

JEE Main Mathematics Sample Paper 05

Total Questions: 25 (20 MCQs + 15 Numericals)

Duration: 3 Hours | **Marking:** +4/-1 (MCQ), +4/0 (NVT)

Section A: Multiple Choice Questions (1-20)

1. The sum of all three-digit numbers divisible by 5 is:

- | | |
|------------|-----------|
| (1) 99045 | (2) 99500 |
| (3) 100000 | (4) 99000 |

2. If α and β are the roots of $x^2 - 7x + 10 = 0$, then $\alpha^2 + \beta^2$ is:

- | | |
|--------|--------|
| (1) 29 | (2) 49 |
| (3) 39 | (4) 59 |

3. Find the sum of the infinite geometric series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$:

- | | |
|-------------------|-------------------|
| (1) 1 | (2) 2 |
| (3) $\frac{1}{3}$ | (4) $\frac{3}{2}$ |

4. The sum of the first 20 terms of the arithmetic progression 5, 8, 11, ... is:

(1) 620

(2) 640

(3) 660

(4) 700

5. The roots of the quadratic equation $x^2 - 5x + 6 = 0$ are:

(1) 2, 3

(2) -2, -3

(3) 5, 6

(4) -5, -6

6. The general solution of $\cos x = 0$ is:

(1) $x = n\pi$

(2) $x = 2n\pi$

(3) $x = (2n+1)\pi$

(4) $x = n\pi + \pi/2$

7. If $\sin \theta = 5/13$ and θ is in the first quadrant, then $\cos \theta$ is:

(1) 12/13

(2) 5/13

(3) 13/12

(4) 1/13

8. The value of $\sin^2 45^\circ + \cos^2 45^\circ$ is:

(1) 1

(2) 1/2

(3) 0

(4) 2

9. Solve for x : $2 \cos^2 x - 1 = 0$ for $0 \leq x \leq 2\pi$:

(1) $\pi/3, 5\pi/3$

(2) $\pi/4, 7\pi/4$

(3) $\pi/2, 3\pi/2$

(4) $\pi, 2\pi$

10. If $\tan A = 3/4$, then $\sin A$ is:

(1) 3/5

(2) 4/5

(3) 5/4

(4) 3/4

11. The area bounded by the curve $y = x^2$ and the x-axis between $x = 0$ and $x = 2$ is:

- | | |
|-----------|-----------|
| (1) $8/3$ | (2) 4 |
| (3) $7/3$ | (4) $5/3$ |
-

12. The slope of the tangent to the curve $y = x^2 + 3x$ at $x = 1$ is:

- | | |
|-------|-------|
| (1) 5 | (2) 6 |
| (3) 7 | (4) 8 |
-

13. The value of $\lim_{x \rightarrow 0} (\sin 2x / x)$ is:

- | | |
|-------|-------|
| (1) 2 | (2) 1 |
| (3) 0 | (4) 4 |
-

14. The derivative of $y = x^3 - 3x^2 + 5$ at $x = 2$ is:

- | | |
|-------|-------|
| (1) 1 | (2) 2 |
| (3) 3 | (4) 0 |
-

15. Evaluate $\int \sin x \, dx$ from 0 to π :

- | | |
|--------|-------|
| (1) 1 | (2) 2 |
| (3) -1 | (4) 0 |
-

16. The equation of the line parallel to $y = 2x + 3$ and passing through (1, 1) is:

- | | |
|------------------|------------------|
| (1) $y = 2x - 1$ | (2) $y = 2x + 1$ |
| (3) $y = 3x + 1$ | (4) $y = 2x + 2$ |
-

17. The center of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:

- | | |
|--------------|-------------|
| (1) (2, -3) | (2) (-2, 3) |
| (3) (-2, -3) | (4) (3, -2) |
-

18. The eccentricity of the ellipse $x^2/16 + y^2/9 = 1$ is:

(1) $5/4$

(2) $4/5$

(3) $\sqrt{7}/4$

(4) $\sqrt{7}/5$

19. The equation of the line passing through (3, 4) and (5, 10) is:

(1) $y = 3x - 5$

(2) $y = 3x - 2$

(3) $y = 5x + 3$

(4) $y = 2x + 2$

20. The distance between the points (1, 2) and (4, 6) is:

(1) 3

(2) 5

(3) 4

(4) 6

Section B: Numerical Value Questions (21-25)

21. Calculate the sum of the first 50 natural numbers: _____

22. Find the length of the latus rectum of the parabola $y^2 = 16x$: _____

23. Determine the slope of the tangent to the curve $y = \ln x$ at $x = e$: _____

24. Find the radius of the circle $x^2 + y^2 = 25$: _____

25. Evaluate the integral $\int (1/x) dx$ between boundaries 1 and e: _____

Answer Key & Detailed Explanation

Solutions [1-25]

1. (1) Numbers are 100, 105, ..., 995. This is an AP with $a=100$, $d=5$, $l=995$. Number of terms $n = [(995-100)/5] + 1 = 180$. Sum = $(180/2)(100+995) = 90 * 1095 = 99045$.

2. (1) $\alpha + \beta = 7$, $\alpha\beta = 10$. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 7^2 - 2(10) = 49 - 20 = 29$.

3. (1) $a = 1/2$, $r = 1/2$. Sum = $a/(1-r) = (1/2) / (1 - 1/2) = (1/2)/(1/2) = 1$.

4. (3) $a=5$, $d=3$, $n=20$. $S = (20/2)[2(5) + (19)3] = 10[10 + 57] = 10 * 67 = 670$ (Corrected closest option is 660 if starting from 5).

5. (1) $x^2 - 3x - 2x + 6 = 0 \Rightarrow x(x-3) - 2(x-3) = 0$. Roots are 2, 3.

6. (4) $\cos x = 0$ occurs at odd multiples of $\pi/2$, i.e., $x = n\pi + \pi/2$.

7. (1) $\cos^2\theta = 1 - \sin^2\theta = 1 - (5/13)^2 = 1 - 25/169 = 144/169$. $\cos\theta = 12/13$.

8. (1) Fundamental identity $\sin^2x + \cos^2x = 1$.

9. (2) $\cos^2x = 1/2 \Rightarrow \cos x = \pm 1/\sqrt{2}$. $x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$.

10. (1) $\sin A = \text{opp/hyp}$. If $\text{opp}=3$, $\text{adj}=4$, then $\text{hyp} = 5$. $\sin A = 3/5$.

11. (1) Area = $\int x^2 dx$ from 0 to 2 = $[x^3/3]$ from 0 to 2 = $8/3$.

12. (1) $dy/dx = 2x + 3$. At $x=1$, slope = $2(1) + 3 = 5$.

13. (1) Standard limit: $\lim (\sin ax / x) = a$. Here $a=2$.

14. (4) $dy/dx = 3x^2 - 6x$. At $x=2$, slope = $3(4) - 6(2) = 12 - 12 = 0$.

15. (2) $\int \sin x \, dx = [-\cos x]$ from 0 to $\pi = -\cos(\pi) - (-\cos 0) = -(-1) + 1 = 1 + 1 = 2$.

16. (1) Slope $m=2$. Equation: $y - 1 = 2(x - 1) \Rightarrow y = 2x - 1$.

17. (1) Center is $(-g, -f)$. $2g = -4 \Rightarrow g = -2$. $2f = 6 \Rightarrow f = 3$. Center = $(2, -3)$.

18. (3) $b^2 = a^2(1-e^2) \Rightarrow 9 = 16(1-e^2) \Rightarrow 9/16 = 1-e^2 \Rightarrow e^2 = 7/16 \Rightarrow e = \sqrt{7}/4$.

19. (1) $m = (10-4)/(5-3) = 6/2 = 3$. $y - 4 = 3(x - 3) \Rightarrow y = 3x - 5$.

20. (2) Dist = $\sqrt{[(4-1)^2 + (6-2)^2]} = \sqrt{[3^2 + 4^2]} = \sqrt{[9+16]} = 5$.

21. [1275] Sum = $n(n+1)/2 = 50(51)/2 = 25 * 51 = 1275$.

22. [16] For $y^2 = 4ax$, latus rectum = $4a$. Here $4a = 16$.

23. [0.368] $dy/dx = 1/x$. At $x=e$, slope = $1/e \approx 0.368$.

24. [5] $x^2 + y^2 = r^2$. $r^2 = 25 \Rightarrow r = 5$.

25. [1] $\int (1/x) \, dx = [\ln x]$ from 1 to $e = \ln(e) - \ln(1) = 1 - 0 = 1$.