MHT CET 2023 Mathematics Answer Key PDF

Answer Key

Mathematics

Question 1 -Option– (A) $\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$
Explanation: The direction ratios of the line are perpendicular to the normal vectors of both given planes, which can be found via the cross product: $\vec{n}_1 \times \vec{n}_2 = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k}) = -3\hat{i} + 5\hat{j} + 4\hat{k}$. Passing through $(1, 2, 3)$, this yields the line equation in -Option (A).

Question 2 -Option– (B) 1
Explanation: Rewriting the limit gives $\lim_{x\to 0} \frac{x}{\tan 4x} \cdot \frac{\tan^2 2x}{\sin^2 x}$. Standard limit rules yield $\frac{1}{4} \cdot \left(\frac{2}{1}\right)^2 = 1$.

Question 3 -Option– (A) $\sqrt{2}\left(\sqrt{\tan x} + \frac{1}{5}(\tan x)^{5/2}\right) + c$
Explanation: Expressing $\sqrt{\sin 2x}$ as $\sqrt{2\tan x}\cos x$ simplifies the integrand to $\frac{\sec^4 x}{\sqrt{2}\sqrt{\tan x}}$. Substituting $t = \tan x$ transforms the integral to $\frac{1}{\sqrt{2}}\int (t^{-1/2} + t^{3/2})dt$, yielding -Option (A).

Question 4 -Option– (A) $\frac{8}{3\sqrt{5}}$
Explanation: Shortest distance $d = \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$. Substituting $\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 4\hat{k}$, $\vec{b}_1 = 3\hat{i} + \hat{j} + 2\hat{k}$, and $\vec{b}_2 = \hat{i} + 2\hat{j}$ yields $d = \frac{8}{3\sqrt{5}}$.

Question 5 -Option– (B) 160
Explanation: Evaluating the objective function $z = 7x + 8y$ at the corner points of the feasible region, which are $(0, 5)$, $(0, 20)$, $(10, 10)$, and $(10, 5)$, shows the maximum value is 160 occurring at the point $(0, 20)$.

Question 6 -Option– (B) $\pi/2$
Explanation: Since $\sin x \geq \frac{2x}{\pi}$ on $[0, \pi/2]$ and $\sin x \leq \frac{2x}{\pi}$ on $[\pi/2, \pi]$, the absolute value integral is split as $\int_0^{\pi/2}(\sin x - \frac{2x}{\pi})dx + \int_{\pi/2}^{\pi}(\frac{2x}{\pi} - \sin x)dx$, evaluating to $\pi/2$.

Question 7 -Option– (D) $1/e$
Explanation: Substituting $u = \log x$ simplifies the function to $f(x) = 2\tan^{-1}(\log x)$ for x close to e. Differentiating gives $f'(x) = \frac{2}{x(1+(\log x)^2)}$, and evaluating at $x = e$ gives $f'(e) = 1/e$.

Question 8 -Option– (C) $\pi/4$
Explanation: Expanding the equation yields $x^2\cos 2\alpha + xy\sin 2\alpha = 0$. Since the lines are perpendicular, the sum of coefficients $A + B = 0 \implies \cos 2\alpha = 0 \implies \alpha = \pi/4$.

Question 9 -Option– (C) $e^y \log y = e^x \sin x + c$
Explanation: Separating variables gives $\int \frac{e^y(1+y\log y)}{y}dy = \int e^x(\sin x + \cos x)dx$. Integrating both sides using standard exponential product integrals gives $e^y \log y = e^x \sin x + c$.

Question 10 -Option– (B) $\frac{x\log_e 2x - \log_e 2}{x}$
Explanation: Taking the natural logarithm and simplifying gives $y = \frac{x + \log_e 2}{1 + \log_e 2x}$. Differentiating with respect to x using the quotient rule yields the expression in -Option (B).

Question 11 -Option– (B) $3\sqrt{3}$ m, $2\sqrt{3}$ m
Explanation: Let the breadth be x and height be y. Area $xy = 18$. Maximizing the print area $A_{\text{print}} = (x-1)(y-1.5)$ yields $x = 2\sqrt{3}$ m and $y = 3\sqrt{3}$ m.

Question 12 -Option– (C) $x + 3y + 8 = 0$
Explanation: The slope of the normal is $-1/3$, meaning the tangent slope is 3. Setting $\frac{dy}{dx} = \frac{3x}{y} = 3 \implies y = x$. Solving with the hyperbola equation gives points $(\pm 2, \pm 2)$, yielding the normal $x + 3y + 8 = 0$ at $(-2, -2)$.

Question 13 -Option– (B) 28
Explanation: For $X \sim \text{Binomial}(5, p)$, $\binom{5}{3}p^3q^2 = 2\binom{5}{2}p^2q^3 \implies p = 2q \implies q = 1/3$. The probability of zero even numbers in a set is $q^5 = 1/243$. Out of 6804 sets, the expected count is $6804/243 = 28$.

Question 14 -Option– (A) 9/2
Explanation: The intersection slope product $m_1m_2 = -1 \implies \frac{3}{y_1}\left(-\frac{9x_1}{by_1}\right) = -1 \implies 27x_1 = by_1^2$. Substituting $y_1^2 = 6x_1$ gives $27x_1 = 6bx_1 \implies b = 9/2$.

Question 15 -Option– (B) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c}$
Explanation: For the circles to touch externally, the distance between their centers $C_1(-a, 0)$ and $C_2(0, -b)$ must equal the sum of their radii: $\sqrt{a^2 + b^2} = \sqrt{a^2 - c} + \sqrt{b^2 - c}$. Squaring and simplifying yields the relation.

Question 16 -Option– (D) 8
Explanation: For continuity at $x = 0$, LHL and RHL must equal $f(0) = a$. LHL $= \lim_{x\to 0^-} \frac{1-\cos 4x}{x^2} =$ 8, and RHL $= \lim_{x\to 0^+} \frac{\sqrt{x}\left(\sqrt{16+\sqrt{x}}+4\right)}{\sqrt{x}} = 8$. Hence, $a = 8$.

Question 17 -Option– (D) orthocentre
Explanation: The vector dot products $(\vec{a}-\vec{d})\cdot(\vec{b}-\vec{c}) = 0$ and $(\vec{b}-\vec{d})\cdot(\vec{c}-\vec{a}) = 0$ mean $AD \perp BC$ and $BD \perp AC$, which shows that D is the point of intersection of the altitudes (orthocentre).

Question 18 -Option– (C) $\sqrt{17}/9$
Explanation: The initial angle θ between the vectors has $\cos\theta = \frac{40}{45} = 8/9$. Since AD' is perpendicular to AB, the rotation angle α satisfies $\alpha = \pi/2 - \theta \implies \cos\alpha = \sin\theta = \sqrt{1 - (8/9)^2} = \sqrt{17}/9$.

Question 19 -Option– (C) $-1/2$
Explanation: Equating $\sin(\cot^{-1} x) = \cos(\tan^{-1}(1+x))$ yields $\frac{1}{\sqrt{1+x^2}} = \frac{1}{\sqrt{x^2+2x+2}} \implies 1+x^2 = x^2 + 2x + 2 \implies 2x = -1 \implies x = -1/2$.

Question 20 -Option– (C) $\frac{-8\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{82}}$
Explanation: The coplanar vector is perpendicular to the normal of the plane $\vec{n} = \vec{a}\times\vec{b} = -\hat{j}+\hat{k}$. Being orthogonal to $\vec{c} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ as well, the vector is along $\vec{n} \times \vec{c} = -8\hat{i} + 3\hat{j} + 3\hat{k}$.

Chemistry

Question 21 -Option– (C) $\Lambda_m^\circ = \lambda_{A^{3+}}^\circ + 3\lambda_{B^-}^\circ$

Explanation: By Kohlrausch's law of independent migration of ions, the molar conductivity of an AB_3 type electrolyte at infinite dilution is the sum of the individual ionic conductivities: $\Lambda_m^\circ = \lambda_{A^{3+}}^\circ + 3\lambda_{B^-}^\circ$.

Question 22 -Option– (B) 81.67 g
Explanation: From $2KClO_3 \rightarrow 2KCl + 3O_2$, producing 1 mole (22.4 dm^3) of O_2 requires 2/3 moles of $KClO_3$. Mass required is $\frac{2}{3} \times 122.5$ g/mol $= 81.67$ g.

Question 23 -Option– (D) Phenol
Explanation: Diazotization of aniline at 273 K yields benzene diazonium chloride (A). Warming this intermediate in water results in hydrolysis to produce phenol (B) alongside N_2 release.

Question 24 -Option– (C) 1.82
Explanation: Using the depression of freezing point relation $\Delta T_f = i \cdot K_f \cdot m$, the van't Hoff factor is calculated as $i = \frac{0.511}{1.86 \times 0.15} \approx 1.83$.

Question 25 -Option– (A) The driving and opposing forces differ by large amount.
Explanation: In a reversible process, the driving and opposing forces must differ only by an infinitesimal (extremely small) amount, enabling the process to be reversed at any step.

Question 26 -Option– (B) PbS
Explanation: Galena is the primary mineral and ore of lead, composed chemically of lead(II) sulfide (PbS).

Question 27 -Option– (A) 1.26×10^{-5} M
Explanation: For a 1 : 1 binary electrolyte like $AgCl$, $K_{sp} = S^2 \implies S = \sqrt{1.6 \times 10^{-10}} \approx 1.26 \times 10^{-5}$ M.

Question 28 -Option– (A) TiO_2
Explanation: Titanium dioxide (TiO_2) nanoparticles are widely utilized as highly active and stable catalysts in photocatalysis applications.

Physics

Question 29 -Option– (A) 5 : 6 Explanation: Dispersive power is $= \text{v} \quad \text{r}$ mean $=$ mean $\text{v} \quad \text{r}$
. For the first prism, $1 = 2\ 10 = 0.2\ 1 = 10\ 2 = 0.2$. For the second prism, $2 = 2\ 12 = 1\ 6\ 2 = 12\ 2 = 6\ 1$
. The ratio is $2 / 1 = 1 / 6\ 2 / 10 = 5 : 6\ 2 / 1 = 2/10\ 1/6 = 5{:}6$.

Question 30 -Option– (A) 2 : 1
Explanation: Equating volumes, $8 \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \implies R = 2r$. The ratio of total surface energy before and after the change is $\frac{8 \cdot 4\pi r^2 \cdot T}{4\pi (2r)^2 \cdot T} = \frac{32}{16} = 2 : 1$.

Question 31 -Option– (D) $A = 1, B = 1$
Explanation: The logic circuit features parallel and series connections of standard logic gates, where the final output Y evaluates to 1 when both inputs A and B are high (1).

Question 32 -Option– (C) $\frac{N_1 T_2}{N_2 T_1}$
Explanation: Activity $A = \lambda N = \frac{\ln 2}{T_{1/2}} N$. The ratio of the activities of the two elements is $\frac{A_1}{A_2} = \frac{\lambda_1 N_1}{\lambda_2 N_2} = \frac{N_1 T_2}{N_2 T_1}$.

Question 33 -Option– (C) 500 m/s
Explanation: Comparing $Y = 5\sin(10\pi t - 0.02\pi x + \pi/3)$ with the general wave equation $Y = \mathcal{A}\sin(\omega t - kx + \phi)$ gives $\omega = 10\pi$ and $k = 0.02\pi$. The wave velocity is $v = \omega/k = \frac{10\pi}{0.02\pi} = 500$ m/s.