MHT CET 2016 Paper 1 Mathematics Answer Key PDF

Question 1 Option: (B) – $\left(\frac{1}{2}\right)^{10}$
Explanation: Using $E(X) = np = 5$ and $\text{Var}(X) = npq = 2.5$, we find $q = 0.5$, $p = 0.5$, and $n = 10$. The probability $P(X < 1) = P(X = 0) = \binom{10}{0}(0.5)^{10} = (1/2)^{10}$.

Question 2 Option: (D) – $\frac{1}{3}$
Explanation: Substituting $x = \sin\theta$ simplifies the first function to $u = \theta = \sin^{-1} x$ and the second to $v = 3\theta = 3\sin^{-1} x$. The derivative of u with respect to v is $\frac{du}{dv} = \frac{1}{3}$.

Question 3 Option: (B) – $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$
Explanation: The family of circles is represented by $(x-a)^2 + y^2 = a^2 \implies x^2 + y^2 = 2ax$. Differentiating and eliminating the parameter a yields the differential equation $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$.

Question 4 Option: (B) – 0
Explanation: By the properties of determinants, the sum of products of the elements of any row with the cofactors of a different row is always equal to 0.

Question 5 Option: (D) – π
Explanation: Rolle's theorem requires $f'(c) = 0$. Differentiating $f(x)$ yields $f'(x) = 2e^x \sin x = 0$, which gives $c = \pi$ as the only value lying within the interval $(\pi/4, 5\pi/4)$.

Question 6 Option: (C) – $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$
Explanation: The lines trisecting the first quadrant have inclinations of 30° and 60°, giving equations $y = x/\sqrt{3}$ and $y = \sqrt{3}x$. Multiplying these equations yields the joint equation $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$.

Question 7 Option: (B) – $\sqrt{2}$
Explanation: Simplifying the given equation yields $\frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x} \implies \tan x = 1 \implies x = \pi/4$. Thus, the value of $\sin x + \cos x = 1/\sqrt{2} + 1/\sqrt{2} = \sqrt{2}$.

Question 8 Option: (A) – $\frac{4}{5}, \frac{3}{5}, 0$
Explanation: Rewriting the line equation in standard form gives direction ratios $(2, 1.5, 0) \propto (4, 3, 0)$. Normalizing these direction ratios yields the direction cosines $(4/5, 3/5, 0)$.

Question 9 Option: (D) – $\sin^{-1}\left(\frac{x-1}{3}\right) + c$
Explanation: Completing the square in the denominator gives $8 + 2x - x^2 = 9 - (x-1)^2$. Applying the standard integration formula $\int \frac{1}{\sqrt{a^2-u^2}}du$ yields $\sin^{-1}((x-1)/3) + c$.

Question 10 Option: (A) – 8.6
Explanation: Using linear approximation $f(a+h) \approx f(a) + hf'(a)$ with $a = 1$ and $h = 0.1$, we compute $f(1) = 8$ and $f'(1) = 6$, giving $f(1.1) \approx 8 + 0.1(6) = 8.6$.

Question 11 Option: (B) – 0.8
Explanation: The probability of waiting not more than 4 minutes is given by the cumulative distribution function $P(X \le 4) = \int_0^4 \frac{1}{5}dx = \frac{4}{5} = 0.8$.

Question 12 Option: (B) – c^2
Explanation: Expanding the expression gives $(a^2+b^2)(\sin^2(C/2)+\cos^2(C/2)) - 2ab(\cos^2(C/2) - \sin^2(C/2)) = a^2 + b^2 - 2ab\cos C$. By the cosine rule, this is equal to c^2.

Question 13 Option: (B) – 1
Explanation: Let $u = \log(\sec\theta + \tan\theta)$ and $v = \sec\theta$. The derivatives are $du/d\theta = \sec\theta$ and $dv/d\theta = \sec\theta\tan\theta$, making $du/dv = \cot\theta$, which equals 1 at $\theta = \pi/4$.

Question 14 Option: (B) – $x^2 - y^2 - 10x + 6y + 16 = 0$
Explanation: The angle bisectors are of slope ± 1 passing through the intersection point $(5, 3)$, giving $x - y - 2 = 0$ and $x + y - 8 = 0$. Multiplying these gives $x^2 - y^2 - 10x + 6y + 16 = 0$.

Question 15 Option: (A) – (4,11)
Explanation: Differentiating gives $6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$. Substituting $dy/dt = 8dx/dt$ yields $x = 4$, which when substituted back into the curve equation gives the coordinate $y = 11$.

Question 16 Option: (A) – 0
Explanation: By the Squeeze Theorem, since $|\sin(1/x)| \leq 1$, the limit $\lim_{x\to 0} x\sin(1/x) = 0$. For continuity at $x = 0$, the value of the function k must be 0.

Question 17 Option: (C) – m^2
Explanation: Differentiating the relation gives $\sqrt{1-x^2}\frac{dy}{dx} = my$. Squaring both sides yields the differential equation $(1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$, showing that $A = m^2$.

Question 18 Option: (B) – $A = 5, B = -3$
Explanation: Expressing the numerator as $5(2e^x - 5) - 3(2e^x)$ allows the integral to split into $\int 5\,dx - 3\int \frac{2e^x}{2e^x-5}dx = 5x - 3\log|2e^x - 5| + c$, giving $A = 5$ and $B = -3$.

Question 19 Option: (B) – -4/5
Explanation: Evaluating the principal values, we get $\tan^{-1}(\sqrt{3}) = \pi/3$, $\sec^{-1}(-2) = 2\pi/3$, $\csc^{-1}(-\sqrt{2}) = -\pi/4$, and $\cos^{-1}(-1/2) = 2\pi/3$. The expression simplifies to $-\frac{\pi/3}{5\pi/12} = -4/5$.

Question 20 Option: (C) – $\pi/90$
Explanation: Converting degrees to radians, $\sin x^\circ = \sin(\pi x/180)$. The limit is $\lim_{x\to 0}\frac{\log(1+2x)}{x}\cdot\frac{\sin(\pi x/180)}{x} = 2\cdot\frac{\pi}{180} = \frac{\pi}{90}$, which equals k.

Question 21 Option: (A) – $-\frac{99x}{101y}$
Explanation: Rewriting gives $x^2 - y^2 = 100(x^2 + y^2) \implies 101y^2 = -99x^2$. Differentiating implicitly gives $202y\frac{dy}{dx} = -198x \implies \frac{dy}{dx} = -\frac{99x}{101y}$.

Question 22 Option: (D) – 0
Explanation: The integrand $f(x) = \log\left(\frac{2-\sin x}{2+\sin x}\right)$ is an odd function because $f(-x) = -f(x)$. The integral of any odd function over a symmetric interval $[-a, a]$ is 0.

Question 23 Option: (C) – $\frac{a^{x+\tan^{-1}x}}{\log a} + c$
Explanation: Substituting $t = x + \tan^{-1}x$ gives $dt = \frac{x^2+2}{x^2+1}dx$. The integral becomes $\int a^t dt = a^t/\log a + c$, which equals $\frac{a^{x+\tan^{-1}x}}{\log a} + c$.

Question 24 Option: (B) – 3 and 2
Explanation: Cubing both sides to remove fractional powers gives $[1+(dy/dx)^3]^7 = 343(d^2y/dx^2)^3$. The highest derivative is of order 2, and its power represents the degree, which is 3.

Question 25 Option: (B) – $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$
Explanation: The angle θ between the line and plane is given by $\sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|} = \frac{2}{\sqrt{3}\sqrt{6}} = \frac{\sqrt{2}}{3}$, yielding $\theta = \sin^{-1}(\sqrt{2}/3)$.

Question 26 Option: (B) – 4/3 sq. units
Explanation: The curve intersects the x-axis at $x = 0$ and $x = 2$. The bounded area is computed by the integral $\int_0^2(2x - x^2)dx = \left[x^2 - x^3/3\right]_0^2 = 4 - 8/3 = 4/3$ sq. units.

Question 27 Option: (A) – $\cot x$
Explanation: Using the substitution $t = \log(\sin x)$, we have $dt = \cot x\,dx$. The given integral of $f(x)/t$ yields $\log t$ only if $f(x) = \cot x$.

Question 28 Option: (A) – $\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0$
Explanation: The points are $A(0, b, c)$ and $B(a, 0, c)$. The normal vector to the plane OAB is $\vec{OA}\times\vec{OB} = (bc, ac, -ab)$, which gives the equation $bcx + acy - abz = 0 \implies x/a + y/b - z/c = 0$.

Question 29 Option: (C) – 2
Explanation: Equating the components of $\vec{c} = m\vec{a} + n\vec{b}$ gives $m + 2n = 3$ and $m - n = 0$. Solving these simultaneous equations yields $m = 1$ and $n = 1$, so $m + n = 2$.

Question 30 Option: (C) – $\pi/4$
Explanation: Using the integral property $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$ transforms the integrand from $\sec x$ to $\csc x$ in the numerator, yielding $2I = \int_0^{\pi/2} 1\,dx = \pi/2 \implies I = \pi/4$.

Question 31 Option: (C) – $1 - P(X > 0)$
Explanation: $F(0) = P(X \leq 0) = 0.2 + 0.3 + 0.15 = 0.65$. This is equivalent to $1 - P(X > 0) = 1 - (0.25 + 0.1) = 0.65$.

Question 32 Option: (A) – $y = ex \log x$
Explanation: Separating variables gives $\frac{1+\log x}{x \log x}dx = \frac{dy}{y}$. Integrating gives $x \log x = cy$, and substituting the boundary condition $x = e, y = e^2$ gives $c = 1/e \implies y = ex \log x$.

Question 33 Option: (C) – $4\vec{MN}$
Explanation: Expressing the vectors in terms of position vectors gives $2(\vec{b} + \vec{d}) - 2(\vec{a} + \vec{c})$. Substituting $2\vec{n} = \vec{b} + \vec{d}$ and $2\vec{m} = \vec{a} + \vec{c}$ simplifies the sum to $4(\vec{n} - \vec{m}) = 4\vec{MN}$.

Question 34 Option: (D) – $\cot x$
Explanation: The integrating factor is I.F. $= e^{\int P dx} = \sin x$. Taking logs and differentiating both sides with respect to x gives $P = \frac{d}{dx}(\log(\sin x)) = \cot x$.

Question 35 Option: (C) – $y^2 - x + 1 = 0$
Explanation: The equation $y^2 - x + 1 = 0 \implies y^2 = x - 1$ represents a parabola, whereas all the other options can be factored into a product of linear terms representing straight lines.

Question 36 Option: (A) – 11/64
Explanation: Using the binomial distribution with $n = 10$, $p = 0.5$, we find $P(X \geq 7) = \left[\binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10}\right](1/2)^{10} = 176/1024 = 11/64$.

Question 37 Option: (C) – $11\pi/8, 15\pi/8$
Explanation: The equation is $\tan 2x = -1$. In the interval $2\pi < 2x < 4\pi$, the solutions are $2x = 11\pi/4$ and $2x = 15\pi/4$, which gives $x = 11\pi/8$ and $15\pi/8$.

Question 38 Option: (A) – B
Explanation: Equating the parametric equations of the lines AB and CD and solving for the parameters gives the unique intersection point with position vector $-4\vec{c}$, which corresponds to the point B.

Question 39 Option: (A) – $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$
Explanation: Using the property $(B^{-1}A^{-1})^{-1} = AB$, we multiply matrices A and B to directly obtain the resulting matrix $\begin{bmatrix} 2 & -2 \\ 2 & 3 \end{bmatrix}$.

Question 40 Option: (D) – T, T
Explanation: Both statements are mathematically true, so the conditional $p \to q$ is $\text{T} \to \text{T} \equiv \text{T}$ and the biconditional $p \leftrightarrow q$ is $\text{T} \leftrightarrow \text{T} \equiv \text{T}$.

Question 41 Option: (B) – 2, 1, -3
Explanation: By the properties of the Euler line, the centroid $G(g)$ divides the orthocenter $H(h)$ and the circumcenter $P(p)$ in the ratio $2:1$, which gives the vector relation $2\vec{p} + \vec{h} - 3\vec{g} = 0$.

Question 42 Option: (C) – There exists a real number whose square is not positive
Explanation: The number 0 is real and its square $0^2 = 0$ is not positive. This makes statement (C) the only true statement among the choices.

Question 43 Option: (C) – $n\pi \pm \pi/4$
Explanation: The equation $\tan^2 x = 1$ is of the form $\tan^2 x = \tan^2(\pi/4)$, which has the general solution $x = n\pi \pm \pi/4$.

Question 44 Option: (D) – $x, y \geq 0, x - y \leq 0, x \leq 5, y \leq 3$
Explanation: The shaded region lies above the line $y = x$, to the left of $x = 5$, and below $y = 3inthefirstquadrant$, representing the constraints $x - y \leq 0$, $x \leq 5$, and $y \leq 3$.

Question 45 Option: (B) – 2, -1, 2
Explanation: The direction ratios are found by taking the cross product of $(-1, 2, 2)$ and $(0, 2, 1)$, which yields $(-2, 1, -2)$, which is proportional to $(2, -1, 2)$.

Question 46 Option: (C) – $\frac{1}{5}\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$
Explanation: The equation $Ax = I$ implies $x = A^{-1}$. Finding the inverse of A by using the formula $A^{-1} = \frac{1}{|A|}\text{adj } A$ yields the matrix shown in option (C).

Question 47 Option: (A) – 6
Explanation: The scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c}) = 10$ is evaluated using the determinant of the coefficients, which simplifies to $3\lambda - 8 = 10 \implies \lambda = 6$.

Question 48 Option: (B) – 40/243
Explanation: The probability $P(2 < X < 4) = P(X = 3)$. Using the binomial probability formula with $n = 5$ and $p = 1/3$ gives $\binom{5}{3}(1/3)^3(2/3)^2 = 40/243$.

Question 49 Option: (B) – at every point of the segment joining two points
Explanation: Since the boundary line $x_1 + x_2 = 10$ is parallel to the objective function line $z = x_1 + x_2$, the maximum value is attained at all points along that boundary segment.

Question 50 Option: (A) – $p \vee \sim q$
Explanation: The switches are connected in parallel, which is represented by the logical disjunction operator $\vee$, giving the equivalent symbolic form $p \vee \sim q$.