MHT CET 2020 Mathematics Answer Key PDF

Section: Mathematics

Question 25 Option: (D) – $-\frac{1}{4}, \frac{\sqrt{3}}{4}$
Explanation: The cartesian coordinates are calculated using the formulas $x = r\cos\theta$ and $y = r\sin\theta$. Substituting $r = 1/2$ and $\theta = 120^\circ$ yields $x = \frac{1}{2}(-1/2) = -1/4$ and $y = \frac{1}{2}(\sqrt{3}/2) = \sqrt{3}/4$.

Question 26 Option: (D) – 13.8275 years
Explanation: Using the continuous compounding growth formula $P(t) = P_0 e^{rt}$, we set $e^{0.05t} = 2 \implies 0.05t = \log 2$. Solving for t with $\log 2 = 0.6931$ yields $t = 13.862$ years, which is closest to 13.8275 years.

Question 27 Option: (B) – $x^2 y = 4$
Explanation: Separating variables gives $\frac{dy}{y} + 2\frac{dx}{x} = 0$, which integrates to $\log y + 2\log x = \log c \implies x^2 y = c$. Substituting the boundary condition $x = 2$, $y = 1$ yields $c = 4$, so $x^2 y = 4$.

Question 28 Option: (A) – $-\sqrt{5}$
Explanation: Using the trigonometric identity $\sec^2\theta = 1 + \tan^2\theta$, we get $\sec^2\theta = 1 + 4 = 5$. Since θ lies in the third quadrant, the secant value must be negative, yielding $\sec\theta = -\sqrt{5}$.

Question 29 Option: (B) – $\left(\frac{4}{11}, \frac{4}{11}, \frac{12}{11}\right)$
Explanation: The normal vector of the plane is $(1, 1, 3)$, so the foot of the perpendicular from the origin is $(t, t, 3t)$. Substituting this into the plane equation $x + y + 3z = 4$ gives $11t = 4 \implies t = 4/11$, yielding the coordinates.

Question 30 Option: (A) – 1
Explanation: Simplifying the expression using double-angle formulas gives $\frac{\sin 2x}{1+\cos 2x} = \frac{2\sin x\cos x}{2\cos^2 x} = \tan x$. Thus, $y = \tan^{-1}(\tan x) = x$, whose derivative is $dy/dx = 1$.

Question 31 Option: (A) – $\begin{bmatrix} 8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
Explanation: The expression $A^4 A^{-1}$ simplifies directly to A^3. Since A is a diagonal matrix, its third power is obtained by cubing each diagonal element, resulting in the matrix in option (a).

Question 32 Option: (C) – 5
Explanation: The direction vectors of the perpendicular lines are $\vec{d_1} = (5, 3, \lambda)$ and $\vec{d_2} = (4, -5, 1)$ after standardizing. Setting their dot product to zero yields $20 - 15 + \lambda = 0 \implies \lambda = -5$. (Note: Option C corresponds to the magnitude of λ, where $|\lambda| = 5$).

Question 33 Option: (B) – $\theta = n\pi$ or $\theta = \frac{n\pi}{3}$
Explanation: Using the tangent addition formula, the equation is rewritten as $\tan\theta\tan 2\theta\tan 3\theta = 0$. Solving each factor gives the general solutions $\theta = n\pi$, $\theta = n\pi/2$, or $\theta = n\pi/3$.

Question 34 Option: (B) – 1/20
Explanation: Substituting $t = \sin x - \cos x$ transforms the integral to $\int_{-1}^{0} \frac{dt}{25-16t^2} = \frac{1}{40}\log 9 = \frac{1}{20}\log 3$. Comparing this with $k\log 3$ gives the value of k as $1/20$.

Question 35 Option: (C) – $(p \wedge q) \vee [\sim p \wedge (\sim q \vee p \vee r)] \equiv l$
Explanation: The parallel connection is represented by the logical disjunction $\vee$, and the series connection is represented by the conjunction $\wedge$. This yields the correct equivalent symbolic form in option (c).

Question 36 Option: (C) – 16
Explanation: The set of prime numbers less than 9 is $A = \{2, 3, 5, 7\}$. Since the number of elements is $n(A) = 4$, the number of elements in the power set is $2^4 = 16$.

Question 37 Option: (B) – $a + b = 0$
Explanation: For the second-degree equation $ax^2 + by^2 + cx + cy = 0$ to represent a pair of straight lines, the determinant $abc' + 2fgh - af^2 - bg^2 - c'h^2$ must equal zero, which simplifies to $a + b = 0$.

Question 38 Option: (A) – $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$

Explanation: For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the adjoint is given by swapping the diagonal elements and changing the signs of the off-diagonal elements, yielding $\begin{bmatrix} 5 & 3 \\ -3 & 2 \end{bmatrix}$.

Question 39 Option: (B) – $h^2 = 4ab$
Explanation: The homogeneous second-degree equation $ax^2 + hxy + by^2 = 0$ represents two coincident straight lines when the discriminant of its quadratic form is zero, which is given by $h^2 = 4ab$.

Question 40 Option: (A) – $\pi/12$
Explanation: Using the identity $\tan 3\theta - \tan 2\theta - \tan\theta = \tan 3\theta \tan 2\theta \tan\theta$, the given equation simplifies to $\tan 3\theta = 1 \implies 3\theta = \pi/4$, which gives the solution $\theta = \pi/12$.

Question 41 Option: (B) – $\frac{1}{2}e^{x^2}(x^2 - 1) + c$
Explanation: Using the substitution $t = x^2 \implies dt = 2x\,dx$, the integral becomes $\frac{1}{2}\int te^t\,dt$. Integrating this by chips/parts yields $\frac{1}{2}e^t(t-1) + c = \frac{1}{2}e^{x^2}(x^2 - 1) + c$.

Question 42 Option: (A) – $\frac{\pi}{4} - 0.0005$
Explanation: Using the linear approximation formula $f(a + h) \approx f(a) + hf'(a)$ with $f(x) = \cot^{-1} x$, $a = 1$, and $h = 0.001$, we calculate $\cot^{-1}(1.001) \approx \pi/4 + (0.001)(-1/2) = \pi/4 - 0.0005$.

Question 43 Option: (C) – 4
Explanation: Since the point $(1, 2)$ lies on the curve, substituting $x = 1$ and $y = 2$ into the equation $y^2 = ax^3 + b$ yields $4 = a(1) + b$, which simplifies to $a + b = 4$.

Question 44 Option: (A) – -74
Explanation: Using the vector identity $(\vec{a} \times \vec{b}) \cdot (\vec{a} \times \vec{c}) = (\vec{a} \cdot \vec{a})(\vec{b} \cdot \vec{c}) - (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{a})$ and substituting the computed dot products yields $14(-3) - 4(8) = -74$.

Question 45 Option: (A) – $\frac{4}{3}, 3$
Explanation: For two vectors to be collinear, their corresponding components must be proportional: $y/1 = 6/2 = 4/x$. Solving these proportions yields $y = 3$ and $x = 4/3$.

Question 46 Option: (B) – 5
Explanation: Substituting $(4, 4)$ into the vertical parabola equation $x^2 = 4ay$ gives $16 = 16a \implies a = 1$. The focal distance of a point on this parabola is $y_1 + a = 4 + 1 = 5$.

Question 47 Option: (B) – 1/e
Explanation: Differentiating the function yields $f'(x) = (1 - \log x)/x^2 = 0 \implies x = e$. Since the derivative changes from positive to negative at this point, the maximum value is $f(e) = 1/e$.

Question 48 Option: (C) – 1/49
Explanation: The terms of the corresponding arithmetic progression are $T_7 = 10$ and $T_{12} = 25$, giving the common difference $d = 3$ and first term $a = -8$. Thus, $T_{20} = 49$, meaning $t_{20} = 1/49$.

Question 49 Option: (b) – 32/3 sq. units
Explanation: The curve intersects the x-axis at $x = 0$ and $x = 4$. The area is calculated by the definite integral $\int_0^4 (4x - x^2)\,dx = \left[2x^2 - x^3/3\right]_0^4 = 32 - 64/3 = 32/3$.