MET 2024 Answer Key PDF

Physics

Ans 1

Correct Option. A

Solution:

The SI unit in which G is measured $= \mathrm{N-m^2/kg^2}$

$\therefore$ Its dimensions will be $\frac{[\mathrm{MLT^{-2}}][\mathrm{L^2}]}{[\mathrm{M^2}]} = [\mathrm{M^{-1}L^3T^{-2}}]$

$\because$ Impulse $=$ Force $\times$ Time $= [\mathrm{MLT^{-2}}][\mathrm{T}] = [\mathrm{MLT^{-1}}]$

$\because$ Dimensions for energy $= [\mathrm{ML^2T^{-2}}]$

Now put the dimensions of G, I, M and E in $\frac{GIM^2}{E^2}$

$$= \frac{[\mathrm{M^{-1}L^3T^{-2}}][\mathrm{MLT^{-1}}][\mathrm{M^2}]}{[\mathrm{ML^2T^{-2}}]^2}$$

$$= \frac{[\mathrm{M^{-1}L^3T^{-2}}][MLT^{-1}][\mathrm{M^2}]}{[\mathrm{M^2L^4T^{-4}}]} = \frac{[\mathrm{M^2L^4T^{-3}}]}{[\mathrm{M^2L^4T^{-4}}]}$$

$$= [\mathrm{T^{-1}}] = [T]$$

The equation $\frac{GIM^2}{E^2}$ has dimension of time.

Ans 2

Correct Option. B

Solution:

The greatest distance between the particles will be at the moment when velocity of both particles gets equal

$\therefore$ Using v = u + at, for acceleratod particle

$$u = 0 + (F)(t)$$
$$t = \frac{u}{(F)}$$

Distance travelled by accelerated in time t,

$$D_1 = O + \frac{1}{2}(F)\left(\frac{u}{F}\right)^2$$
$$D_1 = \frac{u^2}{2F}$$

Distance travelled by constantly moving particle,

$$D_2 = (u)\left(\frac{u}{F}\right) + \frac{1}{2}(0)\left(\frac{u}{F}\right)^2$$
$$D_2 = \frac{u^2}{F}$$

Greatest distance between two particles

$$D = \frac{u^2}{f} - \frac{u^2}{2f}$$
$$D = \frac{u^2}{2f}$$

Ans 3

Correct Option. B

Solution:

Given, $r = 300\ \mathrm{m}, \mu = 0.3$,

$g = 10\ \mathrm{m/s^2}$

If v is the velocity of the vehicle while turning on the road, then

$$\frac{mv^2}{r} \leq \mu mg$$

$$\Rightarrow v \leq \sqrt{\mu r g}$$

$$\Rightarrow v_{\max} = \sqrt{\mu r g}$$

$$= \sqrt{0.3 \times 300 \times 10} = 30\ \mathrm{m/s}$$

Ans 4

Correct Option. 935

Solution:

Using conservation of mechanical energy.

Decrease in kinetic energy = Increase in gravitational potential energy

$$\Rightarrow \frac{1}{2}mv^2 = \Delta U$$
$$\Rightarrow \frac{1}{2}mv^2 = \frac{mgh}{1+\frac{h}{R}}$$
$$\Rightarrow h = \frac{v^2}{2g-\frac{v^2}{R}}$$

Substituting the values, we get

$$h = \frac{\left(4.0 \times 10^3\right)^2}{2 \times 9.8 - \frac{\left(4\times10^3\right)^2}{6.4\times10^6}}$$
$$= 9.35 \times 10^5 \text{ m} = 935 \text{ km}$$

Ans 5

Correct Option. C

Solution:

We know that the slope of the line in a stress-strain curve represents the young's modulus for a wire. We also know that the slope of a line can be calculated as the tangent of the angle the line makes with the positive x -axis of the graph.

So, for wire A, the stress-strain line of the material is at an angle of 60^0 from the positive x-axis. So the slope of the line (m_A) will be

$$m_A = \tan 60^\circ$$
$$\Rightarrow m_A = \sqrt{3}$$

Hence the young's modulus of wire A will also be $\sqrt{3}$.

Similarly, for wire B , the stress-strain line of the material is at an angle of 30° from the positive x axis. So, the slope of the line (m_B) will be

$$m_B = \tan 30^\circ$$
$$\Rightarrow m_B = \frac{1}{\sqrt{3}}$$

Hence the young's modulus of wire B will also be $1/\sqrt{3}$.

Then taking the ratio of the young's modulus for wire A and B , we get

$$\frac{Y_A}{Y_B} = \frac{\sqrt{3}}{1/\sqrt{3}}$$
$$\therefore \frac{Y_A}{Y_B} = 3$$

Ans 6

Correct Option. A

Solution:

General equation of oscillation are

$$y_1 = A\sin\left(\omega_1 t + \frac{\pi}{2}\right)$$
$$y_2 = A\sin\left(\omega_2 t + \frac{\pi}{2}\right)$$

Now, $\omega_1 t + \frac{\pi}{2} = \omega_2 t - \frac{\pi}{2}$

or

$$t = \frac{\pi}{\omega_2 - \omega_1}$$
$$= \frac{\pi}{(2\pi/T_2) - (2\pi/T_1)}$$
$$= \frac{T_1 T_2}{2(T_1 - T_2)}$$
$$= \frac{3 \times 7}{2(7-3)} = \frac{21}{8} \text{ s}$$

Ans 7

Correct Option. A

Solution:

Fundamental frequency,

$$n = \frac{v}{2l} = \frac{\sqrt{T/\mu}}{2l} = \frac{\sqrt{T/\rho S}}{2l} = \frac{\sqrt{T/\rho\pi r^2}}{2l}$$
$$\therefore n \propto \frac{\sqrt{T}}{2l}$$

Given that tension, diameter and length all are made 3 times.

$\therefore n$ will become $\frac{1}{3\sqrt{3}}$ times.

Ans 8

Correct Option. 1.8

Solution:

From figure, electric dipole moment,

$$p = 2ql = q(2l)$$
$$= 2 \times 10^{-6} \times (1 \times 10^{-9})$$
$$\Rightarrow p = 2 \times 10^{-15} \text{ cm}$$

Electric potential at point p due to dipole

$$V_p = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2}$$
$$V_p = 9 \times 10^9 \times \frac{2 \times 10^{-15} \times \cos 60^\circ}{(1 \times 10^{-2})^2}$$
$$= \frac{18 \times 10^{-6} \times \frac{1}{2}}{10^{-4}} = 9 \times 10^{-2} = 0.09 \text{ V}$$

Similarly, electric potential at point Q due to electric dipole

$$V_Q = \frac{9 \times 10^9 \times (2 \times 10^{-15}) \times \cos 120^\circ}{(1 \times 10^{-2})^2}$$
$$= -0.09 \text{ V}$$

$\therefore$ Potential difference between point Q and P.

$$V_{QP} = V_P - V_Q = 0.09 - (-0.09) = 0.18 \text{ V}$$

$\therefore$ Work done to move the charge of 1μC from point Q to P.

$$W = V_{QP} \cdot q = 0.18 \times 1 \times 10^{-6}$$
$$= 1.8 \times 10^{-7} \text{ J}$$

Ans 9

Correct Option. D

Solution:

1. Parallel Combination:

The $2\mu F$ and $6\mu F$ capacitors are in parallel, so their equivalent capacitance:

$$C_{eq} = 2 + 6 = 8\mu F$$

2. Series Combination:

The $4\mu F'$ capacitor is in series with the $8\mu F$ equivalent capacitor. The total capacitance is:

$$\frac{1}{C_{total}} = \frac{1}{4} + \frac{1}{8} = \frac{3}{8} \Longrightarrow C_{total} = \frac{8}{3}\mu F$$

3. Charge Calculation:

The total charge in the circuit is:

$Q = C_{total} \cdot V = \frac{8}{3} \cdot 12 = 32\mu C$

4. Voltage Across $4\mu F$ Capacitor:

Since the charge is the same in series, the voltage across the $4\mu F$ capacitor is:

$V = \frac{Q}{C} = \frac{32}{4} = 8 \text{ V}$

Final answer: 8V.

Ans 10

Correct Option. 400

Solution:

Heat produced in a resistance R in time t is given by

$H = \frac{V^2}{R} t [\text{ where }, V = \text{ source voltage }]$

For first coil,

$H_1 = \frac{V^2}{R_1} t_1 = \frac{V^2}{R_1} \times 10 \times 60 \, [\because t_1 = 10 \times 60 \text{ s}]$

For second coil, $H_2 = \frac{V^2}{R_2} t_2 \, [\because t_2 = 20 \times 60 \text{ s}]$

$= \frac{V^2}{R_2} \times 20 \times 60$

Given that, $H_1 = H_2$

$$\Rightarrow \quad \frac{V^2}{R_1} \times 10 \times 60 = \frac{V^2}{R_2} \times 20 \times 60$$
$$\Rightarrow \quad R_2 = 2R_1 \ldots (i)$$

Let t be the time taken by electric kettle to boil water when both coil are used in parallel, then

$$H_p = \left(\frac{V^2}{R_1} + \frac{V^2}{R_2}\right) \times t = V^2 t \left(\frac{1}{R_1} + \frac{1}{2R_1}\right) [\text{ from Eq. (i) }]$$
$$H_p = \frac{3V^2 t}{2R_1}$$

But, $H_P = H_1$

$\therefore \frac{3V^2 t}{2R_1} = \frac{V^2}{R_1} \times 10 \times 60$

$\Rightarrow t = 400 \text{ s}$

Ans 11

Correct Option. 550

Solution:

When DC is applied across a solenoid, then resistance, i.e. $R=\frac{V}{I}$

$R=\frac{100}{1}=100\Omega$

When AC is applied, then

$Z=\frac{100}{0.5}=200\Omega$

Net impedance,

$$Z=\sqrt{R^2+X_L^2}$$
$$200=\sqrt{(100)^2+X_L^2}$$
$$\Rightarrow(200)^2=(100)^2+X_L^2$$
$$\Rightarrow 30000=X_L^2 \quad \Rightarrow X_L=173.205\Omega$$

Now, $X_L=\omega L$

$$\Rightarrow 173.205=2\pi f L \Rightarrow \frac{173.205}{2\pi\times f}=L$$
$$\Rightarrow \frac{173.205}{2\times 3.14\times 50}=L\Rightarrow 0.55\Omega=L$$

$L=0.55\mathrm{H}=550\mathrm{mH}$

Ans 12

Correct Option. C

Solution:

If R be the magnitude of radius of curvature of equiconvex lens, then its focal length $f$is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R}$$
$$\Rightarrow \frac{1}{f} = \frac{2(\mu - 1)}{R} \ldots (i)$$

When equiconvex lens is cut into two halves along XOX', then

$R_1 = R, R_2 = -R$

$\therefore$ Focal length f' is given as

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R} = \frac{1}{f} \quad [\text{ from (i) }]$$
$$\Rightarrow f' = f$$

Ans 13

Correct Option. C

Solution:

According to Einstein's photoelectric equation,

$$hf = W_0 + eV_0 \ldots (i)$$

If frequency is doubled, then

$$h(2f) = W_0 + eV_0'$$
$$2hf = W_0 + eV_0' \ldots (ii)$$

Substituting hf from Eq. (i) in Eq. (ii), we have,

or $V_0' = 2V_0 + \frac{W_0}{e}$

or $V_0' > 2V_0$

Ans 14

Correct Option. A

Solution:

$$E = \frac{-13.6}{n^2} \mathrm{eV}$$

$$E_4 - E_3 = 13.6\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right]$$

$$= 13.6\left[\frac{7}{144}\right] \mathrm{eV} = 0.66 \mathrm{eV}$$

$$\lambda = \frac{1}{R\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right]} = \frac{144}{1.09 \times 10^7 \times 7} \mathrm{m}$$

$$= 1.88 \times 10^{-6} \mathrm{m}$$

Ans 15

Correct Option. 9

Solution:

Diode will be in forward bias when V_i is greater than 10 V and it conducts till V_i is 12 V .

Total circuit resistance

$= 200 + 20 = 220\Omega$

When V_i is less than 10 V , the diode will not conduct due to its reverse biasing. Thus, initial current, $i_1 = 0$.

When $V_i = 12 \mathrm{~V}$, then diode will be in forward bias with effective voltage, $V' = 12 - 10 = 2 \mathrm{~V}$

$\therefore$ Maximum current, $I_2 = \frac{V'}{R} = \frac{2}{220} = 0.009 \mathrm{~A} = 9 \mathrm{~mA}$

$\therefore$ Change in current, $\Delta I = I_2 - I_1 = 9 \mathrm{~mA}$

Chemistry

Ans 1

Correct Option. 109700

Solution:

Given, $R = 109700\ \text{cm}^{-1}$

Shortest wavelength mean transition is from infinity to $n = 1$.

Using the formula,

$$\bar{v} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$n_t = \infty, n_i = n = 1$$

$$\bar{v} = 109700\left[\frac{1}{1} - \frac{1}{\infty}\right]$$

$$\bar{v} = 109700\ \text{cm}^{-1}$$

Ans 2

Correct Option. C

Solution:

As electronegativity increases across the period, therefore, electronegativity of nitrogen (N) is more than carbon (C) and electronegativity of P is more than Si .

Also electronegativity decreases down the group, therefore electronegativity of carbon is more than Si and electronegativity of N is more than P. Hence, the correct increasing order will be Si, P, C, N.

Ans 3

Correct Option. A

Solution:

The correct match is A-IV, B-III, C-I, D-II.

Hybridisation involved in the given complexes are

$$\mathrm{CuCl_5^{3-}, Cu(II)} = sp^3d(4s, 4p, 4d)$$
$$\mathrm{MnCl_5^{3-}, Mn(II)} = dsp^3(3d, 4s, 4p)$$
$$\mathrm{XeOF, Xe(VI)} = sp^3d^2(5s, 5p, 5d)$$
$$\mathrm{Fe(CO)_5, Fe(0)} = d^2sp^3$$

Ans 4

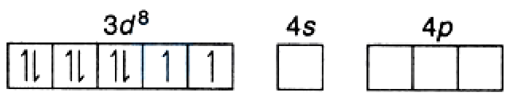
Correct Option. 3

Solution:

$\mathrm{Ni} : 3d^8 4s^2$

for the compound $[\mathrm{NiCl_4}]^{2-}$, Ni is +2 oxidation state. Hence, the electronic configuration of Ni^{2+} is given by $\mathrm{Ni}^{2+} : 3d^8$

Since, Cl^{-1} is a weak field ligand, so it will not cause pairing of electrons. So, for Ni^{2+} the configuration is



Hence, $n = 2$

Spin only magnetic moment is given by

$$\mu_s = \sqrt{n(n+2)} = \sqrt{2(2+2)}\mathrm{BM} = \sqrt{8} = 2.82\mathrm{BM}$$
$$\mu_s \simeq 3\ \mathrm{BM}$$

Ans 5

Correct Option. A

Solution:

Decrease in the value of Gibbs energy (−ΔG) gives the maximum useful work or net work done by the system for the given change whereas decrease in Helmholtz free energy or work function (−ΔA) gives the maximum work done for the given change.

Ans 6

Correct Option. 5

Solution:

Given reaction,

$$A(g) + 2B(g) \rightleftharpoons 3C(g) + D(g)$$
$$\Delta n_g = 4 - 3 = 1$$

Formula for K_p,

$$K_p = K_c(RT)^{\Delta n}$$
$$0.05 = K_c(R \times 1000)^1$$
$$K_c = \frac{5 \times 10^{-5}}{R}$$

Ans 7

Correct Option. 127

Solution:

Elevation in boiling point of water required (ΔT_b).

$= 100 - 99.63 = 0.37°\mathrm{C}$

Mass of solvent (water), $W_1 = 500\ \mathrm{g}$

Molar mass of solvent, $M_1 = 18\ \mathrm{g/mol}$

Molar mass of solute (sucrose)

$(\mathrm{C_{12}H_{22}O_{11}}) = 342\ \mathrm{g/mol}$

$$M_2 = \frac{1000 \times K_b \times w_2}{w_1 \Delta T_b}$$

or $w_2 = \frac{M_2 \times W_1 \times \Delta T_b}{1000 \times K_b} = \frac{342 \times 500 \times 0.37}{1000 \times 0.5}$

$w_2 = 126.54 \approx 127$

Ans 8

Correct Option. B

Solution:

$\mathrm{Cu}(s) + 2\mathrm{Ag}^+(aq) \longrightarrow \mathrm{Cu}^{2+}(aq) + 2\mathrm{Ag}(s)$

$n = 2, E^\circ_{\text{cell}} = 0.46\ \mathrm{V}$

Now using the equation for E°_{cell}

$$E^\circ_{\text{cell}} = \frac{0.0591}{n} \log K_c$$
$$0.46 = \frac{0.0591}{n} \log K_c$$
$$K_c = \text{Antilog } 15.5668$$
$$K_c = 3.92 \times 10^{15}$$

Ans 9

Correct Option. 2

Solution:

$$t_{99\%}=\frac{2.303}{k}\log\frac{(a)}{a-0.99a}$$
$$=\frac{2.303}{k}\log 10^2 \Rightarrow 2\times\frac{2.303}{k}\ldots(i)$$
$$t_{90\%}=\frac{2.303}{k}\log\frac{a}{a-0.90a}$$
$$=\frac{2.303}{k}\log 10=\frac{2.303}{k}\ldots(ii)$$

Take the ratio of (i) and (ii)

$$\frac{t_{99\%}}{t_{90\%}}=2$$
$$t_{99\%}=2\times t_{90\%}$$

Hence, 2 times is the final answer.

Ans 10

Correct Option. A

Solution:

K_2MnF_6 react with SbF_5 to form F_2. In this reaction stronger Lewis acid SbF_5 displace the weaker one (MnF_4) from its salt. MnF_4 being unstable, readily decompose to give MnF_3 and fluorine.

The complete reaction is as follows, $K_2MnF_6+2SbF_5\longrightarrow 2KSbF_6+MnF_4$ (unstable)

$$2MnF_4\longrightarrow 2MnF_3+F_2$$

Overall reaction,

$$2\ K_2MnF_6+4SbF_5\longrightarrow 4KSbF_6+2MnF_3+F_2$$

Ans 11

Correct Option. C

Solution:

Gadolinium does not absorb light in the visible region of spectrum, hence it is colourless inspite of the presence of seven unpaired electrons.

Ans 12

Correct Option. C

Solution:

Oxidation state of Cr ,

$In\ [\mathrm{Cr(H_2}O)_6]\ \mathrm{Cl_3}$
$x + 0 \times 6 - 1 \times 3 = 0$
$x = 3 \ldots (i)$

$\ln[\mathrm{Cr(C_6H_6)_2}]$

$x + 0 \times 2 = 0$
$x = 0 .. (ii)$

$\mathrm{In}\ \mathrm{K_2}\ [\mathrm{Cr(CN)_2(O)_2}\ \mathrm{(O_2)}\ \mathrm{(NH_3)}]$

$(1 \times 2) + x - (1 \times 2) - (2 \times 2) - (1 \times 2) - (0 \times 1) = 0$

$x - 6 = 0$

$x = +6 \ldots (iii)$

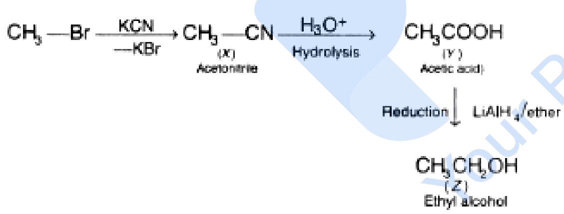
Therefore, oxidation states are $+3, 0, +6$.

Ans 13

Correct Option. D

Solution:

The complete reaction is as follows,



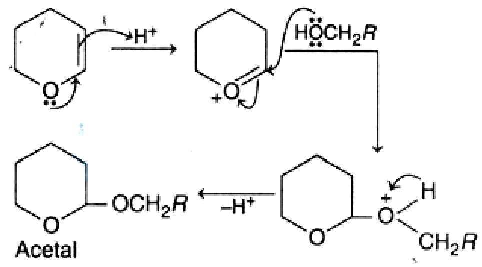
Ans 14

Correct Option. B

Solution:

Dihydropyran reacts with alcohols in presence of anhydrous H^+ to form acetal.

The complete reaction is as follows



Ans 15

Correct Option. A

Solution:

Among the given amino acids, only two i.e. lysine and arginine are basic amino acid. While all other are neutral amino acid.

Mathematics

Ans 1

Correct Option. C

Solution:

$$\because \log(\log_4(\sqrt{x+4}+\sqrt{x}))=0$$
$$\Rightarrow \log_4(\sqrt{x+4}+\sqrt{x})=1$$
$$\Rightarrow \sqrt{x+4}+\sqrt{x}=4^1=4$$

From the given options, $x=\frac{9}{4}$ satisfy the equation.

Ans 2

Correct Option. B

Solution:

$\because \frac{a}{b}=\frac{1}{3}\Rightarrow b=3a$ and $\frac{b}{c}=\frac{3}{4}\Rightarrow\frac{3a}{c}=\frac{3}{4}$

$\Rightarrow \frac{a}{c}=\frac{1}{4}\Rightarrow c=4a$

Then, $\frac{a+2b}{b+2c}=\frac{a+2(3a)}{(3a)+2(4a)}$

$=\frac{a+6a}{3a+8a}=\frac{7a}{11a}=\frac{7}{11}$

Ans 3

Correct Option. C

Solution:

$\because 2079000 = 2^3 \times 3^3 \times 5^3 \times 7 \times 11$

for the divisor to be even and divisible by 15;2,3 and 5 must be occur once.

There, the total number of required divisors are

$3 \times 3 \times 3 \times 2 \times 2 = 108$

Ans 4

Correct Option. 5

Solution:

N = Number of 8 digit number with exactly four 9.

$$\begin{aligned} \Rightarrow N = & \text{ Number with } [\{1^{\text{st}} \text{ digit is } 9\} + \{1^{\text{st}} \text{ digit is not } 9\}] \\ & = {}^7C_3 \cdot 9^4 + 8 \cdot {}^7C_4 \cdot 9^3 \\ & = 9^3\left(9 \cdot \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} + 8 \cdot \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1}\right) \\ & = 729 \times (315 + 280) = 729 \times 595 \end{aligned}$$

Hence, unit digit of N is 5 .

Ans 5

Correct Option. 2

Solution:

$\because x + \frac{1}{x^2} = \frac{x}{2} + \frac{x}{2} + \frac{1}{x^2}$

and $\mathrm{AM} \geq \mathrm{GM}$

$\Rightarrow \frac{x}{2} + \frac{x}{2} + \frac{1}{x^2} \geq \left(\frac{x}{2} \times \frac{x}{2} \times \frac{1}{x^2}\right)^{\frac{1}{3}}$

$\Rightarrow \frac{x}{2} + \frac{x}{2} + \frac{1}{x^2} \geq \left(\frac{1}{4}\right)^{\frac{1}{3}}$

Minimum value of $x + \frac{1}{x^2} = \sqrt[3]{\frac{1}{4}}$

and for minimum value

$\frac{x}{2} = \frac{1}{x^2} \Rightarrow x^3 = 2$

Hence, $\alpha^3 = 2$

Ans 6

Correct Option. B

Solution:

$$\begin{aligned} & \mathrm{T}_{r+1} = {}^{200}C_r(x \cdot \sqrt{180})^{200-r} \cdot (\sqrt[3]{432})^r \\ & = {}^{200}C_r 2^{200-r} \cdot 3^{200-r} \cdot 5^{\frac{200-r}{2}} \cdot x^{200-r} 2^r 3^r 2^{\frac{r}{3}} \\ & \therefore \text{ Coefficient } = {}^{200}C_r 2^{200} \cdot 3^{200} \cdot 5^{100} \cdot \frac{2^{\frac{r}{3}}}{5^{\frac{r}{2}}} \end{aligned}$$

r must be multiple of 6 and takes values between 0 to 200.

$\therefore 34$ terms $= n$

$\left[\frac{n}{6}\right] = \left[\frac{34}{6}\right] = 5$

Ans 7

Correct Option. B

Solution:

$$\because T_{r+1} = {}^9C_r (2x)^{\frac{9-r}{2}} \left(\frac{3}{x^2}\right)^{\frac{r}{3}}$$
$$= {}^9C_r 2^{\frac{9-r}{2}} \cdot 3^{\frac{r}{3}} \cdot x^{\left(\frac{9-r}{2} - \frac{2}{3}\right)}$$
$$= {}^9C_r 2^{\frac{9-r}{2}} \cdot 3^{\frac{r}{3}} \cdot x^{\frac{27-7r}{6}}$$

So, $r = 3$

$$\text{Then, coefficient } = {}^9C_3 \cdot 2^{\frac{9-3}{2}} \cdot 3^1$$
$$= \frac{9 \cdot 8 \cdot 7}{3 \cdot 2} \times 2^3 \times 3$$
$$= 2016$$

Ans 8

Correct Option. A

Solution:

$$\because x^2 + y^2 = 4 \Rightarrow x^2 + y^2 - 4(1)^2 = 0 \ldots (i)$$
$$\text{and } y = 3x + c \Rightarrow \frac{y - 3x}{c} = 1$$

Put the value of ' 1 ' in Eq. (i)

$$\Rightarrow x^2 + y^2 - 4\left(\frac{y - 3x}{c}\right)^2 = 0$$
$$\Rightarrow x^2 + y^2 - 4\left(\left(\frac{y}{c}\right)^2 + \left(\frac{3x}{c}\right)^2 - \frac{6xy}{c^2}\right) = 0$$
$$\Rightarrow x^2\left(1 - \frac{36}{c^2}\right) + y^2\left(1 - \frac{4}{c^2}\right) + \frac{24xy}{c^2} = 0$$

According to the question,

$$\left(1 - \frac{36}{c^2}\right) + \left(1 - \frac{4}{c^2}\right) = 0 \Rightarrow 2 = \frac{40}{c^2}$$
$$\Rightarrow \quad c^2 = 20$$

Ans 9

Correct Option. C

Solution:

Given circle is $x^2 + y^2 - 6x - 10y + 33 = 0$

Centre of the circle $= (3, 5)$

Radius of required circle is same as of given circle $= \sqrt{(3)^2 + (5)^2 - 33} = 1$

Mirror image of centre about $y = x$ is $(5, 3)$

Hence, Required equation $(x - 5)^2 + (y - 3)^2 = 1$

$\Rightarrow x^2 + y^2 - 10x - 6y + 33 = 0$

Ans 10

Correct Option. C

Solution:

Let the equation of tangent is

$y = mx + \frac{16}{m} [\because a = 16]$

It passes through (h, k)

Then, $k = mh + \frac{16}{m} \Rightarrow m^2 h + 16 - mk = 0$

According to the question, roots of the equation will be m_1 and $8m_1$.

So, $m_1 + 8m_1 = \frac{k}{h} \Rightarrow 9m_1 = \frac{k}{h} \ldots (i)$

and $m_1 \cdot 8m_1 = \frac{16}{h} \Rightarrow 8m_1^2 = \frac{16}{h} \ldots (ii)$

By solving these equations, we get

$\frac{k^2}{2h} = 81$

Ans 11

Correct Option. 375

Solution:

As we know,

$\lim_{x \to 0} \frac{\sin x}{x} \simeq 1$

but it is actually less than 1 .

So, $\lim_{x \to 0} \left[\frac{\sin x}{x}\right] = 0 = 1^2 - 1$

Similarly, $\lim_{x \to 0} \left[2\frac{\sin 2x}{x}\right] = \lim_{x \to 0} \left[4\frac{\sin 2x}{2x}\right]$

$= 3 = 2^2 - 1$

Hence, $\lim_{x \to 0} \left[n\frac{\sin nx}{x}\right] = n^2 - 1$

Therefore,

$$\begin{aligned}
&\lim_{x \to 0} f(x) = (1^2 - 1) + (2^2 - 1) + \ldots + (10^2 - (1) \\
&= (1^2 + 2^2 + \ldots + 10^2) - (1 + 1 + \ldots + 1 \text{ times }) \\
&= \frac{10 \times 11 \times 21}{6} - 10 \\
&= 385 - 10 = 375
\end{aligned}$$

Ans 12

Correct Option. B

Solution:

$\sin^2 A + \sin^2 B + \sin^2 C = 2$

$= \frac{1-\cos 2A}{2} + \frac{1-\cos 2B}{2} + \frac{1-\cos 2C}{2} = 2$

$\Rightarrow \cos 2A + \cos 2B + \cos 2C = -1$

$\Rightarrow 2\cos(A+B)\cos(A-B) = -2\cos^2 C$

$\Rightarrow 2\cos(\pi - C)\cos(A-B) = -2\cos^2 C \; [\because A+B+C = \pi]$

$\Rightarrow 2\cos C[\cos C - \cos(A-B)] = 0$

$\Rightarrow -2\cos C[\cos(A+B) + \cos(A-B)] = 0$

$\Rightarrow -\cos A \cdot \cos B \cdot \cos C = 0$

$\Rightarrow \cos A \cdot \cos B \cdot \cos C = 0$

Therefore, one of them should be equal to zero.

So, exactly one angle is 90°.

Ans 13

Correct Option. D

Solution:

As we know,

$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

and $\sin A, \sin B$ and $\sin C$ are in AP.

So, a, b, c will also be in AP

$$\therefore 2b=a+c$$

$$\text{and } \cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{b^2+c^2-(2b-c)^2}{2bc}$$

$$=\frac{b^2+c^2-\left(4b^2+c^2-4bc\right)}{2bc}$$

$$=\frac{-3b^2+4bc}{2bc}$$

$$=\frac{b(4c-3b)}{2bc}=\frac{4c-3b}{2c}$$

Ans 14

Correct Option. A

Solution:

$$\because D=\begin{vmatrix} n & n^2 & n^3 \\ n^2 & n^3 & n^5 \\ 1 & 2 & 3 \end{vmatrix}$$

Then, $M_{11}=\begin{vmatrix} n^3 & n^5 \\ 2 & 3 \end{vmatrix}=3n^3-2n^5$

and $C_{33}=\begin{vmatrix} n & n^2 \\ n^2 & n^3 \end{vmatrix}=n^4-n^4=0$

$$M_{13}=\begin{vmatrix} n^2 & n^3 \\ 1 & 2 \end{vmatrix}=2n^2-n^3$$

$$\therefore \lim_{n\to\infty}=\frac{M_{11}+C_{33}}{\left(M_{13}\right)^2}=\lim_{n\to\infty}\frac{n^3\left(3-2n^2\right)+0}{n^4(2-n)^2}$$

$$=\lim_{n\to\infty}\frac{3-2n^2}{n\left[4+n^2-4n\right]}=\lim_{n\to\infty}\frac{n^2\left(\frac{3}{n^2}-2\right)}{n^3\left(\frac{4}{n^2}+1-\frac{4}{n}\right)}$$

put $n=\infty$

$$=\frac{\left(\frac{3}{\infty^2}-2\right)}{\infty\left(\frac{4}{\infty^2}+1-\frac{4}{\infty}\right)}=\frac{-2}{1\times\infty}=0$$

Ans 15

Correct Option. D

Solution:

we have $x=\sin\left(2\tan^{-1}2\right)$,

$$\Rightarrow x=\sin(2\theta), \tan^{-1}2=\theta$$
$$\Rightarrow x=\frac{2\tan\theta}{1+\tan^2\theta}, \tan\theta=2$$
$$\Rightarrow x=\frac{4}{5}$$

Also, $y=\sin\left(\frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)\right)$

$$\Rightarrow \quad y=\sin(\theta), \tfrac{1}{2}\tan^{-1}\left(\tfrac{4}{3}\right)=\theta$$
$$\Rightarrow \quad y=\sin(\theta), \tan(2\theta)=\tfrac{4}{3}$$
$$\Rightarrow \quad y=\sin(\theta), \tan(\theta)=\tfrac{1}{2}$$
$$\Rightarrow \quad y=\tfrac{1}{\sqrt{5}}$$

Hence, $y^2=\frac{1}{5}=1-\frac{4}{5}=1-x$

Ans 16

Correct Option. B

Solution:

$$\because f(n)=\begin{cases}\frac{n+1}{2}, & n \text{ is odd}\\ \frac{n}{2}, & n \text{ is even}\end{cases}$$

$\therefore f(1)=\frac{1+1}{2}=1$

and $f(2)=\frac{2}{2}=1$

$\because f(1)=f(2)=1$

So, f is not injective

and Range of $f(n)$ is all natural numbers.

So, it is surjective

Hence, f is surjective but not injective.

Ans 17

Correct Option. D

Solution:

Let, $f(x) = 1 + x^n$ or $f(x) = 1 - x^n$

$\because f(3) = 10$

So, $f(x) = 1 + x^2$

Then, $\int f(x)dx = \int \left(1 + x^2\right) dx = x + \frac{x^3}{3} + c$

$\therefore \quad g(x) = x + \frac{x^3}{3} + c$
$\because \quad g(1) = \frac{4}{3}$
$\Rightarrow \quad \frac{4}{3} = 1 + \frac{1}{3} + c$
$\Rightarrow \quad C = 0$

Therefore, $g(x) = x + \frac{x^3}{3}$

and $g(3) = 3 + \frac{3^3}{3} = 12$

Ans 18

Correct Option. B

Solution:

$f(x) + f(y) + 2xy = f(x + y) \ldots (i)$

differentiating the equation,

$\Rightarrow f'(y) + 2x = f'(x + y)$

at $y = 0$

$f'(0) + 2x = f'(x)$

$\Rightarrow f'(x) = 2x$

Integrating both sides.

$\int f'(x) = \int 2x \cdot dx$

$\Rightarrow f(x) = x^2$

$\therefore I = \int_0^{\frac{\pi}{2}} f(\sin x)dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x \cdot dx \ldots (ii)$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2\left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \cos^2 x \cdot dx \ldots (iii)$

Adding Eqs. (ii) and (iii)

$2l = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx = \int_0^{\frac{\pi}{2}} 1 \cdot dx$

$\Rightarrow \quad 2l = \frac{\pi}{2}$

$\Rightarrow \quad l = \frac{\pi}{4}$

Ans 19

Correct Option. 8

Solution:

$\frac{dy}{dx} + 2y \tan x = \sin x$

The given differential equation is of the form

$\frac{dy}{dx} + Py = Q$

By comparing, we get

$P = 2\tan x$ and $Q = \sin x$

$\therefore$ I.F. $= e^{\int \cdot 2 \tan \cdot dx} = e^{2 \log \sec x} = e^{\log \sec^2 x} = \sec^2 x$

Then, solution of the differential equation

$$y \sec^2 x = \int \sin x \cdot \sec^2 x \cdot dx + c$$
$$\Rightarrow y \sec^2 x = \int \frac{\sin x}{\cos^2 x} dx + c$$
$$\Rightarrow y \sec^2 x = \int \tan x \cdot \sec x dx + c$$
$$\Rightarrow y \sec^2 x = \sec x + c$$
$$\Rightarrow y = \frac{1}{\sec x} + \frac{c}{\sec^2 x} = \cos x + \left(\cos^2 x\right) \cdot c$$
$$\text{at } x = \frac{\pi}{3}, y = 0$$
$$\therefore 0 = \cos \frac{\pi}{3} + \left(\cos^2 \frac{\pi}{3} \cdot c\right)$$
$$\Rightarrow 0 = \frac{1}{2} + \frac{c}{4} \Rightarrow c = -2$$
$$\therefore y = \cos x - 2\cos^2 x$$

$\Rightarrow \frac{y}{2} = -\cos^2 x + \frac{1}{2}\cos x = -\left(\cos^2 x - \frac{1}{2}\cos x\right)$

$\Rightarrow \frac{y}{2} = -\left[\left(\cos x - \frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2\right] = -\left[\left(\cos x - \frac{1}{4}\right)^2\right] + \frac{1}{16}$

So, maximum value of $\frac{y}{2} = \frac{1}{16}$

$\Rightarrow$ Maximum value of $y = \frac{1}{8} \Rightarrow k = 8$

Ans 20

Correct Option. 3

Solution:

Let, $\mathbf{a} + \mathbf{b} = \lambda\mathbf{c} \ldots (i)$

and $b + c = \mu a \ldots (ii)$

Subtracting Eq. (ii) from Eq. (i),

$$\begin{aligned} & \mathbf{a} - \mathbf{c} = \lambda\mathbf{c} - \mu\mathbf{a} \\ \Rightarrow \quad & (1+\mu)\mathbf{a} = (1+\lambda)\mathbf{c} \end{aligned}$$

$\because$ a and c are non-collinear.

So, $1 + \mu = 0$ and $1 + \lambda = 0$

$\Rightarrow \lambda = \mu = -1$

Therefore, $\mathbf{a} + \mathbf{b} = -\mathbf{c}$

$$\begin{aligned} & \Rightarrow \mathbf{a}+\mathbf{b}+\mathbf{c}=0 \Rightarrow |\mathbf{a}+\mathbf{b}+\mathbf{c}|^2 = 0 \\ & \Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) = 0 \\ & \Rightarrow 2+2+2+2(\mathbf{b}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) = 0 \\ & \Rightarrow \mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a} = -3 \\ & \text{Hence, } |\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}| = 3 \end{aligned}$$