MHT CET 2021 Mathematics Answer Key PDF

Answer Key

Question 1 Option (A) – an isosceles triangle
Explanation: Applying the sine rule a = 2 R sin A a=2RsinA and b = 2 R sin B b=2RsinB to the equation a cos B = b cos A acosB=bcosA yields sin A cos B = sin B cos A sin (A B) = 0 A = B sinAcosB=sinBcosAsin(AB)=0A=B , indicating two equal angles. code Code

Question 2
Option (A) – $\frac{5}{14}$
Explanation: Dividing both the numerator and the denominator by $\cos\theta$ simplifies the expression to $\frac{5\tan\theta-3}{\tan\theta+2}$. Substituting $\tan\theta = 4/5$ yields the value $\frac{4-3}{4/5+2} = \frac{5}{14}$.

Question 3
Option (C) – $-1 \le x < \frac{1}{\sqrt{2}}$
Explanation: Using $\cos^{-1} x = \pi/2 - \sin^{-1} x$, the inequality becomes $\pi/2 - \sin^{-1} x > \sin^{-1} x \implies \sin^{-1} x < \pi/4$. Considering the domain $[-1, 1]$ of the function, this restricts x to $[-1, 1/\sqrt{2})$.

Question 4
Option (A) – $\tan 3A \tan 2A \tan A$
Explanation: Expanding $\tan 3A = \tan(2A + A)$ using the tangent addition formula and rearranging the terms algebraically leads directly to the identity $\tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A$.

Question 5
Option (C) – $a + b + c = 0$
Explanation: For the three lines to be concurrent (concerning the typographical term "collinear"), the determinant of their coefficients must be zero. Factoring the expansion $a^3+b^3+c^3-3abc = 0$ yields the condition $a + b + c = 0$.

Question 6
Option (B) – $(2 - \sqrt{3})x - y - 4 + 2\sqrt{3} = 0$
Explanation: Rotating the line clockwise by 15° from its initial 30° angle results in a new inclination of 15° with slope $m = \tan 15^\circ = 2 - \sqrt{3}$. Using the point-slope form with $A(2, 0)$ gives the equation.

Question 7
Option (D) – $\sqrt{\frac{1}{26}}$
Explanation: Since x is in the third quadrant, $x/2$ is in the second quadrant where cosine is negative. However, as the options are positive magnitudes, we find $\cos(x/2) = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-12/13}{2}} = \sqrt{1/26}$.

Question 8
Option (D) – $5x^2 + 5y^2 + 60x + 7 = 0$
Explanation: Letting $P(x, y)$ be the moving point, the ratio of the lengths of the tangents being $2 : 3$ gives $\frac{x^2+y^2+4x+3}{x^2+y^2-6x+5} = \frac{4}{9}$. Cross-multiplying and simplifying the terms yields the required locus.

Question 9
Option (D) – touch each other externally
Explanation: The centers of the two circles are $C_1(-3,-3)$ and $C_2(6,6)$, with radii $R_1 = 3\sqrt{2}$ and $R_2 = 6\sqrt{2}$. Since the distance between the centers $C_1C_2 = 9\sqrt{2}$ equals $R_1 + R_2$, they touch externally.

Question 10
Option (D) – 16
Explanation: A variance of zero implies all n observations are identical to the mean, so $x_i = 5$ for all i. Substituting this into $\sum x_i^2 = 400$ gives $25n = 400 \implies n = 16$.

Question 11
Option (D) – $\frac{n^2-1}{12}$
Explanation: The variance of the first n natural numbers is a standard statistical result derived using the sum of the first n natural numbers and their squares, giving $\sigma^2 = \frac{n^2-1}{12}$.

Question 12
Option (C) – $\frac{1}{5}$
Explanation: The restricted sample space S containing tickets where the sum of digits is 8 is $\{08, 17, 26, 35, 44\}$. Out of these 5 tickets, only 17 has a digit product of 7, yielding a probability of 1/5.

Question 13
Option (C) – $\frac{1}{4}$
Explanation: For India's second win to happen on the third match, they must win exactly one of the first two matches and then win the third match. This probability is calculated as $\binom{2}{1}(1/2)^2 \times (1/2) = 1/4$.

Question 14
Option (D) – All of the above
Explanation: The conditional implication "if p then q" ($p \to q$) can be equivalently stated as "p only if q", "q is a necessary condition for p", or via its contrapositive "$\sim q \to \sim p$".

Question 15
Option (A) – 7 is not greater than 4 and 6 is not less than 7
Explanation: Using De Morgan's laws, the negation of a disjunction $\sim (p \vee q)$ is logically equivalent to the conjunction of the negations $\sim p \wedge \sim q$, resulting in option (A).

Question 16
Option (A) – 20
Explanation: Applying the matrix identity $A(\text{adj } A) = |A|I$ and comparing it with the given matrix equation $\begin{bmatrix} 20 & 0 \\ 0 & 20 \end{bmatrix}$ directly yields $|A| = 20$.

Question 17
Option (A) – Null matrix
Explanation: Finding $A^2 = \begin{bmatrix} -1 & -4 \\ 8 & 7 \end{bmatrix}$ and substituting it into the matrix polynomial $A^2 - 4A + 5I$ results in a zero-filled $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ null matrix.

Question 18
Option (B) – $\frac{3\pi}{10}$
Explanation: For continuity at $x = 0$, the limit $\lim_{x\to 0} f(x) = f(0) \implies \frac{3}{5}\lim_{x\to 0} \frac{\sin \pi x}{x} = 2k \implies \frac{3\pi}{5} = 2k$, which simplifies to $k = \frac{3\pi}{10}$.

Question 19
Option (B) – $\frac{3}{5}$

Explanation: Applying the standard limit approximations $\sin\theta \approx \theta$, $\log(1+x) \approx x$, $\tan^{-1}\theta \approx \theta$, and $e^x - 1 \approx x$ as $x \to 0^+$ simplifies the limit to $\lim_{x\to 0} \frac{x^{3/2}\cdot 3x}{x\cdot 5\sqrt{x}\cdot x} = \frac{3}{5}$.

Question 20
Option (C) – $x\frac{dy}{dx} = 2y$
Explanation: The equation of such a family of parabolas is $x^2 = 4ay$. Differentiating both sides with respect to x gives $2x = 4a(dy/dx)$, and substituting $4a = x^2/y$ yields $x(dy/dx) = 2y$.

Question 21
Option (D) – 1 and 2
Explanation: Rearranging the differential equation to $\sqrt{dy/dx} = 4(dy/dx) + 7x$ and squaring both sides removes the fractional power. The highest derivative order is 1, and its power (degree) in polynomial form is 2.

Question 22
Option (A) – $y + \log y + e^x \cos^2 x = 2$
Explanation: Separating variables gives $\int(1 + 1/y)dy + \int e^x(\cos^2 x - \sin 2x)dx = 0 \implies y + \log|y| + e^x \cos^2 x = C$. Substituting $x = 0, y = 1$ yields $C = 2$.

Question 23
Option (C) – $\frac{4}{3}$
Explanation: Splitting the integrand into odd and even parts, the odd part integrates to 0 on $[-1, 1]$. The even part simplifies to $\frac{1-x^4}{1+x^2} = 1 - x^2$, whose definite integral evaluates to $4/3$.

Question 24
Option (B) – 80
Explanation: Total permutations of 'HAVANA' is $6!/3! = 120$. Permutations where V and N are adjacent (treating them as one unit) is $2! \times 5!/3! = 40$. Subtracting these gives $120 - 40 = 80$.

Question 25
Option (C) – 25200
Explanation: The number of ways to select the letters is $\binom{7}{3}\binom{4}{2} = 35 \times 6 = 210$. The selected 5 letters can be arranged in $5! = 120$ ways, giving $210 \times 120 = 25200$ words.

Question 26
Option (A) – 1
Explanation: Simplifying the trigonometric expression yields $f(x) = \frac{5}{4}\sin^2 x + \frac{5}{4}\cos^2 x = \frac{5}{4}$ for all x. Therefore, the composition $g(f(x)) = g(5/4) = 1$.

Question 27
Option (C) – $[-1/2, 1/2]$
Explanation: Let $y = \frac{x}{1+x^2} \implies yx^2 - x + y = 0$. For real x, the discriminant of the quadratic equation must satisfy $1 - 4y^2 \geq 0 \implies y^2 \leq 1/4 \implies -1/2 \leq y \leq 1/2$.

Question 28
Option (C) – $1 \leq x \leq 4$
Explanation: For the function to be defined, the domain of arcsine requires $-1 \leq \log_2(x/2) \leq 1 \implies 2^{-1} \leq x/2 \leq 2^1 \implies 1 \leq x \leq 4$.

Question 29
Option (D) – None of these
Explanation: Summing the probabilities to 1 gives $k\sum_{x=0}^{4}(1/2)^x = 1 \implies k(31/16) = 1 \implies k = 16/31$. Since $16/31$ is not in the options, "None of these" is correct.

Question 30
Option (D) – $\frac{3}{2}$
Explanation: Expanding the numerator as $x \to 0$ gives $e^{x^2} - \cos x \approx (1+x^2) - (1-x^2/2) = \frac{3}{2}x^2$. Thus, the limit evaluates directly to $3/2$.

Question 31
Option (A) – $\frac{8}{5}$
Explanation: Factoring out $(x - \sqrt{2})$ from both the numerator and the denominator simplifies the limit to $\lim_{x \to \sqrt{2}} \frac{(x+\sqrt{2})(x^2+2)}{x+4\sqrt{2}}$, which evaluates to $\frac{2\sqrt{2}\times 4}{5\sqrt{2}} = 8/5$.

Question 32
Option (A) – 1
Explanation: The direction vector of the line is $\vec{b} = (-4, -7, 4)$ with magnitude 9. The dot product of $\vec{a} = (4, -3, 1)$ and $\vec{b}$ is 9. The projection is $|\vec{a} \cdot \vec{b}|/|\vec{b}| = 9/9 = 1$.