MET 2022 Answer Key PDF

Physics

Ans 1

Correct Option. C

Solution:

As, spring constant, $(k) \propto \frac{1}{\text{length of spring } (l)}$

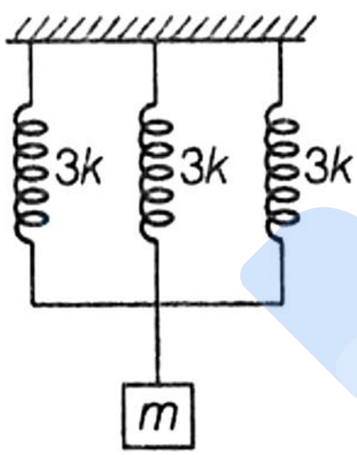
$\because$ The spring cut into three equal parts

$\therefore k' = 3k$ (For each spring)

We know that, time period of a body connected with spring.

$T = 2\pi\sqrt{\frac{m}{k}}$ ...(i)

The parallel arrangement of the spring is given as



$T' =$ Time period of springs $(k' = 3k)$ connected in parallel

$$= 2\pi\sqrt{\frac{m}{3k'}} = 2\pi\sqrt{\frac{m}{3 \times 3k}}$$

$$= \frac{1}{3}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{T}{3} \quad \text{[From Eq. (i)]}$$

Ans 2

Correct Option. B

Solution:

According to work-energy theorem,

$$W_{\text{external}} + W_{\text{internal}} = K_f - K_i = \Delta K$$

Ans 3

Correct Option. 8

Solution:

$$(8)\ \phi = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta = BA\cos\omega t, (\because \theta = \omega t)$$
$$\text{Now, emf } (e) = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t)$$
$$= \omega BA\sin\omega t$$
$$i = \frac{e}{R} = \frac{\omega BA\sin\omega^{t}}{R}$$
$$\therefore \quad P_{\text{ins}} = i^2R = \left(\frac{\omega^2B^2A^2}{R^2}\sin^2\omega t\right)\cdot R$$
$$= \frac{\omega^2B^2A^2}{R}\sin^2\omega t$$
$$\therefore \quad P_{\text{avg}} = \frac{\int_0^T P_{\text{ins}} \times dt}{\int_0^T dt}$$
$$= \frac{\omega^2B^2A^2}{R}\cdot\frac{\int_0^T \sin^2\omega t dt}{\int_0^T dt}$$
$$= \frac{1}{2}\cdot\frac{\omega^2B^2A^2}{R} \quad \left(\because < \sin^2\omega t >_{t=T} = \frac{1}{2}\right)$$
$$= \frac{(B\pi r^2\omega)^2}{8R} \quad \left(\because A = \frac{\pi r^2}{2}\right)$$
$$\therefore \quad x = 8$$

Ans 4

Correct Option. B

Solution:

By the conservation of energy,

$$E_i = E_f$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH = mg\left(\frac{7v^2}{10g}\right)$$

$$\Rightarrow \quad \frac{1}{2}mv^2 + \frac{1}{2}I \cdot \frac{v^2}{R^2} = \frac{7}{10}mv^2 \quad (\because v = \omega R)$$

$$\Rightarrow \quad \frac{1}{2}I \cdot \frac{v^2}{R^2} = \left(\frac{7}{10} - \frac{1}{2}\right)mv^2$$

$$= \frac{2}{10}mv^2 = \frac{1}{5}mv^2$$

$$\therefore \quad I = \frac{2}{5}mR^2$$

The above value of moment of inertia is equal to moment of inertia of solid sphere.

Ans 5

Correct Option. B

Solution:

The escape velocity of a planet is given by,

$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R} \times \frac{4}{3}\pi R^3 \cdot \rho}$$

$$\therefore \quad V \propto R\sqrt{\rho}$$

$$\therefore \quad \frac{v_P}{v_E} = \frac{R_P\sqrt{\rho_P}}{R_E\sqrt{\rho_E}} = \frac{R_P}{R_E} \quad (\because \rho_P = \rho_E)$$

$$\therefore \quad \frac{v_P}{v_E} = \frac{3R_E}{R_E} = 3 \quad (\because R_P = 3R_E)$$

$$\therefore \quad v_P = 3v_E$$

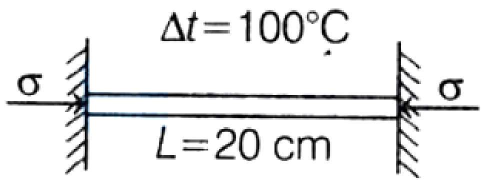
Ans 6

Correct Option. 8

Solution:

From Hook's law,

$Y = \frac{\text{Stress}}{\text{Strain}}$



$\therefore \quad \text{Stress} = Y \times \text{Strain}$

Now, $\quad \text{Strain} = \frac{\Delta L}{L} = \frac{L\alpha\Delta T}{L} = \alpha\Delta T$

$\therefore \quad \text{Stress} = Y \times \alpha\Delta T$

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^{8}$ Pa

According to question,

$2.2 \times 10^{x} = 2.2 \times 10^{8}$

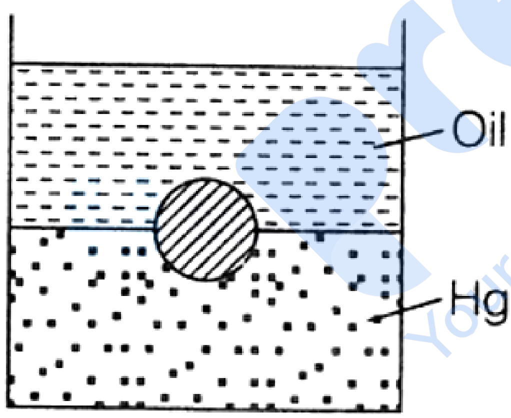
$x = 8$

Ans 7

Correct Option. 7.2

Solution:

For sphere to be float



$\Rightarrow \quad V\rho g = \rho_{\text{oil}} \times \frac{V}{2} \times g + \rho_{\text{Hg}} \times \frac{V}{2} \times g$

$\Rightarrow \quad \rho = \frac{\rho_{\text{oil}}}{2} + \frac{\rho_{\text{Hg}}}{2} = \frac{0.8}{2} + \frac{13.6}{2}$

$= 0.4 + 6.8$

$\therefore \rho =$ density of sphere $= 7.2 \text{ g/cm}^3$

$\therefore x \text{ g/cm}^3 = 7.2 \text{ g/cm}^3$

$\therefore \quad x = 7.2$

Ans 8

Correct Option. C

Solution:

As, $\frac{C}{100}=\frac{F-32}{180}$
Now, $\quad C=F$ (given)

$$\begin{aligned}
\therefore & \quad \frac{C}{100}=\frac{C-32}{180} \\
\Rightarrow & \quad 180 C=100 C-3200 \\
\Rightarrow & \quad 180 C-100 C=-3200 \\
\Rightarrow & \quad 80 C=-3200 \\
\therefore & \quad C=-\frac{3200}{80}=-40 \\
\therefore & \quad C=F=-40
\end{aligned}$$

Ans 9

Correct Option. B

Solution:

The electric potential due to an electric dipole at a distance r from its centre is given by

$$1 \mathrm{~V}=\frac{1}{4 \pi \varepsilon_0} \cdot \frac{p \cos \theta}{r^2}$$
$[\text{where, } p= \text{ electric dipole moment}]$

$$\therefore \quad V \propto \frac{1}{r^2}$$

Ans 10

Correct Option. B

Solution:

$$\text{As, } R=\rho \cdot \frac{L}{A}=\rho \cdot \frac{L^2}{A \cdot L}=\rho \cdot \frac{L^2}{V} ;(V= \text{ Volume })$$
$$\therefore \quad R \propto L^2 \Rightarrow \frac{R_2}{R_1}=\left(\frac{L_2}{L_1}\right)^2=\left(\frac{2 L_1}{L_1}\right)^2=4$$
$$\therefore \quad R_2=4 R_1=4 R$$

Ans 11

Correct Option. 300

Solution:

By Faraday's law of electromagnetic induction,

$$\varepsilon = \frac{d\phi}{dt}, \text{ Also, } \varepsilon = iR$$

$$\therefore \quad \frac{d\phi}{dt} = iR \Rightarrow d\phi = iRdt$$

$$\Rightarrow \quad \int d\phi = R\int idt$$

$$\therefore \quad \phi = R\times \text{ Area of current versus time}$$

$$= 150 \times \frac{1}{2} \times 0.4 \times 10 = 300 \text{ Wb}$$

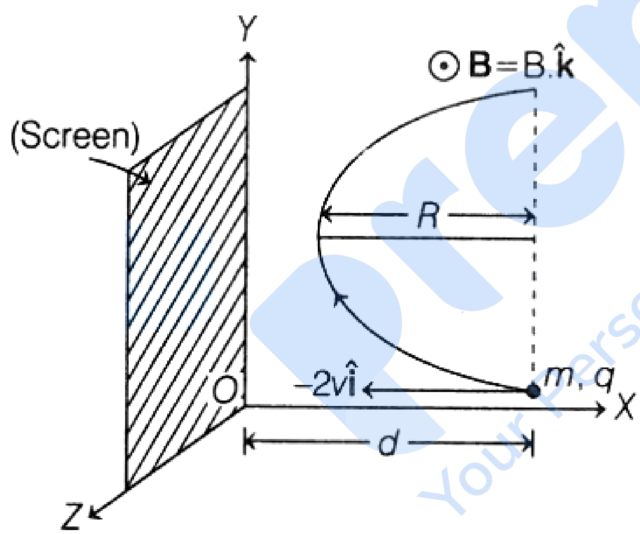
Ans 12

Correct Option. B

Solution:

The charged particle will not strike the screen, if d > R (Radius of path)

$$\Rightarrow \quad d > \frac{m(2v)}{B_0 q} \Rightarrow v < \frac{qdB_0}{2m}$$



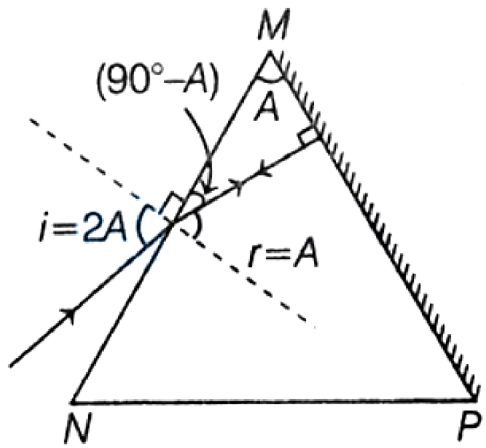
$$\therefore \quad v_{\max} = \frac{qdB_0}{2m}$$

Ans 13

Correct Option. B

Solution:

If reflected ray from silvered surface retrace its path, then it falls normally on the silvered surface.



∴ From the geometry,

$$i = 2A, r = A$$

$$\therefore \quad \mu = \text{Refractive index } = \frac{\sin i}{\sin r} \text{ (From Snell's law)}$$

$$= \frac{\sin 2A}{\sin A} = \frac{2 \sin A \cdot \cos A}{\sin A}$$

$$= 2 \cos A$$

Ans 14

Correct Option. A

Solution:

de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk}}$$

$$\lambda \propto \frac{1}{\sqrt{m}} \quad (\because k_e = k_p = k_{\mathrm{He}^{++}})$$

$$\text{As, } m_{\mathrm{He}^{++}} > m_p > m_\theta$$

$$\therefore \lambda_{\mathrm{He}^{++}} < \lambda_p < \lambda_\theta$$

$$\lambda_\theta > \lambda_p > \lambda_{\mathrm{He}^{++}}$$

Ans 15

Correct Option. 0.98

Solution:

$E = E_0 \cos(kx - \omega t)$

$\therefore \quad \omega = 2\pi \times 6 \times 10^{14}$

$\therefore \quad f = \frac{\omega}{2\pi} = \frac{2\pi \times 6 \times 10^{14}}{2\pi} = 6 \times 10^{14}$ Hz

$\phi =$ Energy of photon $= \frac{1240}{\lambda(\text{ in nm})}$eV

$\therefore \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 0.5 \times 10^{-6}$ m $= 500$ nm

$\therefore \quad \phi = \frac{1240}{500}$eV $= 2.48$eV

Now, by Einstein photoelectric equation,

$$\begin{aligned} \phi &= \phi_0 + \text{KE} = \phi_0 + eV_s \quad (\because \text{KE} = eV_s) \\ \Rightarrow \quad 2.48 &= 1.5 + eV_s \\ eV_s &= 2.48 - 1.5 = 0.98\text{eV} \\ V_s &= 0.98 \text{ V} \end{aligned}$$

Chemistry

Ans 1

Correct Option. A

Solution:

(i) Volume occupied by 1 mole of $\mathrm{CO_2} = 22.4\ \mathrm{L}$

Volume occupied by 1.5 moles of $\mathrm{CO_2}$

$$= 1.5 \times 22.4$$
$$= 33.6\ \mathrm{L}$$

(ii) Given, mass of nitrogen $(m) = 14\ \mathrm{g}$

Molar mass of $\mathrm{N_2}(w) = 28\ \mathrm{g}$

Number of moles of $\mathrm{N_2} = \frac{m}{w} = \frac{14}{28} = \frac{1}{2}$

$= 0.5$ mole

Volume occupied by 1 mole of $\mathrm{N_2}$ at STP

$= 22.4\ \mathrm{L}$

Volume occupied by 0.5 mole of $\mathrm{N_2}$ at STP

$$= 0.5 \times 22.4$$
$$= 11.2\ \mathrm{L}$$

(iii) Given, number of molecules of $\mathrm{O_2} = 10^{21}$

Number of molecules of $\mathrm{O_2}$ in 1 mole

$$\begin{aligned} &= 6.022 \times 10^{23} \text{ molecules} \\ \text{Number of moles } &= \frac{10^{21}}{6.022 \times 10^{23}} \\ &= 0.166 \times 10^{-2} \text{ moles ;} \end{aligned}$$

Volume occupied by 0.166×10^{-2} mole of

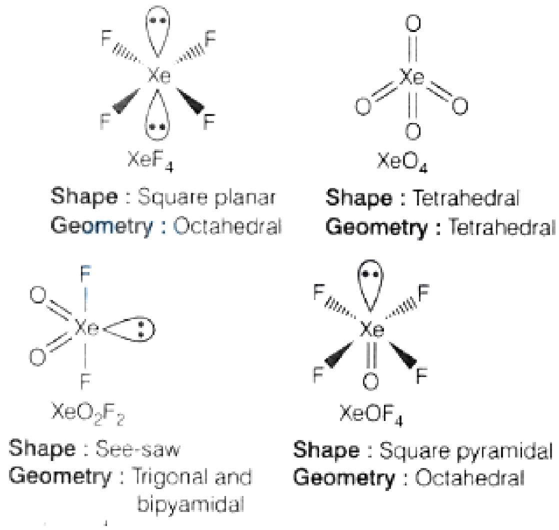
$$\begin{aligned} \mathrm{O_2} &= 0.166 \times 10^{-2} \times 22.4\ \mathrm{L} \\ &= 3.72 \times 10^{-2} \\ &= 0.0372\ \mathrm{L} \end{aligned}$$

Therefore, correct increasing order of volume occupied by the given amount of molecules at STP is : (iii) < (ii) < (i).

Ans 2

Correct Option. C

Solution:



Ans 3

Correct Option. B

Solution:

Given, $q = 150\ \mathrm{J}$

We know,

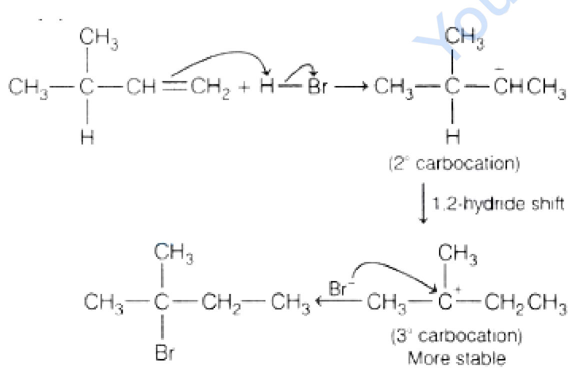
$$\begin{aligned} w = p\Delta V &= 2 \times 10^5 \mathrm{Nm}^{-2} \times 450 \times 10^{-6}\ \mathrm{m}^3 \\ &= 90 \mathrm{Nm} = 90\ \mathrm{J} \\ \Delta U &= q - w = 150 - 90 = 60\ \mathrm{J} \end{aligned}$$

Also,

Ans 4

Correct Option. D

Solution:



Ans 5

Correct Option. A

Solution:

In case of positive deviation from Raoult's law, the intermolecular attractive forces between the solute-solvent molecules are weaker than those

between the solute-solute and solvent-solvent molecules. Therefore, it becomes easy for the solute/solvent molecules to escape from the solution than in pure state. This will increase the vapour pressure and results in positive deviation.

Ans 6

Correct Answer. 232.6

Solution:

$$\Lambda^{\circ}_{m(NHH_4OH)=} = \Lambda^{\circ}_{NH_4^+} +$$
$$= \Lambda^{\circ}_{OH^-}$$
$$= \left(\Lambda^{\circ}_{NH_4^+} + \Lambda^{\circ}_{Cl^-}\right) + \frac{1}{2}\left(\Lambda^{\circ}_{Ba^{2+}} + 2\Lambda^{\circ}_{OH^-}\right)$$
$$- \frac{1}{2}\left(\Lambda^{\circ}_{Ba^{2+}} + 2\Lambda^{\circ}_{Cl^-}\right)$$
$$= \Lambda^{\circ}_{m(NH_4Cl)} + \frac{1}{2}\Lambda^{\circ}_{m[Ba(OH)_2]} - \frac{1}{2}\Lambda^{\circ}_{m(BaCl_2)}$$
$$= 130 + \frac{1}{2} + 446.8 - \frac{1}{2} \times 241.6$$
$$= 232.6\Omega^{-1}\ cm^2\ mol^{-1}.$$

Ans 7

Correct Answer. 0.0103

Solution:

(1.03×10^{-2}) For the given reaction,

$$N_2O_5(g) \longrightarrow 2NO_2(g) + \frac{1}{2}O_2(g)$$
$$-\frac{d[N_2O_5]}{dt} = \frac{1}{2}\frac{d[NO_2]}{dt} = \frac{2d[O_2]}{dt}$$
$$\therefore \quad \frac{d[NO_2]}{dt} = -2\frac{d[N_2O_5]}{dt} = 2 \times 5.15 \times 10^{-3}$$
$$= 1.03 \times 10^{-2}\ mol\ L^{-1}\ s^{-1}.$$

Ans 8

Correct Option. D

Solution:

Mond's process is used for refining nickel. Whereas, van Arkel method is used for refining zirconium and titanium.

Ans 9

Correct Answer. 4

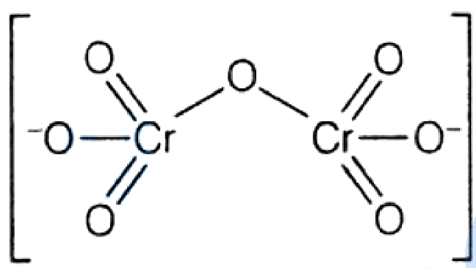
Solution:

Nitrogen does not form pentahalide due to non-availability of the d-orbitals in its valence shell.

Ans 10

Correct Answer. 4

Solution:



Dichromate ion

4 oxygen atoms are directly attached to one chromium ion.

Ans 11

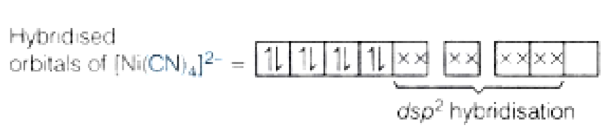
Correct Option. B

Solution:

Both $[Ni(CN)_4]^{2-}$ and $[Ni(CO)_4]$ are diamagnetic.

$[Ni(CN)_4]^{2-}$
Electronic configuration of $Ni^{2+} = [Ar]3d^8$.

CN^- being strong field ligand cause pairing of electrons.



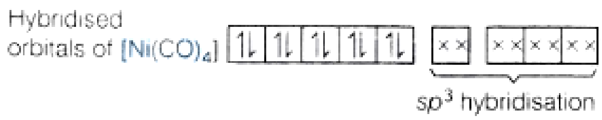
There are no unpaired electrons. Therefore, it is diamagnetic.

Shape : square planar

$[Ni(CO)_4]$

Electronic configuration of Ni $= [Ar]3d^8 4s^2$

CO being strong field ligand cause pairing of electrons.



No unpaired electrons are there. Therefore, it is diamagnetic in nature.

Shape : Tetrahedral.

Ans 12

Correct Option. B

Solution:

Alcohols are soluble in Lucas' reagent (conc. HCl and $ZnCl_2$) while their halides are immiscible and produce turbidity in solution. Tertiary alcohols react the fastest and form halides easily. Primary alcohols do not produce turbidity at room temperature. Therefore, alcoholic group at the 4th position is tertiary, therefore it reacts the fastest.

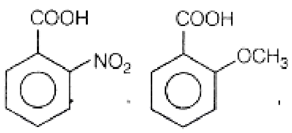
Ans 13

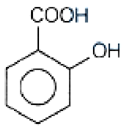
Correct Answer. 3

Solution:

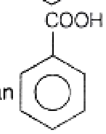
Attached electron withdrawing groups increase the acidity of benzoic acid by −/ and −M-effect (whenever applicable) whereas electron releasing groups decrease the acidity of benzoic acid.

Due to *othro*-effect,



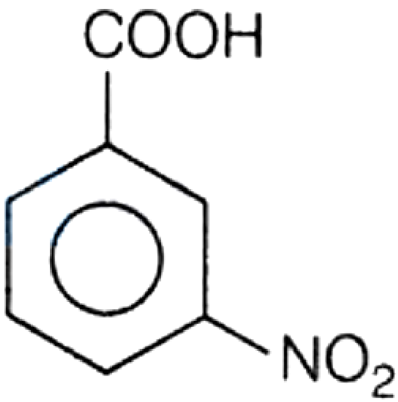


are stronger acids than



even though they have electron releasing groups (i.e. OCH_3) attached.

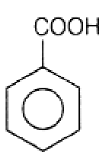
In



one electron withdrawing group is attached, due to this reason it is more acidic than benzoic acid.

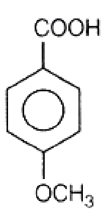
Moreover, aliphatic carboxylic acid is a weaker acid than aromatic carboxylic acid because benzene itself is electron withdrawing in nature that makes the release of proton from - COOH easy.

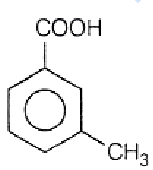
Therefore, acids which are weaker than



are

CH_3COOH,

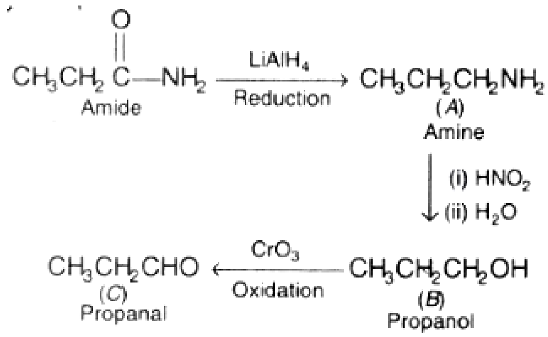




Ans 14

Correct Option. D

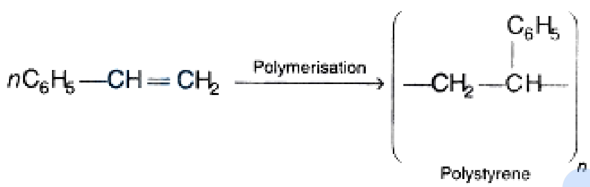
Solution:



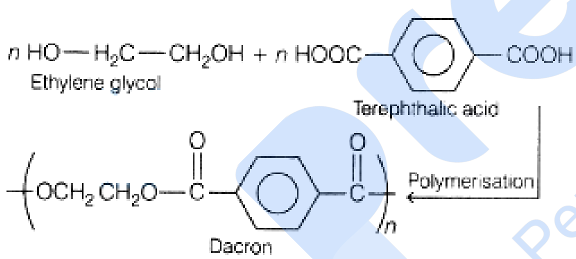
Ans 15

Correct Option. B

Solution:



Polystyrene is an addition and chain growth polymer. It is thermoplastic. Addition polymers have unsaturated monomers.



Dacron is a condensation and step-growth polymer. It is a fibre.

Condensation polymerisation involves a repetitive condensation reaction between two bi-functional or tri-functional monomeric units.

Mathematics

Ans 1

Correct Option. D

Solution:

We can write

$$|z| = \left|z + \frac{2}{|z|} - \frac{2}{|z|}\right| \leq \left|z + \frac{2}{z}\right| + \left|\frac{2}{|z|}\right|$$
$$= 2 + \frac{2}{|z|}$$
$$\therefore \quad |z|^2 - 2|z| - 2 \leq 0$$

Here, $|z|$ lies between $\frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$

$\therefore$ Minimum value of $|z| = 1 - \sqrt{3}$

Ans 2

Correct Option. 2454

Solution:

Given word is 'EXAMINATION' having letters (AA), (II), (NN), EXMOT, we have to form 4 letter words, then following cases are possible,

(I) 2 same, 2 same and number of words are ${}^3C_2 \times \frac{4!}{2!2!} = 18$

(II) 2 same, 2 different and number of words are

$${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} = 3 \times \frac{7 \times 6}{2} \times \frac{4 \times 3 \times 2}{2}$$
$$= 21 \times 36 = 756$$

(III) All are different and number of words are

${}^8C_4 \times 4! = \frac{8 \times 7 \times 6 \times 5}{4!} 4! = 1680$

So, total number of 4 letter words

$18 + 756 + 1680 = 2454$

Ans 3

Correct Option. B

Solution:

Since, α, β and γ are in AP.

$\therefore \quad 2\beta = \alpha + \gamma$ ...(i)

Also, $\tan^{-1}\alpha, \tan^{-1}\beta$ and $\tan^{-1}\gamma$ are also in AP.

$$\begin{aligned} \therefore \quad 2\tan^{-1}\beta &= \tan^{-1}\alpha + \tan^{-1}\gamma \\ \Rightarrow \quad \tan^{-1}\left(\frac{2\beta}{1-\beta^2}\right) &= \tan^{-1}\left(\frac{\alpha+\gamma}{1-\alpha\gamma}\right) \\ \Rightarrow \quad \frac{2\beta}{1-\beta^2} &= \frac{\alpha+\gamma}{1-\alpha\gamma} \\ \Rightarrow \quad \frac{\alpha+\gamma}{1-\beta^2} &= \frac{\alpha+\gamma}{1-\alpha\gamma} \quad \text{b y using} \\ \Rightarrow \quad \beta^2 &= \alpha\gamma \end{aligned}$$

[by using Eq. (i)]

Since, α, β and γ are in AP as well as in GP.

$\therefore \quad \alpha = \beta = \gamma$

Ans 4

Correct Option. C

Solution:

The coefficient of the middle term in power of

$x(1+\alpha x)^4 = {}^4C_2\alpha^2$

The coefficient of the middle term in power of

$x(1-\alpha x)^6 = {}^6C_3(-\alpha)^3$

According to given condition,

$$\begin{aligned} {}^4C_2\alpha^2 &= {}^6C_3(-\alpha)^3 \\ \Rightarrow \quad \frac{4!}{2!2!}\alpha^2 &= -\frac{6!}{3!3!}\alpha^3 \\ \Rightarrow \quad 6\alpha^2 &= -20\alpha^3 \\ \Rightarrow \quad \alpha &= -\frac{6}{20} \\ \therefore \quad \alpha &= -\frac{3}{10} \end{aligned}$$

Ans 5

Correct Option. C

Solution:

General equation of all such circles which passing through the origin whose centre lie on X-axis is

$x^2+y^2+2gx=0$ ...(i)

On differentiating w.r.t. ' x ', we get

$$2x+2y\frac{dy}{dx}+2g=0$$

$$\Rightarrow \quad 2g=-\left(2x+2y\frac{dy}{dx}\right)$$

On putting the value of $2g$ in Eq. (i),

$$x^2+y^2+\left(-2x-2y\frac{dy}{dx}\right)x=0$$

$$\Rightarrow \quad x^2+y^2-2x^2-2xy\frac{dy}{dx}=0$$

$$\Rightarrow \quad y^2=x^2+2xy\frac{dy}{dx}$$

which is the required equation.

Ans 6

Correct Option. C

Solution:

Let $I'=\lim_{x\to 0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ $\left[\frac{0}{0}\text{ form}\right]$

Using L'Hospital's rule,

$=\lim_{x\to 0}\frac{\sec^2 x^2\cdot 2x}{x\cos x+\sin x}$ $\left[\frac{0}{0}\text{ form}\right]$

Again using L'Hospital's rule,

$$\lim_{x\to 0}\frac{2x\cdot 2\sec^2 x^2\cdot\tan x^2\cdot 2x+2\sec^2 x^2}{-x\sin x+\cos x+\cos x}$$

$$=\frac{0+2\sec^2 0}{0+2\cos 0}=1$$

Ans 7

Correct Option. 12

Solution:

We have,

$\lim_{x\to2}\frac{2^x+2^{2-x}-5}{2^{-x/2}-2^{1-x}}$ $\left[\frac{0}{0}\text{ form}\right]$

So, apply L' Hospital's rule

$$=\lim_{x\to2}\frac{\frac{d}{dx}\left(2^x+2^{2-x}-5\right)}{\frac{d}{dx}\left(2^{-x/2}-2^{1-x}\right)}$$

$$=\lim_{x\to2}\frac{2^x\ln2-2^{2-x}\ln2}{2^{1-x}\ln2-\frac{2^{-\frac{x}{2}}\ln2}{2}}$$

$$=\lim_{x\to2}\frac{2^{x+1}-2^{3-x}}{2^{2-x}-2^{-\frac{x}{2}}}$$

$$=\frac{2^3-2^1}{2^0-2^{-1}}=\frac{8-2}{2-\frac{1}{2}}=\frac{12}{2-1}=12$$

Ans 8

Correct Option. D

Solution:

We have,

$\sum_{i=1}^{10}(x_i-3)=7$ and $\sum_{i=1}^{10}(x_i-3)^2=27$

$\therefore$ Standard deviation is remain unchanged, if observation are added or subtracted by a fixed number.

$\therefore$ Standard deviation

$$=\sqrt{\frac{\sum_{i=1}^{10}(xi-3)^2}{10}-\left(\frac{\sum_{i=1}^{10}(xi-3)}{10}\right)^2}$$

$$=\sqrt{\frac{27}{10}-\left(\frac{7}{10}\right)^2}$$

$$=\sqrt{\frac{270}{100}-\frac{49}{100}}=\frac{\sqrt{221}}{10}=\frac{14.86}{10}=1.486$$

Ans 9

Correct Option. A

Solution:

Here,

$$\begin{aligned} A \times C &= \{1,2\} \times \{5,6\} \\ &= \{(1,5),(1,6),(2,5),(2,6)\} \\ \text{and } B \times D &= \{1,2,3,4\} \times \{5,6,7,8\} \\ &= \{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6), \\ &\quad (2,7),(2,8),(3,5),(3,6),(3,7),(3,8), \\ &\quad (4,5),(4,6),(4,7),(4,8)\} \\ \therefore \quad (A \times C) &\subset (B \times D) \end{aligned}$$

Ans 10

Correct Option. B

Solution:

If A is non-singular matrix, then $|A| \neq 0$

$$\begin{aligned} & AA^T = A^{-1}A \text{ and } B = A^{-1}A^T \\ \Rightarrow \quad & BB' = (A^{-1}A^T)(A^{-1}A^T)^T \\ & = A^{-1}A^T \cdot A(A^{-1})^T \quad [\because (AI)' = A] \\ & = A^{-1}AA^T(A^{-1})^T \\ & = IA^T(A^{-1})^T \\ & = A^T(A^{-1})^T \\ & = (A^{-1}A)^T \quad [\because (AB)^T = B^TA^T] \\ & = I^T = 1 \end{aligned}$$

Ans 11

Correct Option. C

Solution:

Given that, $f(x) = \sqrt{1+x^2}$

$$\begin{aligned} \therefore \quad & f(y) = \sqrt{1+y^2} \\ \text{and} \quad & f(xy) = \sqrt{1+x^2y^2} \\ \text{Now,} \quad & \| f(xy) = f(x)f(y) \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad \sqrt{1+x^2y^2} &= \sqrt{1+x^2}\sqrt{1+y^2} \\ &= \sqrt{(1+x^2)(1+y^2)} \end{aligned}$$

On squaring both sides, we get

$$\begin{aligned} & 1+x^2y^2 = (1+x^2)(1+y^2) \\ \Rightarrow \quad & 1+x^2y^2 = 1+y^2+x^2+x^2y^2 \end{aligned}$$

$$\begin{aligned} \therefore \quad & 1+x^2y^2 \leq 1+x^2+y^2+x^2y^2 \\ \text{Hence,} \quad & f(xy) \leq f(x) \cdot f(y) \end{aligned}$$

Ans 12

Correct Option. B

Solution:

We have, $\sin\theta\cos^3\theta > \sin^3\theta\cos\theta$

$$\Rightarrow \quad \sin\theta\cos^3\theta - \sin^3\theta\cos\theta > 0$$
$$\Rightarrow \quad \sin\theta\cos\theta\left(\cos^2\theta - \sin^2\theta\right) > 0$$
$$\Rightarrow \quad \sin 2\theta \cdot \cos 2\theta > 0$$
$$\Rightarrow \quad \sin 4\theta > 0$$
$$\Rightarrow \quad \theta \in \left(0, \frac{\pi}{4}\right) \quad [\because 4\theta \in (0, \pi)]$$

Ans 13

Correct Option. D

Solution:

Given that, $\int f(x)dx = g(x)$

Let $I = \int x^9 \int\left(x^5\right) dx$

Put $x^5 = t \Rightarrow 5x^4dx = dt$

$$= \frac{1}{5}\int t f(t)dt$$
$$= \frac{1}{5}\left[t\int f(t)dt - \int[f(t)dt]\right.$$

[integrating by part]

$$= \frac{1}{5}\left[tg(t) - \int g(t)dt\right]$$
$$= \frac{1}{5}\left[x^2 g\left(x^5\right) - 5\int g\left(x^5\right) x^4 dx\right] + C$$
$$= \frac{x^5}{5} g\left(x^5\right) - \int x^4 g\left(x^5\right) dx + C$$

Ans 14

Correct Option. A

Solution:

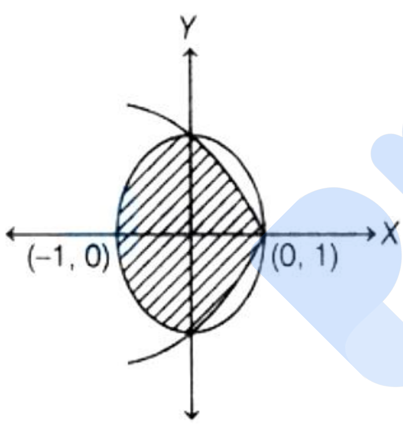
$$
\begin{aligned}
\because \text{ Required area } &= \int_1^3 y\,dx \\
&= \int_1^3 |x-2|\,dx \\
&= \int_1^2 -(x-2)\,dx + \int_2^3 (x-2)\,dx \\
&= \int_1^2 (2-x)\,dx + \int_2^3 (x-2)\,dx \\
&= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3 \\
&= \left[4 - 2 - \left(2 - \frac{1}{2}\right)\right] + \left[\frac{9}{2} - 6 - (2-4)\right] \\
&= 2 - \frac{3}{2} - \frac{3}{2} + 2 \\
&= 1 \text{ sq unit}
\end{aligned}
$$

Ans 15

Correct Option. A

Solution:

Given, $A = \left\{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\right\}$



$$
\begin{aligned}
\because \text{ Required area } &= \frac{1}{2}\pi r^2 + 2\int_0^1 (1-y^2)\,dy \\
&= \frac{1}{2}\pi(1)^2 + 2\left[y - \frac{y^3}{3}\right]_0^1 \\
&= \frac{\pi}{2} + \frac{4}{3}
\end{aligned}
$$

Ans 16

Correct Option. -1

Solution:

We can rewrite,

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$
$$= \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$
$$= (1+abc) \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$$
$$= (1+abc) = 0 \quad \left[\because \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0 \right]$$
$$\Rightarrow \quad abc = -1$$

Ans 17

Correct Option. B

Solution:

The given lines are

$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ ...(i)

and

$\frac{x-1}{2k} = \frac{y-3}{2} = \frac{z-5}{1}$ ...(ii)

Condition for two lines are coplanar

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are points on lines (i) and (ii) and ($l_1, m_1, n_4)$ and (l_2, m_2, n_2) are direction cosines of lines (i) and (ii), respectively.

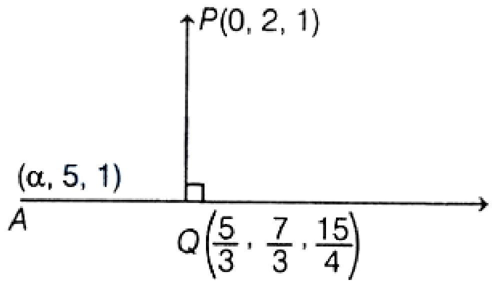
$$\therefore \begin{vmatrix} 2-1 & 3-3 & 4-5 \\ 1 & 1 & -k \\ 2k & 2 & 1 \end{vmatrix} = 0$$
$$\Rightarrow \begin{vmatrix} 1 & 0 & -1 \\ 1 & 1 & -k \\ 2k & 2 & 1 \end{vmatrix} = 0$$
$$\Rightarrow \quad 1(1+2k) - 0 - 1(2-2k) = 0$$
$$\Rightarrow \quad 1 + 2k - 2 + 2k = 0$$
$$\Rightarrow \quad 114k - 1 = 0 \Rightarrow k = \frac{1}{4}$$

Ans 18

Correct Option. 10.893

Solution:

It is given that point $Q\left(\frac{5}{3}, \frac{7}{3}, \frac{15}{4}\right)$ is foot of perpendicular of point $P(0,2,1)$ to a line passes through point $A(\alpha, 5, 1)$. So, $PQ \perp AQ$.



$\because$ Direction ratios of line segments PQ is $\left(\frac{5}{3}, \frac{1}{3}, \frac{11}{4}\right)$ and direction ratios of line AQ is $\left(\alpha - \frac{5}{3}, \frac{8}{3}, -\frac{11}{4}\right)$.

$$\therefore \quad \frac{5}{3}\left(\alpha - \frac{5}{3}\right) + \frac{1}{3} \times \frac{8}{3} + \frac{11}{4} \times \left(-\frac{11}{4}\right) = 0$$

$$\Rightarrow \quad 15\alpha - 25 + 8 + \frac{99(-11)}{16} = 0$$

$$\Rightarrow \quad 15\alpha - 17 = \frac{99 \times 11}{16}$$

$$\Rightarrow \quad 5\alpha = \frac{1089 - 817}{16}$$

$$\Rightarrow \quad 5\alpha = 54.466 \Rightarrow \alpha = 10.893$$

Ans 19

Correct Option. C

Solution:

Probability of guessing a correct answer, $p = \frac{1}{4}$ and probability of guessing a wrong answer $q = \frac{3}{4}$

So, the probability of guessing 4 or more correct answer

$$P(X \geq 4) = {}^5C_4\left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right) + {}^5C_5\left(\frac{1}{4}\right)^5$$

$$= 5 \cdot \frac{1}{4^4} \cdot \frac{3}{4} + \frac{1}{4^5}$$

$$= \frac{15}{4^5} + \frac{1}{4^5} = \frac{16}{4^5} = \frac{4^2}{4^5} = \frac{1}{4^3}$$

Ans 20

Correct Option. 33

Solution:

Total possibilities to make a 6-digit number using only 1 and 8

$\overline{2}\ \overline{2}\ \overline{2}\ \overline{2}\ \overline{2}\ \overline{2}$

Every position has 2 possibility either 1 or 8 $\therefore 2^6$

Number divisible by 21 :

1. for divisibility by 3

A (sum of digits must be divisible by 3)

(a) Number of 1 's = Number of 8

(b) Every digit is 1

(c) Every digit is 8.

2. for divisiblity by 7.

|2 (last digit) - remaining númber |=7k

$K \in Z$

(a) all are 1 s $\Rightarrow$ 1 no

(b) all are 8s $\Rightarrow$ 1 no

(c) 31 's and 38 s $\Rightarrow \frac{6!}{3!3!}$

Total number divisible by 21 = number divisible by 3 and 7

$$= \frac{6!}{3!3!} + 2 = \left(\frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2}\right) + 2 = 22$$

$$\therefore \text{ Probability } = \frac{22}{2^6} = p$$

$$96p = 96 \times \frac{22}{2^6} = 96 \times \frac{22}{2 \times 2 \times 2 \times 2 \times 2 \times 2} = 33$$