\log 2}{\log 10}\right)=\frac{5}{2}\frac{\log 10}{\log 10}=\frac{5}{2}$$

Ans 9

Correct Option. C

Solution:

We know, $(a+b+c)^2 \geq 0$

$\Rightarrow a^2+b^2+c^2+2ab+2bc+2ca \geq 0$
$\Rightarrow 1+2(ab+bc+ca) \geq 0$

$\Rightarrow \quad (ab+bc+ca) \geq \frac{-1}{2}$ ...(i)

Also, $\quad a^2+b^2+c^2=1$

$\Rightarrow a^2+b^2+c^2-(ab+bc+ca)$
$=\frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right] \geq 0$
$\Rightarrow \quad 1-(ab+bc+ca) \geq 0$

$\Rightarrow \quad ab+bc+ca \leq 1$...(ii)

Hence, from Eqs. (i) and (ii),

$-\frac{1}{2} \leq ab+bc+ca \leq 1$

Ans 10

Correct Option. C

Solution:

$a<b \Rightarrow \frac{a+b}{2}<\frac{b+b}{2}$

$\Rightarrow \quad \frac{a+b}{2}<b$

Ans 11

Correct Option. C

Solution:

Since, roots are complex.

$$\begin{aligned} \therefore \quad a^2 - 36 < 0 \quad (\because \text{ discriminant } < 0) \\ \Rightarrow \quad a^2 < 36 \\ \Rightarrow \quad |a| < 6 \end{aligned}$$

Ans 12

Correct Option. A

Solution:

Since, the given equation has real roots.

∴ Discriminant,

$$\begin{aligned} D = \cos^2 p - 4(\cos p - 1)\sin p \geq 0 \\ \Rightarrow \quad D = (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0 \\ \Rightarrow \quad 0 < p < \pi \end{aligned}$$

Ans 13

Correct Option. C

Solution:

Since, α and β are the roots of the equation

$$4x^2 + 3x + 7 = 0$$

$$\therefore \quad \alpha + \beta = -3/4, \alpha\beta = 7/4$$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$$= -\frac{3 \cdot 4}{4 \cdot 7} = -\frac{3}{7}$$

Ans 14

Correct Option. D

Solution:

If ω is a cube root of unity, then

$1+\omega+\omega^2=0$ and $\omega^3=1$.

$$\begin{aligned}\therefore \left(1+\omega-\omega^2\right)\left(1-\omega+\omega^2\right) &= \left(-\omega^2-\omega^2\right)(-\omega-\omega)\\ &= 2\omega^2\cdot 2\omega=4\end{aligned}$$

Ans 15

Correct Option. B

Solution:

Let $z=x+iy$

$\therefore \quad i+z=x+i(1+y)$

and $\quad i-z=-x+i(1-y)$

$\because \quad \left|\frac{i+z}{i-z}\right|=1$

$\Rightarrow |i+z|=|i-z|$

$\Rightarrow x^2+(1+y)^2=x^2+(1-y)^2$

$\Rightarrow x^2+y^2+2y+1=x^2+y^2+1-2y$

$\Rightarrow 4y=0$

$\Rightarrow y=0$

ie, x-axis.

Ans 16

Correct Option. C

Solution:

Number of three digits number when two 1's occur = 9 + 8 + 9 = 26.

Similarly, for two 2's ... 9's

We have, 26 × 9 numbers = 234 numbers

Number of numbers when two zero's occur = 9

Number of numbers when all three digits are same = 9

∴ Total three digit numbers having at least two identical digits =234 + 9 + 9 = 252

Ans 17

Correct Option. C

Solution:

Let T_n be the nth term of a given series:

$$\therefore \quad T_n=\frac{(2n-1)}{(n-1)!}; n=1,2,3,\ldots,\infty$$

$$\begin{aligned}\Rightarrow \quad T_n &= \frac{2n}{(n-1)!}-\frac{1}{(n-1)!}\\ &= 2\left[\frac{(n-1)}{(n-1)!}+\frac{1}{(n-1)!}\right]-\frac{1}{(n-1)!}\\ &= \frac{2}{(n-2)!}+\frac{1}{(n-1)!}\\ \therefore \sum_{n=1}^{\infty} T_n &= \sum_{n=1}^{\infty}\frac{2}{(n-2)!}+\sum_{n=1}^{\infty}\frac{1}{(n-1)!}\\ &= 2e+e=3e\end{aligned}$$

Ans 18

Correct Option. A

Solution:

Even multiples of 9 are of the form $18n$.

$18n$ for $n=1,\ldots,27$ lies between 1 to 500

and $18n$ for $n=1,\ldots,16$ lies between 1 to 300 .

$$\begin{aligned}\therefore \text{ Required sum } &= 18\left[\sum_{n=1}^{27} n-\sum_{n=1}^{16} n\right]\\ &= 18\left[\frac{27\times 28}{2}-\frac{16\times 17}{2}\right]=4356\end{aligned}$$

Ans 19

Correct Option. B

Solution:

$$\begin{array}{ll}\text{Let} & S_{m_1}=\frac{n}{2}[2a_1+(n-1)d_1]\\ \text{and} & S_{n_2}=\frac{n}{2}[2a_2+(n-1)d_2]\\ \Rightarrow & \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]}=\frac{2n+3}{3n-1}\end{array}$$

(given)

For 5th term, replace n by $2\times 5-1=9$

$$\begin{aligned}\Rightarrow \quad \frac{2a_1+8d_1}{2a_2+8d_2} &= \frac{18+3}{27-1}\\ \Rightarrow \quad \frac{a_1+4d_1}{a_2+4d_2} &= \frac{21}{26}\end{aligned}$$

Ans 20

Correct Option. D

Solution:

$$\sum_{r=1}^{\infty}\left[3\cdot 2^{-r}-2\cdot 3^{1-r}\right]=3\sum_{r=1}^{\infty}\frac{1}{2^r}-2\sum_{r=1}^{n}3^{1-r}$$

$$=3\left[\frac{1}{2}+\frac{1}{2^2}+\ldots\infty\right]-2\left[1+\frac{1}{3}+\frac{1}{3^2}+\ldots\infty\right]$$
$$=\frac{3}{2}\left[1+\frac{1}{2}+\frac{1}{2^2}+\ldots\right]-2\left[1+\frac{1}{3}+\frac{1}{3^2}+\ldots\infty\right]$$
$$=\frac{3}{2}\left(\frac{1}{1-\frac{1}{2}}\right)-2\left(\frac{1}{1-\frac{1}{3}}\right)=\frac{3}{2}\times 2-\frac{2\times 3}{2}=0$$

Ans 21

Correct Option. B

Solution:

Sum $=\frac{2}{10}+\frac{4}{10^3}+\frac{6}{10^5}+\frac{8}{10^7}+\ldots\infty$

It is an arithmetic-geometric series.

$$\therefore\quad S-\frac{S}{10^2}=\frac{2}{10}+\frac{2}{10^3}+\frac{2}{10^5}+\ldots\infty$$
$$\Rightarrow\ \frac{99S}{100}=\frac{20}{99}\Rightarrow S=\frac{2000}{9801}$$

Ans 22

Correct Option. B

Solution:

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{n+1}=-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\infty$$
$$=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\infty-1$$
$$=\log 2-1$$

Ans 23

Correct Option. B

Solution:

$\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in AP.

$$\Rightarrow \log_3(2^x - 5) = \frac{\log_3(2^x - \frac{7}{2}) + \log_3 2}{2}$$
$$\Rightarrow \quad (2^x - 5)^2 = 2 \cdot \left(2^x - \frac{7}{2}\right)$$
$$\Rightarrow \quad (2^x)^2 + 25 - 10 \cdot 2^x = 2 \cdot 2^x - 7$$
$$\Rightarrow (2^x)^2 - 12 \cdot 2^x + 25 + 7 = 0$$
$$\Rightarrow \quad (2^x)^2 - 12 \cdot 2^x + 32 = 0$$
$$\Rightarrow \quad 2^x = 8 \text{ or } 2^x = 4$$
$$\Rightarrow x = 3 \text{ or } 2$$

$\Rightarrow x = 3 (\because x = 2$ does not satisfy the given series $)$

Ans 24

Correct Option. A

Solution:

Given, ${}^{2n}C_3 : {}^{n}C_2 = \frac{44}{3}$

$$\Rightarrow \quad \frac{2n!}{(2n-3)!3!} \times \frac{2!(n-2)!}{n!} = \frac{44}{3}$$
$$\Rightarrow \frac{2n(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{44}{3}$$
$$\Rightarrow \quad \frac{2}{3} \times 2(2n-1) = \frac{44}{3}$$
$$\Rightarrow \quad 2n - 1 = 11 \Rightarrow 2n = 12 \Rightarrow n = 6$$

Ans 25

Correct Option. B

Solution:

Middle term $= T_6 = T_{5+1}$

$$= {}^{10}C_5(x)^5\left(-\frac{1}{2}y\right)^5 = \frac{10!}{5! \times 5!}\frac{(x)^5}{(-2)^5}y^5$$
$$= -\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 2 \times 3 \times 2^5}x^5y^5$$
$$= -\frac{63}{8}x^5y^5$$

Ans 26

Correct Option. B

Solution:

The general term is

$$T_{r+1} = {}^{12}C_r(2x^4)^r\left(-\frac{1}{x^2}\right)^{12-r}$$
$$= {}^{12}C_r(2)^r x^{4r}(-1)^{12-r}(x)^{-2(12-r)}$$

For term independent of x, we put

$$4r - 24 + 2r = 0 \Rightarrow 6r = 24 \Rightarrow r = 4$$
$$T_5 = {}^{12}C_4(2)^4(-1)^{12-4}$$
$$= \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} \times (2)^4 = 7920$$

Ans 27

Correct Option. D

Solution:

$$\frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} + \ldots + \frac{{}^nC_n}{n+1} = \sum_{r=0}^{n} \frac{{}^nC_r}{r+1} - {}^nC_0$$
$$= \sum_{r=0}^{n} \frac{1}{n+1} \frac{n+1}{r+1} {}^nC_r - {}^nC_0$$
$$= \frac{1}{n+1} \sum_{r=0}^{n} {}^{n+1}C_{r+1} - {}^nC_0$$
$$= \frac{1}{n+1}\left[\left({}^{n+1}C_0 + {}^{n+1}C_1 + \ldots + {}^{n+1}C_{n+1}\right) - {}^{n+1}C_0\right] - C_0$$
$$= \frac{1}{n+1}\left(2^{n+1} - 1\right) - 1$$
$$= \frac{2^{n+1} - n - 1 - 1}{n+1} = \frac{2^{n+1} - n - 2}{n+1}$$

Ans 28

Correct Option. A

Solution:

Middle term is $\left(\frac{n}{2} + 1\right)$ th term for n even.

$\therefore$ Middle term $= \left(\frac{6}{2} + 1\right) = 4$ th term

$$\therefore \quad T_4 = {}^6C_3(\sqrt{x})^3\left(-\frac{1}{\sqrt{x}}\right)^{6-3}$$
$$= \frac{6!}{3!3!}(\sqrt{x})^3 \frac{(-1)^3}{(\sqrt{x})^3}$$
$$= \frac{-6 \times 5 \times 4}{6}$$
$$= -20$$

Ans 29

Correct Option. D

Solution:

$$\left(1+x+x^2+x^3\right)^{-3}=\left[\frac{\left(x^4-1\right)}{x-1}\right]^{-3}$$

$$\begin{aligned}
&=\left(x^4-1\right)^{-3}(x-1)^3\\
&=(-1)^{-3}\left(1-x^4\right)^{-3}(x-1)^3\\
&=(-1)\left(1+3x^4+\ldots\right)\left(x^3-3x^2+3x-1\right)
\end{aligned}$$

$\therefore$ Coefficient of x in $\left(1+x+x^2+x^3\right)^{-3}$ is -3 .

Ans 30

Correct Option. C

Solution:

Now, $\frac{{}^nC_2}{(n+1)!}=\frac{\frac{n!}{(n-2)!2!}}{(\theta+1)!}$

$$\begin{aligned}
&=\frac{n!}{2(n+1)!(n-2)!}=\frac{n(n-1)}{2(n+1)!}\\
&=\frac{n^2-1+1-n}{2(n+1)!}\\
&=\frac{1}{2}\left[\frac{(n-7)}{n!}-\frac{1}{n!}+\frac{2}{(n+1)!}\right]\\
&=\frac{1}{2}\left[\frac{1}{(n-1)!}-\frac{2}{n!}+\frac{2}{(n+1)!}\right]
\end{aligned}$$

Put $n=0,1,2,3,\ldots$, we get

$$\begin{aligned}
&=\frac{1}{2}[e-2(e)+2(e-1)]\\
&=\frac{1}{2}[e-2]=\frac{e}{2}-1
\end{aligned}$$

Ans 31

Correct Option. B

Solution:

$$\text{Since, } \sin\theta = -\frac{24}{25}$$
$$\therefore \text{ Base } AB = \sqrt{(25)^2 - (24)^2} = 7$$
$$\therefore \quad \cos\theta = \frac{7}{25}$$
$$\text{and } \tan\theta = -\frac{24}{7} \quad (\because \theta \in 4 \text{ th quadrant })$$
$$\therefore 7\tan\theta + 25\cos\theta = -24 + 7 = -17$$

Ans 32

Correct Option. B

Solution:

$$\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta}$$
$$= \frac{\cos^2\theta}{\cos\theta - \sin\theta} + \frac{\sin^2\theta}{\sin\theta - \cos\theta}$$
$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta - \sin\theta} = \cos\theta + \sin\theta$$

Ans 33

Correct Option. D

Solution:

$$\cos\tfrac{\pi}{12} + \cos\tfrac{17\pi}{12} + \cos\tfrac{11\pi}{12}$$
$$= \cos\frac{\pi}{12} + \cos\left(\frac{3\pi}{2} - \frac{\pi}{12}\right) + \cos\left(\pi - \frac{\pi}{12}\right)$$
$$= \cos\frac{\pi}{12} - \cos\frac{\pi}{12} - \sin\frac{\pi}{12}$$
$$= -\sin\frac{\pi}{12} = \frac{1-\sqrt{3}}{2\sqrt{2}}$$

Ans 34

Correct Option. A

Solution:

$$\cot^2\left(\tfrac{\pi}{4} + \tfrac{\theta}{2}\right)$$
$$= \left[\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right]^2 = \left[\frac{\cot\frac{\theta}{2}\cot\frac{\pi}{4} - 1}{\cot\frac{\theta}{2} + \cot\frac{\pi}{4}}\right]^2$$
$$= \left[\frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}\right]^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

Ans 35

Correct Option. A

Solution:

Now, $\cos B+\cos C+(1-\cos A)$

$$=2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)+2\sin^2\frac{A}{2}$$
$$=2\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)+2\sin^2\frac{A}{2}$$
$$=2\sin\frac{A}{2}\left[\cos\left(\frac{B-C}{2}\right)+\sin\frac{A}{2}\right]$$
$$=2\sin\frac{A}{2}\left[\cos\left(\frac{B-C}{2}\right)+\cos\left(\frac{B+C}{2}\right)\right]$$
$$=2\sin\frac{A}{2}\cdot 2\cos\frac{B}{2}\cdot\cos\frac{C}{2}$$
$$=4\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

Ans 36

Correct Option. B

Solution:

Total degree = number of vertices $\times$ degree

$= 4 \times 7$ or 2×14

If there are 14 vertices each of degree 2, then graph is a cycle.

Hence, there are 7 vertices and each of degree 4.

Ans 37

Correct Option. B

Solution:

Given, $\sec A - \tan A = -a$

$$\Rightarrow \quad \frac{1-\sin A}{\cos A} = -a$$
$$\Rightarrow \quad \frac{1-\sin A}{\sqrt{1-\sin^2 A}} = -a$$
$$\Rightarrow \quad 1+\sin^2 A - 2\sin A = a^2\left(1-\sin^2 A\right)$$
$$\Rightarrow \left(1+a^2\right)\sin^2 A - 2\sin A + \left(1-a^2\right) = 0$$
$$\Rightarrow \quad (\sin A - 1)\left[\sin A - \frac{\left(1-a^2\right)}{\left(1+a^2\right)}\right] = 0$$
$$\Rightarrow \quad \sin A = 1 \text{ or } \sin A = \frac{1-a^2}{1+a^2}$$

Ans 38

Correct Option. C

Solution:

Let the required point is (a, a).

$\therefore (4-a)^2 + a^2 = (5-a)^2 + (1-a)^2$

$$\Rightarrow \quad 16 + a^2 - 8a + a^2 = 25 + a^2 - 10a + 1 + a^2 - 2a$$
$$\Rightarrow \quad 16 - 25 - 1 + a^2 + a^2 - a^2 - a^2 - 8a + 10a + 2a = 0$$
$$\Rightarrow \quad 4a = 10$$
$$\Rightarrow \quad a = \frac{10}{4} = \frac{5}{2}$$

$\therefore$ Required point is $\left(\frac{5}{2}, \frac{5}{2}\right)$.

Ans 39

Correct Option. B

Solution:

Diagonals of parallelogram bisects each other.

$\Rightarrow$ Diagonal BD passes through mid point of AC.

$ie,$

$\left(\frac{5}{2}, \frac{7}{2}\right)$
$\therefore$ Equation of diagonal passing through

$(5,1)$ and $\left(\frac{5}{2}, \frac{7}{2}\right)$ is

$$y-1=\frac{\frac{7}{2}-1}{\frac{5}{2}-5}(x-5)$$
$$\Rightarrow \quad y-1=-1(x-5)$$
$$\Rightarrow \quad y+x-6=0$$

Ans 40

Correct Option. A

Solution:

Let the equation of line be $\frac{x}{a}+\frac{y}{b}=1$.

Then $a+b=4$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\therefore \quad \frac{x}{a}+\frac{y}{4-a}=1$

$\Rightarrow \quad x(4-a)+ya$

$\Rightarrow a(4-a)$

This passes through $\left(\frac{9}{2},-5\right)$.

$$\therefore \quad \frac{9}{2}(4-a)-5a=a(4-a)$$
$$\Rightarrow \quad 36-9a-10a=8a-2a^2$$
$$\Rightarrow \quad 2a^2-27a+36=0$$
$$\Rightarrow \quad a=12 \text{ or } \frac{3}{2} \text{ but } a \neq 12$$
$$\therefore \quad a=\frac{3}{2} \Rightarrow b=\frac{5}{2}$$

$\therefore$ Required equation is

$$\frac{2x}{3}+\frac{2y}{5}=1$$
$$\Rightarrow \quad 10x+6y=15$$

Ans 41

Correct Option. C

Solution:

Suppose the segment $2x - 3y + 4 = 0$ divides line segment joining A(2,1) and B(0,−2) in the ratio k:1 at point C.

$\therefore$ C is $\left(\frac{2}{k+1}, \frac{-2k+1}{k+1}\right)$

Since, C lies on $2x - 3y + 4 = 0$

$$\begin{aligned} \Rightarrow & \quad \frac{4}{k+1} - 3\frac{(-2k+1)}{k+1} + 4 = 0 \\ \Rightarrow & \quad 4 + 6k - 3 + 4k + 4 = 0 \\ \Rightarrow & \quad 10k + 5 = 0 \\ \Rightarrow & \quad k = -\frac{1}{2} \\ \therefore & \quad \text{Ratio is } -1:2. \end{aligned}$$

Ans 42

Correct Option. A

Solution:

Point of intersection of $x - y + 1 = 0$ and $3x + 2y + 4 = 0$ is Slope of required line is -4 .

$\therefore$ Required line is $\left(y + \frac{1}{5}\right) = -4\left(x + \frac{6}{5}\right)$

$$\begin{aligned} \Rightarrow & \quad 5y + 1 = -20x - 24 \\ \Rightarrow & \quad 5y + 20x = -25 \\ \Rightarrow & \quad y + 4x + 5 = 0 \end{aligned}$$

Ans 43

Correct Option. C

Solution:

Given conic is $9x^2 - 16y^2 = 144$

$$\begin{aligned} \Rightarrow & \quad \frac{x^2}{16} - \frac{y^2}{9} = 1 \\ \therefore & \quad e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4} \end{aligned}$$

Ans 44

Correct Option. B

Solution:

$\lim_{\theta \to 0} \frac{\tan \theta}{\theta}$. Applying L'Hospital's rule,

$$= \lim_{\theta \to 0} \frac{\sec^2 \theta}{1} = 1$$

Ans 45

Correct Option. B

Solution:

$$\lim_{x \to 0} \frac{\sin 3x - \sin x}{\sin x}$$
$$= \lim_{x \to 0} \frac{2 \cos 2x \sin x}{\sin x} = 2$$

Ans 46

Correct Option. C

Solution:

The only wrong statement is option(c).

∵ It is not necessary that A – B = I,∀A, B square matrix of same order.

Ans 47

Correct Option. D

Solution:

$$\begin{bmatrix} a & 2 & 3 \\ b & 5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} a+6-3 & 2a+8+3 \\ b+15+1 & 2b+20-1 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} a+3 & 2a+11 \\ b+16 & 2b+19 \end{bmatrix} = \begin{bmatrix} 4 & 13 \\ 12 & 11 \end{bmatrix} \text{ (given)}$$
$$\Rightarrow \quad a+3=4 \Rightarrow a=1$$
$$\text{and } b+16=12 \Rightarrow b=12-16=-4$$
$$\therefore \quad (a,b)=(1,-4)$$

Ans 48

Correct Option. A

Solution:

Given that, $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ 1 & 1 & 1+1 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} + ab \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (ab+1) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad (ab+1)\left[a\left(b^2-1\right) - a^2(b-1) + 1\left(b-b^2\right)\right] = 0$$

$$\Rightarrow \quad (ab+1)(b-1)[a(b+1) - a - b)] = 0$$

$$\Rightarrow \quad (ab+1)(b-1)[ab+a-a-b] = 0$$

$$\Rightarrow \quad b(ab+1)(b-1)(a-1) = 0$$

$$\Rightarrow \quad b = 0, 1; a = 1, ab = -1$$

Ans 49

Correct Option. C

Solution:

In a triangle ABC,

$A + B + C = \pi$,

$\Rightarrow \sin(A+B+C) = 0, \cos(A+B) = -\cos C$

$$\therefore \begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix}$$

=0 $\because$ It is the determinant of a skew symmetric matrix of odd order.

Ans 50

Correct Option. A

Solution:

$$\text{Given, } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a-a & b-b & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ans 51

Correct Option. C

Solution:

$$\text{Given, determinant } \begin{vmatrix} 3 & x & -1 \\ 2 & -1 & 4 \\ 1 & y & -3 \end{vmatrix}$$

Here, cofactor of $x = 10$ and cofactor of $y = -14$

$\therefore$ Their sum $= -4$

Ans 52

Correct Option. C

Solution:

Given, system of equations are

$ax + y + z = 0, x + ay + z = 0$
and $x + y + z = 0$ possesses non-zero solution.

$$\therefore \quad \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\begin{aligned} \Rightarrow \quad & a(a-1) - 1(1-1) + 1(1-a) = 0 \\ \Rightarrow \quad & a^2 - a + 0 + 1 - a = 0 \\ \Rightarrow \quad & a^2 - 2a + 1 = 0 \\ \Rightarrow \quad & (a-1)^2 = 0 \\ \Rightarrow \quad & a = 1 \end{aligned}$$

Ans 53

Correct Option. C

Solution:

Given system of equations are

$x+y+z=0, 2x+y-z=0, 3x+2y=0$

Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & 0 \end{vmatrix} = 1(0+2) - 1(0+3) + 1(4-3)$

$= +2 - 3 + 1 = 0$

Hence, it has infinitely many solutions.

Ans 54

Correct Option. B

Solution:

$\sin^{-1}\frac{1}{\sqrt{5}} + \cos^{-1}\frac{3}{\sqrt{10}}$

$$\text{Now, } \sin^{-1}\frac{1}{\sqrt{5}} = \tan^{-1}\left(\frac{1/\sqrt{5}}{\sqrt{1-\frac{1}{5}}}\right) = \tan^{-1}\frac{1}{2}$$

$$\text{and } \cos^{-1}\frac{3}{\sqrt{10}} = \tan^{-1}\left(\frac{\sqrt{1-\frac{9}{10}}}{\frac{3}{\sqrt{10}}}\right) = \tan^{-1}\frac{1}{3}$$

$$\therefore \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

Ans 55

Correct Option. B

Solution:

Given, $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$$\Rightarrow \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$$

$\Rightarrow x+y+z-xyz = 0$

Ans 56

Correct Option. D

Solution:

Let $y = e^x$

$\frac{d}{dx}(y) = \frac{d}{dx}(e^x) \Rightarrow \frac{dy}{dx} = e^x$

Ans 57

Correct Option. C

Solution:

$$y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots}}}$$

$$\Rightarrow \quad y=\sqrt{\sin x+y}$$
$$\Rightarrow \quad y^2=\sin x+y$$

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx}=\cos x+\frac{dy}{dx} \Rightarrow \frac{dy}{dx}(2y-1)=\cos x$$
$$\Rightarrow \quad \frac{dy}{dx} \doteq \frac{\cos x}{2y-1}$$

Ans 58

Correct Option. D

Solution:

$$(\text{ Let } z=\sin x(1+\cos x)$$
$$\text{or } z=\sin x+\frac{1}{2}\sin 2x$$

On differentiating w.r.t. x, we get

$$\frac{dz}{dx}=\cos x+\cos 2x$$

For maximum or minimum, put $\frac{dz}{dx}=0$

$$\begin{aligned}
\Rightarrow \quad & \cos x+\cos 2x && =0 \\
\Rightarrow \quad & 2\cos^2 x-1+\cos x && =0 \\
\Rightarrow \quad & (\cos x+1)(2\cos x-1) && =0 \\
\Rightarrow \quad & \cos x && =-1, \tfrac{1}{2} \\
\Rightarrow \quad & x && =\tfrac{\pi}{3}, \pi
\end{aligned}$$

But $0 \leq x \leq \frac{\pi}{2}$, therefore we take only $x=\frac{\pi}{3}$.

$\therefore \quad \frac{d^2z}{dx^2}=-\sin x-2\sin 2x=-$ ve, for $x=\pi/3$

$\therefore$ It is maximum at $x=\pi/3$.

Ans 59

Correct Option. C

Solution:

$$\int_0^1 x(1-x)^{12}dx = \left[\frac{x(1-x)^{13}}{-13} + \int_0^1 \frac{(1-x)^{13}}{13}dx\right]_0^1$$

$$= \left[\frac{x(1-x)^{13}}{-13}\right]_0^1 + \left[\frac{(1-x)^{14}}{-14\times 13}\right]_0^1$$

$$= [0-0] + \left[0 - \frac{1}{-14\times 13}\right] = \frac{1}{182}$$

Ans 60

Correct Option. A

Solution:

$$\int_{-\pi/2}^{\pi/2} |\sin x|dx = 2\int_0^{\pi/2} \sin x dx$$

$$= 2[-\cos x]_0^{\pi/2} = 2$$

Ans 61

Correct Option. A

Solution:

Let $I = \int_1^e \frac{\log x}{x}dx$

Put $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore \quad I = \int_0^1 tdt = \left[\frac{t^2}{2}\right]_0^1 = \frac{1}{2}$

Ans 62

Correct Option. A

Solution:

Let $I = \int_0^1 \sin^{-1} x dx$

$$= [x\sin^{-1} x]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$$

Put $1-x^2 = t^2 \Rightarrow -2xdx = 2tdt$

$\therefore \quad I = \left[\frac{\pi}{2} - 0\right] + \int_1^0 \frac{tdt}{t}$

$= \frac{\pi}{2} + \int_1^0 dt = \frac{\pi}{2} + [t]_1^0 = \frac{\pi}{2} - 1$

Ans 63

Correct Option. B

Solution:

The point of intersection of the curves $y^2 = 2x$ and $x^2 = 2y$ are $O(0,0), A(2,2)$.

$\therefore$ Required area $= \int_0^2 \left(\sqrt{2x} - \frac{x^2}{2}\right) dx$

$$= \left[\sqrt{2}\frac{x^{3/2}}{3/2} - \frac{x^3}{6}\right]_0^2$$
$$= \left[\frac{2}{3} \cdot 2^2 - \frac{2^3}{6}\right] = \left[\frac{8}{3} - \frac{4}{3}\right]$$
$$= \frac{4}{3} \text{ sq unit}$$

Ans 64

Correct Option. C

Solution:

Given, $y = a\cos\mu x + b\sin\mu x$ ...(i)

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = -a\mu\sin\mu x + b\mu\cos\mu x$$

Again, differentiating, we get

$$\frac{d^2y}{dx^2} = -a\mu^2\cos\mu x - b^2\mu\sin\mu x$$

$= -\mu^2 y$ [from Eq. (i)]

$$\Rightarrow \frac{d^2y}{dx^2} + \mu^2 y = 0$$

Ans 65

Correct Option. C

Solution:

$$|\vec{a}\times\vec{b} + \vec{b}\times\vec{a}| = |\vec{a}\times\vec{b} - \vec{a}\times\vec{b}| = 0$$

Ans 66

Correct Option. C

Solution:

Given, $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k}), \vec{b} = \frac{1}{7}(3\hat{i} - \lambda\hat{j} + 2\hat{k})$

Since, the vectors $\vec{a}$ and $\vec{b}$ are perpendicular.

$\therefore \quad \vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$

$\Rightarrow \frac{1}{7}(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}) \cdot \frac{1}{7}(3\hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 0$

$\Rightarrow \quad \frac{1}{49}(6 - 3\lambda + 12) = 0$

$\Rightarrow \quad 3\lambda = 18 \Rightarrow \lambda = 6$

Ans 67

Correct Option. B

Solution:

$$\left\{(\vec{\mathbf{a}} \times \vec{\mathbf{b}})^2 + (\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})^2\right\} \div a^2b^2$$

$$= \left\{(|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\sin\theta\hat{\mathbf{n}})^2 + (|\vec{\mathbf{a}}||\vec{\mathbf{b}}|\cos\theta)^2\right\} \div a^2b^2$$

$$= a^2b^2\left(\sin^2\theta + \cos^2\theta\right) \div a^2b^2 = 1$$

Ans 68

Correct Option. C

Solution:

$$[\vec{a} + \vec{b}\vec{b} = +\vec{a}$$

$$= (\vec{a} + \vec{b}) \cdot [(\vec{b} + \vec{c}) \times (\vec{c} + \vec{a})]$$

$$= (\vec{a} + \vec{b}) \cdot [(\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + 0 + \vec{c} \times \vec{a})]$$

$$= [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] + 0 + 0 + 0 + 0 + [\vec{\mathbf{b}}\ \vec{\mathbf{c}}\ \vec{\mathbf{a}}]$$

$$= [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}] + [\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$$

$$= 2[\vec{\mathbf{a}}\ \vec{\mathbf{b}}\ \vec{\mathbf{c}}]$$

Ans 69

Correct Option. C

Solution:

Given, $\vec{a} = 3\vec{\alpha} - \vec{\beta}, \vec{b} = \vec{\alpha} + 3\vec{\beta}$

and $|\vec{\alpha}| = |\vec{\beta}| = 2$

$$\begin{aligned} \therefore |\vec{a} + \vec{b}|^2 &= |4\vec{\alpha} + 2\vec{\beta}|^2 \\ &= 16|\vec{\alpha}|^2 + 4|\vec{\beta}|^2 + 16|\vec{\alpha}||\vec{\beta}| \cos \frac{\pi}{3} \\ &= 16(4) + 4(4) + 16(2 \times 2) \times \frac{1}{2} \\ &= 64 + 16 + 32 = 112 \\ \Rightarrow |\vec{a} + \vec{b}| &= 4\sqrt{7} \end{aligned}$$

Ans 70

Correct Option. B

Solution:

Since, a ray makes angles α, β, γ with the positive direction of the axes.

$$\begin{aligned} \therefore \quad & \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \\ \Rightarrow \quad & 1 - \sin^2 \alpha + 1 - \sin^2 \beta + 1 - \sin^2 \gamma = 1 \\ \Rightarrow \quad & \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2 \end{aligned}$$