MHT CET 2018 Paper 1 Mathematics Answer Key PDF

Question 1 Option: (C) – $\frac{1}{3}$
Explanation: Integrating $\frac{1}{2(1+9x^2)}$ yields $\frac{1}{6}\tan^{-1}(3x)$. Evaluating this from 0 to K and equating to $\frac{\pi}{24}$ gives $\tan^{-1}(3K) = \frac{\pi}{4}$, which simplifies to $K = \frac{1}{3}$.

Question 2 Option: (D) – $(-1, 4)$
Explanation: Comparing $y^2 = -16x$ with $y^2 = -4ax$ gives $a = 4$. Substituting $a = 4$ and $t = \frac{1}{2}$ into the parametric coordinates $x = -at^2$ and $y = 2at$ yields the point $(-1, 4)$.

Question 3 Option: (D) – $\sec x + \log|\csc x - \cot x| + c$
Explanation: By writing the numerator as $\sin^2 x + \cos^2 x$, the integral splits into $\int \sec x \tan x\, dx + \int \csc x\, dx$. Integrating both terms gives the correct solution $\sec x + \log|\csc x - \cot x| + c$.

Question 4 Option: (D) – $\frac{y}{x}$
Explanation: Expressing the equation as $\frac{x^3-y^3}{x^3+y^3} = 100$ and cross-multiplying leads to $101y^3 = -99x^3$. Differentiating both sides implicitly and substituting the relation yields $\frac{dy}{dx} = \frac{y}{x}$.

Question 5 Option: (C) – $\mathbb{R} - \{4\}$
Explanation: For $x \neq 2$, the function simplifies to $f(x) = x + 2$. Since x cannot equal 2, the function value $f(x)$ cannot equal 4, meaning the range of the function is $\mathbb{R} - \{4\}$.

Question 6 Option: (C) – $\frac{25}{8}$
Explanation: Continuity at $x = 0$ requires $\alpha = 2 + \beta$. Using $f(1/2) = 2$ gives $\alpha = \frac{7}{4}$, which leads to $\beta = -\frac{1}{4}$, yielding $\alpha^2 + \beta^2 = \frac{25}{8}$.

Question 7 Option: (B) – 2
Explanation: Differentiating $y = (\tan^{-1} x)^2$ gives $(1+x^2)\frac{dy}{dx} = 2\tan^{-1} x$. Differentiating again and multiplying the resulting equation by $(1 + x^2)$ yields the required value of 2.

Question 8 Option: (C) – 11
Explanation: Comparing coefficients of $5x^2+xy-kx-2y+2 = 0$ factored as $(5x+y-1)(x-2) = 0$ gives $k = 11$. This satisfies all conditions for the lines to coincide.

Question 9 Option: (B) – $\begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$
Explanation: Simplifying $(A^2 - 5A)A^{-1}$ yields $A - 5I$. Subtracting five times the identity matrix from A gives the correct matrix representation of the resulting transformation.

Question 10 Option: (C) – $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$
Explanation: The direction vector is obtained by the cross product of the direction ratios $(2, -2, 1)$ and $(1, -2, 2)$, giving direction ratios $(2, 3, 2)$. Using the point $(3, -1, 2)$ yields the required symmetric equation of the line.

Question 11 Option: (C) – $\frac{3}{28}$
Explanation: Treating the three L's as a single unit reduces the problem to arranging 6 units, giving 30 favorable arrangements. Dividing this by the total of 280 possible permutations gives a probability of $\frac{3}{28}$.

Question 12 Option: (B) – $\frac{100}{9}(10^9 - 1)$
Explanation: The sum is rewritten as $\sum_{r=1}^{10}(10^r - 1) = \frac{10(10^{10}-1)}{9} - 10$. Simplifying this expression algebraically leads to the correct option of $\frac{100}{9}(10^9 - 1)$.

Question 13 Option: (B) – 1
Explanation: Using the identity $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ for angles of a triangle, dividing both sides by the product of tangents directly yields $\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$.

Question 14 Option: (D) – $\frac{13}{12}$
Explanation: Integrating $\frac{1}{\sqrt{16-9x^2}}$ yields $\frac{1}{3}\sin^{-1}(\frac{3x}{4})+C$. Identifying $A=\frac{1}{3}$ and $B=\frac{3}{4}$, their sum is calculated as $\frac{13}{12}$.

Question 15 Option: (A) – $e^x\tan x+c$
Explanation: Simplifying the integrand gives $e^x\left[\frac{2+2\sin x\cos x}{2\cos^2 x}\right]=e^x(\sec^2 x+\tan x)$. Since this is in the form $e^x[f'(x)+f(x)]$, the integral evaluates to $e^x\tan x+c$.

Question 16 Option: (D) – $\frac{3}{4}$
Explanation: An absolute difference of 1 between heads and tails occurs in outcomes with either 2 heads and 1 tail, or 1 head and 2 tails. This yields 6 favorable outcomes out of 8, giving a probability of $\frac{3}{4}$.

Question 17 Option: (D) – $-\sqrt{3}$
Explanation: Expanding both sides using trigonometric sum-to-product formulas gives $2(\frac{1}{2}\sin\theta+\frac{\sqrt{3}}{2}\cos\theta)=\frac{\sqrt{3}}{2}\cos\theta+\frac{1}{2}\sin\theta$. Simplifying the terms yields $\tan\theta=-\sqrt{3}$.

Question 18 Option: (B) – $\frac{28}{3}$
Explanation: The area is calculated by evaluating the integral $\int_1^4\sqrt{4y}\,dy=2\int_1^4 y^{1/2}\,dy$. Computing the antiderivative at the limits gives $\frac{4}{3}(8-1)=\frac{28}{3}$ square units.

Question 19 Option: (D) – $\frac{3}{2}$
Explanation: Since the function is continuous at $x=0$, $f(0)=\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$. Using Taylor expansions for e^{x^2} and $\cos x$ simplifies the limit to $\frac{3}{2}$.

Question 20 Option: (A) – 12
Explanation: Evaluating the objective function $2x+y$ at the feasible region's corner points $(0,0)$, $(6,0)$, $(0,5.2)$, and $(4.5,2.5)$ reveals that the maximum value of 12 is attained at the vertex $(6,0)$.

Question 21 Option: (C) – 12
Explanation: The scalar triple product simplifies to $2[\vec{a}\ \vec{b}\ \vec{c}]$. Since the vectors are mutually perpendicular, the product is $2(1\times 2\times 3)=12$.

Question 22 Option: (D) – 4
Explanation: Collinearity of the points implies that their direction ratios are proportional: $\frac{-1}{2}=\frac{y-5}{8-y}=\frac{4-x}{-4}$. Solving these equations yields $x=2$ and $y=2$, so $x+y=4$.

Question 23 Option: (A) – $8h^2=9ab$
Explanation: Using $m_1+m_2=-\frac{2h}{b}$ and $m_1m_2=\frac{a}{b}$ with $m_1=2m_2$, we substitute m_2 to find $\frac{8h^2}{9b^2}=\frac{a}{b}$, which simplifies directly to $8h^2=9ab$.

Question 24 Option: (A) – $x+y+2=0$
Explanation: The angle bisectors of the coordinate axes have slopes of ± 1. Choosing slope $m=-1$ and passing the line through $(-3,1)$ yields $y-1=-1(x+3)$, which simplifies to $x+y+2=0$.

Question 25 Option: (B) – Hema does not get above 95% marks and she gets admission in good college
Explanation: The statement is of the form $p\implies q$. Its negation is $p\wedge\sim q$, which translates to "Hema gets admission in good college and does not get above 95% marks."

Question 26 Option: (A) – 0
Explanation: Since the series of product terms includes $\cos 90°$, and $\cos 90°=0$, the entire product evaluates to 0 regardless of the other values.

Question 27 Option: (C) – $1 - 2abc$
Explanation: Since the planes intersect in a straight line, the determinant of the coefficients must be zero. Expanding the determinant $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$ yields $a^2 + b^2 + c^2 = 1 - 2abc$.

Question 28 Option: (C) – $(-\frac{1}{2}, \frac{3}{2})$
Explanation: The point of intersection of the lines represented by a joint equation is found by solving $\frac{\partial f}{\partial x} = 2x + 1 = 0$ and $\frac{\partial f}{\partial y} = -2y + 3 = 0$, yielding $(-\frac{1}{2}, \frac{3}{2})$.

Question 29 Option: (B) – $\{1, 2, 3, 4\}$
Explanation: The possible outcomes $\{1, 2, 3, 4, 5, 6\}$ have 1, 2, 2, 3, 2, 4 positive divisors respectively. Thus, the range of the random variable representing the number of divisors is $\{1, 2, 3, 4\}$.

Question 30 Option: (B) – $\frac{65}{81}$
Explanation: The probability of not getting a perfect square (1 or 4) on one throw is $\frac{4}{6} = \frac{2}{3}$. Thus, the probability of at least one perfect square in four throws is $1 - (\frac{2}{3})^4 = \frac{65}{81}$.

Question 31 Option: (B) – $\frac{\pi}{4} - \log\sqrt{2}$
Explanation: Using integration by parts, $\int_0^{\pi/4} x \sec^2 x \, dx = [x \tan x]_0^{\pi/4} - \int_0^{\pi/4} \tan x \, dx$. This evaluates directly to $\frac{\pi}{4} - \log(\sec(\pi/4)) = \frac{\pi}{4} - \log\sqrt{2}$.

Question 32 Option: (C) – $\frac{3b}{2}$
Explanation: Using half-angle formulas, the expression becomes $\frac{1}{2}(a + c + a\cos C + c\cos A)$. By projection rule, $a\cos C + c\cos A = b$, and since $a + c = 2b$, the expression simplifies to $\frac{3b}{2}$.

Question 33 Option: (A) – 1
Explanation: Computing derivatives with respect to θ gives $\frac{dx}{d\theta} = 2e^\theta \sin\theta$ and $\frac{dy}{d\theta} = 2e^\theta \cos\theta$. Thus, $\frac{dy}{dx} = \cot\theta$, which equals 1 at $\theta = \frac{\pi}{4}$.

Question 34 Option: (C) – 4
Explanation: Grouping terms gives $\sin 3x(2\cos 2x + 1) = 0$. In $[0, \pi]$, $\sin 3x = 0$ yields $\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi\}$, while $\cos 2x = -\frac{1}{2}$ duplicates two of these solutions, leaving 4 distinct solutions.

Question 35 Option: (C) – $\frac{1}{6}$
Explanation: Applying the identity for $\tan^{-1} A + \tan^{-1} B$ gives $\frac{5x}{1-6x^2} = 1 \implies 6x^2 + 5x - 1 = 0$. Solving this gives $x = \frac{1}{6}$ (discarding the negative root as it does not satisfy the original equation).

Question 36 Option: (C) – -1
Explanation: The sum of the products of elements of any row with their corresponding co-factors equals the determinant of the matrix. Calculating the determinant $|A|$ directly yields -1.

Question 37 Option: (B) – If my friends do not come or we do not go for picnic then weather will not be fine
Explanation: The contrapositive of $p \implies (q \wedge r)$ is $\sim (q \wedge r) \implies \sim p$, which simplifies using De Morgan's laws to $(\sim q \vee \sim r) \implies \sim p$.

Question 38 Option: (D) – $(-1, 1)$
Explanation: A function is increasing when $f'(x) > 0$. Differentiating gives $f'(x) = \frac{1-x^2}{(x^2+1)^2} > 0$, which simplifies to $1 - x^2 > 0$, yielding the interval $x \in (-1, 1)$.

Question 39 Option: (A) – X
Explanation: Using binomial expansion, $4^n - 3n - 1 = (1 + 3)^n - 3n - 1$ is always a multiple of 9. Since Y contains all non-negative multiples of 9, X is a subset of Y, meaning $X \cap Y = X$.

Question 40 Option: (B) – a contradiction
Explanation: Using associative laws, the statement simplifies to $(p \wedge \sim p) \wedge q$. Since $p \wedge \sim p$ is always false (F), the entire pattern simplifies to $F \wedge q \equiv F$, which is a contradiction.

Question 41 Option: (A) – 0
Explanation: Equating slopes at $(2, 3)$ gives $2a = 4 \implies a = 2$. Substituting $(2, 3)$ into the curve's equation yields $b = -7$. Thus, the value of $7a + 2b = 14 - 14 = 0$.

Question 42 Option: (B) – $x^2 + y^2 = a^2 + b^2$
Explanation: The vertices of the rectangle are $(\pm a, \pm b)$. A circle passing through these points is centered at the origin with radius squared equal to a^2+b^2, giving the equation $x^2+y^2 = a^2+b^2$.

Question 43 Option: (A) – $-\frac{1}{e}$
Explanation: Differentiating $f(x) = x \log x$ gives $f'(x) = 1 + \log x = 0 \implies x = \frac{1}{e}$. Since the second derivative is positive, the minimum value is $f(1/e) = -\frac{1}{e}$.

Question 44 Option: (D) – 18.4
Explanation: We use the identity $E(X^2) = \text{Var}(X) + [E(X)]^2$. Substituting $E(X) = np = 4$ and $\text{Var}(X) = npq = 2.4$ yields $E(X^2) = 2.4 + 16 = 18.4$.

Question 45 Option: (A) – $\tan(\frac{x+y}{2}) = y + c$
Explanation: Substituting $x + y = v$ transforms the differential equation into $\frac{dv}{dy} = 1 + \cos v = 2\cos^2(v/2)$. Separating variables and integrating directly gives the solution $\tan(\frac{x+y}{2}) = y + c$.

Question 46 Option: (D) – 3
Explanation: Using the cosine formula for the angle $\frac{\pi}{3}$ between the normal vectors of the planes, we solve $\frac{1}{2} = \frac{|3p-2|}{p^2+5}$, which simplifies to the quadratic equation $(p-3)^2 = 0$, giving $p = 3$.

Question 47 Option: (D) – two
Explanation: The general equation of a parabola with axis parallel to the x-axis and latus rectum $4a$ is $(y - \beta)^2 = 4a(x - \alpha)$. Since there are two arbitrary constants (α, β), the order of the differential equation is 2.

Question 48 Option: (A) – $\frac{9}{2}$
Explanation: For intersecting lines, the coplanarity condition of the direction vectors and the vector joining points on both lines gives a determinant equal to zero. Solving this determinant equation yields $k = \frac{9}{2}$.

Question 49 Option: (C) – 135°
Explanation: Using direction cosines, $\cos^2(120^\circ) + \cos^2\beta + \cos^2(60^\circ) = 1 \implies \frac{1}{4} + \cos^2\beta + \frac{1}{4} = 1$. This yields $\cos\beta = \pm\frac{1}{\sqrt{2}}$, which corresponds to $\beta = 135^\circ$.

Question 50 Option: (C) – $5\vec{b}$
Explanation: Applying the external section formula with ratio $2 : 1$, the position vector is $\vec{r} = \frac{2(\vec{a}+2\vec{b})-1(2\vec{a}-\vec{b})}{2-1}$. Simplifying this expression directly yields the vector $5\vec{b}$.