NG 22 (GROUP A)

PART I — ENGINEERING MATHEMATICS

(Common to all candidates)

(Answer ALL questions)

1. The eigenvalues of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ are

   a. $-2, -3, 6$

   b. $-2, 3, 6$

   c. $2, -3, -6$

   d. $2, 3, 6$

2. For what value of k, the matrix $A = \begin{bmatrix} 2 & k \\ 3 & 5 \end{bmatrix}$ has no inverse?

   a. 3

   b. $-10/3$

   c. 2

   d. 10/3

3. The index and signature of the quadratic form $x_1^2 + 2x_2^2 - 3x_3^2$ are

   a. Index = 2 ; Signature = –1

   b. Index = 2 ; Signature = 1

   c. Index = 1 ; Signature = 2

   d. Index = 1 ; Signature = –2

4. If $u = x^2 + y^2$ then $\frac{\partial^2 u}{\partial x\, \partial y}$ is equal to

   a. 2

   b. 0

   c. $2xy$

   d. $2(x + y)$

5. If $x = u(1 - v)$ and $y = uv$ then the Jacobian $J\left(\frac{u, v}{x, y}\right)$ is

   a. $x + y$

   b. $\frac{1}{x + y}$

   c. 1

   d. xy

6. The Taylor's expansion of the function $f(x) = \frac{1}{1 + x^2}$ is

   a. $\sum_{n=0}^{\infty} (-1)^n x^{2n}$ for $-1 < x < 1$

   b. $\sum_{n=0}^{\infty} x^{2n}$ for $-1 < x < 1$

   c. $\sum_{n=0}^{\infty} (-1)^n x^n$ for $-1 < x < 1$

   d. $\sum_{n=0}^{\infty} (-1)^n x^{2n}$ for any real x

7. If $\vec{F} = x^2yz\,\hat{i} + xy^2z\,\hat{j} + xyz^2\,\hat{k}$ then $div\,\vec{F}$ at (1, 2, 3) is

   a. 1

   b. 6

   c. 12

   d. 24

8. The work done in moving a particle by the force $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$ from (1, 1) to (2, 8) along $y = x^3$ is

a. 24

b. 35

c. 48

d. 70

9. The value of $\oint_{|z|=1} \frac{z^2}{(2z-1)^2}$ is

a. $2\pi i$

b. πi

c. $\pi i/2$

d. $4\pi i$

10. The fixed points of the mapping $w = \frac{5z+4}{z+5}$ are

a. 2, 2

b. 2, –2

c. –2, –2

d. –4/5, 5

11. The residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ at $z = 1$ is

a. 4/9

b. 5/9

c. 1/3

d. 1/9

12. The inverse Laplace transform of $\frac{1}{(s+a)^2}$ is

a. e^{at}

b. e^{-at}

c. te^{-at}

d. te^{at}

13. Using Parsevel's identity of Fourier transform, the value of $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)}$ is

a. $\frac{\pi}{a+b}$

b. $\frac{\pi}{ab(a+b)}$

c. $\frac{\pi}{2ab(a+b)}$

d. $\frac{\pi}{2(a+b)}$

14. The Z transform of $\frac{1}{(n+1)!}$ is

a. $e^{1/z}$

b. $ze^{1/z}$

c. $ze^{1/z} - 1$

d. $z(e^{1/z} - 1)$

15. The approximate value of the roots of the equation $x^3 + x - 1 = 0$ lying in the interval (0, 1) using the method of false position with two iteration is given by

a. 0.61

b. 0.72

c. 0.74

d. 0.64

16. The fourth divided difference of the polynomial $3x^3 + 11x^2 + 5x + 11$ over the points $x = 0, 1, 4, 6, 7$ is

a. 18

b. 11

c. 3

d. 0

17. Simpson's rule for evaluation of $\int_a^b f(x)\,dx$ requires the interval (a, b) to be divided into

a. $3n$ intervals

b. $(2n + 1)$ intervals

c. $2n$ intervals

d. $(3n + 1)$ intervals

18. Given that E and F are events such that $P(E) = 0.6$, $P(F) = 0.3$ and $P(E \cap F) = 0.2$, the value of $P(E / F)$ is

a. 2/3

b. 1/3

c. 1/2

d. 1/5

19. If X has uniform distribution in (–1, 3) and Y has exponential distribution with parameter λ, then the value of λ such that $Var(X) = Var(Y)$ is

a. 4/3

b. 3/4

c. $\frac{2}{\sqrt{3}}$

d. $\frac{\sqrt{3}}{2}$

20. If the correlation coefficient is zero then the two lines of regression are

a. parallel

b. perpendicular

c. coincident

d. inclined at 45° to each other