MET 2023 Answer Key PDF

Physics

Ans 1

Correct Option. D

Solution:

Maximum height, $H_1 = \frac{u^2 \sin^2\theta}{2g} \ldots$ (i)

and $H_2 = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} \ldots$ (ii)

From Eqs. (i) and (ii), we get

$$\frac{H_1}{H_2} = \frac{\sin^2\theta}{\sin^2(90^\circ - \theta)}, \frac{H_1}{H_2} = \frac{\tan^2\theta}{1}$$

$$\therefore \quad H_1 : H_2 = \tan^2\theta : 1$$

Ans 2

Correct Option. B

Solution:

The work done in small displacement from x to x + dx is

$$dW = Fdx = (2 + x)dx$$

Hence,

$$W = \int_1^2 dW = \int_1^2 (2 + x)dx = \int_1^2 2dx + \int_1^2 xdx$$

$$= \left[2x + \frac{x^2}{2}\right]_1^2 = 3.5 \text{ J}$$

Ans 3

Correct Answer. 375

Solution:

Here, mass of the disc, M = 5 kg

Radius of the disc, R = 50 cm = 1/2 m Linear velocity of the disc, v = 10 ms^{-1}

As, v = Rω

$\therefore 10 = 1/2\omega$ or $\omega = 10 \times 2 = 20 rads^{-1}$

Also, moment of inertia of disc about an axis through its centre,

$$I = \frac{1}{2}MR^2 = \frac{1}{2} \times 5 \times \left(\frac{1}{2}\right)^2 = \frac{5}{8}\ \mathrm{kg}^2 - \mathrm{m}$$

$$\therefore \text{KE of translation } = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times (10)^2 = 250\ \mathrm{J}$$

$$\text{Rotational kinetic energy } = \frac{1}{2}/\omega^2 = \frac{1}{2}\left(\frac{5}{8}\right)(20)^2 = 125\ \mathrm{J}$$

$$\begin{aligned}\text{Total kinetic energy } &= \text{ Translational } + \text{ Rotational} \\ &= (250 + 125) = 375\ \mathrm{J}\end{aligned}$$

Ans 4

Correct Option. A

Solution:

The acceleration due to gravity,

$g = \frac{GM}{R^2} \ldots$ (i)

At a height h above the surface of the earth, the acceleration due to gravity is

$g' = \frac{GM}{(R+h)^2} \ldots$ (ii)

From Eqs. (i) and (ii), we get

$$\frac{g}{g'} = \left(\frac{R+h}{R}\right)^2 = \left(1 + \frac{h}{R}\right)^2$$

$$\Rightarrow \quad \frac{g'}{g} = \left(1 + \frac{h}{R}\right)^{-2} = \left(1 - \frac{2h}{R}\right)^2$$

Here,

$g' = \frac{g}{2}$

$$\therefore \quad \frac{g/2}{g} = \left(1 - \frac{2h}{R}\right) \Rightarrow \frac{2h}{R} = \frac{1}{2} \text{ or } h = \frac{R}{4}$$

Ans 5

Correct Answer. 2

Solution:

Elastic potential energy stored in a loaded'wire,

$U=\frac{1}{2}\times(\text{ Stress }\times\text{ Strain }\times\text{ Volume })$

$\therefore$ Energy stored per unit volume,

$U=\frac{U}{\text{Volume}}=\frac{1}{2}\times\text{ Stress }\times\text{ Strain }=\frac{1}{2}\left(\frac{F}{A}\right)^2\times\frac{1}{Y}$

$\frac{U_A}{U_B}=\frac{\frac{1}{2Y}\cdot\frac{F^2}{A_A^2}}{\frac{1}{2Y}\cdot\frac{F^2}{A_B^2}}=\frac{A_B^2}{A_A^2},\frac{U_A}{U_B}=\frac{d_B^4}{d_A^4}$

Here, $\frac{U_A}{U_B}=\frac{1}{4},\frac{d_B^4}{d_A^4}=\frac{1}{4}$ or $\frac{d_B}{d_A}=\frac{1}{\sqrt{2}}$

$\frac{d_A}{d_B}=\sqrt{2}:1=\sqrt{k}:1$ (given)

$\therefore\quad k=2$

Ans 6

Correct Answer. 0.4

Solution:

Given, $h=7.5\text{ cm}=7.5\times10^{-2}\text{ m}$

$S=7.5\times10^{-2}\text{Nm}^{-1}$
$\theta=\sigma^{\circ},r=?$

As, $h=\frac{2S\cos\theta}{r\rho g}\Rightarrow 2r=\frac{4S\cos\theta}{h\rho g}$

$=\frac{4\times7.5\times10^{-2}\times\cos0^{\circ}}{7.5\times10^{-2}\times10^{3}\times10}=4\times10^{-4}=0.4\text{ mm}$

Ans 7

Correct Option. B

Solution:

This is an example of a periodic motion. It can be noted that each term represents a periodic function with different angular frequency. Since, period is the least interval of time after which a function repeats its value, sin ωt has a period

$T_0=\frac{2\pi}{\omega}$

cos $2\omega t$ has a period $\frac{\pi}{\omega}=\frac{T_0}{2}$ and sin $4\omega t$ has a period $\frac{2\pi}{4\omega}=\frac{T_0}{4}$

The period of the first term is a multiple of the periods of the last two terms. Therefore, the smallest interval of time after which the sum of the three terms repeats is T_0 and thus, the sum is a periodic function with a period $2\pi/\omega$.

Ans 8

Correct Option. D

Solution:

When charging battery is disconnected then charge remains constant, V becomes 1/K times and E becomes 1/K times.

So, new value of charge is Q_0.

New value of $V = \frac{V_0 \times 3}{9} = \frac{V_0}{3}$

New value of $E = \frac{E_0 \times 3}{9} = \frac{E_0}{3}$

Ans 9

Correct Option. D

Solution:

Resistance, $R_t = R_0(1 + \alpha t)$

$20 = R_0(1 + 20\alpha)$ and $60 = R_0(1 + 500\alpha)$

From this equation, we can find

$R_0 = 18.33\Omega$

$\alpha = 4.54 \times 10^{-3}$

$R_t = 25\Omega$

Again, $\quad R_t = R_0(1 + \alpha t)$

$25 = 18.33\left(1 + 4.54 \times 10^{-3} t\right)$

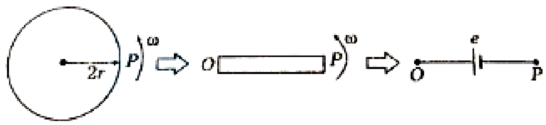
We find $t = 80^\circ \mathrm{C}$

Ans 10

Correct Answer. 1

Solution:

As we know, disc is equivalent to a rod of length r. Divide disc into rods, each rod can be replaced by a battery and these batteries are would be paralial in arrangement.



According to the question, radius of disc, $r = \frac{10}{2} = 5\ \mathrm{cm} = 0.05\ \mathrm{m}$

Angular velocity, $\omega = 20 \times 2\pi = 40\pi \mathrm{rad/s}$

Magnetic field, $B = 10^{-1}\ \mathrm{T}$

The emf of a disc is given as

$e = \frac{1}{2}B\omega r^2 = \frac{1}{2}(10^{-1})\ (40\pi)(0.05)^2$

$\Rightarrow \quad V_0 - V_A = 1.57 \times 10^{-2}\ \mathrm{V}$

$= \frac{\pi}{2} \times 10^{-2}\ \mathrm{V} = \frac{\pi}{x} \times 10^{-p}\ \mathrm{V}$ (given)

$\therefore \quad \frac{x}{p} = \frac{2}{2} = 1$

Ans 11

Correct Option. B

Solution:

Angle i at both focus will be 45^0. For TIR to take place,

$i > \theta_c$ or $\sin i > \sin \theta_c$

$\therefore \quad \frac{1}{\sqrt{2}} > \frac{1}{\mu}$ or $\mu > \sqrt{2}$

Ans 12

Correct Option. B

Solution:

We know that in Young's double slit experiment, fringe width $\beta = \frac{D\lambda}{d}, \beta \propto \lambda$

Since, $\lambda_{\text{blue}} < \lambda_{\text{orange}}$

Thus, $\beta_{\text{blue}} < \beta_{\text{orange}}$

$\therefore$ In Young's double slit experiment, if the source of light changes from orange to blue, then distance between the consecutive fringes will decrease.

Ans 13

Correct Option. D

Solution:

As, $\lambda = \frac{h}{\sqrt{2qVm}} \propto \frac{1}{\sqrt{qVm}}$

$\therefore \quad \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{q_p V_p m_p}{q_\alpha V_\alpha m_\alpha}} = \sqrt{\frac{1\times100\times1}{2\times800\times4}} = \frac{1}{8}$

$\therefore \quad \lambda_\alpha = \frac{\lambda_p}{8} = \frac{\lambda_0}{8}$

Ans 14

Correct Answer. 4.5

Solution:

Radius of circular path described by a charged particle in a magnetic field B is given by

$r = \sqrt{\frac{2mK}{qB}} \Rightarrow K = \frac{q^2B^2r^2}{2m} = \left(\frac{e}{m}\right)\frac{eB^2r^2}{2}$

where, $K =$ kinetic energy of a particle

$q =$ charge on a particle

$r =$ radius of circular path

$= \frac{1}{2} \times 1.7 \times 10^{11} \times 1.6 \times 10^{-19} \times \left(\frac{1}{\sqrt{17}} \times 10^{-5}\right)^2 \times (1)^2$

$= 8 \times 10^{-20}\ \mathrm{J} = 0.5\mathrm{eV}$

Using the relation,

$E = \phi + K_{\max} \Rightarrow \phi = E - K_{\max}$

where, $E = hv = \frac{hc}{\lambda} = \frac{12375}{\lambda(\text{ in } \mathring{A})}$

$= \left(\frac{12375}{2475}\right)\mathrm{eV} - 0.5\mathrm{eV} = 4.5\mathrm{eV}$

Ans 15

Correct Option. C

Solution:

Final energy of electron

$= -13.6 + 12.1 = -1.51\mathrm{eV}$

This energy corresponds to third level i.e., n = 3

Hence, number of spectral lines emitted

$$= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

Chemistry

Ans 1

Correct Option. C

Solution:

100 mL of $0.001 \mathrm{M NaCl}^{-2}$ contain Cl^{-1} ions

$$= \frac{0.001}{1000} \times 100 = 10^{-4} \text{ mol}$$
$$= 10^{-4} \times (6.022 \times 10^{23}) = 6.022 \times 10^{19} \mathrm{Cl}^{-} \text{ ions}$$

Ans 2

Correct Option. A

Solution:

Energy of photon depends upon frequency and not on intensity. As on increasing the intensity of light of a particular frequency v, the number of electrons ejected increases but not their kinetic energy.

Ans 3

Correct Option. B

Solution:

$\mathrm{H_2}$, $\mathrm{B_2}$, $\mathrm{O_2^{2-}}$ have bond order equal which is to 1 and $\mathrm{Be_2}$ and $\mathrm{N_2}$ have 0 and 3 bond order respectively.

Ans 4

Correct Option. C

Solution:

I. → Isothermal expansion

II. → Adiabatic expansion

III. → Isothermal compression

IV. → Adiabatic compression

Ans 5

Correct Answer. 3.37

Solution:

For a weak acid,

$$pH = \frac{1}{2}[pK_a - \log C]$$
$$= \frac{1}{3}[4.74 - \log 10^{-2}]$$
$$= \frac{1}{2} \times 6.74 = 3.37$$

Ans 6

Correct Option. C

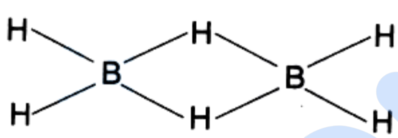
Solution:

As we move down the group, solubility increases. So, $Be(OH)_2$ is insoluble and $Ba(OH)_2$ is highly soluble in water.

Ans 7

Correct Answer. 6

Solution:



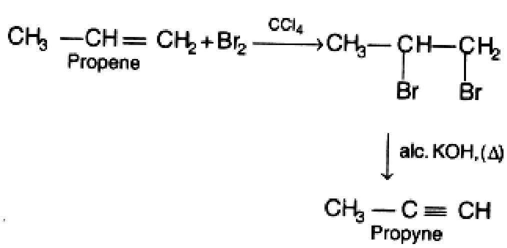
Number of $3c - 2e^{-1}$ and $2c - 2e^{-}$ bond present in diborane are 2 and 4, respectively.

Thus, total number of $3c - 2e^{-1}$ and $2c - 2e^{-}$ bonds are 6.

Ans 8

Correct Option. B

Solution:



Ans 9

Correct Answer. 200

Solution:

Solution with the same osmotic pressure are isotonic.

Let the molar mass of the substance be M.

$\therefore \quad \pi_1 = \pi_2$

$C_1RT = C_2RT$

$C_1 = C_2$

As density are also same

So, $\frac{5}{M} = \frac{1.5}{60}$

$\Rightarrow \quad M = \frac{5\times60}{1.5} = 200 \text{ g mol}^{-1}$

Ans 10

Correct Answer. 0.77

Solution:

$\Delta G_3 = \Delta G_1 + \Delta G_1$

$n_3FE_3^{\circ} = n_1FE_1^{\circ} + n_2FE_2^{\circ}$

$3 \times 0.036 = 2 \times 0.439 + 1 \times E_2^{\circ}$

$\Rightarrow \quad E_2^{\circ} = 0.108 - 0.878$

$= -0.77 \text{ V}$

Ans 11

Correct Answer. 3

Solution:

Cu^{2+}, Fe^{3+}, Ti^{3+} have unpaired electron which impart colour to the ions whereas Zn^{2+} has no unpaired, thus it does not impart any colour.

Ans 12

Correct Option. B

Solution:

$[Ni(CO)_4]$ – sp^3-hybridisation

$[Ni(CN)_4]^{2-}$ – dsp^2-hybridisation

$[Ni(NH_3)_6]^{2+}$ – sp^3d^2-hybridisation

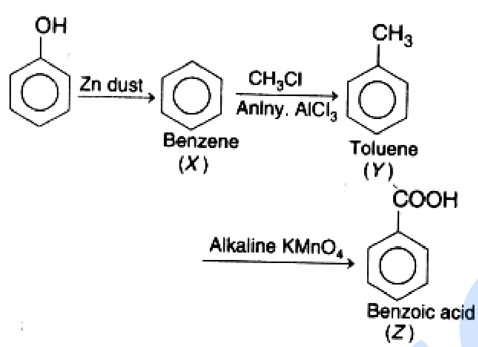
$[Fe(CN)_6]^{3-}$ – d^2sp^3-hybridisation

The correct answer is A-2, B-1, C-4, D-3.

Ans 13

Correct Option. C

Solution:

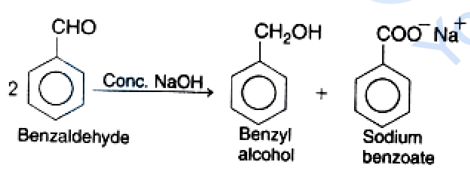


Ans 14

Correct Option. D

Solution:

In Cannizzaro reaction benzaldehyde acts as reactant not the product.



Ans 15

Correct Option. A

Solution:

Styrene exist as liquid at room temperature.

Mathematics

Ans 1

Correct Option. D

Solution:

Here, we have

Radius of circle = 3 units we find the maximum and minimum value of |z + 1|

for z lying in the interier or boundary of the circle

|z| = 3

For $\mathrm{Max}\,|z+1|$, z must lies on the bondary of the circle, then $|z|=3$ and $z=3e^{\theta}$, where θ is the argument of z.

Now, $|z+1|=\left|3e^{i\theta}+1\right|$

$$=|(3\cos\theta+1)+(3i\sin\theta)|$$
$$=\sqrt{(3\cos\theta+1)^2+(3\sin\theta)^2}$$

$$=\sqrt{9+1+6\cos\theta}$$
$$=\sqrt{10+6\cos\theta}$$

$\sin\theta, \cos\theta$ ranges from -1 to 1 , the maximum value of $|z+1|$ occur, when $\cos\theta=1$. Therefore, $\mathrm{Max}\,|z+1|=\sqrt{10+6}=\sqrt{16}=4$

For minimum value The minimum value of $z+1$ occures, when z is at the centre of the circle $|z|=3$. In this case $z=0$.

On putting $z=0$ into the expression, we have $\mathrm{Min}(|z+1|)=|0+1|=1$

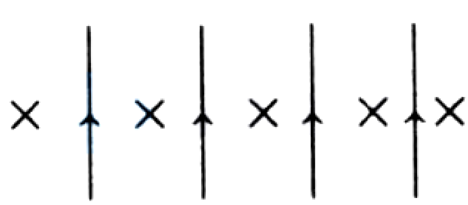
Hence, maximum value of $|z+1|=4$ minimum value of $|z+1|=1$

Ans 2

Correct Answer. 120

Solution:

Given, word is LETTER, having vowels E, E and consonants $\mathbf{L}, \mathbf{T}, \mathbf{T}$ and R . Now, the number of ways to arrange the consonants are $\frac{4!}{2!}=12$ Now, we have five place to put vowels E, E.



So, number of ways to arrange vowels is ${}^5C_2=10$ So, number of required words $=12\times10=120$

Ans 3

Correct Option. C

Solution:

We have, a^2, b^2 and c^2 are in AP adding $ab+bc+ca$ to each of these terms, we get $a^2+ab+bc+ca, b^2+ab+bc+ca$ and $c^2+ab+bc+ca$ are in AP.

$\Rightarrow (a+b)(a+c), (b+c)(b+a)$ and $(c+a)$

$(c+b)$ are in AP.

Dividing each term by $(b+c)(c+a)(a+b)$, we get

$\frac{1}{b+c}, \frac{1}{c+a}$ and $\frac{1}{a+b}$ are in AP

$\Rightarrow b+c, c+a$ and $a+b$ are in HP.

Ans 4

Correct Option. B

Solution:

In the expansion of $\left(1+ax+bx^2\right)(1-2x)^{18}$ coefficient of x^3 in $\left(1+ax+bx^2\right)(1-2x)^{18}$

$=$ Coefficient of x^3 in $(1-2x)^{18}+$ coefficient of x^2 in $a(1-2x)^{18}+$ coefficient of x in $b(1-2x)^{18}$.

$= -{}^{18}C_3 \cdot 2^3 + a^{18}C_2 \cdot 2^2 - b^{18}C_4 \cdot 2$

$\because \quad -{}^{18}C_3 \cdot 2^3 + a^{18}C_2 \cdot 2^2 - b^{18}C_4 \cdot 2 = 0$

$\Rightarrow \frac{18\times17\times16}{3\times2} \cdot 8 + a \cdot \frac{18\times17}{2} 2^2 - b \cdot 18 \cdot 2 = 0$

$\Rightarrow 17a - b = \frac{34\times16}{3} \ldots$ (i)

Similarly, coefficient of x^4

${}^{18}C_4 \cdot 2^4 - a \cdot {}^{18}C_3 2^3 + b \cdot {}^{18}C_2 \cdot 2^2 = 0$

$\therefore \quad 32a - 32b = 240 \ldots$ (ii)

From Eqs. (i) and (ii), we get

$a = 16$ and $b = \frac{272}{3}$

Ans 5

Correct Answer. 01

Solution:

$\because (9)^{50} = (10-1)^{50}$

$= [{}^{50}C_0(10)^{50}(-1)^0 - {}^{50}C_1(10)^{49}(1) + \ldots .] - {}^{50}C_{49}(10)^1 + {}^{50}C_{50}(10)^0$

$\because$ All the terms in the bracket are multiples of 10^2.

So, we can write as 100λ

$= 100\lambda - 50 \times 10 + 1 = 100\lambda - 500 + 1$

so, last 2 digits will be $= 01$.

Ans 6

Correct Option. C

Solution:

The equation of family of circles which passess through the origin and whose centrer lies on Y-axis is

$(x-0)^2 + (y-k)^2 = k^2$

$\Rightarrow x^2 + y^2 + k^2 - 2yk = k^2$
$\Rightarrow x^2 + y^2 - 2yk = 0$

where, k is parameter.

On differentiating w.r.t . x, we get

$2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx}k = 0$

$\Rightarrow \quad x + (y-k)\frac{dy}{dx} = 0$
$\Rightarrow \quad x + \left\{y - \frac{x^2+y^2}{2y}\right\}\frac{dy}{dx} = 0$

$\Rightarrow \quad x + \frac{1}{2y}\left(y^2 - x^2\right)\frac{dy}{dx} = 0$
$\Rightarrow \quad 2xy = \left(x^2 - y^2\right)\frac{dy}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{2xy}{x^2-y^2},$

which is required equation.

Ans 7

Correct Option. A

Solution:

Let $I = \lim_{x \to 3} \int_6^{f(x)} \frac{2t}{x-2} dt \left[\frac{0}{0}\text{ form, as } f(3) = 6\right]$

On applying the L' Hospital rule, we get

$I = \lim_{x \to 3} \int_6^{f(x)} \frac{2f(x)f'(x)}{1} \; (\because f(3) = 6)$

So, $I = 3f(3) \cdot f'(3) = 18f'(3)$

$\therefore \quad \lim_{x \to 3} \int_6^{f(x)} \frac{2t dt}{x-2} = 18f'(3)$

Ans 8

Correct Answer. 16

Solution:

$y = \lim_{x \to 0} \left(\frac{8^x + 2^x + 4^x}{3}\right)^{2/x}$

$$\Rightarrow \log y = \lim_{x \to 0} \frac{2}{x} \log\left(\frac{8^x + 2^x + 4^x}{3}\right)$$
$$= 2 \lim_{x \to 0} \frac{1}{x} [\log(8^x + 2^x + 4^x) - \log 3]$$

$= 2 \lim_{x \to 0} \frac{[\log(8^x + 2^x + 4^x) - \log 3]}{x} \quad \left[\frac{0}{0}\text{ form}\right]$

On applying L' Hospital rule, we get

$$= 2 \lim_{x \to 0} \frac{[8^x \log 8 + 2^x \log 2 + 4^x \log 4]}{8^x + 2^x + 4^x}$$
$$= 2\left[\frac{\log 8 + \log 2 + \log 4}{3}\right]$$

$= \frac{2}{3}[3 \log 2 + \log 2 + 2 \log 2] \quad [\log(m)^n = n \log m]$

$$= \frac{2}{3} \times 6 \log 2$$

$\log y = 4 \log 2 = \log 16 \Rightarrow y = 16$

Ans 9

Correct Option. D

Solution:

Given, $\sum_{i=1}^{y}(x_i - 5) = 9$

$\sum x_i - 45 = 9 \Rightarrow \sum x_i = 54$

Also, $\sum_{i=1}^{9}(x_i - 5)^2 = 45$

$$\Rightarrow \sum x_i^2 - 10 \times 54 + 25 \times 9 = 45$$
$$\sum x_i^2 = 360$$

$\therefore$ Standard deviation $(\sigma) = \sqrt{\frac{360}{9} - \left(\frac{54}{9}\right)^2} = 2.$

Ans 10

Correct Option. A

Solution:

$A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{3, 6, 7, 9\}$.

Total subset of $A = 2^7 = 128$ $C \cap B = \phi$, when set C contains the elements 1,2 , 4,5 .

$S = \{C \subseteq A; C \cap B \neq \phi\}$

Total $-(C \cap B = \phi) = 128 - 2^4 = 112$

Ans 11

Correct Option. B

Solution:

$B^T A^T = (AB)^T$

$$= \left(\begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 8-7 & 2-2 \\ -28+28 & -7+8 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Ans 12

Correct Option. B

Solution:

Given that

$f(x)=\frac{1}{\sqrt{1+x^2}}$

$\therefore \quad f(y)=\frac{1}{\sqrt{1+y^2}}$

and $f(x, y)=\frac{1}{\sqrt{1+x^2 y^2}}$

Now, $f(x, y)=f(x) \cdot f(y)$

$\frac{1}{\sqrt{1+x^2 y^2}}=\frac{1}{\sqrt{1+x^2}} \times \frac{1}{\sqrt{1+y^2}}$

$\frac{1}{\sqrt{1+x^2 y^2}}=\frac{1}{\sqrt{(1+x^2)(1+y^2)}}$

On squaring both sides, we get

$\frac{1}{1+x^2 y^2}=\frac{1}{(1+x^2)(1+y^2)}$

$\frac{1}{1+x^2 y^2} \geq \frac{1}{(1+x^2)(1+y^2)}$

$[\because 1+x^2 y^2 \leq 1+x^2+y^2+x^2 y^2]$

Hence, $f(x, y) \geq f(x) \cdot f(y)$

Ans 13

Correct Option. A

Solution:

$|\sin x|>2 \sin ^2 x$

$\Rightarrow|\sin x|(2|\sin x|-1)<0 \Rightarrow 0<|\sin x|<\frac{1}{2}$

$\Rightarrow x \in\left(0, \frac{\pi}{6}\right) \cup\left(\frac{5 \pi}{6}, \pi\right) \cup\left(\pi, \frac{7 \pi}{6}\right) \cup\left(\frac{11 \pi}{6}, 2 \pi\right)$

Ans 14

Correct Option. B

Solution:

Here, $\int f(x)dx = g(x)$

Now, $\int \cos x f(\sin x)dx$

Let, $\sin x = k \Rightarrow \cos x dx = dk$

$\therefore \int f(k)dk = g(k) + C = g(\sin x) + C$

Ans 15

Correct Option. B

Solution:

Required area $= \left|\int_1^6 |5 - x| dx\right|$

$= \left|\int_1^5 -(5 - x)dx + \int_5^6 (5 - x)dx\right|$

$= \left|\left[-5x + \frac{x^2}{2}\right]_1^5 + \left[5x - \frac{x^2}{2}\right]_5^6\right|$

$= \left|\left[-25 + \frac{25}{2} + 5 - \frac{1}{2}\right] + \left[30 - 18 - 25 + \frac{25}{2}\right]\right|$

$= \left[\frac{-50+25+10-1}{2}\right] + \left[\frac{-26+25}{2}\right]$

$= \left|-8 - \frac{1}{2}\right| = \frac{17}{2}$ sq units

Ans 16

Correct Option. C

Solution:

The given curves, $y = \tan x$ and $y = \cot x$ meet at $x = \frac{\pi}{4}$.

$\therefore$ Required area $= \int_0^{\pi/4} \tan x dx + \int_{\pi/4}^{\pi/2} \cot x dx$

$= (\ln \sec x)_0^{\pi/4} + (\ln \sin x)_{\pi/4}^{\pi/2}$

$= \ln \sqrt{2} - \ln \frac{1}{\sqrt{2}} = 2 \ln \sqrt{2} = \ln 2$

Ans 17

Correct Answer. -0.33

Solution:

Given, vectors are

$$\mathbf{p} = (a+1)\hat{\mathbf{i}} + a\hat{\mathbf{j}} + a\hat{\mathbf{k}}$$
$$\mathbf{q} = a\hat{\mathbf{i}} + (a+1)\hat{\mathbf{j}} + a\hat{\mathbf{k}}$$

and $\mathbf{r} = a\hat{\mathbf{i}} + a\hat{\mathbf{j}} + (a+1)\hat{\mathbf{k}}$

$[pqr] = 0$

$$\Rightarrow \begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$
$$\Rightarrow (a+1)\left[(a+1)^2 - a^2\right] - a\left[a(a+1) - a^2\right] + a\left[a^2 - a(a+1)\right] = 0$$

$$\Rightarrow (a+1)[2a+1] - 2a[a] = 0$$
$$\Rightarrow 2a^2 + 3a + 1 - 2a^2 = 0$$
$$\Rightarrow a = -\frac{1}{3} = -0.33$$

Ans 18

Correct Option. C

Solution:

Given, two lines

$L_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

and $L_2 : \frac{x-3}{1} = \frac{y-k}{2} = \frac{z-0}{1}$

Let $L_1 : \frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = p$

and $L_2 : \frac{x-3}{1} = \frac{y-k}{2} = \frac{z-0}{1} = q$

Any point P on line L_1 is of type

$P(2p+1, 3p-1$ and $4p+1)$ and any point Q on the line L_2 is of type $Q(q+3, 2q+k$ and $q)$

Since, L_1 and L_2 are intersecting each other, hence, both point P and Q should coincide at the point of intersection. Corresponding co-ordinates of P and Q should be same.

$2p+1=q+3, 3p-1=2q+k$ and $4p+1=q$

On solving, $2p+1=q+3$ and $4p+1=q$, we get

$p=-\frac{3}{2}$ and $q=-5$

On substituting the values of p and q in the third equation $3p-1=2q+k$, we get

$3\left(-\frac{3}{2}\right)-1=2(-5)+k$

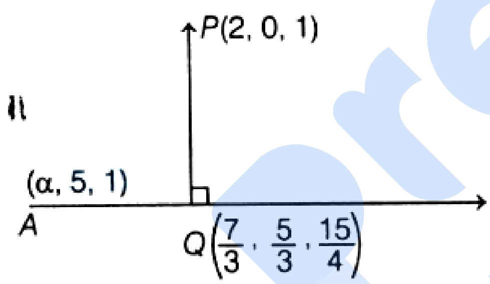
$\therefore \quad k=\frac{9}{2}$

Ans 19

Correct Answer. 4.372

Solution:

It is given that $Q\left(\frac{7}{3}, \frac{5}{3}, \frac{15}{4}\right)$ is foot of perpendicular of point $P(2,0,1)$ to a line passes through point $A(\alpha, 5, 1)$. So $PQ \perp AQ$



$\because$ Direction ratios of line segments PQ is $\left(\frac{7}{3}, \frac{5}{3}, \frac{15}{4}\right)$ and direction ratios of line AQ is

$\left(\alpha-\frac{7}{3}, \frac{10}{3}, -\frac{11}{4}\right)$

Since, PQ and AQ is perpendicular to each other,

so $PQ \cdot AQ=0$

$\frac{7}{3}\left(\alpha-\frac{7}{3}\right)+\frac{5}{3} \times \frac{10}{3}+\frac{15}{4} \times\left(-\frac{11}{4}\right)=0$

$\Rightarrow \quad \frac{7(3\alpha-7)}{9}+\frac{50}{9}-\frac{165}{16}=0$

$$\Rightarrow \quad \frac{336\alpha-784+800-1485}{144}=0$$

$$\Rightarrow \quad 336\alpha=1469 \Rightarrow \alpha=\frac{1469}{336}=4.372$$

Ans 20

Correct Option. C

Solution:

The coin is tossed n times.

Let x denotes the number of heads in n trials and it is given that p(x = 5) = p(x = 6)

$$\Rightarrow {}^{n}C_5\left(\frac{1}{2}\right)^n = {}^{n}C_6\left(\frac{1}{2}\right)^n \Rightarrow n = 11$$

$$\therefore p(x = 3) = {}^{11}C_3\left(\frac{1}{2}\right)^{11}$$